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Flavor structure of soft supersymmetry-breaking parameters is studied in a certain type of effective supergravity theory derived from moduli- or dilaton-dominated supersymmetry breaking. Some interesting sum rules for soft scalar masses are presented. They constrain their flavor structure and predict some interesting patterns appearing in soft scalar masses in the nonuniversal case. We also study the alignment phenomena in the flavor space of soft breaking parameters due to Yukawa couplings and discuss their phenomenological consequences.

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I. INTRODUCTION

In supersymmetric theories soft supersymmetry-breaking parameters play the crucial role [1]. Their phenomenological features are completely dependent on those parameters. This means that low energy phenomenology can put strong constraints on these parameters and may also give some information on the fundamental theory at the high energy region, which determines the structure of soft breaking parameters. Especially, constraints coming from rare phenomena such as flavor-changing neutral currents (FCNC's) [2] and the electric dipole moment of neutron (EDMN) [3] are very strong. Because of these reasons, the universality and reality of the soft supersymmetry-breaking parameters are usually assumed when an analysis of various phenomenological aspects of the minimal supersymmetric standard model (MSSM) is done. Up to now in many works the flavor structure of soft breaking parameters has been discussed on the basis of suitable fundamental frameworks [4]. They mainly treat how the universality of soft scalar masses is realized at the low energy region.

Recently it has been noticed that in superstring theory soft supersymmetry-breaking parameters are generally nonuniversal [5–7] and their various phenomenological consequences have been studied in that framework [8,9]. These works show that nonuniversal soft breaking parameters bring rather different phenomenological features in comparison with universal ones. In this situation it seems to be very interesting to study the more detailed flavor structure of these parameters on the basis of certain fundamental frameworks. The flavor structure of soft breaking parameters seems not to have been studied enough beyond whether or not they are universal. This is, in part, because of the above-mentioned phenomenological constraints and also the predictivity of the theory. However, there can be various flavor structures even if the phenomenological constraints are imposed. FCNC constraints, indeed, only require mass degeneracy among squarks with the same quantum numbers. This point should be kept in mind when we consider the FCNC constraints. Nonuniversality among squark masses with different quantum numbers can bring various interesting results as suggested in [9].

In this paper we study the flavor structure of soft breaking parameters in a certain type of effective supergravity theory which is derived from moduli- or dilaton-dominated supersymmetry breaking in superstring [6,7]. This recently proposed framework has some advantages. We can use it without knowledge of the origin of supersymmetry breaking. Furthermore, within this framework it can give us rather detailed information on the soft supersymmetry-breaking parameters. As a result it allows us to extract their concrete flavor structure as seen in the following discussion. Taking account of these aspects, the study of soft supersymmetry-breaking parameters based on this framework now seems worthy to be done more extensively from various points of view. The results we present in this paper are derived under strong assumptions which, however, are expected to be applicable to a rather wide class of superstring effective models.

In the following at first we briefly review the derivation of soft breaking parameters in the case of moduli- or dilaton-dominated supersymmetry breaking, which we take as the basis of our argument. After that we present our basic assumptions. Next under these assumptions we derive soft breaking parameters, referring to their flavor structure. Using these results, we give interesting sum rules on their flavor structure and discuss the consequences derived from them. Finally we study their alignment phenomena due to Yukawa couplings and discuss their effects on the low energy physics briefly. The last section is devoted to the summary.

II. SOFT SUSY-BREAKING PARAMETERS

A. General formulas

We begin with a brief review of the general structure of soft breaking parameters in the case of moduli- or dilaton-dominated supersymmetry breaking. Various works based on superstring theory and also general supergravity theory suggest that soft supersymmetry-breaking parameters are generally nonuniversal [5–7]. Low energy effective supergravity theory is characterized in terms of the Kähler potential K, the superpotential W, and the gauge kinetic function f_a. Each of these is a function of ordinary massless chiral matter super-

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fields Ψ^I and gauge singlet fields Φ^i called moduli,[1] whose potential is perturbatively flat as far as supersymmetry is unbroken.

Usually it is assumed that nonperturbative phenomena such as a gaugino condensation occur in the hidden sector. After integrating out the fields relevant to these phenomena, the Kähler potential and the superpotential are expanded by the low energy observable matter fields Ψ^I as

$$K=\kappa^{-2}\hat{K}(\Phi,\bar{\Phi})+Z(\Phi,\bar{\Phi})_{I\bar{J}}\Psi^I\bar{\Psi}^{\bar{J}} +[\tfrac{1}{2}Y(\Phi,\bar{\Phi})_{IJ}\Psi^I\Psi^J+\text{H.c.}]+\cdots, \tag{1}$$

$$W=\hat{W}(\Phi)+\tfrac{1}{2}\tilde{\mu}(\Phi)_{IJ}\Psi^I\Psi^J+\tfrac{1}{3}\tilde{h}(\Phi)_{IJK}\Psi^I\Psi^J\Psi^K+\cdots, \tag{2}$$

where $\kappa^2=8\pi/M_{\rm pl}^2$. The ellipsis stands for the higher order terms in Ψ^I. In Eq. (2), $\hat{W}(\Phi)$ and $\tilde{\mu}(\Phi)_{IJ}$ are considered to be induced by nonperturbative effects in the hidden sector. Using these functions the scalar potential V can be written as [10]

$$V=\kappa^{-2}e^G[G_\alpha(G^{-1})^{\alpha\bar{\beta}}G_{\bar{\beta}}-3\kappa^{-2}]+(D\ \ \text{term}), \tag{3}$$

where $G=K+\kappa^{-2}\ln\kappa^6|W|^2$ and the indices α and β denote Ψ^I as well as Φ^i. The gravitino mass $m_{3/2}$ which characterizes the scale of supersymmetry breaking is expressed as

$$m_{3/2}=\kappa^2 e^{\hat{K}/2}|\hat{W}|. \tag{4}$$

In order to get soft supersymmetry-breaking terms in the low energy effective theory from Eq. (3), we take the flat limit $M_{\rm pl}\to\infty$, preserving $m_{3/2}$ fixed. Through this procedure we obtain the effective superpotential $W^{\rm eff}$ and soft supersymmetry-breaking terms $\mathcal{L}_{\rm soft}$.

In the effective superpotential $W^{\rm eff}$, Yukawa couplings are rescaled as $h_{IJK}=e^{\hat{K}/2}\tilde{h}_{IJK}$ and the μ_{IJ} parameter is effectively expressed as

$$\mu_{IJ}=e^{\hat{K}/2}\tilde{\mu}_{IJ}+m_{3/2}Y_{IJ}-F^{\bar{j}}\partial_{\bar{j}}Y_{IJ}. \tag{5}$$

Soft breaking terms $\mathcal{L}_{\rm soft}$ corresponding to $W_{\rm eff}$ are defined by

$$\mathcal{L}_{\rm soft}=-\tilde{m}^2_{I\bar{J}}\psi^I\bar{\psi}^{\bar{J}}-(\tfrac{1}{3}A_{IJK}\psi^I\psi^J\psi^K+\tfrac{1}{2}B_{IJ}\psi^I\psi^J+\text{H.c.}), \tag{6}$$

where ψ^I represents the scalar component of Ψ^I. Each soft breaking parameter is expressed by using K and W as [6][2]

$$\tilde{m}^2_{I\bar{J}}=m^2_{3/2}Z_{I\bar{J}}-F^i\bar{F}^{\bar{j}}[\partial_i\partial_{\bar{j}}Z_{I\bar{J}}-(\partial_{\bar{j}}Z_{N\bar{J}})Z^{N\bar{L}}(\partial_i Z_{I\bar{L}})] +\kappa^2V_0Z_{I\bar{J}}, \tag{7}$$

$$A_{IJK}=F^i[(\partial_i+\tfrac{1}{2}\hat{K}_i)h_{IJK}-Z^{\bar{M}L}\partial_i Z_{\bar{M}(I}h_{JK)L}], \tag{8}$$

$$B_{IJ}=F^i[(\partial_i+\tfrac{1}{2}\hat{K}_i)\mu_{IJ}-Z^{\bar{M}L}\partial_iZ_{\bar{M}(I}\mu_{J)L}]-m_{3/2}\mu_{IJ} +[F^i(\partial_i+\tfrac{1}{2}\hat{K}_i)F^{\bar{j}}-2m_{3/2}F^{\bar{j}}]\partial_{\bar{j}}Y_{IJ}, \tag{9}$$

where F^i is an F term of Φ^i and ∂_i denotes $\partial/\partial\Phi^i$. V_0 is the cosmological constant expressed as $V_0=\kappa^{-2}(F^i\bar{F}^{\bar{j}}\partial_i\partial_{\bar{j}}\hat{K}-3m^2_{3/2})$. From these expressions we find that these soft breaking parameters are generally nonuniversal and their structure is dependent on the form of the Kähler potential, especially, the functional form of $Z_{I\bar{J}}$.

The gaugino mass M_a is derived through the formula [10]

$$M_a=\tfrac{1}{2}(\text{Re}f_a)^{-1}F^j\partial_j f_a, \tag{10}$$

where the subscript a represents a corresponding gauge group. In the superstring effective theory it is well known that $f_a=k_aS$ at the tree level, where k_a is the Kac-Moody level. It has a dependence on M^i through one-loop effects [11]. This fact and Eq. (10) entail an important result. That is, if the dilaton contribution to the supersymmetry breaking is large, the gaugino masses become large. On the other hand, if the gaugino masses are large enough, the difference among the squark masses disappears at the low energy region due to the radiative effects of heavy gauginos. In this paper we are mainly interested in the nonuniversal soft scalar masses. Thus in the following discussion we assume the small gaugino masses implicitly.

Here it is necessary to make some comments on the application of these formulas to the MSSM. The chiral superfields Ψ^I represent quarks and leptons Q^α, $\bar{U}^\alpha$, $\bar{D}^\alpha$, L^α, and $\bar{E}^\alpha$ where α is a generation index. If only a pair of Higgs doublets are included, Ψ^K in Yukawa couplings and the corresponding A terms should be identified with H_1 and H_2. For this reason we will abbreviate the index K of A_{IJK} in Eq. (8). From the gauge invariance, allowed terms such as $\mu_{IJ}\Psi^I\Psi^J$ in $W^{\rm eff}$ and $Y_{IJ}\Psi^I\Psi^J$ in the Kähler potential K are only μH_1H_2 and YH_1H_2, respectively. Taking account of these, the effective superpotential $W^{\rm eff}$ and soft supersymmetry-breaking terms $\mathcal{L}_{\rm soft}$ in the MSSM can be written as

$$W^{\rm eff}=h^U_{\alpha\beta}\bar{U}^\alpha H_2Q^\beta+h^D_{\alpha\beta}\bar{D}^\alpha H_1Q^\beta+h^E_{\alpha\beta}\bar{E}^\alpha H_1L^\beta +\mu H_1H_2, \tag{11}$$

$$\mathcal{L}_{\rm soft}=-\sum_{I,J}z^{I\dagger}\tilde{m}^2_{I\bar{J}}z^J-\Big(A^U_{\alpha\beta}\bar{U}^\alpha H_2Q^\beta+A^D_{\alpha\beta}\bar{D}^\alpha H_1D^\beta +A^E_{\alpha\beta}\bar{E}^\alpha H_1L^\beta+BH_1H_2+\sum_a\tfrac{1}{2}M_a\bar{\lambda}_a\lambda_a+\text{H.c.}\Big). \tag{12}$$

The first term of Eq. (12) represents the mass term of all scalar components ($z^I=Q^\alpha$, U^α, D^α, L^α, E^α, H_1, H_2) in

[1]Here we are using the terminology ‘‘moduli’’ in the generalized meaning. A dilaton $S(\equiv\Phi^0)$ is included in Φ^i other than the usual moduli $M^i[\equiv\Phi^i\ \ (1\leqslant i\leqslant N)]$. Throughout this paper we will be using this terminology when it is not stated.

[2]It should be noted that these soft breaking parameters are not canonically normalized because the kinetic term of ψ^I is expressed as $Z_{I\bar{J}}\partial^\mu\psi^I\partial_\mu\bar{\psi}^{\bar{J}}$.

the MSSM. In the last term λ_a are the gaugino fields for the gauge groups specified by $a(a=3,2,1)$.

As seen from the general expressions of soft breaking parameters (7)–(9), their structure is determined by the moduli dependence[3] of $Z_{I\bar{J}}$ and W. In order to apply these general results to Eq. (12) and proceed further to investigate their flavor structure, it is necessary to make the model more definite by introducing some assumptions.

B. Assumptions

Our basic assumptions are the following.

(i) We impose the simplest target space duality SL(2,**Z**) invariance

$$M^i \to \frac{a_i M^i - i b_i}{i c_i M^i + d_i} \quad (a_i d_i - b_i c_i = 1, a_i, b_i, c_i, d_i \in \mathbf{Z}) \tag{13}$$

for each usual modulus M^i. Under this target space duality transformation (13) the chiral superfields Ψ^I are assumed to be transformed as

$$\Psi^I \to (i c_i M^i + d_i)^{n_I^i} \Psi^I, \tag{14}$$

where n_I^i is called the modular weight and takes a suitable negative value [5]. This requirement also causes invariance under the Kähler transformation

$$K \to K + f(M^i) + \overline{f(M^i)}, \tag{15}$$

$$W \to e^{-f(M^i)} W, \tag{16}$$

(ii) The Kähler metric and then the kinetic terms of chiral superfields are flavor diagonal:

$$Z_{I\bar{J}} = Z_I \delta_{IJ}. \tag{17}$$

(iii) The coefficient functions $\tilde{h}_{IJK}, \tilde{\mu}_{IJ}, Y_{IJ}$ of the superpotential and Kähler potential are independent of the moduli fields whose F terms contribute the supersymmetry breaking: that is,

$$\partial_i \tilde{h}_{IJK} = \partial_i \tilde{\mu}_{IJ} = \partial_i Y_{IJ} = 0 \quad \text{for} \quad F^i \neq 0. \tag{18}$$

The second assumption is satisfied in almost all known superstring models as suggested in [7]. The third one is a pure assumption at this level. Under this assumption the superpotential W, except for a $\hat{W}$ part, can depend only on moduli which do not contribute the supersymmetry breaking and as a result, for example, Yukawa couplings $\tilde{h}_{IJK}$ can be dynamical variables at the low energy region as discussed in Ref. [12]. These assumptions are rather strong ones but they may be expected to be satisfied in many known superstring models. Moreover, they can induce very interesting features to the soft breaking parameters as seen in the following sections.

[3]The known exception is the dilaton-dominated supersymmetry breaking.

In order to parametrize the direction of supersymmetry breaking in the moduli space, we introduce the parameters Θ_i which correspond to the generalized Goldstino angles in moduli space [7]. They are defined as

$$F^i \sqrt{\hat{K}_{i\bar{j}}} = \sqrt{3} C m_{3/2} \Theta_i, \quad \sum_{i=0}^{N} \Theta_i^2 = 1, \tag{19}$$

where we take the $\kappa = 1$ unit. N is the number of usual moduli M^i in the model. A constant C satisfies $V_0 = 3\kappa^{-2}(|C|^2 - 1) m_{3/2}^2$. In the following we assume $C=1$ and then $V_0 = 0$. The introduction of these parameters makes it possible to discuss the soft breaking parameters without needing to know the origin of supersymmetry breaking.

C. Sum rules for soft scalar masses

In the superstring models studied by now, $\hat{K}$ can be generally written as

$$\hat{K} = -\sum_{i=0}^{N} \ln(\Phi^i + \overline{\Phi^i}). \tag{20}$$

On the other hand, if we apply assumptions (i) and (ii) to $Z_{I\bar{J}}$ in the Kähler potential K, we can constrain the functional form of Z_I as

$$Z_I = \prod_{i=1}^{N} (M^i + \overline{M^i})^{n_I^i}. \tag{21}$$

Using these facts in Eqs. (7)–(9) and normalizing them canonically, we can write down the soft breaking parameters as[4]

$$\tilde{m}_I^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 n_I^i \right), \tag{22}$$

$$A_{IJ} = -\sqrt{3} m_{3/2} h_{IJ} \sum_{i=0}^{N} \Theta_i (n_I^i + n_J^i + n_{H_{1,2}}^i + 1), \tag{23}$$

$$B = -m_{3/2}\mu \left[\sqrt{3} \sum_{i=0}^{N} \Theta_i (n_{H_1}^i + n_{H_2}^i + 1) + 1 \right], \tag{24}$$

where the indices I and J represent the flavors Q_α, $\bar{U}_\alpha$, $\bar{D}_\alpha$, L_α, and $\bar{E}_\alpha (\alpha = 1-3)$. In these formulas $n_I^0 = 0$ should be understood since we do not consider the transformations such as Eqs. (13) and (14) for a dilaton.

Taking account of the functional form of $\hat{K}$ in Eq. (20), assumption (i) requires $f(M^i) = -\ln(i c_i M^i + d_i)$ in Eq. (15). As a result, Eq. (16) shows that the superpotential W and then its coefficient functions $\tilde{h}_{IJ}$ and $\tilde{\mu}_{IJ}$ are transformed as the modular forms under the duality transformation of moduli fields M^i,

[4]It should be noted that these are the tree-level results. However, the introduction of string one-loop effects will not change the qualitative features discussed here.

$$W \to \prod_{i=1}^{N} (ic_i M^i + d_i)^{-1} W, \tag{25}$$

$$\tilde{h}_{IJ} \to \prod_{i=1}^{N} (ic_i M^i + d_i)^{(-n_I^i - n_J^i - n_{H_{1,2}}^i - 1)} \tilde{h}_{IJ}, \tag{26}$$

$$\tilde{\mu}_{IJ} \to \prod_{i=1}^{N} (ic_i M^i + d_i)^{(-n_{H_1}^i - n_{H_2}^i - 1)} \tilde{\mu}_{IJ}. \tag{27}$$

Assumption (iii) requires that the modular weights of $\tilde{h}_{IJ}$ and $\tilde{\mu}_{IJ}$ for moduli M^i which are relevant to the supersymmetry breaking $(F_i \neq 0)$ should be equal to zero. This results in the relations

$$n_I^i + n_J^i + n_{H_{1,2}}^i + 1 = 0, \tag{28}$$

$$n_{H_1}^i + n_{H_2}^i + 1 = 0, \tag{29}$$

for each $i(\neq 0)$. Flavor indices I and J in Eq. (28) should be taken as the ones composing each Yukawa coupling in Eq. (11). After substituting these relations into Eqs. (22)–(24), we obtain the formulas for soft breaking parameters:

$$\tilde{m}_{Q_\alpha}^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 n_{Q_\alpha}^i \right), \tag{30}$$

$$\tilde{m}_{\bar{U}_\alpha}^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 (-n_{Q_\alpha}^i - n_{H_2}^i - 1) \right), \tag{31}$$

$$\tilde{m}_{\bar{D}_\alpha}^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 (-n_{Q_\alpha}^i - n_{H_1}^i - 1) \right), \tag{32}$$

$$\tilde{m}_{L_\alpha}^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 (-n_{\bar{E}_\alpha}^i - n_{H_2}^i - 1) \right), \tag{33}$$

$$\tilde{m}_{\bar{E}_\alpha}^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 n_{\bar{E}_\alpha}^i \right), \tag{34}$$

$$A_{IJ} = -\sqrt{3} m_{3/2} h_{IJ} \Theta_0, \tag{35}$$

$$B = -(\sqrt{3}\Theta_0 + 1) m_{3/2} \mu. \tag{36}$$

The results for parameters A_{IJ} and B are similar to the ones obtained in the case of dilaton-dominated supersymmetry breaking. These features are brought about by assumption (iii). Although soft scalar masses are flavor diagonal, those values are nonuniversal unlike the dilaton-dominated case. Their nonuniversality is determined by the modular weights of the relevant fields, which are completely dependent on the models. This feature seems to make it difficult to practice the model-independent study of the flavor structure of soft scalar masses in the present model. However, it is remarkable that we can easily extract some information on the flavor structure of soft scalar masses in a model-independent way by constructing sum rules from these formulas. We present here two sum rules[5] at $M_{\rm pl}$:

$$\tilde{m}_{Q_\alpha}^2 + \tilde{m}_{\bar{D}_\alpha}^2 = \tilde{m}_{L_\alpha}^2 + \tilde{m}_{\bar{E}_\alpha}^2, \tag{37}$$

$$2\tilde{m}_{Q_\alpha}^2 + \tilde{m}_{\bar{U}_\alpha}^2 + \tilde{m}_{\bar{D}_\alpha}^2 = m_{3/2}^2 \left(4 - 3 \sum_{i=1}^{N} \Theta_i^2 \right), \tag{38}$$

where for these derivations we used the above constraints (28) and (29) on the modular weights. These sum rules for the flavors are satisfied in each generation $(\alpha = 1-3)$.

Although these sum rules become trivial in the universal case, they can give us useful information on the flavor structure of soft scalar masses in the nonuniversal situation. Especially, the latter sum rule (38) gives us very interesting insights into the flavor structure in the quark sector.

At first it shows that we cannot impose the relation $\tilde{m}_{f_\alpha}^2 < \tilde{m}_{f_\beta}^2$ $(\alpha < \beta)$ on all flavors $f = Q$, $\bar{U}$, and $\bar{D}$, simultaneously. This means that soft scalar masses at least for one flavor in the squark sector must decrease according as the generation number increases though scalar masses in other flavor sectors increase with the generation. For example, we assume that soft scalar masses of the first two generations with the same charges cause each other to degenerate and

$$\tilde{m}_{Q_1}^2 = \tilde{m}_{Q_2}^2 < \tilde{m}_{Q_3}^2, \quad \tilde{m}_{\bar{D}_1}^2 = \tilde{m}_{\bar{D}_2}^2 < \tilde{m}_{\bar{D}_3}^2. \tag{39}$$

In this situation, from Eq. (38) we have

$$\tilde{m}_{\bar{U}_1}^2 = \tilde{m}_{\bar{U}_2}^2 > \tilde{m}_{\bar{U}_3}^2. \tag{40}$$

This feature can bring nontrivial results at the weak scale through dynamical effects of the low energy region as seen in the next section.

Furthermore, since the sum of squark masses is constrained to be constant independently of the generation, if the soft scalar mass for one flavor, for example, $\tilde{m}_{Q_\alpha}^2$ becomes larger, soft scalar masses $\tilde{m}_{\bar{U}_\alpha}^2$ and $\tilde{m}_{\bar{D}_\alpha}^2$ must be smaller in comparison with[6] $\tilde{m}_{Q_\alpha}^2$. In such a case Eq. (37) shows that in the present model all left-handed soft scalar masses can be larger than all right-handed ones,

$$\tilde{m}_{Q_\alpha}^2, \ \tilde{m}_{L_\alpha}^2 \gg \tilde{m}_{\bar{U}_\alpha}^2, \ \tilde{m}_{\bar{D}_\alpha}^2, \ \tilde{m}_{\bar{E}_\alpha}^2. \tag{41}$$

Soft scalar masses with this feature have been shown to bring various interesting implications in the phenomenology of the MSSM [9].

In the next section we will study the low energy implication of these structures. In this study we will consider the alignment of soft scalar masses in the flavor space which has recently been proposed in Ref. [15].

[5]Similar sum rules are derived in [13]. However, the flavor structure is not explicitly discussed there.

[6]It can be shown that this kind of hierarchical soft scalar mass can be realized if we consider three moduli case in orbifold models [14].

III. ALIGNMENT OF SOFT SCALAR MASSES

A. Flavor symmetry

We start this section with a discussion of the flavor symmetry of the system defined by the Kähler potential (1) and the superpotential (2). What we refer to here as the flavor symmetry is the invariance under the transformation

$$\Psi^{f_\alpha} \to S^{(f)}_{\beta\alpha}\Psi^{f_\alpha}, \tag{42}$$

where Ψ^{f_α} stands for Q_α, U_α, D_α, L_α, and E_α $(\alpha=1\sim3)$ and $S^{(f)}$ is an element of U(3). Thus the full flavor symmetry of our present model is $\mathrm{U}(3)^5$. Needless to say, Yukawa couplings in the superpotential break this symmetry. Even if we switch off these couplings, this symmetry is also broken by the kinetic terms unless $Z_{I\bar{J}}$ is proportional to the unit matrix. This condition for the kinetic terms is not generally satisfied in the present model. However, there may be an alternative possibility. If there are some relations between moduli space and flavor space, the moduli dependence of $Z_{I\bar{J}}$ may restore the symmetry. That is, it may be expected that $Z_{I\bar{J}}(\equiv Z^{(f)}_{\alpha\bar{\beta}})$ is also transformed simultaneously under the transformation (42) as

$$Z^{(f)}_{\alpha\bar{\beta}} \to S^{(f)}_{\gamma\alpha} Z^{(f)}_{\alpha\bar{\beta}} S^{(f)\dagger}_{\bar{\beta}\bar{\delta}}, \tag{43}$$

and then this can be the symmetry of the system.

If such a situation is realized in the present supersymmetry-breaking scenario, vacuum expectation values (VEV's) of moduli F terms and then the soft supersymmetry-breaking parameters will cause the breaking of this flavor symmetry as well as supersymmetry. In this breaking process the dynamical degrees of freedom corresponding to the Goldstone modes of this spontaneously broken flavor group will remain undetermined unless explicit breakings exist. However, there are Yukawa couplings in the real world. As a result the phenomenon known as alignment occurs to fix the remaining undetermined degrees of freedom in the soft breaking parameters at the low energy region. Recently, this possibility has been suggested in the general framework of supersymmetric models assuming the existence of the corresponding flavor symmetry [15]. In the remaining part we study the alignment of soft breaking parameters in our model. In this study we assume the transformation property (43) in the moduli sector as the starting point of our argument.[7]

[7]It should be noted that this assumption is crucial for the realization of the alignment of soft breaking parameters in flavor space due to Yukawa couplings. Although the contributions from the Goldstone boson loops to the effective potential are sufficiently suppresssed by the symmetry-breaking scale Λ in the case of explicit breakings due to Yukawa couplings, their contributions through the breakings due to $Z_{I\bar{J}}$ in the kinetic terms cannot be suppressed by Λ and make the scenario in [15] ineffective. For this reason we will adopt this assumption, although the existence of such a proprety has not been known in superstring models up to now.

B. Alignment in the flavor space

As seen from Eqs. (30)–(36), soft scalar masses and A terms are produced in flavor diagonal form in the present model. In the limit that Yukawa couplings are negligible, however, the degrees of freedom corresponding to the rotation $(\mathrm{SU}(3)/\mathrm{U}(1)^2)^5$ in flavor space remain undetermined under the above assumption of the flavor symmetry. If we represent these degrees of freedom with 3×3 matrices $S^{(f)}$ in the basis where Yukawa couplings $h^F(F=U,D,E)$ are diagonal, soft breaking parameters $\tilde{m}_f^2$ and $A^F(\equiv A_{IJ})$ can be written as[8]

$$\tilde{m}_f^2 = S^{(f)\dagger}\Sigma^{(f)}S^{(f)}, \quad A^F = S^{(f')\dagger}\Delta^F S^{(f)}. \tag{44}$$

From the definition of the Yukawa couplings, Eq. (11), the index f' in the representation of A^F stands for the flavor $f'=U,D,E$ which can compose the Yukawa couplings with the flavor $f=Q,L$. Both $\Sigma^{(f)}$ and Δ^F correspond to the ones derived in the previous section and they are defined at the scale $\Lambda=M_{\rm pl}$. The Goldstone degrees of freedom $S^{(f)}$ are determined through the physics below the scale Λ. To study this determination process we adopt a Wilsonian approach to the low energy effective theory. Following Ref. [15], the low energy effective potential which can be derived by such a prescription is

$$V_{\rm eff} = V_\Lambda + \frac{\Lambda^2}{32\pi^2}\mathrm{Str}\left(\mathcal{M}^2 - \frac{1}{32\pi^2}\beta^{(1)}_{\mathcal{M}} + \cdots\right), \tag{45}$$

where V_Λ is an $S^{(f)}$-independent part and $\mathcal{M}$ represents a mass matrix of the fields in the theory at the scale Λ. The ellipses stand for higher order correction terms which are irrelevant in the present approximation. One-loop β functions $\beta^{(1)}_{\mathcal{M}}$ for the masses of the relevant scalar fields are given in Ref. [16]:

$$\mathrm{Tr}\beta^{(1)}_{\tilde{m}_Q^2} = \mathrm{Tr}[2(h^{U\dagger}h^U + h^{D\dagger}h^D)\tilde{m}_Q^2 + 2h^{U\dagger}\tilde{m}_{\bar{U}}^2 h^U + 2h^{D\dagger}\tilde{m}_{\bar{D}}^2 h^D + 2A^{U\dagger}A^U + 2A^{D\dagger}A^D] + \cdots,$$

$$\mathrm{Tr}\beta^{(1)}_{\tilde{m}_L^2} = \mathrm{Tr}[\tilde{m}_L^2 h^{E\dagger}h^E + 2h^{E\dagger}\tilde{m}_{\bar{E}}^2 h^E + h^{E\dagger}h^E\tilde{m}_L^2 + 2A^{E\dagger}A^E] + \cdots,$$

$$\mathrm{Tr}\beta^{(1)}_{\tilde{m}_{\bar{U}}^2} = \mathrm{Tr}[2\tilde{m}_{\bar{U}}^2 h^U h^{U\dagger} + 4h^U\tilde{m}_Q^2 h^{U\dagger} + 2h^U h^{U\dagger}\tilde{m}_{\bar{U}}^2 + 4A^U A^{U\dagger}] + \cdots,$$

$$\mathrm{Tr}\beta^{(1)}_{\tilde{m}_{\bar{D}}^2} = \mathrm{Tr}[2\tilde{m}_{\bar{D}}^2 h^D h^{D\dagger} + 4h^D\tilde{m}_Q^2 h^{D\dagger} + 2h^D h^{D\dagger}\tilde{m}_{\bar{D}}^2 + 4A^D A^{D\dagger}] + \cdots,$$

[8]It should be noted that $S^{(f)}$ for the flavors $\bar{U}$, $\bar{D}$, and $\bar{E}$ are defined as the ones for the right-handed U, D, and E in Eq. (42).

$$\mathrm{Tr}\beta^{(1)}_{\tilde m^2_{\bar E}} = \mathrm{Tr}[2\tilde m^2_{\bar E} h^E h^{E\dagger} + 4h^E \tilde m^2_L h^{E\dagger} + 2h^E h^{E\dagger}\tilde m^2_{\bar E} + 4A^E A^{E\dagger}] + \cdots,$$

$$\mathrm{Tr}\beta^{(1)}_{\tilde m^2_{H_2}} = 6\ \mathrm{Tr}[\tilde m^2_Q h^{U\dagger} h^U + h^{U\dagger}\tilde m^2_{\bar U} h^U + A^{U\dagger}A^U] + \cdots,$$

$$\mathrm{Tr}\beta^{(1)}_{\tilde m^2_{H_1}} = \mathrm{Tr}[6\tilde m^2_Q h^{D\dagger}h^D + 2\tilde m^2_L h^{E\dagger}h^E + 6h^{D\dagger}\tilde m^2_{\bar D} h^D + 2h^{E\dagger}\tilde m^2_{\bar E} h^E + 6A^{D\dagger}A^D + 2A^{E\dagger}A^E] + \cdots, \tag{46}$$

where the ellipsis stands for the $S^{(f)}$ independent contributions. Following Eq. (44), the soft scalar masses in these formulas are expressed in the Yukawa coupling diagonal basis by using the nonlinearly realized Goldstone modes $S^{(f)}$ as follows:

$$\tilde m^2_{Q_U} = S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)},$$

$$\tilde m^2_{Q_D} = \mathcal{K}^\dagger S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}\mathcal{K},$$

$$\tilde m^2_{\bar U} = S^{(U)\dagger}\Sigma^{(U)}S^{(U)},$$

$$\tilde m^2_{\bar D} = S^{(D)\dagger}\Sigma^{(D)}S^{(D)}, \tag{47}$$

where $\mathcal{K}$ is the Kobayashi-Maskawa matrix. Using these, we can write down the $S^{(f)}$-dependent part of this effective potential:

$$V(S^{(f)}) = -\frac{\Lambda^2}{(32\pi^2)^2}\mathrm{Tr}[12S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}(h^{U\dagger}h^U + \mathcal{K}h^{D\dagger}h^D\mathcal{K}^\dagger) + 8S^{(L)\dagger}\Sigma^{(L)}S^{(L)}h^{E\dagger}h^E + 12S^{(U)\dagger}\Sigma^{(U)}S^{(U)}h^U h^{U\dagger} + 12S^{(D)\dagger}\Sigma^{(D)}S^{(D)}h^D h^{D\dagger} + 8S^{(E)\dagger}\Sigma^{(E)}S^{(E)}h^E h^{E\dagger}]. \tag{48}$$

Here it should be noted that the A^F contribution to V_{eff} disappears in Eq. (48) because A^F appears as the unitary-invariant form in the effective potential in this approximation level. The flavor structure of soft breaking parameters at the weak scale is determined through Eq. (44) by using $S^{(f)}$ which minimizes this effective potential.

As easily proved for the diagonal matrices X and Y, $\mathrm{Tr}(XS^{(f)}YS^{(f)\dagger})$ is maximized when $S^{(f)}$ is aligned so as for their eigenvalues X_i and Y_i to be ordered in such a way that $X_i \leqslant X_j$ and $Y_i \leqslant Y_j$ are simultaneously realized for the diagonal component labels $i<j$. We are considering the effective potential (48) in the basis where the Yukawa coupling matrices are diagonal and also their eigenvalues are ordered in such a way that their values increase. So the potential minimization makes $S^{(f)}(f=L,U,D,E)$ a unit matrix if the eigenvalues of $\Sigma^{(f)}$ are ordered in a suitable way. Thus soft scalar masses for these flavors at the weak scale are completely aligned to Yukawa couplings in flavor space. In the $f=Q$ sector the situation is different. Because of the existence of the Kobayashi-Maskawa matrix $\mathcal{K}$, $S^{(Q)}$ deviates from the unit matrix even if the eigenvalues of $\Sigma^{(Q)}$ are ordered according to their magnitude. From the investigation of the effective potential it is easily found that $S^{(Q)}$ should be determined so that $S^{(Q)}(h^{U\dagger}h^U + \mathcal{K}h^{D\dagger}h^D\mathcal{K}^\dagger)S^{(Q)\dagger}$ is proportional to $\Sigma^{(Q)}$. As found from Eq. (44), these facts show that A parameters in the squark sector are not generally proportional to the Yukawa couplings, although A^L in the lepton sector is proportional to h^L.

Here we should remember again that in our model the following condition for $\Sigma^{(f)}$ is satisfied independently of the generation,

$$2\Sigma^{(Q_\alpha)} + \Sigma^{(U_\alpha)} + \Sigma^{(D_\alpha)} = \mathrm{const}, \tag{49}$$

and then the eigenvalues of $\Sigma^{(f)}$ are not necessarily ordered in such a way that their magnitude increases for all f as mentioned before. In that case the minimization of the effective potential determines $S^{(f)}(\neq 1)$ so as to exchange the ordering of the eigenvalues according to their magnitude. The existence of the Kabayashi-Maskawa matrix $\mathcal{K}$ makes this situation more subtle. These make the flavor structure of A parameters somehow complex, at least in the squark sector.

C. Squark sector

We study the squark sector in more detail. The squark mass matrices in u-squark and d-squark sectors can be written as

$$\begin{pmatrix} |m_U|^2 + \tilde m^2_{Q_U} + m_Z^2\cos 2\beta(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W) & A^U\langle H_2\rangle + m_U\mu^*\cot\beta \\ A^{U\dagger}\langle H_2\rangle^* + m_U^\dagger\mu\ \cot\beta & |m_U|^2 + \tilde m^2_{\bar U} + \frac{2}{3}m_Z^2\cos 2\beta\ \sin^2\theta_W \end{pmatrix}, \tag{50}$$

$$\begin{pmatrix} |m_D|^2 + \tilde m^2_{Q_D} - m_Z^2\cos 2\beta(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W) & A^D\langle H_1\rangle + m_D\mu^*\tan\beta \\ A^{D\dagger}\langle H_1\rangle^* + m_D^\dagger\mu\ \tan\beta & |m_D|^2 + \tilde m^2_{\bar D} - \frac{1}{3}m_Z^2\cos 2\beta\ \sin^2\theta_W \end{pmatrix}, \tag{51}$$

where m_f and $\widetilde{m}_f$ are masses of the f quark and the corresponding left- and the right-handed squark mass matrices, respectively. These squark mass matrices can be explicitly determined in our model. We show this by using a typical example.

As discussed above, no matter how eigenvalues of $\Sigma^{(f)}$ are ordered in the squark sector, under the condition of Eq. (49) we obtain the nontrivial results for $S^{(Q)}$, $S^{(U)}$, and $S^{(D)}$ as far as the $\Sigma^{(f)}$ has nondegenerate eigenvalues. We are interested in the nontrivial case where the eigenvalues of $\Sigma^{(f)}$ are not ordered according to their magnitude. As a typical example, we take

$$\Sigma^{(Q)}=\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_1^2 & 0 \\ 0 & 0 & m_2^2 \end{pmatrix},$$

$$\Sigma^{(U)}=\Sigma^{(D)}=\begin{pmatrix} m_3^2 & 0 & 0 \\ 0 & m_3^2 & 0 \\ 0 & 0 & m_4^2 \end{pmatrix}, \qquad (52)$$

where $m_1^2>m_2^2$ and $m_4^2>m_3^2$ following Eq. (49). For the $\overline{U}$ and $\overline{D}$ sectors we get $S^{(U)}=S^{(D)}=1$ trivially. If $\mathcal{K}=1$, for the Q sector we obtain

$$S^{(Q)}=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}. \qquad (53)$$

For this $S^{(Q)}$, $S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}$ becomes diagonal with increasingly ordered eigenvalues in the same way as the case of $\overline{U}$ and $\overline{D}$ sectors. If $\mathcal{K}\neq 1$, however, $S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}$ cannot be diagonal anymore. After some algebra for the minimization of the effective potential, we obtain

$$S^{(Q)}=\begin{pmatrix} 0 & 1 & X \\ Y & X & -1 \\ 1 & 0 & Y \end{pmatrix}, \qquad (54)$$

where $X\sim -V_{ts}m_b^2/m_t^2$ and $Y\sim -V_{td}m_b^2/m_t^2$ This results in

$$S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}\sim\begin{pmatrix} Y^2m_1^2+m_2^2 & XYm_1^2 & -Ym_1^2+Ym_2^2 \\ XYm_1^2 & m_1^2+X^2m_1^2 & 0 \\ -Ym_1^2+Ym_2^2 & 0 & X^2m_1^2+m_1^2+Y^2m_2^2 \end{pmatrix}. \qquad (55)$$

This shows that the effect of $\mathcal{K}\neq 1$ is very small and $S^{(Q)\dagger}\Sigma^{(Q)}S^{(Q)}$ can be regarded as diagonal in the good approximation even in the Q sector. However, it should be noted that the degeneracy between the first and second generation squarks in the Q sector at $M_{\rm pl}$ is lost at the low energy region.

If we use Eq. (44), we can explicitly calculate A^U and A^D in this example as,

$$A^U=\Delta S^{(U)\dagger}h^U S^{(Q)}$$
$$\sim\frac{e\Delta}{\sqrt{2}m_W\sin\beta\sin\theta_W}\begin{pmatrix} 0 & m_u & Xm_u \\ Ym_c & Xm_c & -m_c \\ m_t & 0 & Ym_t \end{pmatrix}, \qquad (56)$$

$$A^D=\Delta S^{(D)\dagger}h^D S^{(Q)}$$
$$\sim\frac{e\Delta}{\sqrt{2}m_W\cos\beta\sin\theta_W}\begin{pmatrix} 0 & m_d & Xm_d \\ Ym_s & Xm_s & -m_s \\ m_b & 0 & Ym_b \end{pmatrix}, \qquad (57)$$

where Δ is defined as $\Delta^F=\Delta h^F$. This result shows that A^U and A^D can have rather large off-diagonal elements unless $S^{(Q)}=S^{(U)}=S^{(D)}=1$, which is realized only when the masses of three generation squarks with the same quantum numbers degenerate. However, the nonuniversal soft scalar masses such as Eq. (52) appear only in the case of moduli-dominated supersymmetry breaking ($\Sigma_{i=1}{}^N\Theta_i^2\sim 1$ and $\Theta_0^2\ll 1$) and then $\Delta^F\ll m_{3/2}$. Thus the components of off-diagonal blocks in squark mass matrices (50) and (51) are small compared with the elements of the diagonal blocks. In our model the left and right mixing squark masses generally bring no serious problems to the FCNC.

From this example we learn that if flavor alignment occurs in the squark masses, the good degeneracy between the masses of the first and second generation squarks at $M_{\rm pl}$ can be lost at the low energy region. This shows that the good degeneracy at $M_{\rm pl}$ among the masses of three squarks with the same quantum numbers may be necessary to satisfy the FCNC constraints.

A consideration like this can give us more insight into soft scalar masses. Finally we present such a typical example in the nondegenerate case. Taking account of the alignment effects discussed above, if we assume

$$\widetilde{m}^2_{Q_1}\simeq\widetilde{m}^2_{Q_2}<\widetilde{m}^2_{Q_3}, \quad \widetilde{m}^2_{\overline{D}_1}\simeq\widetilde{m}^2_{\overline{D}_2}<\widetilde{m}^2_{\overline{D}_3}, \qquad (58)$$

at the scale Λ, these relations are preserved at the weak scale. This situation is favorable to satisfy the FCNC constraints. On the other hand, the sum rule at Λ and the alignment effects predict

$$\widetilde{m}^2_{\overline{U}_1}<\widetilde{m}^2_{\overline{U}_2}\simeq\widetilde{m}^2_{\overline{U}_3} \qquad (59)$$

at the low energy region. This suggests that in the present model it is difficult to make only $\widetilde{m}^2_{\bar{U}_3}$ light enough in comparison with other right-handed U sector squarks.

IV. SUMMARY

We studied the structure of soft supersymmetry-breaking parameters in the effective theory derived from moduli- or dilaton-dominated supersymmetry breaking. In particular, we focused our attention on the flavor structure of soft scalar masses. Under certain assumptions about the moduli dependence of the Kähler potential and superpotential we obtained sum rules for soft scalar masses, which gave interesting relations among different flavors in a model-independent way. We showed that we could extract some typical features from these sum rules when the scalar masses were nonuniversal.

We also applied these results as the initial conditions at $M_{\rm pl}$ to the recently proposed alignment scenario for soft scalar masses in flavor space. In this discussion we pointed out that the degeneracy of the masses among three generation squarks with the same quantum numbers might be required at $M_{\rm pl}$ to satisfy the FCNC constraints. Only the good degeneracy between the first and second generation squark masses at $M_{\rm pl}$ may not be necessarily sufficient to avoid excessive FCNC's.

The consequences induced from the nonuniversality of soft supersymmetry-breaking parameters have been studied from various viewpoints by now. Although FCNC constraints require the degeneracy of squark masses among the species with the same quantum numbers, it does not require universality more than that. It seems to be necessary to take account of this important point when we study the flavor structure of soft breaking parameters. This viewpoint may open new possibilities in the study of supersymmetric theory.

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