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Optimum Design of Aluminum Beverage Can Ends Using Structural Optimization Techniques

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Abstract. This paper has tried to apply the response surface approximate method in the structural optimization techniques to develop aluminum beverage can ends. Geometrical parameters of the end shell are selected as design variables. The analysis points in the design space are assigned using an orthogonal array in the design-of-experiment technique. Finite element analysis code is used to simulate the deforming behavior and to calculate buckling strength and central panel displacement of the end shell under internal pressure. On the basis of the numerical analysis results, the response surface of the buckling strength and panel growth are approximated in terms of the design variables. By using a numerical optimization program, the weight of the end shell is minimized subject to constraints of the buckling strength, panel growth suppression and other design requirements. A numerical example on 202 end shell optimization problem has been shown in this paper.

INTRODUCTION

A large cost saving may be achieved through the mass production of beverage containers even if only a small saving is achieved in one container, therefore cost reductions in cans & ends together with openability and drinkability improvements are major subjects of can-makers [1, 2]. Since the material of can ends (AA5000 series) is more expensive than that of can bodies (AA3000 series), light weighting of ends is more attractive. The combination of smaller ends and thinner gauges has been the principal method of decreasing end costs. Light weighting has largely been achieved through a reduction in diameter from 211 through 209, 207.5, 206, 204 and now to 202 diameter. The diameter specification for an end, such as 211, means the outside diameter of the double seam is approximately 2 and 11/16 inches (68.3 mm). Similarly, a 202 end would measure 2 and 2/16 inches (54.0 mm) on the outside of the double seam. With the necking technology being improved, the diameter of

ends may be reduced further. Beverage can ends and bodies are transported from can-making plants to can-filling plants and are seamed together after a beverage filling process. Therefore, in developing the lightweight ends, design requirements such as the formability, stackability, seamability, buckling strength, panel growth suppression, etc. must be taken into consideration. Although many can-makers, die manufacturers and aluminum rolling companies have been developing their own lightweight end profiles as next generation candidates [3], difficulties still remain not only on meeting the end design requirements, but also on the huge investments in tooling that are necessary in can-making plants and the changes of seaming and filling systems required in can-filling plants. In Japan, 204 diameter ends have been applied to 211 diameter bodies, while 202 diameter ends are under research and development.

On the other hand, the Finite Element Method, as a cost effective tool, has been applied to predict

beverage can performance and to simulate the can forming process for one decade [4-6]. In recent years, structural optimization techniques based on finite element analyses have been developed rapidly and been applied widely in industry. The response surface approximation (RSA) method in the structural optimization technique has been used to carry out design optimization for aluminum beverage cans & bottles [7-9]. As crushable beverage cans, cylindrical shells have been triangulated and optimized for recycling after drinking [7]. In order to optimize effectively a design problem with many design variables in complex geometrical relations, the Design Variable Progressive Optimization method based on RSA is proposed and applied to optimize the bottom of two-piece aluminum beverage bottles [8]. Moreover, in developing an aluminum beverage bottle for being served hot, the RSA method and Weighted Sum Approach in multi-objective optimization techniques have been applied to optimize the rib-shape embossed bottle body so that it may have temperate touch feeling and good embossing formability as expected by designers [9]. However, no effort is found on optimizing the aluminum can end using the structural optimization technique.

Since greater light weighting can be achieved by modifying the geometry of the ends, this paper describes a shape optimum design for the end shell of aluminum beverage cans using RSA method based on design-of-experiment techniques. The finite element code MSC.MARC is used to simulate deforming process of the end shell under internal pressure, and the response surface approximation technique is then applied to construct the approximate functions of the buckling strength and the panel growth in terms of design variables. The geometric parameters of end shells are then optimized using Design Optimization Tools (DOT). The objective of the shape optimization is to minimize the weight of the end shell subject to

constraints of the buckling strength, the panel growth suppression and other design requirements

DESIGN AND ANALYSES MODEL OF END SHELL

The cross section of the end shell double seamed with the can body is illustrated in Fig.1(a), and the finite element analysis model of the end shell is shown in Fig.1(b). The circular end shell is discretized into three-node axisymmetric shell elements. All freedoms of the nodes on the cover hook are fixed and the internal pressure q is applied to the inner surface of the shell except the seaming panel. The material of the end shell is assumed as aluminum with the thickness $t = 0.215$ mm, Young's modulus $E = 68.6$ GPa, Poisson's ratio $\nu = 0.33$, and yield stress $\sigma_y = 275$ MPa. Work hardening of deformation is taken into consideration by adopting work hardening experimental data.

Figure 2 illustrates the base design shape of the shell considered in this paper, and Table 1 lists nomenclature. The base design shape has one curving point M on the chuck wall and a fixed seaming panel with outline dimensions a_1 and b_1 , hence, a constant surface area S_0 . Figure 3 shows a typical plot of internal pressure versus displacement of the shell center (panel growth). The maximum pressure Q in the curve is the so-called buckling strength. When light weighting the end by modifying the shell geometrical profile, this paper considers the following four basic design requirements.

- (1) Buckling strength requirement: The buckling strength of the shell is required to be larger than its allowable minimum value $Q_{\min}$.
- (2) Panel growth suppression: The panel growth of the shell when internal pressure becomes Q_b

FIGURE 1. Analysis model of the end shell

FIGURE 2. Base design shape (central lines in thickness direction).

(so-called bulge strength) is expected to be smaller than its allowable maximum value. In other words, a clearance H (Fig.4) when $q = Q_b$ is required to be larger than its allowable minimum value $H_{\min}$ in order to keep a space for the pull-tab between the panel and the top of the seaming part.

- (3) End stackability requirement: As shown in Fig. 5(a), the end stackability is required in order to reduce the volume for transportation and stock. Since the seamed end is considered in this paper, an alternative method, maintaining clearance H_1 between panels of two ends, is applied to meet the end stackability requirement. As shown in Fig. 5(b), clearance H_1 is maintained to be larger than its minimum value $H_{1\min} = 2$ mm so that the seaming panels rather than other parts contact each other, hence, the inner surface of the end can be kept clean.
- (4) Countersink forming clearance requirement: The allowable minimum clearance $H_{2\min} = t + 0.05$ mm is required to be maintained in the countersink forming process as shown in Fig.6.

Under the consideration of the end design requirements and various restrictions in the practical production, the design space and the panel wall angle $A_2 = 6^\circ$, the countersink depth $h_1 = h_2 + b_1 + 1.4125$ mm are decided in this paper. From a study on geometrical parameter effectiveness, it is observed that when upper chuck wall angle A_1 is greater than 45° , the buckling deformation occurs at the intersection N between the seaming panel and chuck wall, and the buckling strength decreases.

FIGURE 3 Internal pressure VS panel growth

TABLE 1. Nomenclature

SYMBOL	Description
h_1 (mm)	Countersink depth
h_2 (mm)	Panel depth
h_3 (mm)	Height of chuck wall curving point
D_S (mm)	Seam diameter
D_P (mm)	Panel diameter
D_C (mm)	Countersink diameter
r_1 (mm)	Panel radius
r_2 (mm)	Inside countersink radius
r_3 (mm)	Outside countersink radius
A_1 (deg)	Upper chuck wall angle
A_2 (deg)	Panel wall angle
A_3 (deg)	Lower chuck wall angle

OPTIMUM DESIGN OF THE SHAPE OF END SHELL

Formulation of Weight Minimization Problem of End Shell

The geometric parameters of the end shell are optimized under the constraints of the buckling strength and panel growth suppression in order to minimize the weight of the end shell. The material and thickness of the end shell are assumed to be uniform, so minimizing the weight of the end shell can be achieved by minimizing the surface area S . The buckling strength Q and the clearance H are restricted to be larger than the allowable minimum values $Q_{\min}$ and $H_{\min}$, respectively. The main geometric parameters of the end shell are selected as design variables. The weight minimization problem of the end shell is then posed as:

$$\text{Find design variables: } X = \{x_i\}, \quad i = 1, \dots, n \quad (1)$$

(n : the number of design variables)

FIGURE 4. Panel growth suppression.

$$\text{minimize } f = S(X), \quad (2)$$

$$\text{subject to } g_1 = 1 - Q(X)/Q_{\min} \leq 0, \quad (3)$$

$$g_2 = 1 - H(X)/H_{\min} \leq 0, \quad (4)$$

$$x_i^L \leq x_i \leq x_i^U, \quad i = 1, \dots, n, \quad (5)$$

where x_i^U, x_i^L are the upper and lower bounds of design variable i , respectively. The RSA method in the structural optimization technique based on the finite element analyses is applied to perform the weight minimization of the end shell. At first, an orthogonal array in the design-of-experiment technique is employed to assign analysis points and MSC.MARC is adopted to simulate the deforming behavior of the end shell under internal pressure for each analysis point. On the basis of the numerical results of the structural behavior, the response surface approximation technique is applied to generate the approximated functions of the buckling strength $Q(X)$ and clearance $H(X)$ in terms of the design variables. The objective function $S(X)$ is then minimized under constraints of the buckling strength and panel growth suppression by the numerical optimization program DOT. This optimization process is repeated until the given convergence condition is satisfied.

Numerical Examples of Optimum Design

As a numerical example, shape optimization is performed for 202 diameter end shell, i.e. the seam diameter $D_S = 53.78$ mm. The dimensions of the end shell, such as the panel depth h_2 , height of chuck wall curving point h_3 , upper chuck wall angle A_1 , lower chuck wall angle A_3 , panel radius r_1 , inside countersink radius r_2 and outside countersink radius r_3 , are taken as design variables. Consequently, the countersink diameter D_C and panel diameter D_P are

specified by the design variables as,

$$D_C = D_S - 2(a_1 + a_2 + a_3 + a_4) \quad (6)$$

$$D_P = D_C - 2(a_5 + a_6 + a_7) \quad (7)$$

where,

$$a_2 = b_2 \tan A_1, \quad a_3 = b_3 \tan A_3, \quad a_4 = r_3 \cos A_3, \quad (8)$$

$$a_5 = r_2 \cos A_2, \quad a_6 = b_4 \tan A_2, \quad a_7 = r_1 \cos A_2,$$

$$\text{and } b_2 = h_1 - h_3 - b_1, \quad b_3 = h_3 - r_3(1 - \sin A_3),$$

$$b_4 = h_2 - (r_1 + r_2) \cdot (1 - \sin A_2). \quad (8)$$

The surface area S of the shell except fixed seaming panel (S_0) is then calculated using Eq. (9),

$$S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7, \quad (9)$$

$$\text{where } S_1 = 2\pi \frac{b_2}{\cos A_1} \left(\frac{D_S}{2} - a_1 - \frac{a_2}{2} \right), \quad (10)$$

$$S_2 = 2\pi \frac{b_3}{\cos A_3} \left(\frac{D_C}{2} + a_4 + \frac{a_3}{2} \right), \quad (11)$$

$$S_3 = 2\pi \cdot r_3 \left(\frac{\pi}{2} - \frac{A_3\pi}{180} \right) \left(\frac{D_C}{2} + \frac{a_4}{2} \right), \quad (12)$$

$$S_4 = 2\pi \cdot r_2 \left(\frac{\pi}{2} - \frac{A_2\pi}{180} \right) \left(\frac{D_C}{2} - \frac{a_5}{2} \right), \quad (13)$$

$$S_5 = 2\pi \frac{b_4}{\cos A_2} \left(\frac{D_C}{2} - a_5 - \frac{a_6}{2} \right), \quad (14)$$

FIGURE 5. End stactability.

FIGURE 6. Countersink forming.

$$S_6 = 2\pi \cdot r_1 \left(\frac{\pi}{2} - \frac{A_2 \pi}{180} \right) \left(\frac{D_P}{2} + \frac{a_7}{2} \right), \quad (15)$$

$$S_7 = \pi \left(\frac{D_P}{2} \right)^2. \quad (16)$$

Considering the end design requirements and various restrictions in the practical production, ranges of design variables are given as:

$$\begin{aligned} 1.43\text{mm} \leq h_2 \leq 3.0\text{mm}, \quad 1.0\text{mm} \leq h_3 \leq 2.5\text{mm}, \\ 15.5^\circ \leq A_1 \leq 30.0^\circ, \quad 5.57^\circ \leq A_3 \leq 15.0^\circ, \\ 0.2\text{mm} \leq r_1 \leq 0.5\text{mm}, \quad 0.3\text{mm} \leq r_2 \leq 1.0\text{mm} \\ 0.2\text{mm} \leq r_3 \leq 0.5\text{mm}. \end{aligned}$$

To construct response surfaces of the problem with seven design variables, the orthogonal array L_{27} in the design-of-experiment technique is adopted to arrange twenty-seven design points as shown in Table 2, in which three design levels of each design variable have equal intervals. These twenty-seven models are analyzed by finite element software MSC.MARC, and the buckling strength Q and the clearance H are estimated numerically at each design sampling point. The response surfaces for the buckling strength $Q(X)$ and the clearance $H(X)$ are then approximated by the orthogonal polynomials as follows,

$$\begin{aligned} Q(h_2, h_3, A_1, A_3, r_1, r_2, r_3) = & 4.75 \\ & + 2.08h_2 - 0.22h_2^2 + 1.10h_3 - 0.20h_3^2 \\ & - 0.04A_1 - 0.16A_3 - 0.54r_1 + 0.30r_1^2 \\ & - 1.55r_2 + 0.05r_2^2 + 4.73r_3 - 8.59r_3^2 \\ H(h_2, h_3, A_1, A_3, r_1, r_2, r_3) = & -0.09 \\ & + 1.37h_2 - 0.24h_2^2 + 0.09h_3 - 0.04h_3^2 \\ & + 0.61r_1 - 0.89r_1^2 + 0.12r_2 - 0.05r_2^2 \\ & + 0.674r_3 - 1.04r_3^2 \end{aligned} \quad (17)$$

The allowable minimum values of the buckling strength Q and the clearance H are given as $Q_{\min} = 0.58 \text{ MPa}$ and $H_{\min} = 1.68 \text{ mm}$ when $Q_b = 0.45 \text{ MPa}$. The optimal solution is obtained by DOT as $h_2 = 1.43 \text{ mm}$, $h_3 = 1.00 \text{ mm}$, $A_1 = 30^\circ$, $A_3 = 5.57^\circ$, $r_1 = 0.50 \text{ mm}$, $r_2 = 0.8 \text{ mm}$, and $r_3 = 0.4 \text{ mm}$. The finite element analysis is performed on this optimum end model 1 to estimate the buckling strength Q_1 and clearance H_1 . Figure 7 shows the relation between the internal pressure and the panel growth, and the

cross section of deformed end shell when buckling occurs. It is observed that the buckling strength Q_1 is 95.0% of $Q_{\min}$ and the clearance H_1 is 97.0% of $H_{\min}$. Two constraint violations are less than 5%, which is small enough to use as an aid for optimum design. The typical buckling deformation occurs at the curving point M on the chuck wall. The weight of the optimized shell model 1 is reduced by about 15% as compared with that of 204 diameter shell baseline.

Aluminum cans used for different kinds of beverages are designed to meet different requirements. Therefore, an optimum design of the shell is also performed when allowable limits are assumed as $Q'_{\min} = 0.71 \text{ MPa}$ and $H_{\min} = 1.68 \text{ mm}$ when $Q_b = 0.45 \text{ MPa}$. The optimal solution is obtained by DOT as $h_2 = 1.43 \text{ mm}$, $h_3 = 1.8 \text{ mm}$, $A_1 = 30^\circ$, $A_3 = 5.57^\circ$, $r_1 = 0.5 \text{ mm}$, $r_2 = 0.3 \text{ mm}$, and $r_3 = 0.3 \text{ mm}$. Deforming behavior of this optimum end model 2 under internal pressure is simulated to calculate the buckling strength Q_2 and clearance H_2 . Figure 8 shows the plot of the internal pressure versus the panel growth and the cross section of the deformed end shell when the internal pressure becomes Q_2 . It is clear that the buckling strength Q_2 is 96% of $Q'_{\min}$ and the clearance H_2 is 100% of $H_{\min}$. The weight of the optimized shell model 2 is about 13 % lighter than that of 204 diameter shell baseline.

CONCLUSIONS

The response surface approximation method in the structural optimization techniques is applied to develop aluminum beverage can end shells. Geometrical parameters of the end shell are selected as design variables. The analysis points are assigned using an orthogonal array in the design-of-experiment technique. Finite element analysis code is used to simulate the deforming behavior and to calculate buckling strength and the panel growth under internal pressure. On the basis of the numerical analysis results, the response surface of the buckling strength and panel growth are approximated in terms of the design variables. By using a numerical optimization program, the weight of the end shell is minimized subject to constraints of the buckling strength, panel growth suppression and other design requirements.

A numerical optimum design example has been shown on designing 202 end shells to meet different requirements for different usage, and the optimization result of at least 14.7% metal saving has been obtained. It is obvious that using RSA method based on the design-of-experiment technique for developing the end shell can save the computational cost on constructing

FIGURE 7. Numerical analysis results of the optimum model 1.

FIGURE 8. Numerical analysis results of the optimum model 2.

TABLE 2. Analysis points.

Analysis points	h_2 mm	h_3 mm	A_1 deg	A_3 deg	r_1 mm	r_2 mm	r_3 mm
1	1.430	1.00	15.50	5.570	0.20	0.30	0.20
2	1.430	1.00	22.75	5.570	0.35	0.65	0.20
3	1.430	1.00	30.00	5.570	0.50	1.00	0.20
4	1.430	1.75	15.50	10.285	0.20	0.65	0.35
5	1.430	1.75	22.75	10.285	0.35	1.00	0.35
6	1.430	1.75	30.00	10.285	0.50	0.30	0.35
7	1.430	2.50	15.50	15.000	0.20	1.00	0.50
8	1.430	2.50	22.75	15.000	0.35	0.30	0.50
9	1.430	2.50	30.00	15.000	0.50	0.65	0.50
10	2.215	1.00	15.50	10.285	0.35	0.30	0.50
11	2.215	1.00	22.75	10.285	0.50	0.65	0.50
12	2.215	1.00	30.00	10.285	0.20	1.00	0.50
13	2.215	1.75	15.50	15.000	0.35	0.65	0.20
14	2.215	1.75	22.75	15.000	0.50	1.00	0.20
15	2.215	1.75	30.00	15.000	0.20	0.30	0.20
16	2.215	2.50	15.50	5.570	0.35	1.00	0.35
17	2.215	2.50	22.75	5.570	0.50	0.30	0.35
18	2.215	2.50	30.00	5.570	0.20	0.65	0.35
19	3.000	1.00	15.50	15.000	0.50	0.30	0.35
20	3.000	1.00	22.75	15.000	0.20	0.65	0.35
21	3.000	1.00	30.00	15.000	0.35	1.00	0.35
22	3.000	1.75	15.50	5.570	0.50	0.65	0.50
23	3.000	1.75	22.75	5.570	0.20	1.00	0.50
24	3.000	1.75	30.00	5.570	0.35	0.30	0.50
25	3.000	2.50	15.50	10.285	0.50	1.00	0.20
26	3.000	2.50	22.75	10.285	0.20	0.30	0.20
27	3.000	2.50	30.00	10.285	0.35	0.65	0.20

approximate response surfaces and can save time on designing the shape of the end shell to meet different levels of the buckling strength requirements.

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