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MAXIMAL SINGULAR INTEGRALS ON PRODUCT HOMOGENEOUS GROUPS

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ABSTRACT. We prove L^p boundedness for $p \in (1, \infty)$ of maximal singular integral operators with rough kernels on product homogeneous groups under a sharp integrability condition of the kernels.

1. INTRODUCTION

Let $\mathbb{R}^d$, $d \geq 2$, be the d -dimensional Euclidean space. We assume that $\mathbb{R}^d$ is also equipped with a homogeneous group structure, where multiplication is given by a polynomial mapping; the underlying manifold is $\mathbb{R}^d$ itself. We also write $\mathbb{R}^d = \mathbb{H}$. Thus, $\mathbb{H}$ is associated with a dilation group $\{A_t\}_{t>0}$ of automorphisms of the group structure such that

$$A_t x = (t^{a_1} x_1, t^{a_2} x_2, \dots, t^{a_d} x_d), \quad x = (x_1, \dots, x_d) \in \mathbb{H},$$

where real numbers $a_1, \dots, a_d$ satisfy $0 < a_1 \leq a_2 \leq \dots \leq a_d$ (see [10], [21], [18], [6] and [11, Section 2 of Chapter 1]). So, we have, for each $t > 0$,

$$A_t(xy) = (A_t x)(A_t y), \quad x, y \in \mathbb{H}.$$

Consequently, $\mathbb{H}$ is endowed with both the Euclidean structure and a homogeneous nilpotent Lie group structure. The group law of $\mathbb{H}$ is given by a polynomial mapping which conforms to the Campbell-Hausdorff formula in a corresponding Lie algebra via an exponential map and the action of the automorphism family $\{A_t\}$. We note that the identity is the origin 0 and $x^{-1} = -x$; furthermore, Lebesgue measure is bi-invariant Haar measure.

Let us recall a norm function $r(x)$ associated with $\{A_t\}$. The function $r(x)$, which is non-negative and vanishes only at the origin, satisfies that $r(A_t x) = tr(x)$ for $t > 0$ and $x \in \mathbb{R}^d$. We assume that $r(x)$ is even, continuous on $\mathbb{R}^d$ and smooth in $\mathbb{R}^d \setminus \{0\}$, and also that the unit sphere $\Sigma_d = \{x \in \mathbb{R}^d : r(x) = 1\}$ defined by the norm function coincides with the unit sphere $S^{d-1} = \{x \in \mathbb{R}^d : |x| = 1\}$, where $|x|$ denotes the Euclidean norm. Let $\gamma = a_1 + \dots + a_d$ (the homogeneous dimension of $\mathbb{H}$). We shall

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use the formula:

$$\int_{\mathbb{R}^d} f(x) dx = \int_0^\infty \int_{\Sigma_d} f(A_t \theta) t^{\gamma-1} dS_d(\theta) dt, \quad dS_d = \omega d\sigma_d,$$

where ω is a strictly positive C^∞ function on Σ_d and $d\sigma_d$ is the Lebesgue surface measure on Σ_d . See [18, 6] and also [3, 10, 11, 14, 19, 20, 21] for more details and related results.

We consider a function Ω which is locally integrable in $\mathbb{R}^d \setminus \{0\}$ and homogeneous of degree 0 with respect to the dilation group $\{A_t\}$, that is, $\Omega(A_t x) = \Omega(x)$ for $x \neq 0$, $t > 0$. We assume the cancellation property:

$$(1.1) \quad \int_{\Sigma_d} \Omega(\theta) dS_d(\theta) = 0.$$

Convolution on $\mathbb{H}$ is defined by

$$f * g(x) = \int_{\mathbb{H}} f(y) g(y^{-1}x) dy.$$

Let

$$(1.2) \quad Tf(x) = \text{p.v.} f * K(x) = \text{p.v.} \int_{\mathbb{H}} f(y) K(y^{-1}x) dy$$

for appropriate functions f , where $K(x) = \Omega(x') r(x)^{-\gamma}$, $x' = A_{r(x)^{-1}} x$ for $x \neq 0$. We also define the maximal singular integral operator

$$(1.3) \quad T_* f(x) = \sup_{\epsilon > 0} \left| \int_{r(y) > \epsilon} f(xy^{-1}) K(y) dy \right|.$$

Then the following results are known.

Theorem A ([21]). *If $\Omega \in L \log L(\Sigma_d)$ with (1.1) and Tf is as in (1.2), then T is bounded on $L^p(\mathbb{H})$ for all $p \in (1, \infty)$.*

Theorem B ([18]). *Let $T_* f$ be defined as in (1.3) with $\Omega \in L \log L(\Sigma_d)$ satisfying (1.1). Let $p \in (1, \infty)$. Then the operator T_* is bounded on $L^p(\mathbb{H})$.*

We refer to [4, 12, 13, 14, 15, 16] for relevant results.

Part of a theory of Duoandikoetxea and Rubio de Francia [8] for singular integrals on the Euclidean spaces has been generalized to the case of homogeneous groups by [18]. The arguments of [18] replace Fourier transform estimates by a variant of Tao's L^2 estimates via convolution (see [21]). As a result, [18] proved Theorem B and some weighted estimates, and also gave another proof of Theorem A.

Also, it has been shown in [6] that the theory of [18] extends to the case of product homogeneous groups to treat multiple singular integrals. Let $\mathbb{R}^n = \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ be a product homogeneous group with $\mathbb{R}^{n_1} = \mathbb{H}_1$, $\mathbb{R}^{n_2} = \mathbb{H}_2$, where $n = n_1 + n_2$ and $\mathbb{H}_1, \mathbb{H}_2$ are homogeneous groups with

dilations $A_t^{(1)}$, $A_t^{(2)}$ and norm functions r_1, r_2 , respectively. We consider a function Ω in $L^1(\Sigma_{n_1} \times \Sigma_{n_2})$ which satisfies

$$(1.4) \quad \int_{\Sigma_{n_1}} \Omega(u, v) dS_{n_1}(u) = 0, \quad \text{for all } v \in \Sigma_{n_2},$$

$$(1.5) \quad \int_{\Sigma_{n_2}} \Omega(u, v) dS_{n_2}(v) = 0, \quad \text{for all } u \in \Sigma_{n_1}.$$

Define

$$K(u, v) = r_1(u)^{-\gamma_1} r_2(v)^{-\gamma_2} \Omega(u', v'), \quad u' = A_{r_1(u)^{-1}}^{(1)} u, v' = A_{r_2(v)^{-1}}^{(2)} v,$$

where γ_1 and γ_2 are the homogeneous dimensions of $\mathbb{H}_1$ and $\mathbb{H}_2$, respectively.

We consider the multiple singular integral

$$(1.6) \quad Tf(x, y) = \text{p.v.} \int_{\mathbb{H}_1 \times \mathbb{H}_2} f(xu^{-1}, yv^{-1}) K(u, v) du dv.$$

Then the following result is proved in [6].

Theorem C. *Let T be defined as in (1.6) with Ω in $L(\log L)^2(\Sigma_{n_1} \times \Sigma_{n_2})$ satisfying (1.4) and (1.5). Let $1 < p < \infty$. The operator T is then bounded on $L^p(\mathbb{H}_1 \times \mathbb{H}_2)$.*

We can find in [2] the optimality of the $L(\log L)^2$ integrability condition for multiple singular integrals with Euclidean convolution.

Let us recall that the maximal singular integral is defined by

$$(1.7) \quad T_* f(x, y) = \sup_{\substack{\epsilon_1 > 0, \\ \epsilon_2 > 0}} \left| \int_{\substack{r_1(u) > \epsilon_1, \\ r_2(v) > \epsilon_2}} f(xu^{-1}, yv^{-1}) K(u, v) du dv \right|.$$

In this note we shall prove the following.

Theorem 1. *Let T_* be defined as in (1.7). Suppose that Ω is in $L(\log L)^2(\Sigma_{n_1} \times \Sigma_{n_2})$ and satisfies (1.4), (1.5). Then T_* is bounded on $L^p(\mathbb{H}_1 \times \mathbb{H}_2)$ for all $p \in (1, \infty)$.*

Previous works concerning singular integrals on product of Euclidean spaces can be found in [1, 2, 7, 9]. Theorem 1 is an analogue of a result of [2] for multiple singular integrals on product homogeneous groups.

Similarly to the proof of Theorem C in [6], we use extrapolation arguments in proving Theorem 1 by applying the following result.

Theorem 2. *Let $1 < s \leq 2$. Suppose that Ω is in $L^s(\Sigma_{n_1} \times \Sigma_{n_2})$ and satisfies (1.4), (1.5). Then, for $1 < p < \infty$ we have*

$$\|T_* f\|_p \leq C_p (s - 1)^{-2} \|\Omega\|_s \|f\|_p$$

with a constant C_p independent of s and Ω .

Let Ω be as in Theorem 1. Then there exist a sequence $\{\Omega_k\}$ of functions in $L^1(\Sigma_{n_1} \times \Sigma_{n_2})$ and a sequence $\{c_k\}$ of non-negative real numbers such that each Ω_k satisfies (1.4) and (1.5), $\sup_{k \geq 1} \|\Omega_k\|_{1+1/k} \leq 1$, $\sum_{k=1}^{\infty} k^2 c_k < \infty$, and

$$\Omega = \sum_{k=1}^{\infty} c_k \Omega_k.$$

Theorem 1 easily follows from this decomposition of Ω and Theorem 2 (see [17, 15, 16]).

In Section 2 we recall some preliminary results from [6]. We shall prove Theorem 2 in Section 3 by using results of [6, 5, 18]; similar arguments, via Fourier transform estimates, for singular integrals with Euclidean convolution can be found in [1, 2].

2. PRELIMINARIES

Let $\rho \geq 2$ and let $\psi_j \in C_0^\infty(\mathbb{R})$, $j \in \mathbb{Z}$ (the set of integers), be such that

$$\begin{aligned} \text{supp}(\psi_j) &\subset \{t \in \mathbb{R} : \rho^j \leq t \leq \rho^{j+2}\}, \quad \psi_j \geq 0, \\ (\log 2) \sum_{j \in \mathbb{Z}} \psi_j(t) &= 1 \quad \text{for } t > 0; \end{aligned}$$

furthermore,

$$|(d/dt)^m \psi_j(t)| \leq c_m |t|^{-m} \quad \text{for } m = 0, 1, 2, \dots,$$

where c_m is a constant independent of ρ ; we note that this is possible since $\rho \geq 2$.

Suppose that F belongs to $L^1(\mathbb{H}_1 \times \mathbb{H}_2)$ with support in D_0 , where $D_0 = D_0^{(1)} \times D_0^{(2)}$,

$$D_0^{(1)} = \{x \in \mathbb{H}_1 : 1 \leq r_1(x) \leq 2\}, \quad D_0^{(2)} = \{y \in \mathbb{H}_2 : 1 \leq r_2(y) \leq 2\}.$$

Let $\delta_s^{(1)} f(x) = s^{-\gamma_1} f((A_s^{(1)})^{-1}x)$, $\delta_t^{(2)} g(y) = t^{-\gamma_2} g((A_t^{(2)})^{-1}y)$. Define $\delta_{s,t} = \delta_s^{(1)} \otimes \delta_t^{(2)}$ and let

$$(2.1) \quad S_{j,k} F(x, y) = \int_0^\infty \int_0^\infty \psi_j(s) \psi_k(t) \delta_{s,t} F(x, y) \frac{ds}{s} \frac{dt}{t}.$$

Then $\sum_{j,k \in \mathbb{Z}} S_{j,k} K_0 = K$, where

$$(2.2) \quad K_0(x, y) = \begin{cases} K(x, y), & (x, y) \in D_0, \\ 0, & \text{otherwise.} \end{cases}$$

For $s \geq 1$, let $L^s(D_0)$ denote the subspace of $L^s(\mathbb{H}_1 \times \mathbb{H}_2)$ consisting of functions F with support in D_0 . Define

$$M_F f(x, y) = \sup_{j,k \in \mathbb{Z}} |f * S_{j,k}(|F|)(x, y)|$$

for $F \in L^s(D_0)$. The following result is Lemma 8 of [6].

Lemma 1. *Let $p > 1$. Suppose that $s \in (1, 2]$, $\rho = 2^{s'}$, $s' = s/(s-1)$, and $F \in L^s(D_0)$. Then,*

$$\|M_F f\|_p \leq C_p (s-1)^{-2} \|F\|_s \|f\|_p$$

with a positive constant C_p independent of s and F .

Also, we need another result of [6].

Lemma 2. *Let $1 < s \leq 2$. Suppose that Ω belongs to $L^s(\Sigma_{n_1} \times \Sigma_{n_2})$ and satisfies (1.4) and (1.5). Let*

$$Rf(x, y) = \sup_{\ell, m \in \mathbb{Z}} \left| \sum_{j=\ell}^{\infty} \sum_{k=m}^{\infty} f * S_{j,k} K_0(x, y) \right|,$$

where K_0 is defined by (2.2) and $S_{j,k} K_0$ is as in (2.1) with K_0 in place of F . Let $\rho = 2^{s'}$. Then, for $p \in (1, \infty)$ there exists a positive constant C_p independent of $s \in (1, 2]$ and $\Omega \in L^s$ such that

$$\|Rf\|_p \leq C_p A(s, \Omega) \|f\|_p,$$

where $A(s, \Omega) = (s-1)^{-2} \|\Omega\|_s$.

This is Proposition 2 of [6].

3. PROOF OF THEOREM 2.

We first note that

$$S_{j,k} K_0(x, y) = r_1(x)^{-\gamma_1} r_2(y)^{-\gamma_2} \Omega(x', y') \int_{1/2}^1 \psi_j(sr_1(x)) \frac{ds}{s} \int_{1/2}^1 \psi_k(tr_2(y)) \frac{dt}{t},$$

where $x' = A_{r_1(x)^{-1}}^{(1)} x$, $y' = A_{r_2(y)^{-1}}^{(2)} y$. From this we easily see that

$$\text{supp}(S_{j,k} K_0) \subset \{\rho^j \leq r_1(x) \leq 2\rho^{j+2}\} \times \{\rho^k \leq r_2(y) \leq 2\rho^{k+2}\}.$$

Thus, if $\ell, m \in \mathbb{Z}$ are determined by the conditions $\rho^{\ell+2} \leq \epsilon < \rho^{\ell+3}$, $\rho^{m+2} \leq \delta < \rho^{m+3}$ and if f is a compactly supported smooth function, we have

$$\begin{aligned} (3.1) \quad & \int_{\substack{r_1(u) > \epsilon, \\ r_2(v) > \delta}} f(xu^{-1}, yv^{-1}) K(u, v) du dv \\ &= \sum_{\substack{j \geq \ell, \\ k \geq m}} \int_{\substack{r_1(u) > \epsilon, \\ r_2(v) > \delta}} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u, v) du dv \\ &= A_{\epsilon, \delta} f(x, y) + B_{\epsilon, \delta} f(x, y) + C_{\epsilon, \delta} f(x, y) + D_{\epsilon, \delta} f(x, y), \end{aligned}$$

where

$$\begin{aligned}
A_{\epsilon,\delta}f(x,y) &= \sum_{\substack{j>\ell+3, \\ k>m+3}} \int_{\mathbb{H}_1 \times \mathbb{H}_2} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u,v) du dv \\
&= \sum_{\substack{j>\ell+3, \\ k>m+3}} f * S_{j,k} K_0(x,y), \\
B_{\epsilon,\delta}f(x,y) &= \sum_{\substack{\ell+3 \geq j \geq \ell, \\ k>m+3}} \int_{\{r_1(u)>\epsilon\} \times \mathbb{H}_2} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u,v) du dv, \\
C_{\epsilon,\delta}f(x,y) &= \sum_{\substack{j>\ell+3, \\ m+3 \geq k \geq m}} \int_{\mathbb{H}_1 \times \{r_2(v)>\delta\}} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u,v) du dv, \\
D_{\epsilon,\delta}f(x,y) &= \sum_{\substack{\ell+3 \geq j \geq \ell, \\ m+3 \geq k \geq m}} \int_{\substack{r_1(u)>\epsilon, \\ r_2(v)>\delta}} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u,v) du dv.
\end{aligned}$$

Let

$$\begin{aligned}
A_*f(x,y) &= \sup_{\epsilon,\delta>0} |A_{\epsilon,\delta}f(x,y)|, & B_*f(x,y) &= \sup_{\epsilon,\delta>0} |B_{\epsilon,\delta}f(x,y)|, \\
C_*f(x,y) &= \sup_{\epsilon,\delta>0} |C_{\epsilon,\delta}f(x,y)|, & D_*f(x,y) &= \sup_{\epsilon,\delta>0} |D_{\epsilon,\delta}f(x,y)|.
\end{aligned}$$

Then, (3.1) implies

$$(3.2) \quad T_*f(x,y) \leq A_*f(x,y) + B_*f(x,y) + C_*f(x,y) + D_*f(x,y).$$

Let $\rho = 2^{s'}$. Since $A_*f \leq Rf$, by Lemma 2 we have

$$(3.3) \quad \|A_*f\|_p \leq C_p A(s, \Omega) \|f\|_p.$$

Also, since $D_*f \leq CM_{K_0}(|f|)$, Lemma 1 implies

$$(3.4) \quad \|D_*f\|_p \leq C \|M_{K_0}(|f|)\|_p \leq C_p A(s, \Omega) \|f\|_p.$$

To estimate B_*f , we note that

$$\begin{aligned}
&|B_{\epsilon,\delta}f(x,y)| \\
&\leq \sum_{\ell+3 \geq j \geq \ell} \int_{\rho^\ell \leq r_1(u) \leq 2\rho^{\ell+5}} \left| \sum_{k>m+3} \int_{\mathbb{H}_2} f(xu^{-1}, yv^{-1}) S_{j,k} K_0(u,v) dv \right| du.
\end{aligned}$$

By changing variables with respect to u , we see that the right hand side is equal to

$$\sum_{\ell+3 \geq j \geq \ell} \int_{\rho^\ell}^{2\rho^{\ell+5}} \int_{\Sigma_{n_1}} \left| \sum_{k>m+3} F_k(x,y,s,\theta) \right| \Psi_j(s) \frac{ds}{s} dS_{n_1}(\theta),$$

where

$$F_k(x,y,s,\theta) = \int_{\mathbb{H}_2} f(x(A_s^{(1)}\theta)^{-1}, yv^{-1}) \Omega(\theta, v') r_2(v)^{-\gamma_2} \Psi_k(r_2(v)) dv,$$

$\Psi_k(t) = \int_{1/2}^1 \psi_k(rt) dr/r$. Thus, since $0 \leq \Psi_j(s) \leq 1$, we have

$$|B_{\epsilon,\delta} f(x, y)| \leq C \int_{\rho^\ell}^{2\rho^{\ell+5}} \int_{\Sigma_{n_1}} \left| \sum_{k>m+3} F_k(x, y, s, \theta) \right| \frac{ds}{s} dS_{n_1}(\theta).$$

We write

$$K_\theta^{(2)}(v) = K_0(\theta, v), \quad S_k^{(2)} K_\theta^{(2)}(v) = \int_0^\infty \psi_k(t) \delta_t^{(2)} K_\theta^{(2)}(v) \frac{dt}{t}.$$

Then

$$F_k(x, y, s, \theta) = f(x(A_s^{(1)}\theta)^{-1}, \cdot) *_{(2)} S_k^{(2)} K_\theta^{(2)}(y),$$

where $*_{(2)}$ denotes the convolution on $\mathbb{H}_2$. Consequently,

$$(3.5) \quad |B_{\epsilon,\delta} f(x, y)| \leq C \int_{\rho^\ell}^{2\rho^{\ell+5}} \int_{\Sigma_{n_1}} \left| \sum_{k>m+3} f(x(A_s^{(1)}\theta)^{-1}, \cdot) *_{(2)} S_k^{(2)} K_\theta^{(2)}(y) \right| \frac{ds}{s} dS_{n_1}(\theta).$$

Let

$$R_\theta^{(2)} g(y) = \sup_{m \in \mathbb{Z}} \left| \sum_{k>m} g *_{(2)} S_k^{(2)} K_\theta^{(2)}(y) \right|$$

for g on $\mathbb{H}_2$. We write $f_x(y) = f(x, y)$ when considering $f(x, y)$ as a function on $\mathbb{H}_2$ fixing x ; similarly, we write $f_y(x) = f(x, y)$. Define

$$F^\theta(x, y) = R_\theta^{(2)} f_x(y).$$

Then, using (3.5), we have

$$(3.6) \quad |B_* f(x, y)| \leq C \sup_{\ell \in \mathbb{Z}} \int_{\rho^\ell}^{2\rho^{\ell+5}} \int_{\Sigma_{n_1}} F^\theta(x(A_s^{(1)}\theta)^{-1}, y) \frac{ds}{s} dS_{n_1}(\theta) \\ \leq C \log \rho \int_{\Sigma_{n_1}} M_\theta^{(1)} F_y^\theta(x) dS_{n_1}(\theta),$$

where

$$M_\theta^{(1)} h(x) = \sup_{t>0} \frac{1}{t} \int_0^t |h(x(A_s^{(1)}\theta)^{-1})| ds$$

for h on $\mathbb{H}_1$. The last inequality of (3.6) can be seen as follows. Take a positive integer d such that $2^d \leq 2\rho^5 < 2^{d+1}$. Then

$$\int_{\rho^\ell}^{2\rho^{\ell+5}} |h(x(A_s^{(1)}\theta)^{-1})| \frac{ds}{s} \leq \sum_{i=0}^d \int_{2^i \rho^\ell}^{2^{i+1} \rho^\ell} |h(x(A_s^{(1)}\theta)^{-1})| \frac{ds}{s} \\ \leq \sum_{i=0}^d 2M_\theta^{(1)} h(x) = 2(d+1)M_\theta^{(1)} h(x) \\ \leq C(\log \rho) M_\theta^{(1)} h(x),$$

since $d \sim \log \rho$. This implies what we need.

By M. Christ [5] $M_\theta^{(1)}$ is bounded on L^p , $p > 1$, with a bound independent of θ . Thus, using (3.6) and the Minkowski inequality we have

$$(3.7) \quad \|B_* f\|_p \leq C(\log \rho) \int_{\Sigma_{n_1}} \|F^\theta\|_p dS_{n_1}(\theta).$$

By Lemma 9 of [18] with $\rho = 2^{s'}$, we have

$$\|R_\theta^{(2)} g\|_p \leq C_p(\log \rho) \left(\int_{\Sigma_{n_2}} |\Omega(\theta, \omega)|^s dS_{n_2}(\omega) \right)^{1/s} \|g\|_p.$$

Thus

$$\|F_x^\theta\|_p \leq C_p(\log \rho) \left(\int_{\Sigma_{n_2}} |\Omega(\theta, \omega)|^s dS_{n_2}(\omega) \right)^{1/s} \|f_x\|_p.$$

Using this in (3.7) and noting $\|F^\theta\|_p = (\int \|F_x^\theta\|_p^p dx)^{1/p}$, we have

$$(3.8) \quad \|B_* f\|_p \leq C_p(\log \rho)^2 \int_{\Sigma_{n_1}} \left(\int_{\Sigma_{n_2}} |\Omega(\theta, \omega)|^s dS_{n_2}(\omega) \right)^{1/s} dS_{n_1}(\theta) \|f\|_p \\ \leq C_p(\log \rho)^2 \|\Omega\|_s \|f\|_p,$$

where the last inequality follows from Hölder's inequality.

Similarly, we have

$$(3.9) \quad \|C_* f\|_p \leq C_p(\log \rho)^2 \|\Omega\|_s \|f\|_p.$$

Combining (3.2), (3.3), (3.4), (3.8) and (3.9), we get the conclusion of Theorem 2.

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