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Nakayama Kenji, Hirano Akihiro, Horita Akihide

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A LEARNING ALGORITHM WITH ADAPTIVE EXPONENTIAL STEPSIZE FOR BLIND SOURCE SEPARATION OF CONVOLUTIVE MIXTURES WITH REVERBERATIONS

Kenji Nakayama Akihiro Hirano Akihide Horita

Dept. of Information and Systems Eng., Faculty of Eng., Kanazawa Univ.
2-40-20, Kodatsuno, Kanazawa, 920-8667, JAPAN
e-mail: nakayama@t.kanazawa-u.ac.jp

First, convergence properties in blind source separation (BSS) of convolutive mixtures are analyzed. A fully recurrent network is taken into account. Convergence is highly dependent on relation among signal source power, transmission gain and delay in a mixing process. Especially, reverberations degrade separation performance. Second, a learning algorithm is proposed for this situation. In an unmixing block, feedback paths have an FIR filter. The filter coefficients are updated through the gradient algorithm starting from zero initial guess. The correction is exponentially scaled along the tap number. In other words, stepsize is exponentially weighted. Since the filter coefficients with a long delay are easily affected by the reverberations, their correction are suppressed. Exponential weighting is automatically adjusted by approximating an envelop of the filter coefficients in a learning process. Through simulation, good separation performance, which is the same as in no reverberations condition, can be achieved by the proposed method.

1. INTRODUCTION

Signal processing including noise cancelation, echo cancelation, equalization of transmission lines, estimation and restoration of signals have been becoming very important technology. In some cases, we do not have enough information about signals and interference. Furthermore, their mixing and transmission processes are not well known in advance. Under these situations, blind source separation (BSS) technology using statistical property of the signal sources have become very important [1]-[7],[13],[14].

Since, in many applications, mixing processes are convolutive mixtures, FIR or IIR filters are required in unmixing processes. Several methods in a time domain and a frequency domain have been proposed. However, when high-order filters are required in the feedbacks, a learning process becomes unstable and separation performance is not enough [8]–[12]. An approach has been proposed taking some practical assumption into account [15]. High-order FIR filters can be used in a unmixing process. Furthermore, reverberations must be taken into account, which causes severe condition in BSS. No efficient method has been proposed.

In this paper, convergence properties are analyzed for convolutive mixtures with reverberations. A learning algorithm with an exponentially weighted stepsize is proposed. The exponential weighting is automatically adjusted in a learning process. Simulation will be shown to confirm usefulness of the proposed method.

2. NETWORK STRUCTURE AND EQUATIONS

Figure 1 shows a fully recurrent BSS model proposed by Jutten et al [3]. The mixing stage has convolutive structure. FIR filters are used in feedback circuits of an unmixing block as shown in Fig.2.

Fig. 1. Block diagram of recurrent BSS.

Fig. 2. FIR filter used for $C_{21}(z)$ and $C_{12}(z)$ in feedback.

The signal sources $s_i(n), i = 1, 2, \cdots, N$ are combined
through the unknown convolutive mixture block, which has
the impulse response \( h_{ji}(m) \), and are sensed at \( N \) points,
resulting in \( x_j(n) \).

\[
x_j(n) = \sum_{i=1}^{N} \sum_{m=0}^{M_{ji}-1} h_{ji}(m)s_i(n-m)
\]  

(1)

The output of the unmixing block \( y_j(n) \) is given by

\[
y_j(n) = x_j(n) - \sum_{i=1, i \neq j}^{N} \sum_{l=0}^{L_{ij}-1} c_{jk}(l)y_k(n-l)
\]

(2)

This relation is expressed using vectors and matrices as follows:

\[
x(n) = H^T s(n)
\]

(3)

\[
y(n) = x(n) - C^T \tilde{y}(n)
\]

(4)

\[
s(n) = [s_1^T(n), s_2^T(n), \ldots, s_N^T(n)]^T
\]

(5)

\[
s_i(n) = [s_i(n), s_i(n-1), \ldots, s_i(n-M_i+1)]^T
\]

(6)

\[
x(n) = [x_1(n), x_2(n), \ldots, x_N(n)]^T
\]

(7)

\[
y(n) = [y_1(n), y_2(n), \ldots, y_N(n)]^T
\]

(8)

\[
\tilde{y}(n) = [y_1^T(n), y_2^T(n), \ldots, y_N^T(n)]^T
\]

(9)

\[
y_k(n) = [y_k(n), y_k(n-1), \ldots, y_k(n-L_{jk}+1)]^T
\]

(10)

\[
H = \begin{bmatrix} h_{11} & h_{21} & \ldots & h_{N1} \\ h_{12} & h_{22} & \ldots & h_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1N} & h_{2N} & \ldots & h_{NN} \end{bmatrix}
\]

(11)

\[
h_{ji} = [h_{ji}(0), h_{ji}(1), \ldots, h_{ji}(M_{ji}-1)]^T
\]

(12)

\[
C = \begin{bmatrix} 0 & c_{e1} & \ldots & c_{e1} \\ c_{12} & 0 & \ldots & c_{e2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1N} & c_{e2N} & \ldots & 0 \end{bmatrix}
\]

(13)

\[
c_{jk} = [c_{jk}(0), c_{jk}(1), \ldots, c_{jk}(L_{jk}-1)]^T
\]

(14)

\[
S(z) = [S_1(z), S_2(z), \ldots, S_N(z)]^T
\]

(15)

\[
Y(z) = (I + C(z))^{-1} X(z)
\]

(16)

\[
Y(z) = (I + C(z))^{-1} H(z) S(z)
\]

(17)

In order to evaluate separation performance, the following matrix is defined.

\[
P(z) = (I + C(z))^{-1} H(z)
\]

(22)

If each row and column of \( P(z) \) has only a single non-zero element, the signal sources \( s_i(n) \) are completely separated at the outputs \( y_k(n) \). However, since equalization of \( H(z) \)

is not guaranteed, the separated signals have the following form.

\[
Y_j(z) = P_{ji}(z)S_i(z)
\]

(23)

3. LEARNING ALGORITHM

The learning algorithm proposed for convolutive BSS is briefly explained here [15]. For simplicity, 2-channel case is taken into account.

There are two cases, in which possible solutions for perfect separation exist, as shown below.

(1) \( C_{21}(z) = \frac{H_{21}(z)}{H_{11}(z)} \) \( C_{12}(z) = \frac{H_{12}(z)}{H_{22}(z)} \)

(24)

\[
y_1(n) = h_{11}^T s_1(n) \quad y_2(n) = h_{22}^T s_2(n)
\]

(25)

(2) \( C_{21}(z) = \frac{H_{22}(z)}{H_{12}(z)} \) \( C_{12}(z) = \frac{H_{11}(z)}{H_{21}(z)} \)

(26)

\[
y_1(n) = h_{12}^T s_2(n) \quad y_2(n) = h_{21}^T s_1(n)
\]

(27)

It is assumed that delay time of \( H_{11}(z) \) and \( H_{22}(z) \) are shorter than that of \( H_{21}(z) \) and \( H_{12}(z) \). This means that in Fig.2, the sensor of \( X_1 \) is located close to \( s_1(n) \), and the sensor of \( X_2 \) close to \( s_2(n) \). From this assumption, the solutions in the case (1) become causal systems. On the other hand, the solutions in the case (2) are noncausal.

From Eq.(21), the outputs are expressed as

\[
[\begin{array}{c} Y_1(z) \\ Y_2(z) \end{array}] = \frac{1}{1 - C_{12}(z) C_{21}(z)} \left[ \begin{array}{c} S_1(z) \\ S_2(z) \end{array} \right]
\]

(28)

\[
\times \left[ \begin{array}{c} H_{11}(z) \\ H_{21}(z) \end{array} \right]
\]

\[
\times \left[ \begin{array}{c} H_{12}(z) \\ H_{22}(z) \end{array} \right]
\]

\[
\times \left[ \begin{array}{c} 1 \\ -C_{21}(z) \end{array} \right]
\]

\[
= \frac{1}{1 - C_{12}(z) C_{21}(z)} \left[ \begin{array}{c} H_{11}(z) - C_{12}(z) H_{21}(z) \\ H_{21}(z) - C_{21}(z) H_{11}(z) \end{array} \right]
\]

(29)

Since Eq.(26) cannot be realized using causal circuits, the diagonal elements of Eq.(29) cannot be zero. On the other hand, the non-diagonal elements can be zero. Therefore, a cost function can be defined as follows:

\[
J_j(n) = E[q(y_j(n))]
\]

(30)

\( q() \) is an even function with a single minimum point. By minimizing this cost function, \( C_{12}(z) \) and \( C_{21}(z) \) can approach to Eq.(24). Instead of \( E[q(y_j(n))] \), the instantaneous
value $g(y_j(n))$ is used, and the gradient method can be applied.

$$\hat{J}_j(n) = q(y_j(n))$$  \hspace{1cm} (31)

The gradient of $\hat{J}_j(n)$ becomes

$$\frac{\partial \hat{J}_j(n)}{\partial c_{jk}(l)} = \frac{\partial q(y_j(n))}{\partial y_j(n)} \frac{\partial y_j(n)}{\partial c_{jk}(l)}$$

$$= \frac{\partial q(y_j(n))}{\partial y_j(n)} y_k(n-l)$$

$$\frac{\hat{y}_j(n)}{= x_j(n) - \sum_{l=0}^{L_{jk}-1} c_{jk}(l)y_k(n-l)}$$  \hspace{1cm} (32)

$\hat{q}(\cdot)$ is a partial derivative, which is an odd function. If $k = 1$, then $j = 2$, and vice versa. Therefore, the update equation of $c_{jk}(l)$ is given by

$$c_{jk}(n+1, l) = c_{jk}(n, l) + \Delta c_{jk}(n, l)$$  \hspace{1cm} (34)

$$\Delta c_{jk}(n, l) = \mu \hat{q}(y_j(n)) y_k(n-l)$$  \hspace{1cm} (35)

The probability density function (pdf) of the signal sources are assumed to be even functions. Furthermore, the signal sources are statistically independent to each other. Then, they satisfy

$$E[f(s_1(n))g(s_2(n))] = E[f(s_1(n))]E[g(s_2(n))]$$

$$= 0$$  \hspace{1cm} (36)

$$f(), g() : \text{odd functions}$$

If a very small stepsize $\mu$ is used in Eq.(35), the correction term can be regarded as $E[\hat{q}(y_j(n)) y_k(n-l)]$. Since, $\hat{q}(y_j(n))$ and $y_k(n-l)$ are also odd functions, then Eq.(37) can be held. This means that as the correction terms are reduced, $y_1(n)$ and $y_2(n)$ can approach to $h_{11}^T s_1(n)$ and $h_{22}^T s_2(n)$, respectively.

4. A LEARNING ALGORITHM FOR CONVOLUTIVE BSS WITH REVERBERATIONS

4.1. Convergence Analysis

When reverberations occur, the assumption on the transmission delay in the mixing process cannot be held. A model including reverberations is shown in Fig.3. $H_{11}(z)$ and $H_{22}(z)$ express transfer functions caused by reverberation, which has a long transmission delay. $H_{12}(z)$ and $H_{21}(z)$ are not shown here for simplicity. By using the learning algorithm described in the previous section, the following two terms can be reduced at $X_1$.

$$H_{11}^r(z)S_1(z) - C_{12}(z)H_{21}(z)S_1(z) \to 0$$  \hspace{1cm} (38)

$$C_{12}(z) \to \frac{H_{11}(z)}{H_{21}(z)}$$  \hspace{1cm} (39)

$$H_{21}(z)S_2(z) - C_{12}(z)H_{22}(z)S_2(z) \to 0$$  \hspace{1cm} (40)

$$C_{12}(z) \to \frac{H_{12}(z)}{H_{22}(z)}$$  \hspace{1cm} (41)

$$H_{11}'(z)/H_{21}(z) \text{ can be also causal. In other words, not only the } S_2(z) \text{ component but also the } S_1(z) \text{ component can be cancelled by the signal through the path of } -C_{12}(z). \text{ However, the optimum forms of } C_{12}(z) \text{ for canceling } S_2(z) \text{ and } S_1(z) \text{ are different.}$$

In the same manner, at $X_2$,

$$H_{22}'(z)S_2(z) - C_{21}(z)H_{12}(z)S_2(z) \to 0$$  \hspace{1cm} (42)

$$C_{21}(z) \to \frac{H_{22}'(z)}{H_{12}(z)}$$  \hspace{1cm} (43)

$$H_{21}(z)S_1(z) - C_{21}(z)H_{11}(z)S_1(z) \to 0$$  \hspace{1cm} (44)

$$C_{21}(z) \to \frac{H_{21}(z)}{H_{11}(z)}$$  \hspace{1cm} (45)

Eqs.(41) and (45) are the ideal solutions. However, $C_{12}(z)$ and $C_{21}(z)$ cannot approach to these solutions due to the reverberations given by Eqs.(39) and (43).

4.2. A Learning Algorithm with Exponential Scaling

Reverberations have a long delay, and from Eqs.(39) and (43), effects of reverberations appear at the latter part of the impulse responses. For this reason, the correction in the latter part is suppressed. This can be done by controlling the stepsize $\mu$ exponentially along a delay line in the FIR filters. The update equation is modified as follows:

$$c_{jk}(n+1, l) = c_{jk}(n, l) + \mu(l)f(y_j(n))g(y_k(n-l))$$  \hspace{1cm} (46)

$$\mu(l) = \mu_0 r^l, \quad 0 < r < 1$$  \hspace{1cm} (47)

$\mu(l)$ should be proportional to the ideal solution. However, it is not known beforehand. Therefore, the exponential scaling is proposed here. $\mu_0$ is the initial stepsize and $r^l$ is an exponential part.
The exponentially weighted stepsize was proposed for NLMS adaptive filters [16]. However, in this method, the geometric ratio $r$ should be estimated in advance taking room impulse responses into account. Therefore, this method is not practical. In this paper, an adaptive method is proposed. The exponentially weighted stepsize is automatically adjusted by approximating an envelop of the filter coefficients in the learning process.

Let $\mu(n,l)$ be the stepsize at the sampling point $n$ and the tap number $l$. The stepsize and the filter coefficients are transferred as follows:

$$
\log \mu(n,l) = \log \mu_0(n) + l \log r(n)
$$

$$
b(n,l) = \log |c_{jk}(n,l)|
$$

(b(n,l) is approximated using $x_1(n) + lx_2(n)$ by the least squares method.

$$
x_1(n) + lx_2(n) = b(n,l)
$$

$$
A \mathbf{x}(n) = b(n)
$$

$$
A = \begin{bmatrix}
1 & l_{\text{max}} \\
1 & \text{max} + 1 \\
\vdots & \vdots \\
1 & L_{jk - 1}
\end{bmatrix}
$$

$$
\mathbf{x}(n) = \begin{bmatrix}
x_1(n) \\
x_2(n)
\end{bmatrix}
$$

$$
b(n) = \begin{bmatrix}
b(n,l_{\text{max}}) \\
b(n,l_{\text{max}} + 1) \\
\vdots \\
b(n,L_{jk - 1})
\end{bmatrix}
$$

$L_{\text{max}}$ means the tap number, where the peak of the filter coefficients appears. The least square solution is given by

$$
\mathbf{x}(n) = A^+ b(n)
$$

$$
A^+ = (A^T A)^{-1} A^T
$$

Using these results, $r(n)$, $\mu_0(n)$ and the stepsize $\mu(n,l)$ are given by

$$
\mu_0(n) = e^{x_1}
$$

$$
r(n) = e^{x_2}
$$

$$
\hat{r}(n) = \alpha r(n) + (1 - \alpha) \hat{r}(n - 1)
$$

$$
0 < \alpha \ll 1
$$

$$
\mu(n,l) = \mu_0(n) \hat{r}(n)^l
$$

The geometric ratio is gradually updated. The initial guess of $\hat{r}(n)$ is 1.

## 6. SIMULATION

### 6.1. Simulation Conditions

Two channel blind separation of speech signals was simulated. The following nonlinear functions are used.

$$
f(y) = \tanh(2.5y) \quad g(y) = \tanh(0.5y)
$$

The separation performance is evaluated by the following SNR, defined by using $P(z)$ in Eq.(22).

$$
\sigma_s^2 = \sum_{i=1}^{2} \frac{1}{2\pi} \int_{-\pi}^{\pi} |P_{ii}(e^{j\omega T})|^2 d\omega T
$$

$$
\sigma_c^2 = \sum_{j \neq i} \frac{1}{2\pi} \int_{-\pi}^{\pi} |P_{ij}(e^{j\omega T})|^2 d\omega T
$$

$$
\text{SNR} = 10 \log \frac{\sigma_s^2}{\sigma_c^2} \ [\text{dB}]
$$

$\sigma_s^2$ expresses power of the selected signals and $\sigma_c^2$ is that of the cross components.

Convolution mixing process with reverberations are shown below.

$$
H_{11} = 1 - 0.4z^{-T} + 0.18z^{-2T}
$$

$$
H_{12} = 0.5z^{-6T} + 0.175z^{-7T} + 0.03z^{-8T}
$$

$$
H_{21} = 0.5z^{-6T} + 0.135z^{-7T} + 0.01z^{-8T}
$$

$$
H_{22} = 1 + 0.4z^{-T} - 0.2z^{-2T}
$$

$$
H'_{11} = 0.1z^{-10T}(1 - 0.5z^{-T} + 0.02z^{-2T})
$$

$$
H'_{12} = 0.1z^{-10T}(1 + 0.38z^{-T} + 0.02z^{-2T})
$$

$$
H'_{21} = 0.1z^{-10T}(1 + 0.32z^{-T} + 0.03z^{-2T})
$$

$$
H'_{22} = 0.1z^{-10T}(1 + 0.33z^{-T} + 0.01z^{-2T})
$$

40 taps, which can cover the impulse response of the ideal solutions are assigned to both $C_{12}(z)$ and $C_{21}(z)$.

### 6.2. Separation Performance with Fixed Stepsizes

$SNR$ are shown in Fig.4, which are obtained by using a constant stepsize $\mu = 0.005$, an inversely controlled stepsize $\mu = 0.02/l$ and the exponential stepsize with $\mu_0 = 0.025$ and $r = 0.83$. $\mu$ and $r$ are is obtained by approximating an envelop of the ideal impulse response. Furthermore, $SNR$ obtained without the reverberations and with a constant stepsize $\mu = 0.005$ is also shown for comparison. From this figure, the reverberations significantly degrade the separation performance with a constant stepsize. The exponentially weighted stepsize can achieve almost the same $SNR$ as under no reverberation condition. Thus, degradation due to the reverberation can be improved by using the exponential stepsize.

### 6.3. Separation Performance with Adaptive Stepsize

Figure 5 shows the adjusting process of the geometric ratios $r_{12}(n)$ and $r_{21}(n)$ used in updating $c_{12}(n)$ and $c_{21}(n)$.
respectively. They are adjusted around the optimum value $r = 0.83$. Thus, it is confirmed that the learning of the geometric ratios is successful.

The final stepsize of the proposed method and the reference stepsize, which is obtained by approximating the envelop of the ideal filter coefficients in the least squares sense, are shown in Fig. 6. They are almost the same. Thus, the proposed adaptive stepsize can reach the envelop of the ideal filter coefficients.

The separation performance is shown in Fig. 7. The reference stepsize and the adaptive stepsize are used. Their separation performance are almost the same. In the proposed method, it is not necessary to estimate the envelop of the ideal filter coefficients in advance. This is a very important point in practical applications.

Figure 8 shows the ideal filter coefficients and the final
filter coefficients obtained by the proposed method for $c_{12}$ and $c_{21}$.

![Figure 8](image_url)

**Fig. 8.** Ideal and trained filter coefficients $c_{12}$ and $c_{21}$.

## 7. CONCLUSIONS

Convergence properties have been analyzed in convolutive BSS with reverberations. Due to the reverberations, the filters used in the unmixing block deviate from the ideal. The effects appear in the latter part of the impulse responses. A learning algorithm using the exponentially weighted step-size has been proposed. The geometric ratio of the stepsize is automatically adjusted. From the simulation results for 2 channel BSS, the proposed method can achieve good separation as in BSS without reverberations.

## 8. REFERENCES


