Abstract For Dissertation

A Study on Multi-Agent Systems for Stable Matching

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1 Motivation and Objectives

The stable marriage problem (SMP) is a combinatorial optimization problem of finding a stable matching between two sets, namely men and women. Each man and woman has their own set of preference lists in which they rank orderly their preferred partners. With this information, the aim of solving the matching problem is to establish a stable partnership without the blocking pairs.

The terminology of SMP was first coined in year 1962 by David Gale and Lyod Shapley in their seminal work of [1]. They introduced a sequential algorithm (as from now on we refer it as G-S algorithm) to establish matching between two bipartite sets, in consideration of matching students with their appropriate colleges, or in the marriage situation per se. The algorithm works by one side of the divides to make the proposal, while the other evaluates either to accept or reject the said proposal. The proposing iteration continues as long as there exists proposer which has yet to be matched with his/her stable partners.

The G-S algorithm works flawlessly to attain stable matching between these two sets without the blocking pairs. However, it only uses partial information from the preference lists. Therefore, there are possibilities that other matching might yield to better matching results suppose full information is used. Moreover, the G-S algorithm favors proposer than the receiver. Hence, the final matching attained by the G-S algorithm will be optimal to the men (if they are the proposer) over the women. These phenomena is commonly known as the *men-optimal, women-pessimal* situation.

The way this algorithm works in sequential manner limits the usage of all the information available in the preference lists. Therefore, in this dissertation we aim to emulate the strategies given by the G-S algorithm, but to introduce dynamical approach in attaining stable matching between the bipartite sets. In the following, we state the main objectives which motivate us in our investigation of the stable marriage problem.

The objectives of this work are as follows:

- To investigate the potential of treating a discrete optimization problem through solving the sets of differential equations in the multi-agents formulation.
- To identify and formulate suitable cost function that best interpret the preference lists of both men and women.
- To attain dynamically stable matching between men and women.

2 Stability Theory

Consider the autonomous system of

$$\dot{x} = f(x(t)) \tag{1}$$

where $f : D \to \mathbb{R}^n$ is locally Lipschitz map from domain $D \subset \mathbb{R}^n$ into $\mathbb{R}^n$. The system (1) is said to be continuous and smooth, since the right hand side is continuous.

On the other hand, let the differential equation with discontinuous right hand side be represented by a differential inclusion

$$\dot{x} \in^{a.e.} K\{f(x,t)\} \tag{2}$$

where $K\{f(x,t)\}$ is the Filippov's differential inclusion and $a.e.$ is the abbreviation for "almost everywhere".

Definition 1 (Filippov solution [2, 3])**.** A vector function $x(\cdot)$ is called a solution of (2) on $[t_0, t_1]$ if $x(\cdot)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1]$

$$\dot{x} \in K\{f(x,t)\} \tag{3}$$

where

$$K\{f(x,t)\} = \bigcap_{\delta>0} \bigcap_{\mu(D)=0} \overline{\text{co}} f(B(x,\delta) - D, t) \tag{4}$$

where $\overline{\text{co}}$ means the closed convex hull; $B(x,\delta)$ is a closed δ neighborhood of x; D is an arbitrary set in R^n; μ is n dimensional Lebesgue measure. Hence, $\cap_{\mu(N)=0}$ means the intersection over all sets D of Lebesgue measure zero.

2.1 Lyapunov Stability

To investigate the stability of a given dynamical system, we may analyze the stability of the solutions of its differential equations near to a point of equilibrium. Hence, by ensuring that the stability conditions are satisfied, then we can conclude about the behavior of those system at this point. The following lemmas provide us with essential tools in analyzing the stability of the dynamical system near its equilibrium points.

Lemma 1 (Lyapunov's second method for stability)**.** Assume that the system (1) has an equilibrium point at $x = x_0$. Then, the system is *asymptotically stable* in the sense of Lyapunov if and only if, there exist a Lyapunov function $V(x) : \mathbb{R}^n \to \mathbb{R}$ such that

- $V(x) > 0$ for all $x \neq x_0$ and $V(x_0) = 0$,
- $\dot{V}(x) < 0$ for all $x \neq x_0$ and $\dot{V}(x_0) = 0$.

Suppose the right-hand-side of (1) is non-smooth (i.e., equation (2)), then the following lemma provides the generalized form of Lemma 1 to discuss the stability of the non-smooth dynamic.

Lemma 2 (Generalized Lyapunov Theorem)**.** Given that (2) is discontinuous on the right-hand-side, and has an equilibrium point $x = x_0$. Then, if there exists

- a $V : \mathbb{R}^n \to \mathbb{R}, V(x_0) = 0, V(x) > 0, \forall x \neq 0$, such that $V(x(t))$ is absolutely continuous on $[t, \infty)$,
- with $\dfrac{d}{dt}[V(x(t))] < -\epsilon < 0$ a.e. on $\{t \mid x(t) \neq x_0\}$,

then x converge to x_0 in finite time. Thus, the system (2) is *generally asymptotically stable* in the sense of Lyapunov.

Proof. See Theorem 2 in [3] for the complete proof. □

3 Problem Formulation

3.1 Gale and Shapley Algorithm

We assume that there exists sets of men $\mathcal{M} := \{m_1,\ m_2, \dots, m_P\}$, and women $\mathcal{W} := \{w_1, w_2, \dots, w_P\}$ with P is the number of pairs. Each of the individuals in these two sets rank orderly their preferred partners in the preference lists. Let us denote the man m is the current man proposing to the woman w in his preference list. Also, if the proposed woman w is already engaged, we denote her current partner as $\bar{m}$. We denote $\bar{w}$ as the next-woman to be proposed from of the man m's preference list. The procedure of the G-S algorithm [1] is presented in Algorithm 1.

Algorithm 1 Gale & Shapley Algorithm (Men-proposer)

procedure STABLEMATCHING(m, w)
    Initialize all men and women to be free
    while $\exists$ free m **do**
        w = m's highest ranked woman to whom he has not yet proposed
        if w is free **then**
            $m = p_M(w)$ & $w = p_M(m)$
            $(m, w) \to M$ ▹ m, w become *partner*
        else if w is already engaged with $\bar{m}$ **then**
            if m precedes $\bar{m}$ in w's preference list **then**
                $w = p_M(m)$
                $(m, w) \to M$ ▹ w choose new *partner*
                $\bar{m}$ becomes free
            else
                $w = p_M(\bar{m})$ remain engaged
                $(\bar{m}, w) \to M$ ▹ $(\bar{m}, w)$ remain *partner*
                m proposes to $\bar{w}$ in his preference list
            end if
        end if
    end while
    Final matching is established.
end procedure

This algorithm is guaranteed to terminate at $O(P \log P)$ iterations[4], and upon termination, stable pairs M will be established. The stable pairs established in M is said to be *Men-Optimal, Women-Pessimal* since the men are the first proposing to the women. The results will be the opposite if women is the first party to propose [1, 5].

Notice that G-S algorithm utilizes the sequential steps of optimizing in seeking for the stable pairs between the bipartite sets. In the following chapters, we attempt to address the same optimization problem as before, but utilize the multi-agent system to achieve the objective.

3.2 Agents Dynamics

We consider N agents moving in an n dimensional Euclidean space. Each of the agents is described by a single integrator as

$$\dot{x}_i = u_i, \tag{5}$$

where $x_i \in \mathbb{R}^n$ is the position vector of agent i and $u_i \in \mathbb{R}^n$ is the (velocity) control input to be designed. To represent N agents, the total state and control vectors are defined as

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \in \mathbb{R}^{nN} \text{ and } u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \in \mathbb{R}^{nN}, \tag{6}$$

respectively. We further rewrite the total system dynamic in the form

$$\dot{x} = u. \tag{7}$$

The group center that is the virtual center state can be obtained by considering the average of all positions as

$$x_c = \frac{1}{N} \sum_{i=1}^{N} x_i. \tag{8}$$

It is also assumed that the desired state trajectory $x_d \in \mathbb{R}^n$ for the group center x_c can be achieved by $u_d \in \mathbb{R}^n$, that is

$$\dot{x}_d = u_d. \tag{9}$$

3.3 Dynamical Stable Marriage Problem

In this section, we introduce the the terminology of the dynamical stable marriage problem. For the case of the dynamical stable marriage problem, we aim to find the optimum stable matching between the bipartite sets by using the dynamic of the multi-agent systems.

We consider a group of agents $\mathcal{I} = \{1, 2, \ldots, N\}$ and the corresponding positions $\mathcal{X} = \{x_1, x_2 \ldots, x_N\}$, where x_i is defined in (5). We assume that $\mathcal{I} = \mathcal{M} \cup \mathcal{W}$ and $\mathcal{M} \cap \mathcal{W} = \emptyset$. Furthermore, we assume that P satisfies $N = 2P$. In addition, let the odd numbering agents belong to the men set and the even numbering agents belong to the women set, respectively. That is,

$$\begin{aligned} i \in \mathcal{M} &\leftrightarrow i \in \mathcal{I}_M := \{1, 3, \ldots, N-1\} \\ i \in \mathcal{W} &\leftrightarrow i \in \mathcal{I}_W := \{2, 4, \ldots, N\}. \end{aligned} \tag{10}$$

The following definition provides the physical interpretation of the dynamical stable matching.

Definition 2 (Dynamical Stable Matching)**.** For the agent x_i, we define an index set denoting opposite gender and same gender as follows.

$$\mathcal{J}_i = \begin{cases} \mathcal{I}_W & \text{if } i \in \mathcal{M} \\ \mathcal{I}_M & \text{if } i \in \mathcal{W}, \end{cases} \qquad \mathcal{K}_i = \begin{cases} \mathcal{I}_M & \text{if } i \in \mathcal{M} \\ \mathcal{I}_W & \text{if } i \in \mathcal{W}. \end{cases} \tag{11}$$

The agents $i \in \mathcal{I}$ are said to dynamically achieve stable matching if

(a) for the stable partner $j^* \in \mathcal{J}_i$ of i, (i.e., $j^* := p_M(i) \mapsto (i, j^*) \in M$), then $\lim_{t\to\infty} \|x_{j^*}(t) - x_i(t)\| = 0$.

(b) for $k \in \mathcal{K}_i$ there exists $t_0 > 0$ such that $\|x_k(t) - x_i(t)\| \geq d$ for all $t > t_0$.

(c) the center trajectory x_c satisfies $\lim\limits_{t\to\infty} \|x_c(t) - x_d(t)\| = 0$, for the desired trajectory x_d.

To achieve the dynamical stable matching, we use the weighted degree w_{ij} defined as follows:

Definition 3 (Weighted Degree)**.** The weighted degree w_{ij} is defined by using the preference rank p_{ij} as

$$w_{ij} = \begin{cases} \dfrac{2}{|\mathcal{J}_i|\left(|\mathcal{J}_i| + 1\right)} \left(|\mathcal{J}_i| - p_{ij} + 1\right) & \text{if } j \in \mathcal{J}_i \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where $|\mathcal{J}_i|$ is the cardinality of $\mathcal{J}_i$.

3.4 Lyapunov Function Minimization

In designing the suitable control law for the dynamical stable matching, we utilize all information available in the preference list. We design a Lyapunov function to be minimized as

$$V(x) = V_T(x) + V_c(x), \tag{13}$$

where $V_T(x) > 0 : \mathbb{R}^{nN} \to \mathbb{R}$ and $V_c(x) > 0 : \mathbb{R}^n \to \mathbb{R}$ are defined to use a multiplier constant $\nu_{ik} > 0 \in \mathbb{R}$ and the weighted degree w_{ij} as

$$V_T(x) = \sum_{i=1}^{N} V_i(x), \tag{14}$$

$$V_i(x) = \exp \sum_{j\in\mathcal{J}_i} w_{ij} \|x_j - x_i\| + \sum_{k\in\mathcal{K}_i} \nu_{ik} \left(d - \|x_k - x_i\|\right)^2 - 1, \tag{15}$$

$$V_c(x) = \frac{1}{2}\|x_c - x_d\|^2. \tag{16}$$

The Lyapunov candidate V_T is designed to take (a) and (b) in Def. 2 into consideration. Minimizing V_T implies to solve an optimization problem:

$$\begin{aligned} &\text{minimize} && \exp \sum_{j\in\mathcal{J}_i} w_{ij} \|x_j - x_i\| \\ &\text{subject to} && \|x_k - x_i\| \geq d, \ \forall k \in \mathcal{K}_i, i = 1, 2, \ldots, N. \end{aligned} \tag{17}$$

Meanwhile, the Lyapunov candidate V_c is designed as the energy dissipation function corresponding to the x_c and x_d to take (c) into consideration in Def. 2. The weight w_{ij} in (17) is defined in Def. 3.

Problem 1. For the dynamic of agent i in (5) and to use the weighted degree (12) obtained from the preferences list of men and women, design the appropriate control action u_i such that the Lyapunov function (13) is minimized.

4 Main Results

4.1 Centralized Control

The following theorem provide the suitable control law to attain dynamical stable matching.

Theorem 1. (Fixed network stable matching theorem) For fixed network agents, starting from $\Omega = \{x : V(x) \leq \gamma\}$, the dynamical stable matching is achieved under the state feedback control

$$u_i = u_i^{SM} + u_i^c \tag{18}$$

$$u_i^{SM} = u_i^a + u_i^r, \tag{19}$$

$$u_i^c = -k^c(x_i - x_d) + u_d, \tag{20}$$

where u_i^a is the attraction of agent i with $j \in \mathcal{J}_i$

$$u_i^a = \sum_{j \in \mathcal{J}_i} (a_i b_{ij} + a_j b_{ji})(x_j - x_i) \tag{21}$$

and u_i^r is the repulsion between agent i with $k \in \mathcal{K}_i$

$$u_i^r = \sum_{k \in \mathcal{K}_i} (c_{ik} + c_{ki})(x_k - x_i), \tag{22}$$

and $k^c > 0$ is an appropriate control gain and

$$a_i = \exp \sum_{j' \in \mathcal{J}_i} w_{ij'} \|x_{j'} - x_i\| \tag{23}$$

$$b_{ij} = \frac{w_{ij}}{\|x_j - x_i\|} \tag{24}$$

$$c_{ik} = 2\nu_{ik}\left(1 - \frac{d}{\|x_k - x_i\|}\right). \tag{25}$$

4.2 Decentralized Control

The proposed control law (19) for each agent i presented in Section 4.1 utilizes the information from all agents. Due to the excess information, the calculated control action tends to be very large. Hence, to overcome this problem we allow each agent i to communicate only with its neighboring agents. As all agents tend to follow the desired trajectory to achieve condition (c) in Def. 2, there will be more agents within i's sensing range, thus permitting for wider selection of agents to be matched with.

Definition 4 (Neighboring Agents)**.** Let R_g be the distance sensing range of agent i. Agent j is called a neighbor of agent i if it belongs to the set

$$\mathcal{N}_i = \left\{ j \in \mathcal{I} \mid r_{ij} = \|x_j - x_i\| \leq R_g \right\}. \tag{26}$$

Due to limited communication between agents, we propose the decentralized control of the same form as (18) given by

$$u_i = u_{i_\mathcal{N}}^{SM} + u_i^c \tag{27}$$

and in the vector form as

$$u = u_\mathcal{N}^{SM} + u^c \tag{28}$$

where $u_\mathcal{N}^{SM}$ is denoted in (29) and $u^c = -(I_N \otimes \mathbf{k}^c)(x - \mathbf{1}_N \otimes x_d) + \mathbf{1}_N \otimes u_d$. The control law $u_{i_\mathcal{N}}^{SM}$ in (27) is due to SMP and has the same form as (19), but with different control gains depending on the communication topology. In the vector form, it is recurrently defined as

$$u_\mathcal{N}^{SM} = -(L^\mathcal{N} \otimes I_n)x \tag{29}$$

where $L^\mathcal{N} = L_\mathcal{N}^a + L_\mathcal{N}^r$ with

$$L_\mathcal{N}^a := (l_{ij}^{a,\mathcal{N}})_{N\times N} = \begin{cases} \sum\limits_{j' \in \mathcal{J}_i \cap \mathcal{N}_i} (a_i^\mathcal{N} b_{ij'}^\mathcal{N} + a_{j'}^\mathcal{N} b_{j'i}^\mathcal{N}) & \text{if } \ j = i \\ -(a_i^\mathcal{N} b_{ij}^\mathcal{N} + a_j^\mathcal{N} b_{ji}^\mathcal{N}) & \text{if } \ j \neq i, j \in \mathcal{J}_i \cap \mathcal{N}_i \\ 0 & \text{otherwise,} \end{cases}$$

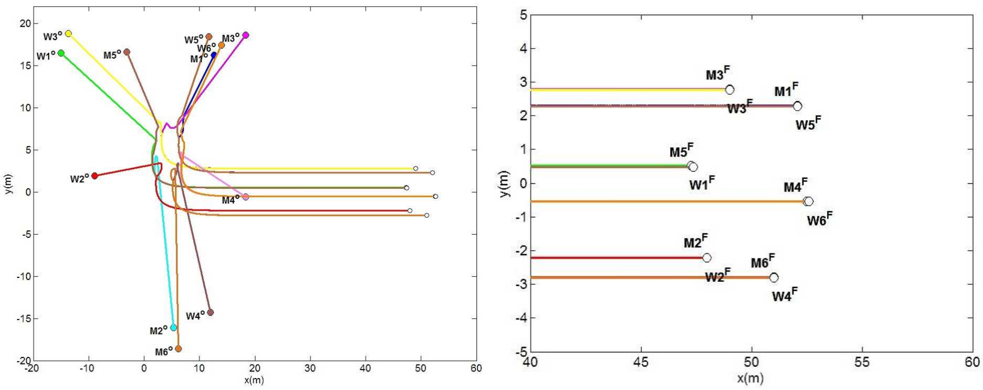
$$L_\mathcal{N}^r := (l_{ij}^{r,\mathcal{N}})_{N\times N} = \begin{cases} \sum\limits_{k' \in \mathcal{K}_i \cap \mathcal{N}_i} (c_{ik'}^\mathcal{N} + c_{k'i}^\mathcal{N}) & \text{if } \ k = i, \\ -(c_{ik}^\mathcal{N} + c_{ki}^\mathcal{N}) & \text{if } \ k \neq i, \ k \in \mathcal{K}_i \cap \mathcal{N}_i \\ 0 & \text{otherwise.} \end{cases}$$

The variables $a_i^\mathcal{N}, b_{ij}^\mathcal{N}$ and $c_{ik}^\mathcal{N}$ of each agent i take the form of equation (23) but only their neighboring agents are taken into accounts. Here, we redefine (23) as

$$\begin{aligned} a_i^\mathcal{N} &= \exp\left(\sum_{j' \in \mathcal{J}_i \cap \mathcal{N}_i} w_{ij'} \|x_{j'} - x_i\| \right) \\ b_{ij}^\mathcal{N} &= \frac{w_{ij}}{\|x_j - x_i\|} \\ c_{ik}^\mathcal{N} &= 2\nu_{ik}\left(1 - \frac{d}{\|x_k - x_i\|}\right). \end{aligned} \tag{30}$$

4.3 Discontinuous Dynamic

Due to the selection of control law (29), the total control dynamic (7) has discontinuous right-hand-side, hence it takes the differential form of (3). Based on Def. 1 and using (28) where $f(x,t) = u = u_\mathcal{N}^{SM} + u^c$, the differential inclusion defined in (4) for



(a) Trajectories (b) Final positions (enlarged)

Figure 1: Agents trajectories in matching with their stable partners when we use the centralized control (18).

the discontinuous differential equation of (3) can be calculated using the calculus in [3]. Hence, the total dynamic is obtained as

$$\begin{aligned} \dot{x} \in K\{f(x,t)\} &= K\{u\} \\ &= K\{u^{SM}\} + K\{u^c\} \\ &= \begin{cases} u_N^{SM} + u^c & \text{if } \overline{\exists} \\ u^c & \text{otherwise,} \end{cases} \end{aligned} \tag{31}$$

where $\overline{\exists} \triangleq \{\forall i,\ \exists j \in \mathcal{J}_i \cap \mathcal{N}_i,\ \exists k \in \mathcal{K}_i \cap \mathcal{N}_i\}$.

4.3.1 Nonsmooth Stability Analysis

In similar form to (13), we use a suitable Lyapunov function candidate for the nonsmooth system as

$$V(x) = V_T(x) + V_c(x) \tag{32}$$

where

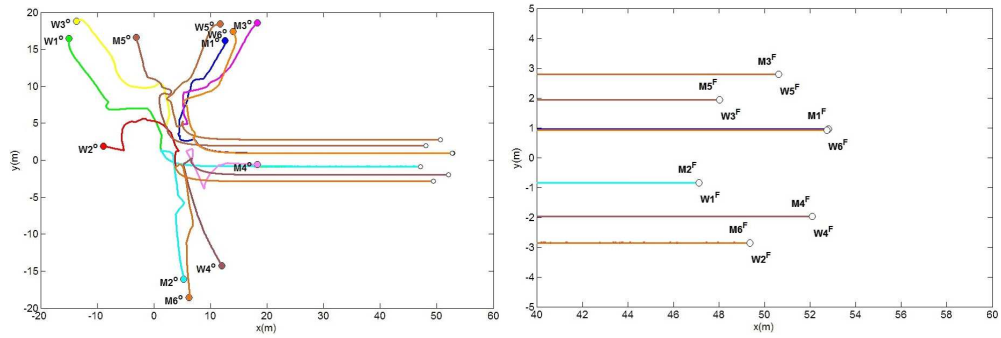
$$V_T(x) = \sum_{i=1}^{N} V_i(x) \tag{33}$$

and $V_c(x)$ is defined in (16), respectively. However, slight modification in V_i is needed so that $\mathcal{J}_i$ is divided into two groups: neighbor $\mathcal{J}_i \cap \mathcal{N}_i$ and not neighbor $\mathcal{J}_i \backslash \mathcal{N}_i$, and $\mathcal{K}_i$ into $\mathcal{K}_i \cap \mathcal{N}_i$ and $\mathcal{K}_i \backslash \mathcal{N}_i$, as well. That is,

$$\begin{aligned} V_i(x) = &\exp\left(\sum_{j \in \mathcal{J}_i \cap \mathcal{N}_i} w_{ij} \|x_j - x_i\|\right) + \exp\left(\sum_{j \in \mathcal{J}_i \backslash \mathcal{N}_i} w_{ij} R\right) \\ &+ \sum_{k \in \mathcal{K}_i \cap \mathcal{N}_i} v_{ik} \left(d - \|x_k - x_i\|\right)^2 + \sum_{k \in \mathcal{K}_i \backslash \mathcal{N}_i} v_{ik} \left(d - R\right)^2 - 1 \end{aligned}$$

where $V_i(\cdot) \in C^0$ corresponds to the local Lyapunov function of agent i.

To achieve dynamical stable matching between agents with time-varying communication topology, we propose the following theorem:



(a) Trajectories (b) Final positions (enlarged)

Figure 2: Agents trajectories in matching with their stable partners when we use the decentralized control (27).

Theorem 2. (Time-varying network stable matching theorem) For time-varying network agents, starting from $\Omega = \{x : V(x) \leq \gamma\}$, the dynamic stable matching between agents as defined in Def. 2 is *generally* asymptotically stable under the state feedback control u_i (27) with appropriate control gain k^c.

5 Numerical Example

We consider a MAS consists of 12 agents (which constitute 6 stable pairs) in such they can be segregated into men and women's sets. We denote the odd-numbering agents as belonging to the men set and the even-numbering agents are assigned to the opposite gender. Each of these agents ranks orderly their preferred partners as listed in Table 1. We assume that the individual agent has access to the information on the preference list of the other agents. We consider the movement of agents in the Euclidean space such that $n = 2$.

The positions of all agents at $t = 0$ were randomly initialized within $[-20m, 20m]$ in both x-y coordinates. The desired trajectory dynamic was chosen as straight line given by $x_d(t) = [5t,\ 0]^T$. Meanwhile, the desired distance separation between stable pairs was set as $d = 5m$. The state feedback control gain and the multiplier constant were chosen as $k_c = 2$ and $\nu_i = 10$, respectively.

We first investigate the overall system's behavior due to the centralized control law (18). Figure 1 shows the agents' trajectories and their final positions, respectively. It can be seen that formation center converged to the desired trajectory which yield to the final matching of $M = \{m_1w_5, m_2w_2, m_3w_3, m_4w_6, m_5w_1, m_6w_4\}$ satisfying all conditions in Def. 2.

Next, we tested the decentralized control law (27) to achieve the stable matching in the multi-agent systems. In contrast to the centralized control structure, each of the agents calculate its control action based on the information obtained from the other agents which are within its sensing range. In this case, we choose the sensing range of the agent i as $R = 10m$. Figure 2 shows the agent trajectories and their final positions when we use the decentralized control law (27). The final matching obtained by this control action is $M = \{m_1w_6,\ m_2w_1,\ m_3w_5,\ m_4w_4,\ m_5w_3,$

$m_6w_2\}$.

6 Conclusion

Motivated by the Gale & Shapley seminal work, we investigated the potential of acquainting the SMP theory into MAS. Two control algorithms have been presented: the centralized and decentralized control structures. Our results indicated that the total system is asymptotically stable in the sense of Lyapunov, but the final matching exhibited a number of blocking pairs. Hence, the final matching is unstable in terms of SMP stability theory.

A Example of Preference Lists

Table 1: Example of Men and Women Preference Lists for 6 pairs SMP

Men's List

	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
m_1	w_3	w_6	w_5	w_1	w_2	w_4
m_2	w_2	w_1	w_5	w_6	w_3	w_4
m_3	w_2	w_3	w_1	w_6	w_5	w_4
m_4	w_1	w_3	w_2	w_4	w_5	w_6
m_5	w_4	w_6	w_2	w_3	w_5	w_1
m_6	w_1	w_6	w_4	w_2	w_5	w_3

Women's List

	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
w_1	m_2	m_3	m_5	m_1	m_4	m_6
w_2	m_6	m_4	m_2	m_3	m_5	m_1
w_3	m_2	m_4	m_3	m_5	m_6	m_1
w_4	m_5	m_4	m_1	m_6	m_3	m_2
w_5	m_6	m_1	m_5	m_2	m_3	m_4
w_6	m_3	m_4	m_2	m_6	m_5	m_1

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学位論文審査報告書（甲）

１．学位論文題目（外国語の場合は和訳を付けること。）

A Study on Multi-Agent Systems for Stable Matching

（安定マッチングのためのマルチエージェントシステムに関する研究）

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３．審査結果の要旨（600～650字）

当該学位論文に関し，平成２７年８月３日に第１回学位論文審査委員会を開催した．同日に口頭発表を実施し，その後に第２回審査委員会を開催した．慎重審議の結果，以下の通り判定した．なお口頭発表における質疑を最終試験に代えるものとした．

本論文は，マルチエージェントシステムの安定マッチング形成に関する研究成果をまとめたものである．二部グラフの安定な一対一マッチングを求める問題は，安定結婚問題として知られ，その解を得る Gale-Shapley アルゴリズムは広く知られている． 本論文では，二次元空間内を移動できる複数の移動体の間で安定マッチングを形成するための制御則を提案している．従来のアルゴリズムではペアを形成するためには離散的なステップを繰り返すが，提案法では連続的に二次元空間内を移動して最適なパートナーを探すことができる方法となっている．この提案手法に従うマルチエージェントシステムは安定結婚問題の動的モデルとも解釈できる．

以上の研究成果は，マルチエージェントシステムの制御問題での新しい知見を与えると共に，ゲーム理論や社会制度設計などへの展開に寄与するとも考えられる．従って，本論文は博士（工学）に値すると判定した．

４．審査結果　(1) 判　　定（いずれかに○印）　合　格　・　不合格

(2) 授与学位　博　士（　工　学　）