Chasing and escaping by three groups of species

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We study group chasing and escaping between three species. In our model, one species acts as a group of chasers for another species and acts as a group of targets for the third species. When a particle is caught by a target, the particle becomes a new chaser. Although the ratio of three species is changed, the total number of particles is conserved. When particles move randomly, the numbers of the three species change periodically but no species seems to become extinct. If particles escape from the nearest chaser and chase the nearest target, the extinction of a species occurs. The extinction induces that of the second species and finally only one species survives.

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I. INTRODUCTION

Problems in chase and escape have been studied using various models. The simplest case is the system with one chaser and one target [1–3], which is, for example, the model of a merchant vessel pursued by a pirate ship. A more complicated case, in which one target escapes from many chasers, was also studied [4–7]. When the chasers perform random walks to search for a target, the survival probability of the immobile target, $p_{\mathrm{imm}}(t)$, is given by $p_{\mathrm{imm}}(t) \sim e^{-\alpha\rho t}$, where ρ is the density of chasers, and α is a constant determined by the diffusion coefficient of chasers. If the target can escape from the chasers, the dependence of the survival probability on the density ρ varies: the survival probability is given by $p_{\mathrm{imm}}(t) \sim e^{-\alpha\rho^3 t}$. When the area in which the target searches for the chasers is wider than the area in which the chasers search for the target, the time dependence changes from t to $t^{1.65}$.

Recently, Kamimura and co-workers used a simple model and studied group chase and escape [8–10]. In the studies, the system consists of two types of species: one is a group of chasers, which try to catch the nearest target, and the other is a group of targets, which escape from the nearest target. In the simulation, although an individual chaser independently tries to catch a target, the chasers form flocks and seem to cooperate to catch flocks of targets. The lifetime of the final target becomes longer upon increasing the initial number of targets, but the typical lifetime becomes longest at a suitable number of the initial number of targets. There is an optimal number of chasers to minimize the cost of catching all targets.

Collective motions of interacting entities are studied in many models [11–17]. The models are simplified and generalized, so that they can be regarded as the basic models of the motions of interacting entities such as human societies, animals, and bacteria. One model in previous studies about group chase and escape [8–10] is one of those simplified and generalized models, so that it may be applied to many problems. Thus, it is useful to expand the model and to study basic properties.

In previous studies [8–10], the system consisted of two types of species, namely targets and chasers. In this paper, we consider a system formed by three species: particles in one species act as chasers for another species and as targets for the third species. Because of these third particles, the system is more complicated.

The relationship of the three species in our model is similar to that of the rock, paper, and scissors in the game of the same name, and chasing and escaping between three species is seen in a kind of game of tag. Similar phenomena may be observed in our society and in nature. Thus, we think that the model we use in this paper can be a prototype of complicated phenomena in our society and nature. In Sec. II, we introduce our model. In Sec. III, we show the results of simulation. In Sec. IV, we summarize the results and present a brief discussion.

II. MODEL

We consider a two-dimensional square lattice with the periodic boundary condition. Initially, we put particles on sites in the lattice at random. We consider three kinds of species: species A, B, and C. Particles in species A act as chasers for species B, those in species B are the chasers for species C, and those in species C are the chasers for species A.

The motion of each particle is similar to that in Refs. [8,9]. In one trial, we choose one of the particles and move the particle. First, the particle behaves as a chaser. The particle tries to catch the nearest target. If there are a few nearest targets, the particle randomly selects a target among them and moves to the direction in order to decrease the distance from the target. When there are two or more sites to which the chaser can move in order to decrease the distance from the target, we choose one site randomly and try to move the chaser to the site. If the selected site is occupied by another particle, which is not a target for the chosen particle, the chosen particle does not move to the selected site and stays at the same site. If a particle which acts a target for the chosen particle is present at the selected site, the particle in the selected site transforms into a chaser. Thus, the number of total particles is conserved, but the numbers of the three types of particles change during a simulation.

After a moving trial as a chaser, a chosen particle tries to move while acting as a target: the particle escapes from the nearest chaser. If two or more nearest chasers are present, the particle chooses one of them randomly and tries to escape from it. If there are some directions to increase the distance from the chaser, the particle chooses one direction randomly. The particle moves to the site if the selected site is empty, but

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does not move and stays in the same site if the selected site is occupied by another particle. The time increase in a trial is $(N_A + N_B + N_C)^{-1}$, where N_A, N_B, and N_C are the number of species A, B, and C, respectively.

III. RESULTS OF SIMULATION

We carry out a Monte Carlo simulation and study the motion of particles. Figure 1 shows the snapshots of the positions of particles with various times.

The system size is $L_x \times L_y = 160 \times 120$, and the initial numbers of particles are $N_A = 2900$, $N_B = 2900$, and $N_C = 200$. Initially, particles are randomly present in the system. Large flocks have not been seen in the initial stage [Fig. 1(a)]. With increasing time, some flocks of each species are formed. They repeat chasing and escaping [Fig. 1(b)] while changing the numbers of the three types of species temporally. Finally, one kind of species vanishes [Fig. 1(c)]. Then, another kind of species vanishes in succession and the other species survives.

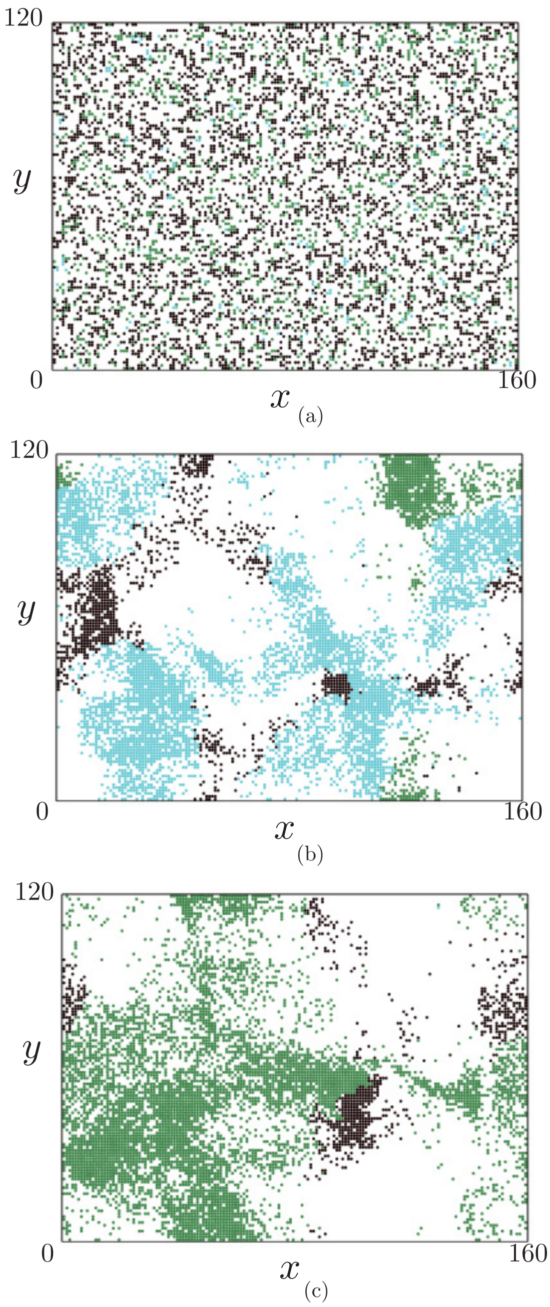

FIG. 1. (Color online) Snapshot of positions of particles (a) in the initial stage ($t = 1.25$), (b) in the middle stage ($t = 110$), and (c) in the last stage ($t = 175$). Red (dark gray) dots, green (gray) dots, and cyan (light gray) dots represent species A, B, and C, respectively.

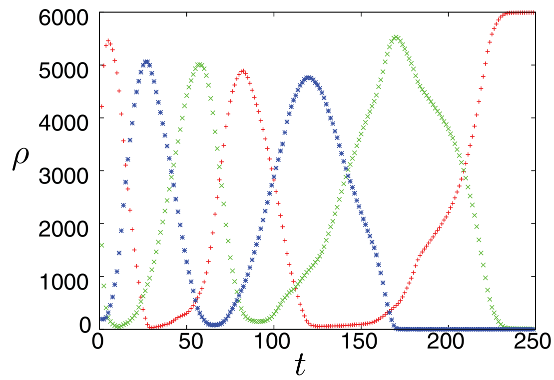

FIG. 2. (Color online) Time evolution of the numbers of particles. Red pluses, green crosses and blue asterisks show the numbers of species A, B, and C, respectively.

Figure 2 shows the time evolution of the numbers of the three species. In the initial stage, during which the flocks of species are not formed, the numbers oscillate periodically. In the middle stage, the period of the oscillation is disturbed. A long interval with a low density of species A appears. However, species A does not become extinct and increases again. During the long time interval with the low density of species A, the system spuriously behaves as it does with two species, B and C. Since particles of species C act as a target for those of species B, the number of species C becomes small. The peak of the number of species B becomes higher than the peaks at the other time. The number of species B is so larger that species B exterminates species C. After the extinction of species C, which is accidentally caused by the fluctuation of the number of species, only species A and B are present in the system. In the system with species A and B, the particles of species A act as chasers and those of species B act as targets. Thus, species B successively vanishes and finally species A survives.

The periodic time evolution of the numbers of particles seems to be unstable for small perturbations, and the extinction of species is accidentally caused by a fluctuation of the time evolution. Thus, we carry out simulations with 500 individual initial conditions, and we investigate how many times each species becomes approximately extinct. Figure 3 shows the time distributions of the frequencies of extinction of each species. The system size and the initial numbers of species A, B, and C are the same as those in Fig. 1. The time interval we use in the simulation is 3334. During the interval, the number of times of extinction of species A, B, and C is 95, 80, and 99 runs, respectively. In 226 runs, no species becomes extinct. Thus, we found that the type of extinct species and the lifetime of species is determined stochastically. The highest peaks of extinction of species A, B, and C appear when t is about 45, 125, and 85, respectively.

From Fig. 2, we find that the density of one of three species becomes a minimum value around the times of extinction.

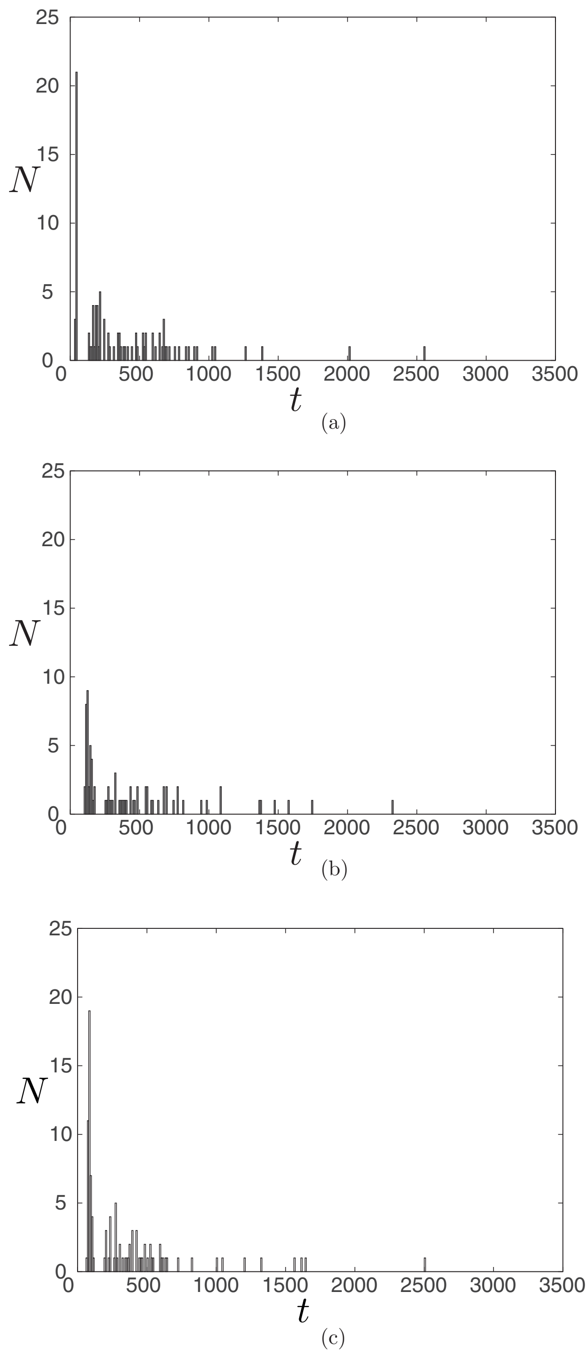

FIG. 3. Time distributions of the number of occurrences of extinction of species (a) A, (b) B, and (c) C.

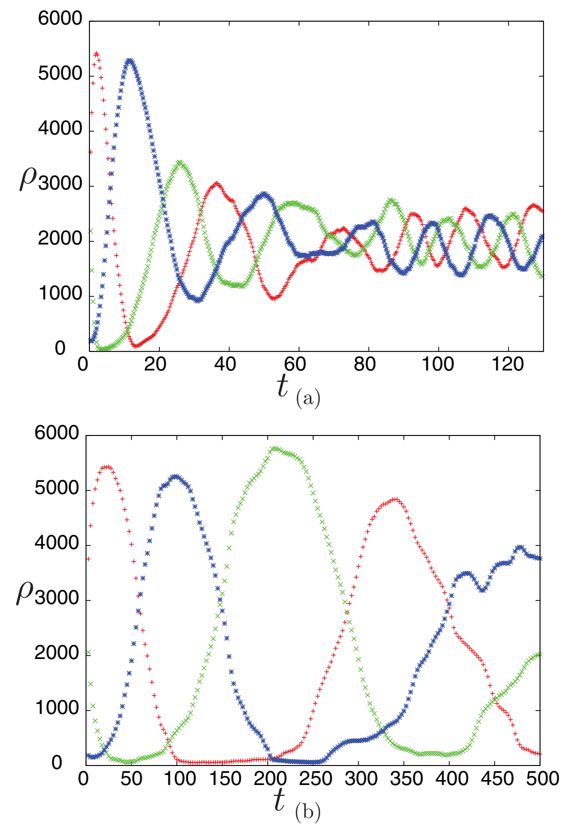

FIG. 4. (Color online) Time evolutions of the numbers of particles. Red pluses, green crosses, and blue asterisks show the numbers of species A, B, and C, respectively.

Even if the species survive at the crisis of extinction in the early stage, the densities of species oscillate and the time with the minimum density appears periodically. Thus, the crisis of extinction of species appears many times during the long time interval, but the frequency of the extinction decreases with increasing time.

We fix the population of three species and carry out simulations with other system sizes. Figure 4 shows the time evolutions of the numbers of particles with the system size 100×75 [Fig. 4(a)] and 320×240 [Fig. 4(b)]. The density of particles of Figs. 4(a) and 4(b) is 2.56 times and 0.25 times as large as that of Fig. 2, respectively. In Fig. 4(a), the period of oscillation of the densities of species is shorter than that in Fig. 2. The form of the oscillation is more symmetric than that in Fig. 2. The amplitude of the oscillation decreases with increasing time. On the other hand, in Fig. 4(b), in which the system size is larger than that in Fig. 2, the period of oscillation is longer and the form of the oscillation is more asymmetric than that in Fig. 2: the period with low density is longer than that with high density and the time interval with a flat bottom appears.

We also investigate how the time distribution of the numbers of extinction of species changes with the change of the density of population. Figure 5 shows the distributions of frequencies of extinction with the system size 100×75 [Fig. 5(a)], 160×120 [Fig. 5(b)], and 320×240 [Fig. 5(c)]. Note that the range of the y axis in Fig. 5(a) is four times longer than that in the other figure parts. The numbers of extinction of the species are 181, 274, and 276, respectively. With small system size, with which the period of oscillation of the density of species is short, the extinction of species occurs in the early stage. Upon increasing the system size, the peak of the extinction becomes broad and low.

The oscillation of the ratios of species can be explained by a mean-field approximation. We assume that the particles are initially present at random. When we select one particle of species A, the number of particles of species B around the selected particle is approximately $4N_B/N_S$ and that of species C is $4N_C/N_S$, where N_S is the total number of sites in the system and the factor 4 comes from the square lattice. Since the particles of species A act as targets for species C and chasers for

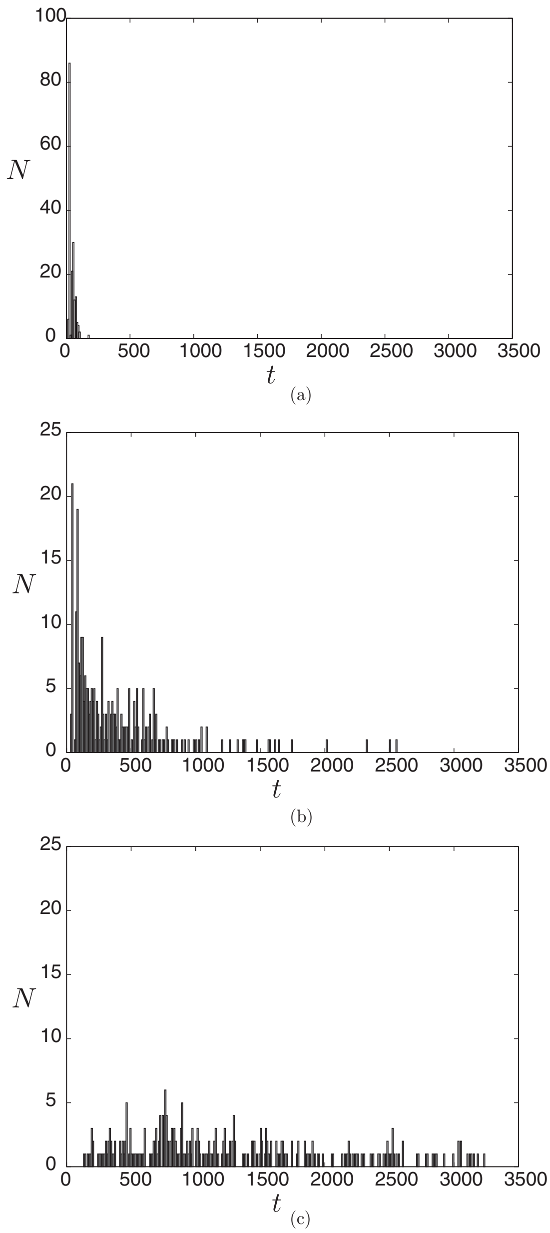

FIG. 5. Time distribution of the numbers of occurrences of extinction of all species. The system size is (a) 100×75, (b) 160×120, and (c) 320×240.

species B, the number of species A increases when species A attaches to species B and decreases when species A attaches to species C. Thus, during unit time, the change of the number of species A is given by $4N_\mathrm{A}N_\mathrm{B}/N_\mathrm{S} - 4N_\mathrm{A}N_\mathrm{C}/N_\mathrm{S}$. Considering the similar relations for the other species, the time evolutions of the ratios of species in the mean-field approximation are given by

$$\frac{d}{dt}\left(\frac{N_\mathrm{A}}{N_0}\right) = 4\frac{N_0}{N_\mathrm{S}}\frac{N_\mathrm{A}}{N_0}\left(\frac{N_\mathrm{B}}{N_0} - \frac{N_\mathrm{C}}{N_0}\right), \tag{1}$$

$$\frac{d}{dt}\left(\frac{N_\mathrm{B}}{N_0}\right) = 4\frac{N_0}{N_\mathrm{S}}\frac{N_\mathrm{B}}{N_0}\left(\frac{N_\mathrm{C}}{N_0} - \frac{N_\mathrm{A}}{N_0}\right), \tag{2}$$

$$\frac{d}{dt}\left(\frac{N_\mathrm{C}}{N_0}\right) = 4\frac{N_0}{N_\mathrm{S}}\frac{N_\mathrm{C}}{N_0}\left(\frac{N_\mathrm{A}}{N_0} - \frac{N_\mathrm{B}}{N_0}\right), \tag{3}$$

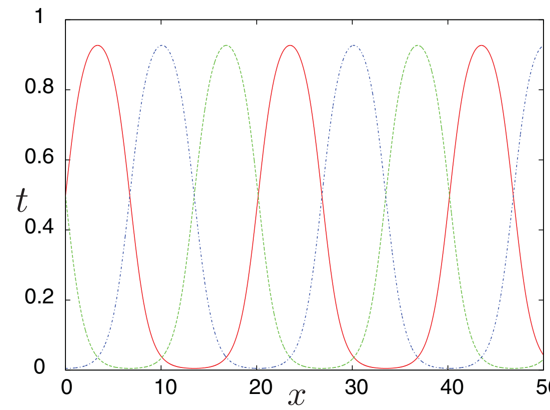

FIG. 6. (Color online) A solution of the simultaneous equations (1)–(3). Red line, green dotted line, and blue dashed line show the time evolution of the ratio of species A, B, and C, respectively. The time is normalized by $4N_0/N_\mathrm{S}$.

where $N_0 = (N_\mathrm{A} + N_\mathrm{B} + N_\mathrm{C})$. We solve the simultaneous equations (1)–(3), and see the time evolution of the ratios.

Figure 6 shows the time evolutions of the ratios. Initially, the ratio of species C, $\rho_\mathrm{C}(\equiv N_C/N_0)$, is 0.005, and that of species A and species B, ρ_A and ρ_B, is $0.5(1-\rho_\mathrm{C})$. In the initial stage, ρ_A and ρ_C increase and ρ_B decreases. When $\rho_\mathrm{B} = \rho_\mathrm{C}$, the increase of ρ_A stops and $\rho_\mathrm{A} > \rho_\mathrm{B}$, so that ρ_C keeps increasing and ρ_A starts decreasing. As a result, the ratio of particle A oscillates regularly. Since the forms of Eqs. (2) and (3) are the same as that of Eq. (1), ρ_B and ρ_C show the same behavior. The oscillation of the tree ratios seems to have the same form and the same period with a phase shift. The time interval with a large value is shorter than that with a small value. If the initial value of ρ_C is large and the difference between ρ_C and ρ_A is smaller than that in Fig. 6, the form of the time evolution of the ratios is more symmetric. We also carry out numerical simulation with other initial values of ρ_A and ρ_B: $\rho_\mathrm{A} = 0.6 - \rho_\mathrm{C}/2$ and $\rho_\mathrm{B} = 0.4 - \rho_\mathrm{C}/2$, and $\rho_\mathrm{A} = 0.4 - \rho_\mathrm{C}/2$ and $\rho_\mathrm{B} = 0.6 - \rho_\mathrm{C}/2$, where the initial value of ρ_C that we used is 0.005. In both cases, the extinction of species does not occur and the oscillation of the ratios repeats as in Fig. 6.

To analyze the effect of the formation of groups on the extinction of species, we investigate the time evolution of ratios of species using another model, in which particles move randomly. Figure 7 shows the snapshots of the positions of species. In the initial stage [Fig. 7(a)], the snapshot is similar to that in Fig. 1(a). The densities of all species are distributed equally in the system. However, in later stages [Figs. 7(b) and 7(c)], the distribution is different from that in Fig. 1. Although the type of species is different locally, the density of particles is uniformly distributed in the system. The flocks of particles, which are formed by a single species with high density, do not appear.

The change in the distribution of particles affects the time evolutions of the numbers of particles. Figure 8 shows the time evolutions of the numbers of particles using the model with random walking species. The initial condition is the same as that in Fig. 2. In the model, the ratios oscillate periodically, which is similar to that in the mean-field model. However, the amplitude of oscillation decreases with time. We also carry out 500 individual runs, in which the time interval is 1667.

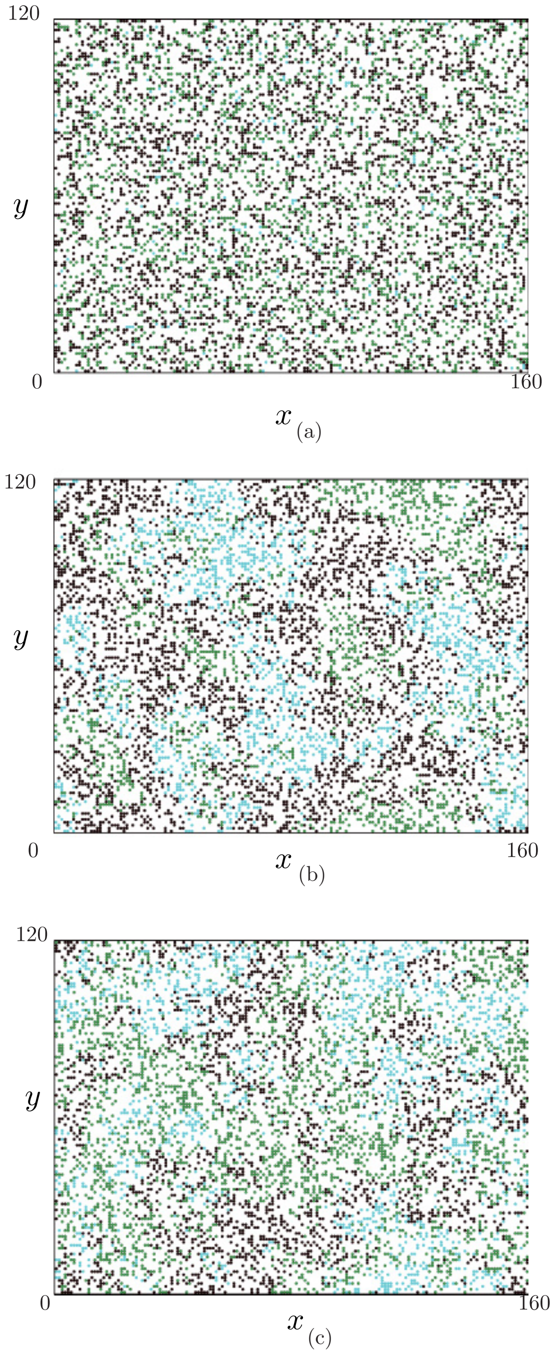

FIG. 7. (Color online) Snapshot of positions of particles moving randomly (a) in the initial stage ($t = 1.25$), (b) in the middle stage ($t = 110$), and (c) in the last stage ($t = 175$).

The extinction of species, which is observed in an early stage in the system with particles reaching targets, does not occur in the simulation with random walking species. Thus, the extinction of species is caused by group chasing.

IV. SUMMARY

In this paper, we studied the chasing and escaping of groups of three species. By carrying out a simulation in which the particles move in the direction of the nearest target, we found that flocks of each species are formed. The densities of each

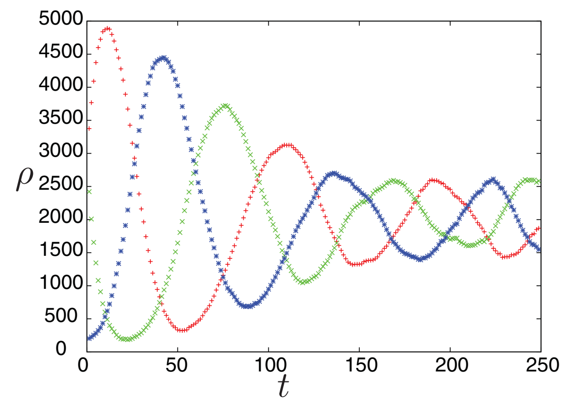

FIG. 8. (Color online) Time evolution of numbers of species with the model in which particles move randomly. Red pluses, green crosses, and blue asterisks show the numbers of species A, B, and C, respectively.

species oscillate in an early stage. Then, the extinction of a species occurs, which induces the extinction of another species, and finally only one species survives. The time at which the extinction of a species occurs and the type of the extinct species are determined stochastically. The density of particles affects the period of the oscillation and the frequency of occurrence of the extinction. When the density of particles is low, the period of the oscillation is long and the occurrence of the extinction of the species occurs gently. Upon increasing the density, the period of the oscillation becomes short and the extinction of the species occurs intensively in an early stage.

The oscillation of the density of the species is explained by a mean-field approximation. In a mean-field approximation, in which the formation of groups is neglected, the density of the species oscillates with a time interval, but the extinction of the species does not occur. In the simulation in which particles move randomly, the large groups made by a single kind of particle with high density do not appear, and the density of particles is uniform. The ratios of each particle oscillate and the extinction of any species does not occur, which agrees with the expectation of the mean-field approximation. However, like a simulation with particles chasing targets, the amplitude of oscillation decreases with time, which does not agree with the expectation of the mean-field approximation. The decrease of the amplitude is probably caused by the formation of domains of species. Due to the formation of domains, the number of species that face another kind of species decreases, so that the frequency of transition of the type of species decreases and the amplitude of the oscillation becomes small.

In this work, we carried out a simulation and studied behaviors qualitatively. We should study the behaviors more quantitatively. For example, the dependence of the lifetime of species on some parameters and the chasing rule should be studied more precisely. We intend to study those problems in detail.

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