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IEEJ Transaction on Electrical and Electronic Engineering

Volume 9, Number 1, Page Range 39-48, Year 2014-01-01

URL: http://hdl.handle.net/2297/36312

doi: 10.1002/tee.21934
Graph Planarization Problem Optimization Based on Triple-Valued Gravitational Search Algorithm

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This article presents a triple-valued gravitational search algorithm (TGSA) to tackle the graph planarization problem (GPP). GPP is one of the most important tasks in graph theory, and has proved to be an NP-hard problem. To solve it, TGSA uses a triple-valued encoding scheme and models the search space into a triangular hypercube quantitatively based on the well-known single-row routing representation method. The agents in TGSA, whose interactions are driven by the gravity law, move toward the global optimal position gradually. The position updating rule for each agent is based on two indices: one is a velocity index which is a function of the current velocity of the agent, and the other is a population index based on the cumulative information in the whole population. To verify the performance of the algorithm, 21 benchmark instances are tested. Experimental results indicate that TGSA can solve the GPP by finding its maximum planar subgraph and embedding the resulting edges into a plane simultaneously. Compared with traditional algorithms, a novelty of TGSA is that it can find multiple optimal solutions for the GPP. Comparative results also demonstrate that TGSA outperforms the traditional meta-heuristics in terms of the solution qualities within reasonable computational times. © 2013 Institute of Electrical Engineers of Japan. Published by John Wiley & Sons, Inc.

Keywords: gravitational search algorithm, triple-valued encoding, graph planarization

Received 12 July 2012; Revised 6 October 2012

1. Introduction

Graph planarization problem (GPP) is a widely researched optimization problem. It has not only theoretical importance related with topological problems but also practical applications in numerous areas, such as circuit board layout, facility layout, automatic graph drawing, and VLSI circuit routing [1]. A graph is said to be planar or embeddable in a plane if it can be drawn on the plane in such a way that no two of its edges intersect except at a common endpoint. Given an $n$-vertex $m$-edge graph $G = (V,E)$ with vertex set $V$ and edge set $E$, the objective of finding the maximum planar subgraph (MPS) is to find a minimum cardinality subset of edges $F \subseteq E$ such that the graph $G' = (V,E\setminus F)$ resulting from the removal of the edges $F$ from $G$ is planar. As a result, the GPP generally needs to fulfill two tasks: MPS acquisition and plane embedding. MPS acquisition is to verify if the input graph is planar, and further to extract the edges of its MPS if it is a nonplanar graph. Plane embedding is to embed the MPS of the input graph into a plane and perform the routing of the vertices and edges in the plane without any cross connections or cross edges. The GPP is very difficult to solve, and it belongs to NP-hard problems [2].

Several studies have been presented to propose algorithms for the GPP using different techniques. Initial works focused on developing sophisticated heuristic algorithms based on planarity testing, aiming to identify the edges of the MPS with low computational complexity [3–7]. Nevertheless, these algorithms were devoted only to the graph planarity test task (i.e. MPS acquisition). In other words, they failed to fulfill the plane embedding task. The reason was that there were no embedding mechanisms in these algorithms. To solve the GPP with the MPS acquiring and embedding tasks simultaneously, some meta-heuristics were proposed. In Ref. [8], the first example illustration was provided, and the proposed neural network was then further developed in Refs [9,10] with the purpose of improving its searching performance. In addition to neural networks, genetic algorithms [11,12] and artificial immune systems [13] also exhibited super applicability on the GPP. Although these meta-heuristics are capable of solving the GPP, their performance is limited with the size of the input graphs; thus seeking other meta-heuristics with better optimization abilities is still an open problem.

In this paper, a triple-valued gravitational search algorithm (TGSA) is proposed to solve the GPP. TGSA is an extended version of the gravitational search algorithm (GSA) [14,15]. The main characteristic of TGSA is its triple-valued encoding scheme, which makes the algorithm more suitable to manipulate the GPP. To realize this, the representation of GPP adopts the single-row routing method. The vertices of input graphs are first placed on a straight line. Then the edges are divided into three subsets: the upper edges over the line (U), the lower ones (L), and the directly eliminated ones (E), which naturally form a triangular...
hypercube search space for the algorithm. Thereafter, the GPP is transferred into an optimization problem, where the objective of the implemented TGSAs is to optimize the combinatorial problem of the edges' placement. It is important to highlight that the GPP is able to produce several optimal planar subgraphs, comparable to those obtained by other approaches. This is the crucial feature of population-based algorithms in general, and of the algorithm, TGSAs, used in this research work in particular. Eventually, in order to show the accuracy of the proposed TGSAs, a fair comparison is made with other six meta-heuristics. Experimental results verify the effectiveness of the proposed algorithm.

This paper is structured as follows. The following section makes a small literature review on the algorithms used when solving the GPP. The review also indicates that gravitational-search-based algorithms have never been used for such a study. Section 3 gives a general description of the GSA. Section 4 illustrates the problem representation, the designs, and the application of TGSAs on the GPP. Simulation results are summarized and discussed in Section 5. Finally, some general remarks are given to conclude this paper.

2. Related Works

For finding the MPS of a given graph, numerous heuristics have been proposed. A widely used standard process is to start with a spanning tree of the input graph \( G = (V, E) \), and then iteratively try to add the remaining edges one by one. In every step, a planarity testing algorithm is called for the obtained graph. If the addition of an edge would lead to a nonplanar graph, then the edge is disregarded; otherwise, the edge is added permanently to the planar graph obtained so far. Within this algorithmic framework, Jayakumar et al. [4] proposed a near-maximal planarity testing algorithm with computational complexity \( O(|E|^2) \) based on the PQ-tree technique [3]. Kant [5] presented a corrected algorithm and more generalized version of Jayakumar’s algorithm. Cai et al. [16] and Battista and Tamassia [17] proposed efficient algorithms with the same complexity bound of \( O(|V| \log |E|) \), which were derived from the Hopcroft–Tarjan planarity testing algorithm [18] and the incremental planarity testing algorithm [17], respectively. Furthermore, linear-time \( O(|V| + |E|) \) algorithms can be considered as in Refs [7,19]. Obviously, the heuristics were developed for the purpose of reducing computational times. In addition, Poranen [20] proposed a hybrid simulated annealing method based on the planarity test [21] for MPS, claiming that the hybrid algorithm outperformed its earlier heuristics in terms of the solution quality and computational times. However, its deficiency was also distinct because the planar test used numerous times during its execution. If an implementation for the planar test is not available, the algorithm could not be used to solve the problem.

In general, the algorithms proposed for MPS cannot be directly adopted for the GPP since finding MPS is just one of its objectives, but their basic functions can motivate (or be incorporated into) the design of the GPP solving algorithms. In the literature, some meta-heuristics were proposed to deal with the MPS finding task and plane embedding task simultaneously. Among these algorithms, neural network learning methods, two-phase graph planarization algorithms (TGPA), genetic algorithms, and artificial immune algorithms exhibited promising performance.

Using the neural network techniques, Takefuji and Lee presented a parallel planarization algorithm (TLNN) for generating a near-maximal planar subgraph within \( O(1) \) time [8,22], and claimed superior performance to previously published algorithms. An improved Hopfield network learning algorithm (WNN) with a gradient ascent technique for alleviating the inherent drawbacks (typically the local optima problem) of Takefuji and Lee’s algorithm was proposed by Wang et al. [9], Zhang and Qin [10] combined the Hopfield network with the simulated annealing strategy (ZNN), aiming to facilitate the flexibility of the algorithm and further to achieve a better performance.

In addition, Goldschmidt and Takvorian [23] presented a TGPA, based on which Resende and Ribeiro [24] proposed a greedy randomized adaptive search procedure (GRASP) by incorporating more randomness into it. GRASP had two phases: solution construction phase and solution improvement phase. GRASP often mainly focused on the randomized generation of high-quality starting solutions by very refined construction phases and sophisticated management of the solutions, while the subsequent solution improvement phase was usually performed by a rather simple local search. The first phase in GRASP was an enhancement version of the TGPA by randomly choosing one of the best candidates in the restricted candidate list. The second phase in GRASP aimed at minimizing the number of edges that needed to be removed to eliminate all edge crossings with respect to the first-phase sequence, which was realized by implementing a simple local search within a slightly restricted neighborhood. Simulation results indicated that GRASP could obtain better or competitive results than TGPA and branch-and-cut algorithms [25,26] for graphs having up to 300 vertices or 1500 edges.

Besides, the applicability of genetic algorithms to solving the GPP had been verified by Cornellas in Ref. [11]. An effective genetic algorithm (EGA) performing crossover and mutation operators conditionally instead of probability could be also referred to as in Ref. [12]. Most recently, the authors [13] proposed a multilayered artificial immune system (MAIS) for the GPP, which took advantages of the technologies of search space minimization and feature extraction, exhibiting excellent performance on all tested benchmark problems. It is to be noted that this section does not aim to provide a comprehensive literature review, but intends to give some insights into the ideas of the proposed algorithms, and further verify that gravitational search-based algorithms have never been studied for solving the GPP.

3. Gravitational Search Algorithms

Meta-heuristic algorithms mimic biological or physical processes. One of the newest meta-heuristic algorithms that has been inspired by the physical laws is the GSA. It was originally presented by Rashedi et al. [14] for optimizing continuous nonlinear functions, and then widely applied to many engineering problems [27–29]. The algorithm is based on the Newtonian gravity: every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This force is given by the equation:

\[
F = \frac{Gm_1m_2}{r^2}
\]

where \( F \) is the gravitational force between two masses, \( m_1 \) and \( m_2 \), separated by a distance \( r \), and \( G \) is the gravitational constant.

Figure 1 depicts the conceptual graph of gravity law used in GSA. In GSA, agents are considered as solutions and their performance is measured by their masses. Each object in GSA has two
specifications: position and mass. The position of the agent corresponds to a solution of the problem on hand, and its mass is determined using a fitness function. To describe the continuous GSA [14] in formulations, consider a system with $S$ agents in which the position of the $i$th agent is defined as follows:

$$X_i = (x_{id}^1, \ldots, x_{id}^d, \ldots, x_{id}^n), \quad i = 1, 2, \ldots, S$$

where $x_{id}^d$ is the position of the $i$th agent in the $d$th dimension and $n$ is the dimension of the search space.

The mass of each agent is calculated after computing the current population’s fitness as follows:

$$q_i(t) = \frac{\text{fit}_i(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)}$$

$$M_i(t) = q_i(t) \sum_{j=1}^{S} q_j(t)$$

where $M_i(t)$ and $\text{fit}_i(t)$ represent the mass and the fitness value of the agent $i$ at the iteration $t$, respectively. Without loss of generality, $\text{best}(t)$ and $\text{worst}(t)$ are defined for a minimization problem as follows:

$$\text{best}(t) = \min_{j \in \{1, \ldots, S\}} \text{fit}_j(t)$$

$$\text{worst}(t) = \max_{j \in \{1, \ldots, S\}} \text{fit}_j(t)$$

To compute the acceleration of an agent, total forces from all the agents with the best fitness value and biggest mass, which is a function of time as shown in (9), initialized to $K_0$ at the beginning and decreasing with time. The gravitational constant $G$ will take an initial value $G_0$, and it will be reduced with time (10) [14]:

$$K = [(\beta + (1 - \frac{t}{T_{\text{max}}}) (1 - \beta)) K_0]$$

$$G(t) = G(G_0, t) = aG(t - 1)$$

The main components of binary GSA (BGSA) [15] are the same as those of continuous GSA (CGSA). Compared to CGSA, the main novelty of BGSA is its position updating rule, where only the switching between 0s and 1s takes place, and thus makes the search space form into a binary hypercube. Correspondingly, (7) and (8) are changed into the following equations, respectively:

$$v_{id}^{d}(t + 1) = w(t)v_{id}^{d}(t) + 2r_1(x_{id}^d - x_{best}^d) + 2r_2(x_{id}^d - x_{worst}^d)$$

$$x_{id}^{d}(t + 1) = \begin{cases} 1 - x_{id}^{d}(t) & \text{if } r < \frac{1}{1 + e^{-w(t)}} \\ x_{id}^{d}(t) & \text{otherwise} \end{cases}$$

where $r_1$, $r_2$, and $r$ are three uniform random variables in the range $[0, 1]$, $w$ is the inertia weight, and $x_{best}^d$ and $x_{worst}^d$ represent the best previous position of the $i$th agent and the best previous position among all the agents in the population, respectively.

To sum up, GSA (either continuous or binary variant) is a memory-less algorithm and the agent direction is calculated based on the overall force obtained by other agents, which is different from other population-based algorithms, such as PSO, ACO[29], GA, etc. In GSA, the force is directly proportional to the fitness value while inversely proportional to the distance between solutions, so that heavy masses have large effective attraction radii and hence great intensities of attraction, revealing that the agents tend to move toward the best agent, which is the main characteristic of the algorithm. Previous works [14,15,29] on solving many optimization problems have demonstrated the superiority of the algorithm. However, these mentioned GSAs cannot be directly applied on the GPP, which motivates us to propose a novel encoding scheme into GSA, and further evaluate its performance on solving the GPP.

4. TGSA to Solve the GPP

4.1. Problem definition

The way of representing the GPP is described. In this paper, we use the single-row routing representation that was originally proposed in Takefuji and Lee’s algorithm [8] and widely adopted by many other studies [12],[13],[23],[24]. According to this representation method, an arbitrary sequence of all the vertices in the given graph is placed along a line, first. Then all the edges are determined to belong to any of the two sets of edges that may be represented without crossings above and below that line, respectively. Consider a simple undirected graph composed of four vertices and six edges as shown in Fig. 2(a). The graph is planar as long as two edges,

![Fig. 2. Single-row routing representation used in GPP. (a) A graph with four vertices and six edges. (b) A planar graph. (c) A possible planar graph. (d) Violation conditions.](image-url)
agents using (5), the distance between agents should be defined is calculated according to (1)–(4). To compute the force during the formation of edges whose values equal \(E\), the resultant planar subgraph. Formally, for a given set. Consequently, under this representation, each edge has three possible states according to whether, in accordance with a determined vertex sequence, the edge is a lower edge, or it is not considered, or it is an upper edge. The upper edges of the agent are defined as:

\[
X_i = (e_{i}^{1}, e_{i}^{2}, \ldots, e_{i}^{n}), \quad \forall e_{i}^{j} \in \{U, L, E\}, j = 1, 2, \ldots, n
\]

Here, it is worth emphasizing that randomly generated position \(X_i\) might become an invalid solution for the GPP, because the resulting placement of edges is likely to satisfy the violation conditions in (13). Thus, a repair operator to ensure the position always corresponding to valid solutions is necessary. To realize this, a very simple repair operator is proposed in this paper as shown in Algorithm 1.

**Algorithm 1 (Repair Operator used in TGSA):**

**Input:** an arbitrary position \(X_i\) of the agent \(i\), and an empty graph with an established vertex sequence.

01: do
02: for \(j = s_1\) to \(s_n\)
03: embed the edges one by one according to the sequence \((e_{i}^{j}, e_{i}^{j+1}, \ldots, e_{i}^{n})\) into the graph
04: if the current selected edge \(e_{i}^{j}\) does not fulfill the violation conditions with respect to the graph, then embed it into the graph as a new edge \(e_{i}^{j} = U\) or \(L\)
05: otherwise set \(e_{i}^{j} = E\)
06: end-for
07: while: the termination condition is satisfied

**Output:** the best repaired position \(X_i\) of the agent, and a resultant planar subgraph.

Obviously, all agents operated by the repair process will generate valid solutions for the GPP. In this paper, the termination condition in the repair operator is when the number of iteration reaches \(T_E\). The computational complexity of the repair operator is \(O(nT_E)\).

The fitness for position \(X_i\) of agent \(i\) is counted as the number of edges whose values equal \(U\) or \(L\). The mass that apply on an agent can be computed based on (5). The acceleration and velocity of an agent are calculated in the same way using \(\nabla f_{i}(t)\) and \(\nabla v_{i}(t)\), respectively.

In (16), the updating of the position for an agent \(i\) is related to two indices. The prime index is a *velocity index*, which is a function of the current velocity, \(tanh(v_{i}^{j}(t+1)) = (1 + e^{-v_{i}^{j}(t+1)})^{-1}\), and determines whether the updating progress takes place or just remains the same as with the previous position. The second index is a *population index*, which is a cumulative index of being an upper edge or a lower edge for the current selected one. The population index is calculated based on the information collection of the whole population. For instance, if more \(d\)-th edges of the agent to be upper ones exist in the population, larger is probability of the current \(d\)-th edge to be updated into an upper one. In addition, the update into \(E\) never happens, since it will lead to a decrease of the fitness.

4.2. Description of TGSA

In this paper, a novel approach based on the TGSA to solve the GPP is presented. Since there are three possibilities of placing the edges over the line, that is, to be the upper (U) edges, the lower (L) edges, and the eliminated (E) ones, the encoding scheme is naturally designed from the triple-valued set \(\{U, L, E\}\). The agents in TGSA are defined as the edges sequences, whose values are taken from the triple-valued set. Formally, for a given \(m\)-vertex \(n\)-edge graph \(G = (V, E)\), the position \(X_i\) of the agent \(i\) is expressed as:

\[
X_i = (e_{i}^{1}, e_{i}^{2}, \ldots, e_{i}^{n}), \quad \forall e_{i}^{j} \in \{U, L, E\}, j = 1, 2, \ldots, n
\]

By doing so, the triangular hypercube forming the search space can be depicted quantitatively. The distance between each pair of basic elements in the triple-valued set \(\{U, L, E\}\) has equal values. For instance, the solution shown in Fig. 2(c) corresponds to a position \(X_i = (L, U, U, U, L)\), and its fitness is 6. The distance between \(X_i\) and \(X_j\) is \(|v_i^U - v_j^U|\). Consequently, under this representation, each edge has three possible states according to whether, in accordance with a determined vertex sequence, the edge is a lower edge, or it is not considered, or it is an upper edge. The upper edges of the agent are defined as:

\[
X_i = (e_{i}^{1}, e_{i}^{2}, \ldots, e_{i}^{n}), \quad \forall e_{i}^{j} \in \{U, L, E\}, j = 1, 2, \ldots, n
\]

Here, it is worth emphasizing that randomly generated position \(X_i\) might become an invalid solution for the GPP, because the resulting placement of edges is likely to satisfy the violation conditions in (13). Thus, a repair operator to ensure the position always corresponding to valid solutions is necessary. To realize this, a very simple repair operator is proposed in this paper as shown in Algorithm 1.

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**Output:** the best repaired position \(X_i\) of the agent, and a resultant planar subgraph.

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A large value of velocity provides a high probability of changing the position of the agent with respect to its previous position (to be U, L, or remain the same), thus facilitating the agents that have large velocities (usually lighter masses) to move, rather than to stay at the original place. On the contrary, a small value of velocity provides a small probability of changing the position.

Realizing the fact that the force acting on a heavier agent (implying to possess a good solution) causes a smaller acceleration of the agent according to (6), the velocity of a heavier agent usually is hard to be changed by the acceleration, indicating that a heavier agent is hard to move. Conversely, a lighter agent is likely to change its position, and move toward better positions at the end.

Controlled by the population index in (16), which is implemented using a roulette wheel selection method, it is clear that if the current agent moves, it will move to the position where most other agents stand. Hence, all agents will move toward the global optimum position, thus finding the best solution eventually.

### 4.4. Algorithm
To solve the GPP, an algorithm based on TGSA is shown in Algorithm 2. Here it should be noted that the vertex sequence is generated based on an improved Hamiltonian cycle generation (HCG) method. For more details of the original HCG method, refer to Ref. [23]. To make the paper self-explanatory, the improve HCG method is described as follows: The first vertex in the sequence is \( \pi(1) = v_k \), where \( v_k \) is a randomly selected vertex in the given graph \( G \). Different vertex sequence can be generated by selecting different \( \beta \). After the first \( k \) vertices of the sequence have been determined, say \( \pi(1), \pi(2), \ldots, \pi(k) \), the next vertex \( \pi(k+1) \) is randomly selected from the vertices adjacent to \( \pi(k) \) in \( G \) having the \( L \) least adjacencies in the subgraph \( G_k \) of \( G \) induced by \( V \setminus \{\pi(1), \pi(2), \ldots, \pi(k)\} \). If \( \pi(k) \) has no neighbors in \( G_k \), we select \( \pi(k+1) \) as a vertex of the \( L \) minimum degree in \( G_k \) with homogeneous probability. Then, the vertices of \( G \) are placed on a line in accordance with the determined vertex sequence \( \pi \).

With respect to the generated vertex sequences, the TGSA is used to find the optimal placements of edges. Thus, the largest MPS can be acquired and embedded into a plane based on the position of the heaviest agent and its corresponding vertex sequence.

**Algorithm 2 (Solve the GPP based on TGSA):**

**Input:** a given nonplanar graph \( G = (V, E) \) with \( |V| = m, |E| = n \)

01: initialize user-specified parameters, and randomly generate \( S \) positions for the agents
02: **do:**
03: **for** each agent \( i = 1 \) to \( S \)
04: generate a vertices \( \pi_i \) for agent \( i \) using the improved HCG method
05: operate the agent \( i \) using Algorithm 1
06: evaluate the fitness of agent \( i \), and update \( G(t), best(t), worst(t) \) and \( M_i(t) \)
07: **end-for**
08: compute the total force in different directions using (5)
09: compute the acceleration and velocity for each agent using (6) and (7)
10: update all agents’ positions using Eq. (16)
11: **while:** the maximum iteration number achieves \( T_{\text{max}} \)
**Output:** an optimal agent with the heaviest mass, and the corresponding planar subgraph \( G^* = (V', E') \)

### 5. Simulation Results and Discussion

To evaluate the performance of TGSA when applied it on the GPP, a set of 21 benchmark problems described in the literature [23] were used in the experiments. The problem instances are summarized in Table I, where the information data are the name of the input graph, the number of vertices \(|V|\), the number of edges \(|E|\), the Euler upper bound \((3|V| - 6)\) on the number of edges in an MPS, and the number of edges in the best known solution for the GPP published earlier. The algorithm is coded using ANSI C language under the Visual Studio 2005 platform running on a personal PC (Intel(R) Core i3, 2.40 GHz with 2 GB RAM). In the experimental analysis, we tried to follow the suggestions given by Johnson [30].

#### 5.1. Parameter sensitivity
The user-specified parameters were first analyzed, and each combination was tested 30 times. Intuitively, the number of agents used in the population directly influences the trade-off between the search performance and the computational times. To make the subsequent comparison with other population-based algorithms [12],[13] fairly, the same size of the population was set, i.e. \( S = 20 \). Besides, the initial attraction scope of the gravity force was set to be the whole population, i.e. \( K_0 = S \), and the associated shrink parameter \( \beta \) was set to be 2\% as in the original work.

The maximum iteration number \( T_{\text{max}} \) determines the termination condition of the algorithm. The larger the value of \( T_{\text{max}} \), the more the computational time the algorithm needs. In preliminary experiments, we found that TGSA converged to global or local optimal solutions for all instances within 1000 iterations. Table II records the smallest, largest, and average iteration number when TGSA found the global optimal solution or had been trapped into a local optimal solution (In this paper, the solution that cannot be improved over 100 iterations is regarded as a local optimum). From this table, we can find that, for the instances with small graph size, TGSA converges fast. Especially, for the instance \( G_1 \), TGSA can always find the global optimum using only one iteration. Thus, \( T_{\text{max}} = 1000 \) was sufficient to evaluate the performance of TGSA in all experiments.

Different from the declaration in Ref. [14] that the gravitational constant parameter \( G \) strongly effects the performance of the algorithm, in this study we found that TGSA was not so sensitive to this parameter. After setting the initial constant \( G_0 = m^2n^2 \), a set of values as \( \{0.8, 0.9, 0.95, 0.99, 0.995\} \) for the declining factor \( \alpha \) was tested on all instances in the experiments. Similar quality results were obtained for different settings, revealing that TGSA was less sensitive to \( G \). The reason for this seems to be the influence of population index in (16), which partly decreases the effects of the gravitational forces.

As to the maximum replication number \( T_R \) used in the repair operator, we set it as \( \{1, 5, 10, 20, 50, 100\} \). Table III summarizes the best and average solution qualities and the computational times on the instances \( G_2 \) and \( G_3 \). From this table, we can find that \( T_R = 10 \) makes significant improvements of the solutions’ quality than \( T_R = 1 \) and \( T_R = 5 \), while possessing competitive solutions when compared with \( T_R = 20, 50, 100 \). However, with the increment of the value of \( T_R \), the computational times steadily increase. Thus, it can be concluded that the setting of \( T_R = 10 \) can well balance the trade-off between the solution quality and computational times.

Compared to the vertex sequence construction method (HCG) used in Ref. [23], the improved HCG method used in this paper is a generalized version of HCG. By incorporating the parameter \( L \) into HCG, more diversity of the vertex sequence can be obtained, thus enabling TGSA to search more different areas in the search space. In the experiments, we set \( L \) to be \{1, 2, 3, 5, 10, \}_m}.
Table I. Problem instances used in the experiments

<table>
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<tr>
<th>Graph name</th>
<th>No. vertices</th>
<th>No. edges</th>
<th>Upper bound</th>
<th>Best known pre. (References)</th>
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<td>24(^a)</td>
<td>24 ([23,24,13])</td>
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<td>69</td>
<td>69(^a)</td>
<td>69 ([24,13])</td>
</tr>
<tr>
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<td>25</td>
<td>70</td>
<td>69(^a)</td>
<td>69 ([24,13])</td>
</tr>
<tr>
<td>G10</td>
<td>25</td>
<td>71</td>
<td>69(^a)</td>
<td>69 ([24,13])</td>
</tr>
<tr>
<td>G11</td>
<td>25</td>
<td>72</td>
<td>69(^a)</td>
<td>69 ([24,13])</td>
</tr>
<tr>
<td>G12</td>
<td>25</td>
<td>90</td>
<td>69(^a)</td>
<td>67 ([23,24])</td>
</tr>
<tr>
<td>G13</td>
<td>50</td>
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<td>135 ([24])</td>
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<td>G14</td>
<td>50</td>
<td>491</td>
<td>144</td>
<td>143 ([24,13])</td>
</tr>
<tr>
<td>G15</td>
<td>50</td>
<td>582</td>
<td>144</td>
<td>144 ([24])</td>
</tr>
<tr>
<td>G16</td>
<td>100</td>
<td>451</td>
<td>294</td>
<td>196 ([24])</td>
</tr>
<tr>
<td>G17</td>
<td>100</td>
<td>742</td>
<td>294</td>
<td>236 ([24])</td>
</tr>
<tr>
<td>G18</td>
<td>100</td>
<td>922</td>
<td>294</td>
<td>246 ([23,24])</td>
</tr>
<tr>
<td>G19</td>
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<td>1064</td>
<td>444</td>
<td>311 ([24,13])</td>
</tr>
<tr>
<td>G20</td>
<td>300</td>
<td>4136</td>
<td>894</td>
<td>627 ([23])</td>
</tr>
<tr>
<td>G21</td>
<td>1000</td>
<td>9991</td>
<td>2994</td>
<td>1496 ([23])</td>
</tr>
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</table>

\(^a\)Actual known optimal size of the planar subgraph.

Table II. Number of iterations that TGSA needs when it converges to optimal (or suboptimal) solutions

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G5</th>
<th>G8</th>
<th>G13</th>
<th>G16</th>
<th>G19</th>
<th>G20</th>
<th>G21</th>
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<td>Smallest</td>
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<td>32</td>
<td>18</td>
<td>54</td>
<td>112</td>
<td>151</td>
<td>278</td>
<td>325</td>
<td>581</td>
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<tr>
<td>Largest</td>
<td>125</td>
<td>32</td>
<td>89</td>
<td>256</td>
<td>287</td>
<td>523</td>
<td>612</td>
<td>735</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>54</td>
<td>26</td>
<td>77</td>
<td>181</td>
<td>235</td>
<td>424</td>
<td>448</td>
<td>642</td>
<td></td>
</tr>
</tbody>
</table>

Table III. The influence of the parameter \(T_R\) used in the experiments

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<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
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<tbody>
<tr>
<td>(G2)</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td>Time</td>
<td>9.8</td>
<td>10</td>
<td>10.4</td>
<td>10.9</td>
<td>12.1</td>
<td>13.9</td>
</tr>
<tr>
<td>(G11)</td>
<td>68</td>
<td>68</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
</tr>
<tr>
<td>Average</td>
<td>66.7</td>
<td>67.5</td>
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<td>8.4</td>
<td>8.9</td>
<td>10.7</td>
<td>12</td>
</tr>
</tbody>
</table>

respectively, where \(m\) is the number of vertices of the input graph. Note that the original HCG method was a specific case of the improved one when \(L = 1\). The case of \(L = m\) indicated that the constraints of edges’ adjacencies were not considered any longer.

The experimental results are shown in Table IV, where the data recorded are the best and average qualities of final solutions, the computational times, and the average number of different vertex sequences per iteration obtained by the algorithm. From this table, we can find that, first, with the increment of the value of \(L\), the computational times made no significant changes (less than 0.1 s) for all cases; second, the best solution qualities were acquired when \(L = 3\); third, the search performance suddenly declines when \(L = m\), which shows that the use of vertex’s degree does facilitate HCG to generate more promising vertices sequences; and last but not least, the average number of different vertex sequences gradually becomes larger with the increment of the value of \(L\). The larger the \(Num\) in Table IV, the more the different the areas visited by the algorithm in the search space. Obviously, \(L = 3\) was the best choice for the algorithm.

Table IV. The influence of the parameter \(L\) used in the experiments

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<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>(m)</th>
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<tr>
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<td>81.8</td>
<td>81.8</td>
<td>81.8</td>
<td>81.8</td>
<td>76.5</td>
</tr>
<tr>
<td>Time</td>
<td>10.4</td>
<td>10.4</td>
<td>10.4</td>
<td>10.4</td>
<td>10.4</td>
<td>10.4</td>
</tr>
<tr>
<td>Num.</td>
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<td>18.5</td>
<td>19.2</td>
<td>19.9</td>
<td>19.9</td>
<td>20.0</td>
</tr>
<tr>
<td>Best</td>
<td>68</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>66</td>
</tr>
<tr>
<td>(G11) Ave.</td>
<td>66.8</td>
<td>67.9</td>
<td>68.2</td>
<td>68.1</td>
<td>68.1</td>
<td>65.5</td>
</tr>
<tr>
<td>Time</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>Num.</td>
<td>17.5</td>
<td>18.3</td>
<td>18.8</td>
<td>19.8</td>
<td>19.9</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Table V. User-specified parameters employed in the experiments

- Number of agents (i.e., population size) \(S = 20\)
- Initial gravitational constant \(G_0 = m^2n^2\)
- Gravitation declining coefficient \(\alpha = 0.95\)
- Initial attraction scope \(K_0 = S\)
- Attraction narrowing coefficient \(\beta = 2\%
- Vertices sequence generation parameter \(L = 3\)
- Maximum replication number in Algorithm 1 \(T_R = 10\)
- Maximum iteration number in Algorithm 2 \(T_{\text{max}} = 1000\)

Based the above analysis, we can say that the performance of TGS is competitive when using the user-specified parameters (for any given input graph \(G = (V,E)\) with \(|V| = m\) and \(|E| = n\)) shown in Table V.

5.2. Characteristics analysis Compared with CGSA and BGSA, TGSAS has the main novelties of the encoding scheme which is based on the triple-valued set \([U,L,E]\), and the position updating rule shown in (16). The triple-valued encoding scheme naturally depicted the three choices (upper, lower, or eliminated) of placing edges upon the vertices line, and then quantitatively forming a triangular hypercube of search space for the algorithm. Afterward, the position updating rule matured the solutions within the search space.
The comparative simulation results during TGSA, TGSA₁, and TGSA₂ are given in Table VI. For all tested instances, TGSA₁ performed slightly worse than TGSA in terms of the best and average qualities of solutions. However, TGSA₂ performed significantly worse than TGSA, showing that only population index would result in a very fast local optimum trapping. From the above analysis regarding (16), we can conclude that both the velocity index and the population index play positive roles in determining the search directions for the algorithm.

For the convenience of describing the progress of solving the GPP using TGSA, we used $G_{11}$ as an illustration. Figure 3(a) shows the input nonplanar graph $G_{11}$ with 25 vertices and 72 edges. After generating a vertices sequence of $[13, 16, 20, ..., 11, 5]$ by the improved HCG method, TGSA optimized the values of the combination of all edges. Finally, 47 edges were set to be the values of $U$, 22 edges to be $L$, and 3 to be $E$. Figure 3(b) depicts its corresponding planar subgraph based on the single-row routing representation method. Clearly, TGSA can solve the GPP with two objectives simultaneously: not only the acquisition of an MPS, but also the embeddness of these edges into a planar subgraph.

In addition, due to the stochastic nature of the algorithm and of the populations evolved during the convergence process, TGSA can produce more than one planar subgraph for the GPP. To show this capacity of TGSA, we give an example for the instance $G_3$, as shown in Fig. 4. Figure 4(a) depicts the input graph $G_3$ with 10 vertices and 24 edges, while Fig. 4(b)–(h) illustrate seven different global optimal solutions of $G_3$ found by TGSA. For all tested instances, the number of different global optimal solutions in the final iteration found by the algorithm was recorded. The index “No.” shown in Table VI counted the total number of such solutions over 30 runs. A major benefit of this feature is that it will help the decision maker to assess and select an engineering-relevant planar subgraph for industrial applications.

5.3. Comparison with other algorithms Finally, Table VII summarizes the comparative results of TGSA with other six meta-heuristics. It should be noted that all the compared algorithms were able to solve the GPP by fulfilling the two objectives simultaneously. Those algorithms that were only capable of finding the MPS for the GPP were not considered in this study, even though they might find better MPSs. In Table VII, the recorded results of the earlier six algorithms were taken from the original literature, while the latter results were taken on the basis of 30 replication runs of TGSA. The symbol “-” means there is no record for the corresponding instance. From this table, it can be easily found that, for all tested instances, the sizes of the acquired MPSs by TGSA were larger than those by the other meta-heuristics. Besides, TGSA can always find the global optima for a part of the input graphs (9 out of 21 instances), because the average quality of solutions are equal to the best one, thus revealing the strong robustness of TGSA.

Regarding the computational times, Fig. 5 illustrates the computational times in seconds versus the number of edges of the input instances. For all the comparative algorithms, the times increased along with the size of the input graphs grown, nearly at polynomial speed. Although TGSA cannot dominate the other algorithms in terms of the computational times, it can be concluded that TGSA is able to find better solutions for the GPP within reasonable times.
Table VII. Simulation results using seven algorithms

<table>
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<tbody>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Best</td>
</tr>
<tr>
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<td>20</td>
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<td>20</td>
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<td>1496</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1903</td>
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</table>

Fig. 4. Global optimum solutions for the instance $G_3$ based on TGSA

Fig. 5. Computational times of all tested algorithms
6. Conclusions

The TGSA has been successfully used to solve the GPP by finding its MPS and embedding the edges into a plane. Based on the single-row routing representation method, once the vertex sequence was determined by the improved HCG method, TGSA was utilized to optimize the placement of edges, wherein all agents interacted based on the gravity law, and the triple-valued encoding strategy of their positions naturally suited the three choices of edges’ placements.

To improve the search performance of TGSA, the position updating rule was manipulated by two important indices. The velocity index, which is a probability function of velocities, is the prime factor controlling the search direction of an agent by indirectly using the weight of its mass and the forces from other agents. The other is the population index, which is based on the cumulative information collected from the whole population, directly influencing the movement direction by calculating the proportion of the selected edges in the population. By doing so, the local exploitation and global exploration of the search can be well balanced, thus enhancing the search ability of TGSA.

Experimental results based on 21 benchmark instances verified the effectiveness of TGSA. Compared to other traditional meta-heuristics, TGSA could always find better solutions for the GPP within reasonable computational times. In addition, due to its stochastic property and the populations evolved during the convergence process, TGSA can find multiple optimal solutions, which enables the decision maker to assess and select an engineering-relevant planar subgraph.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China under Grants 61203325 and 61003205, Genguang Project of Shanghai Educational Development Foundation (No. 12CG35), and the Programs for Young Excellent Talents in Tongji University (No. 2011KJ054) and Donghua University (No.104-07-0053012), respectively.

References


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