A Theoretical Analysis of the Optical Feedback Noise Based on Multimode Model of Semiconductor Lasers

Sazzad M.S. Imran, Minoru Yamada, *Fellow, IEEE,* and Yuji Kuwamura

***Abstract*—An improved theoretical model to analyze dynamics and operation of semiconductor lasers under optical feedback has been presented in this paper. A set of rate equations are formulated, in which the self and mutual gain saturation effects among lasing modes, re-injection of delayed feedback light reflected at surface of connecting optical device and Langevin noise sources for the intensity and phase fluctuations are taken into account. The proposed model is applied to 850nm GaAs lasers operating under optical feedback. The rate equations are calculated by tracing time variation, and frequency spectra of intensity noise are determined by help of the fast Fourier transformation. In this paper, numerical simulations based on our theoretical model confirmed that the feedback noise is classified into two types based on profiles of the frequency spectrum, where one is the low frequency type and another is the flat type. These properties are in good agreement with those previously obtained in the experimental measurements. This evidence of agreement between experimental results and numerical simulations supports the accuracy of our model.**

***Index Terms*—Feedback noise, gain suppression, intensity noise, Langevin noise source, mode competition, multimode, semiconductor laser.**

I. Introduction

SEMICONDUCTOR lasers are used as light sources in the optical disc system and the fiber optic communication systems, and are required to reveal the lower noise for the higher performance. The intensity noise of the semiconductor lasers consists of the quantum noise and the optical feedback noise. The quantum noise is generated by intrinsic property of the quantum mechanical fluctuation of the laser, which is very difficult to control in principle [1], [2]. This quantum noise is also called the optical shot noise, whose origin is introduced with the Langevin noise sources in the rate equations [3].

Semiconductor lasers tend to show an excess noise called the optical feedback (OFB) noise, which is induced by the re-injection of output light into the laser followed by reflection at surface of connecting optical device such as the lens, the optical disc and the optical fiber. The OFB noise is generated even though amount of the OFB is very small [4]-[7].

Now a day, it may be well known that the OFB noise is generated by the mode competition phenomena among the internal cavity modes (those are the lasing modes of a solitary laser) and/or external cavity modes formed by the output facet and the reflected surface [8]-[11]. The latter effect has been alternatively analyzed in terms of phase distortion effect in the reflected light called as the coherence collapses [11]-[14].

Theoretical analyses of the noise in the semiconductor lasers with external optical feedback are classified into three groups. First one is the so called small signal (variation) analysis representing dynamical behaviors on frequency domain where the Langevin noise sources are taken into account [9], [14]. Dynamic effects of semiconductor lasers based on differential analysis of the rate equations using the Langevin method to determine laser's relative intensity noise (RIN) has been presented in Ref. [9]. The effects of optical feedback on the quantum mechanical amplitude noise properties of laser has also been described in [10] and [11]. However, affect of the OFB on the mode competition has been firstly analyzed in Ref. [8]. The second group is the single mode model, where time delay of the feed-backed light is taken into account with or without introduction of the Langevin noise sources [12], [15]-[17]. Generation of chaotic phenomena by the OFB was well explained by this model. H. Haken has proposed some theoretical models for single mode lasers based on instability hierarchies of laser light, i.e., chaos and routes to chaos, using semiclassical approach [17]. This second model is effective to apply on the DFB (distributed feedback) laser or laser with external DBR (distributed Bragg reflector) mirrors which are also called the dynamically single mode lasers used in the high capacity optical communication systems. The third group is the multimode model which counts the mode competition phenomena among the lasing mode in the solitary laser, but has not counted the Langevin noise sources yet [18].

Model in this paper is an extension of the third group. We start from a set of multimode rate equations for a solitary laser, where effects of the OFB are taken into account as delayed light. Additionally, the Langevin noise sources caused with photon generation are taken into account. Introduction of the noise sources is essential to show the noise characteristics.

Our model is applied to 850nm GaAs lasers, and characteristics of the OFB noise are expressed in terms of the relative intensity noise (RIN).

This paper is organized as follows. In Sec. II, our theoretical

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model is presented. In Sec. III, procedure of calculation is explained. In Sec. IV, simulated results and discussions are given with comparison to experimentally reported data. Finally, this work is concluded in Sec. V.

II. Theoretical Model

Operation of a semiconductor laser under optical feedback (OFB) is illustrated in Fig. 1. The cavity length and the effective refractive index of the laser are L and n_r, respectively. Distance between the laser and a reflecting mirror is l. Light emitted from the laser front facet with reflectivity R_f is assumed to travel single round-trip between the front facet and the external reflecting mirror with optical feedback ratio Γ, then re-injects into the laser cavity. The round-trip time is $\tau=2l/c$, where c is the speed of light in vacuum.

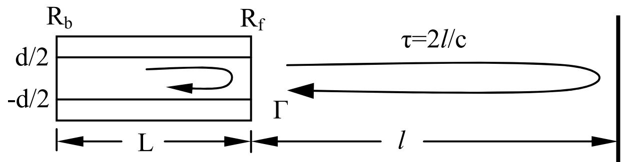


Fig. 1. Operation of a semiconductor laser under optical feedback.

In this model, the modal behavior of longitudinal modes is analyzed considering a stable laser in which only fundamental transverse mode exists. The detailed analysis to obtain stable fundamental transverse mode operation in semiconductor injection lasers was explained in Ref. [19]. Also, our model is based on the assumption that spatial distribution of the electron is uniform in the active region.

The electric component of the lasing field oscillating at frequency ω_p is expressed by [20]

$$E(r,t)=\sum_p \tilde{E}_p(t)\phi_p(r)\exp(j\omega_p t)+c.c. \tag{1}$$

where $\tilde{E}_p(t)$ is a slowly time-varying complex amplitude which is defined with an optical phase $\theta_p(t)$ as

$$\tilde{E}_p(t)=\left|\tilde{E}_p\right|\exp(j\theta_p)\ . \tag{2}$$

$\Phi_p(r)$ is a field spatial distribution function normalized as

$$\int_o^L\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|\phi_p(r)\right|^2 dxdydz=1\ . \tag{3}$$

p=0, ±1, ±2, ±3, . . . is an index of the mode number.

Electric field amplitude $\left|\tilde{E}_p(t)\right|$ can be transformed to photon number $S_p(t)$ using the following relationship postulated by quantization of the lasing field [20], [21]

$$2\varepsilon\left|\tilde{E}_p\right|^2=\begin{cases}(S_p+1)\hbar\omega & \textit{for optical emission}\\ S_p\hbar\omega & \textit{for optical absorption}\end{cases} \tag{4}$$

where ε is the dielectric constant of the active region.

Therefore, the following rate equations of the modal photon number $S_p(t)$, modal phase $\theta_p(t)$ and number of injected electrons $N(t)$ are obtained as [15], [21], [22]

$$\frac{dS_p}{dt}=\left(G_p-G_{tho}+\frac{c}{n_r L}\ln\left|U_p\right|\right)S_p+\frac{a\xi N/V}{\left[2\dfrac{(\lambda_p-\lambda_0)}{\delta\lambda}\right]^2+1}+F_{Sp}(t) \tag{5}$$

$$\frac{d\theta_p}{dt}=\frac{1}{2}\left[\frac{\alpha a\xi}{V}(N-\overline{N})-\frac{c}{n_r L}\varphi\right]+F_{\theta_p}(t) \tag{6}$$

$$\frac{dN}{dt}=-\sum_p A_p S_p-\frac{N}{\tau_S}+\frac{I}{e} \tag{7}$$

where G_p is the gain of mode p whose wavelength is λ_p, G_{tho} is the threshold gain of the solitary laser, U_p is a function counting contribution of the OFB to the instantaneous photon number $S_p(t)$ of the mode p. These parameters are defined as

$$G_p=A_p-B_p S_p-\sum_{q\neq p}(D_{p(q)}+H_{p(q)})S_q \tag{8}$$

$$G_{tho}=\frac{c}{n_r}\left[k+\frac{1}{2L}\ln\frac{1}{R_f R_b}\right] \tag{9}$$

$$U_p=1-(1-R_f)\sqrt{\frac{\eta\Gamma}{R_f}}\exp(-j\omega_p\tau)\sqrt{\frac{S_p(t-\tau)}{S_p(t)}}\exp(j\{\theta_p(t-\tau)-\theta_p(\tau)\})$$
$$=\left|U_p\right|\exp(-j\varphi) \tag{10}$$

$$\varphi=-\tan^{-1}\frac{\mathrm{Im}U_p}{\mathrm{Re}U_p}+m\pi \qquad (-\pi\le\varphi\le\pi) \tag{11}$$

$F_{Sp}(t)$ and $F_{S\theta}(t)$ are the noise sources by inclusion of the fluctuated spontaneous emission determined as the Langevin noise sources and are given by [13]

$$F_{sp}(t)=\sqrt{\frac{V_{S_pS_p}}{\Delta t}}g_s \tag{12}$$

$$F_{\theta p}(t)=\sqrt{\frac{V_{S_pS_p}}{\Delta t}}\cdot\frac{g_\theta}{2(S_p+1)} \tag{13}$$

The noise sources are originally defined as Poisson random processes, and are well approximated as Gaussian distributions with zero mean values and satisfy the following correlations:

$$\left\langle F_{sp}(t)F_{sq}(t')\right\rangle=V_{S_pS_p}\delta_{pq}\delta(t-t') \tag{14}$$

$$\left\langle F_{\theta p}(t)F_{\theta q}(t')\right\rangle=V_{\theta_p\theta_p}\delta_{pq}\delta(t-t') \tag{15}$$

where V_{SpSp} and $V_{\theta p\theta p}$ are variances of the correlations given by

$$V_{S_pS_p}=\left[\frac{a\xi}{V}(N+N_g)+G_{tho}\right]S_p+\frac{a\xi N}{V} \tag{16}$$

$$V_{\theta_p\theta_p}=\frac{V_{S_pS_p}}{4(S_p+1)^2} \tag{17}$$

Here, δ is the Kronicher delta, A_p is the linear gain, B_p is the coefficient of self-saturation, and $D_{p(q)}$ and $H_{p(q)}$ are the coefficients of the symmetric and the asymmetric mutual-saturations, respectively. These coefficients are given by [15]

$$A_p=\frac{a\xi}{V}[N-N_g-bV(\lambda_p-\lambda_0)^2] \tag{18}$$

$$B_p=\frac{9}{4}\frac{\hbar\omega}{\varepsilon_0 n_r^2}\left(\frac{\xi\tau_{in}}{\hbar V}\right)^2 aR_{cv}^2(N-N_s) \tag{19}$$

$$D_{p(q)}=\frac{4}{3}\frac{B_p}{\left(\dfrac{2\pi c\tau_{in}}{\lambda_p^2}\right)^2(\lambda_p-\lambda_q)^2+1} \tag{20}$$

$$H_{p(q)} = \frac{3\lambda_p^2}{8\pi c}\left(\frac{a\xi}{V}\right)^2 \frac{\alpha(N-N_g)}{\lambda_q - \lambda_p} \tag{21}$$

In (5), a is the differential gain coefficient, ξ is the field confinement factor, V is the volume of the active region, λ_0 is the peak wavelength and $\delta\lambda$ is the half-width of spontaneous emission. In (6), α is the linewidth enhancement factor which gives the ratio of (refractive index change)/(gain change) and $\bar{N}$ is the time averaged value of $N(t)$. In (7), τ_s is the electron lifetime, I is the injection current and e is the electron charge. In (9), k is the internal loss in the laser cavity. In (10), η is a coupling coefficient from the reflected light to the lasing mode in the solitary laser, Γ is the optical feedback ratio to the laser facet, $\omega_p=2\pi c/\lambda_p$ is the angular frequency of mode p, $\omega_p\tau$ is the phase delay of the field in each roundtrip time. In (12) and (13), g_s and g_θ are independent Gaussian random variables with zero mean values and variances of unity in ranges of [15], [21]

$-1 \le g_s \le 1$ and $-1 \le g_\theta \le 1$

and Δt is a time-step of the calculation. In (16)-(21), N_g is the electron number to achieve transparency, b is width of the linear gain coefficient, $\hbar$ is the reduced Planck constant, $\omega=2\pi c/\lambda_0$ is central angular frequency, τ_{in} is the intraband relaxation time, R_{cv} is the dipole moment and N_s is the electron number characterizing the self-saturation coefficient.

The central mode, p=0 with wavelength λ_0, is assumed to lie at the centre of the gain spectrum, and the wavelength of the other modes is defined as

$$\lambda_p = \lambda_0 + p\Delta\lambda = \lambda_0 + p\frac{\lambda_0^2}{2n_r L} \qquad p = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{22}$$

The noise sources are added to the rate equations to describe the intrinsic fluctuations in the lasing field due to inclusion of the incoherent spontaneous emission into the stimulated emission. The intensity noise is the fluctuation in the photon number $S_p(t)$. RIN (Relative Intensity Noise) is evaluated from fluctuations $\delta S(t_i) = S(t_i) - \bar{S}$ of the total photon number $S(t_i) = \sum_p S_p(t_i)$ according to the definition of [15]

$$\begin{aligned} RIN &= \frac{1}{\bar{S}^2}\left\{\frac{1}{T}\int_0^T\left[\int_o^\infty \delta S(t)\delta S(t+\tau)\exp(j\omega\tau)d\tau\right]dt\right\} \\ &= \frac{1}{\bar{S}^2}\left\{\frac{1}{T}\left|\int_0^T \delta S(\tau)\exp(-j\omega\tau)d\tau\right|^2\right\} \end{aligned} \tag{23}$$

RIN is then computed directly from the obtained values of $S(t)$ by employing the fast Fourier transform (FFT) to integrate the discrete version of equation (23) as

$$RIN = \frac{1}{\bar{S}^2}\frac{\Delta t^2}{T}\left|FFT[\delta S(t_i)]\right|^2 \tag{24}$$

where T is the total time period of calculation and $\bar{S} = \sum_p \bar{S}_p$ in which $\bar{S}_p$ is the time average photon number of $S_p(t)$.

We might add another Langevin noise source in (7) caused by the extinction of the electron number. However, we neglected this noise source on the electron extinction in this paper, because analysis with counting all possible noise sources takes several days in a numerical calculation. Also we already know that the Langevin noise source for the electron extinction less affects in calculated results than those for the photon generation [21]. The electron density $N(t)$ suffers sufficient fluctuation from the Langevin noise sources $F_p(t)$ through (5), (7) and (18).

TABLE I
VALUES OF THE SIMULATION PARAMETERS FOR 850nm GaAs SEMICONDUCTOR LASER

Symbol	Definition	Value	Unit
a	tangential gain coefficient	2.75×10^{-12}	m^3s^{-1}
B	dispersion parameter of the linear gain spectrum	3×10^{19}	m^3A^{-2}
$\lvert R_{cv}\rvert^2$	squared absolute value of the dipole moment	2.8×10^{-57}	C^2m^2
$\Delta\lambda$	half-width of spontaneous emission	23	nm
Α	Linewidth enhancement factor	2.6	-
Ξ	confinement factor of field	0.2	-
τ_{in}	electron intraband relaxation time	0.1	ps
τ_S	average electron lifetime	2.79	ns
N_S	electron number characterizing non-linear gain	1.7×10^8	-
N_g	electron number at transparency	2.1×10^8	-
V	volume of the laser active region	100	μm^3
D	thickness of the laser active region	0.11	μm
L	length of the laser active region	300	μm
n_r	refractive index of laser active region	3.6	-
Κ	internal loss in the laser cavity	10	cm^{-1}
R_f	reflectivity of front facet	0.30	-
R_b	reflectivity of back facet	0.60	-

III. Numerical Calculation

The proposed model is applied to investigate the effect of external optical feedback on the characteristics of intensity noise in GaAs lasers. The systems of rate equations (5)-(7) are solved numerically by means of the fourth-order Runge-Kutta method [23], [24]. The time-step of integration is set as short as Δt=5ps. This short time step corresponds to a cutoff Fourier frequency of 100GHz that is high enough to guarantee fine resolution of the OFB-induced dynamics. Thirteen longitudinal modes are considered here in the calculation. The distance to the reflecting mirror is l=15cm, which corresponds to external cavity modes whose frequency spacing is $f_{ex}=1/\tau$=1GHz. The integration is first made without OFB for solitary laser from time t=0 until the round-trip time $t=\tau$. The calculated values of $S_p(t=0$ to $\tau)$ and $\theta_p(t=0$ to $\tau)$ of mode p are then stored for use as time delayed values $S_p(t-\tau)$ and $\theta_p(t-\tau)$, including OFB terms, for the rest of the calculations. Dynamics are examined after 0.5μs passed from $t=0$, which is long enough to ensure a steady state of laser operation. The integration is then proceeded over a long time period of $T \ge 2\mu s$, which can give intensity noise for $f \ge 250$KHz. The RIN is then computed from the obtained values of $S(t)=\sum S_p(t)$ by employing the FFT to integrate the equation (24). Calculated spectra are smoothed by running an adjacent averaging of spectral components. The numerical values of 850nm GaAs laser parameters, listed in

Table-I, are employed in the calculations. Each of the independent Gaussian random variables g_S and g_θ are generated using Polar Rejection Method [25] and restricted the values between [-1, 1].

IV. Results and Discussions

As the first, we show the results of applying our model to solitary laser, without any external feedback, to simulate the quantum noise. The results are plotted in Fig. 2 to investigate the effect of current injection on the characteristics of quantum noise. When the laser is in pure single-mode operation or in stable multimode operation, the noise coincides to the quantum noise obtained by D.E. McCumber in [1]. Noise with injection current below threshold is mainly caused by fluctuations of the electron density, not by fluctuations of the photons. The peak shown around the threshold current is attributed to the maximum contribution of the spontaneous emission to light amplification, which rapidly decreases above threshold compared with the contribution of the stimulated emission [8].

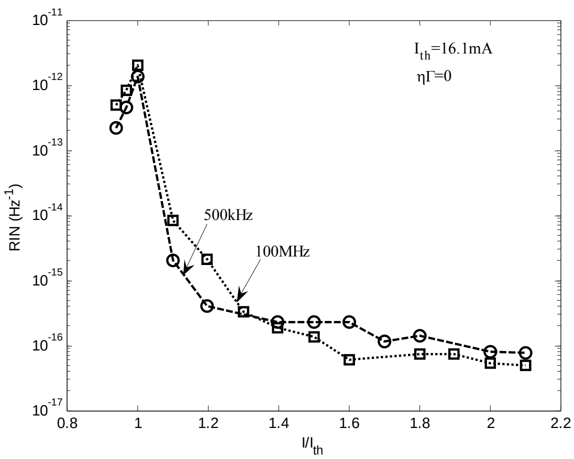


Fig. 2. The simulated characteristics of quantum noise with normalized current. The quantum noise reveals a peak value at the threshold current and reduces with increasing of the current.

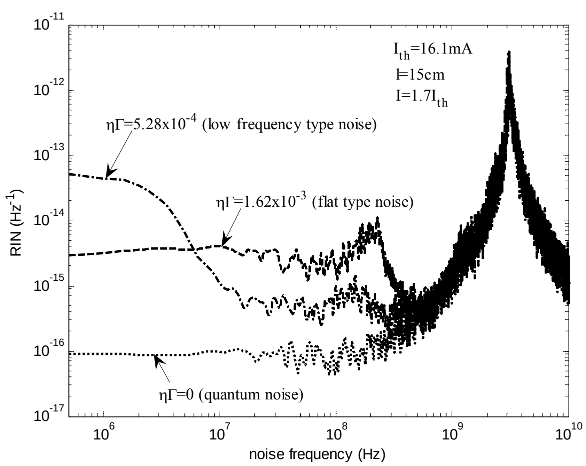


Fig. 3. The simulated spectra of RIN profiles for different OFB strengths. The OFB noise is classified into the low frequency type and the flat type based on noise frequency profile.

Typical noise spectra in the 850nm GaAs laser under the OFB are shown in Fig. 3. The noise level is around 10^{-16} Hz^{-1} when there is no feedback; that corresponds to the quantum noise of the solitary laser. By increasing $\eta\Gamma$, where $\eta\Gamma$ is the effective feedback ratio measuring the OFB strength, from 0 to $5.28\text{x}10^{-4}$ the RIN was increased in lower frequency region below 10MHz. We call here this type of noise to be low frequency type noise. When OFB strength was increased more, the RIN profile became flat for wide frequency range from very low frequency to several 100MHz. We call here this type of noise to be flat type noise.

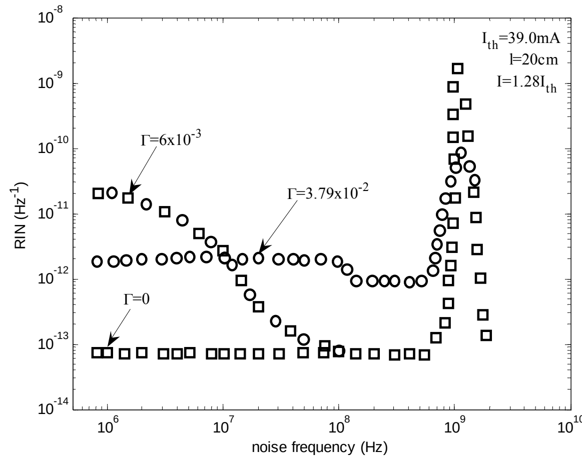
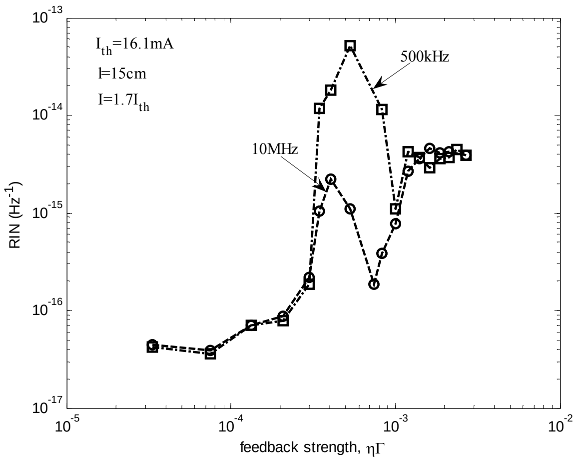


Fig. 4. Experimentally observed frequency spectra of the noise cited from Ref. [6].

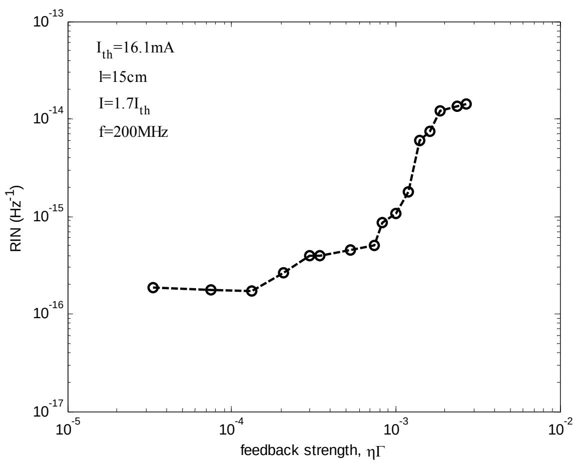
The low frequency type noise must be caused by the mode competition among the lasing modes in the solitary laser [8], [22], and the flat type noise must be caused by the phase distortion between the internal reflected light and the external feed-backed light [12], [15]. Cause of the flat type noise is also explained in terms mode competition among external modes whose lasing frequency is decided by the space between the laser facet and the reflecting mirror [6], [8].

The peak around at 3.2GHz in Fig. 3 indicates the relaxation oscillation. The beating signal $\Delta f = c/2\ell$ corresponding to the external modes must be $\Delta f =$ 1GHz with ℓ =15cm, but is almost hidden in the broadened spectrum of the relaxation oscillation. The numerical results also show a secondary peak around at 250MHz for larger optical feedback. This peak must be a subharmonic of Δf.

Experimentally observed frequency spectra of the noise in 780nm HL7801E AlGaAs laser are cited in Fig. 4 from Ref. [6]. Experimental data are given with the Γ not $\eta\Gamma$ because determination of η in the experiment is difficult. We can find good correspondence between the simulated results in Fig. 3 and the experimental data in Fig. 4. Different feature between the theoretical calculation and the experimental data is on the height of the noise and the detailed profile. The data by the theoretical calculation show lower levels than those by experiment. The difference may be caused by different selection of parameters for the laser material and structure as well as additional fluctuation phenomena such as on the electron diffusion, the inhomogeneous electron injection mechanism in the real device [26]



(a)



(b)

Fig. 5. Simulated results of the variation of the noise with feedback strength. (a) Low frequency type, (b) Flat type. The low frequency type noise reveals the maximum peak with certain feedback ratio.

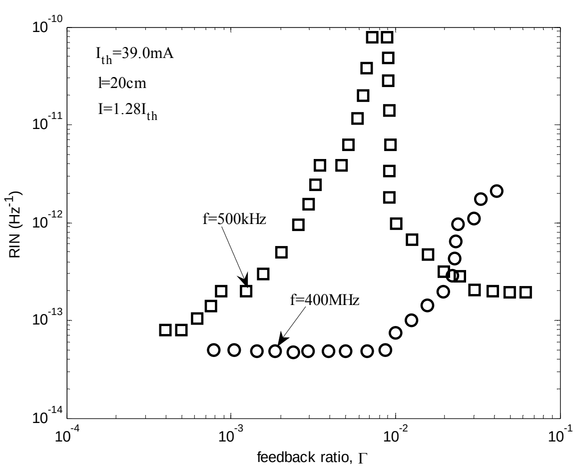
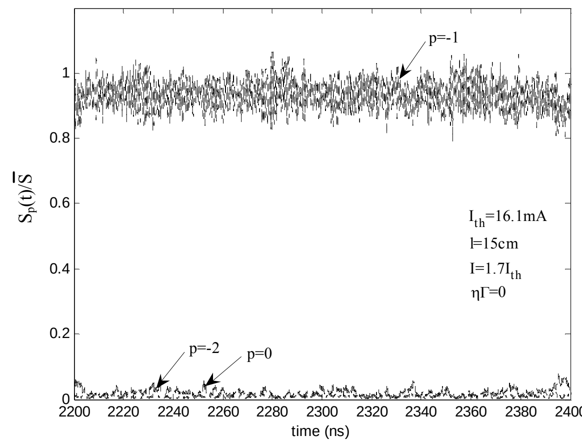
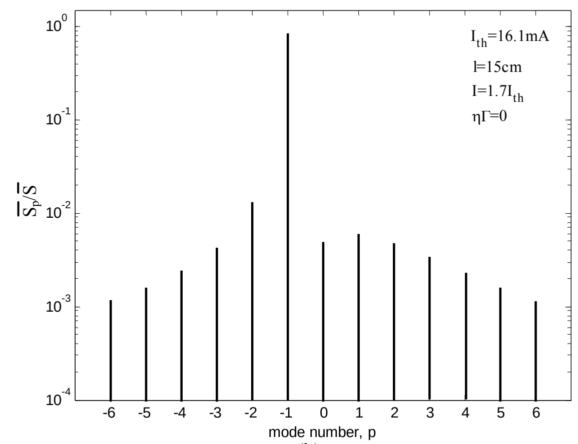


Fig. 6. Experimentally observed variation of the noise with the feedback ratio cited from Ref. [6].

Variations of the RIN with feedback strength are shown in Fig. 5(a) and 5(b), for f=500kHz and f=10MHz representing the low frequency type noise and for f=200MHz representing the flat type noise, respectively. The RIN increased with the feedback strength for $\eta\Gamma$>7.41x10^{-4}. However, the RIN at 500kHz had a peak value at $\eta\Gamma$=5.28x10^{-4} and the RIN at 10MHz also shown a peak at around $\eta\Gamma$=4.04x10^{-4}. Experimental results of noise variations for f=500kHz and f=400MHz are also cited in Fig. 6 from Ref. [6] which show good correspondences to the numerical simulations.



(a)



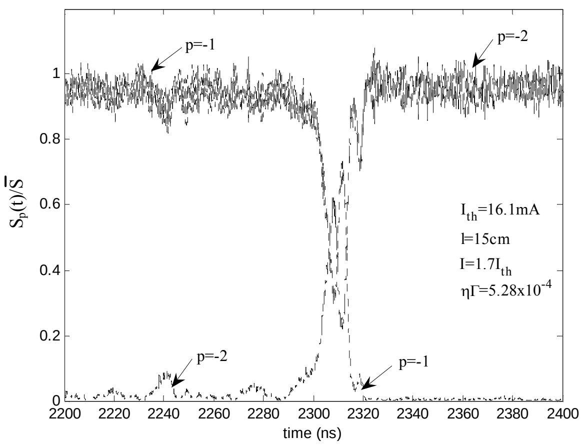
(b)

Fig. 7. Modal behavior without OFB. (a) Temporal variation of lasing modes. (b) Time-averaged modal spectrum. Stable single mode operation is achieved with low noise.

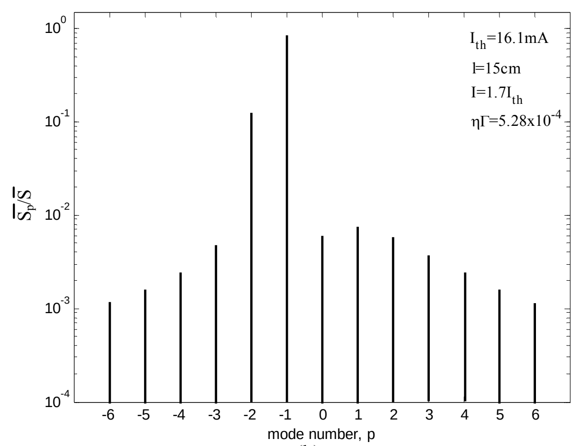
The temporal variations and time averaged profiles of the lasing modes are shown in Figs. 7 to 9. Fig. 7 is the case that the OFB is zero. The laser show stable single mode operation with a dominant mode p=-1. The dominant mode has been shifted from p=0 to p=-1 by increase of the lasing power due to the asymmetric mutual-gain saturation. Other side modes are well suppressed lower than 1/100 of the dominant mode as shown in Fig. 7(b), achieving the low noise level corresponding to the quantum noise.

Fig. 8 is the case showing unstable mode hopping between p=-2 and -1 due to the OFB of $\eta\Gamma$= 5.28x10^{-4} with which the RIN shows the highest value of the OFB noise in form of the low frequency type noise. All previous analyses based on the single mode model never reveal such low frequency type noise

[12], [15]. This result gives clear evidence that the low frequency type noise is caused by the mode hopping phenomena among the lasing modes. The RIN becomes the highest when the mode hopping is the most unstable. We need to pay attention here that the time averaged modal spectrum looks like a multimode operation as shown in Fig. 8(b). However, this is not the true multimode operation but the mode hopping phenomena between bi-stable states of the single mode operation with *p=-2* or *-1*.



(a)

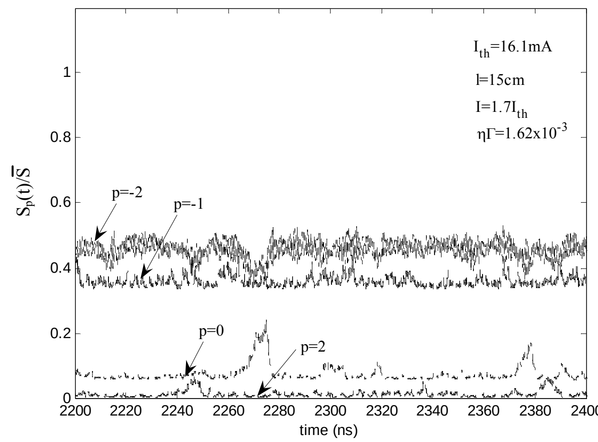


(b)

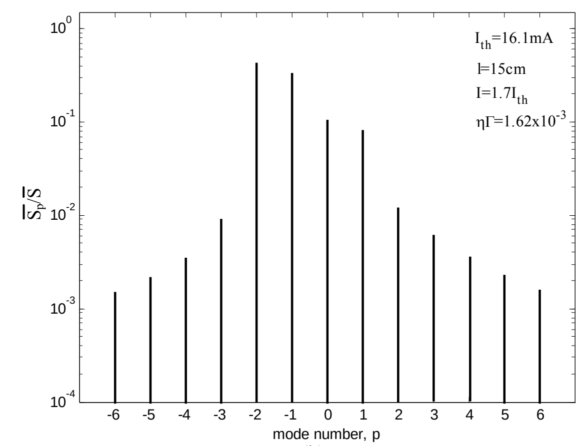
Fig. 8. Modal behavior when the RIN becomes the highest with form of the low frequency type noise by the OFB. (a) Temporal variation of lasing modes. (b) Time-averaged modal spectrum. The lasing modes show unstable mode hopping between *p=-2* and *-1*. The time-averaged modal spectrum looks like a multimode operation but is not true multimode operation.

<u>It has been well known that when the optical feedback noise raise up operation of the laser becomes unstable as firstly pointed out by Lang and Kobayashi in Ref. [4]. This instability must come from the unstable mode hopping which start form $\eta\Gamma$= 3.46x10^{-4} in our calculation.</u>

By further increase of the OFB, the operation changes to a stable multimode operation, resulting in reduction of the low frequency type noise. Fig. 9 is the case the OFB is rather high as $\eta\Gamma$= 1.62x10^{-3}. Temporal variations of the lasing modes are stabilized as in Fig. 9(a) and the modal spectrum is spread to wider wavelength as in Fig. 9(b). The flat type noise increases with increase of the feedback ratio. Since the flat type noise is obtained even in the single mode model as in Refs [12] and [15], generating mechanism of the flat type noise is independent from the mode competition phenomena among the lasing modes in the solitary laser. The flat type noise is explained in terms of the phase distortion of the lasing modes or mode competition among the external cavity modes.



(a)



(b)

Fig. 9. Modal behavior with rather high OFB ratio. (a) Temporal variation of lasing modes. (b) Time-averaged modal spectrum. The operation changes to a stable multimode operation with reduction the low frequency type noise, while the flat type noise increase with increase of the feedback ratio.

V. CONCLUSION

We have proposed a new model to analyze generation of noise in semiconductor lasers under influence of the optical feedback (OFB) which is reflected from a surface of connecting optical device. Newly introduced factors in this analysis are multimode properties and noise generating sources in the lasers. The self and mutual gain saturation effects among the lasing modes, phase delay on the feed-backed light and Langevin noise sources on the photon generation and phase fluctuations are taken into account in form of the rate equations.

Temporal variations of the photon numbers, the optical phases and the electron density were traced by numerical calculation. The noise has been expressed in terms of the RIN (relative intensity noise) by help of the first Fourier transformation from the variant photon numbers.

The RIN is classified into two groups of the low frequency type and the flat type based on the noise frequency profile. Features of these two types noise are well simulated by this model with good correspondence to experimentally obtained data.

Detailed variations of the noise characteristics with the optical feedback ratio are given with discussion on the modal behavior.

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