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A method for portfolio evaluation based on linear expansion and its application to an empirical portfolio

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Abstract This paper introduces a new scheme for portfolio evaluation founded on a linear expansion in a basis of economic indicators. The core idea is to express the price movement of a complex set of financial instruments by several carefully selected indicators whose behaviour is by far more understandable than the complicated relations of financial derivatives contained in the portfolio. The principles of selection of the economic indicators for a given portfolio are discussed in detail and the method is tested on a basic portfolio containing one stock.

Key words: Portfolio analysis, Economic indicator, Expansion method.

1 Introduction

The property of financial institutions, such as banks, should be evaluated based on international standard law, Basel II accord. According to this agreement, financial institutions are required to quantify the operational risks. The property, or financial portfolio, of the institution is a collection of financial instruments, such as stocks, bonds, derivatives, etc., and is often very complex. Therefore, it is necessary to find a suitable model for evaluation of such complex portfolios. Moreover, the decision on which assets should be included in the portfolio, when and how many of these assets should be purchased or sold, always involves the expected return on the portfolio, and the risk associated with this return [8].

In 1933, Cowles examined return but ignored any consideration of risk [9], while

Markowitz developed techniques [10] for evaluating portfolio performance based on risk and return. He showed that the risk on a portfolio return depends not only on the riskiness of the individual components associated with the portfolio but also on the relationship among the components. On the basis of this context, Sharpe [4] developed the capital asset price model. This model assumes that the risk-return profile of a portfolio can be optimized, that the asset return is normally distributed and that there is no transaction cost.

In the early 1990s, Value at Risk (VaR) was the only common risk measure that could be defined for all types of business. In 1994, JPMorgan Chase & Co., an American multinational banking corporation of securities, first published the methodology of VaR. VaR describes the losses that can occur over a given period of time at a given confidence level due to market risks [5]. Having the ability to express the market risk of an entire portfolio in one number, it is a well established and accepted risk measure in the risk management practice. In recent years, it has become not only a risk measurement method but also a process that helps decision-making and investment guiding.

According to Hendricks [6] there are several methods that can be used for portfolio's VaR calculation. Those methods which are based on historical information can be classified into three groups: Variance-covariance method, Monte-Carlo simulation method, and Historical simulation method.

According to the variance-covariance approach, returns on the portfolio are normally distributed and the portfolio is a linear function of its risk factors. Its main problem is that the returns distribution may have fat tails and thus the VaR will underestimate the expected losses. The Monte Carlo method generates large number of random market scenarios. The fundamental assumption in this method is that the portfolio price changes are normally distributed and the distribution is generated from the market scenarios. Then for each scenario, the profits and the losses of the portfolio are computed and the corresponding volatility is calculated. In case of complex portfolios including derivatives like options, Monte Carlo simulation can be used to evaluate the VaR, even though it requires a longer time to be performed. Historical simulation method uses the historical distribution of returns of a portfolio to simulate volatility which measures the VaR. On the other hand, the historical simulation method does not assume that the asset returns are normally distributed. Also, it is relatively easy to implement. It assumes that the asset returns are independent and identically-distributed and it applies equal weight to all returns over the whole period.

The main purpose of this study is to evaluate a given financial portfolio through the historical simulation approach based on certain important economic indicators, which improves the volatility (risk) estimations provided by the standard historical simulation approach. Based on a detailed analysis of the relationship between the portfolio and economic indicators, economic indicators relevant to the portfolio are chosen from the current market. The value of the portfolio is then approximated by a linear combination of the values of the selected indicators. The coefficients of the linear expansion are calculated by a fitting method using known historical data for both the indicators and the portfolio. By extrapolation we can then construct an approximate distribution of expected portfolio price changes from which the VaR and other properties can be deduced.

The main difficulty of portfolio evaluation up to now has been the fact that the portfolio consists of a large number of instruments that are correlated to each other. In many cases, especially for financial institutions, it is usual to have more than 10,000 instruments in a portfolio. This implies the necessity for a huge number of calculations, even just for the initial calculation of the correlations between the instruments. Such calculations are time-consuming even when modern supercomputers are employed. Moreover, considering a large number of instruments separately entails the growth of error and possibility of VaR overestimation.

On the other hand, the new method uses only a few representative economic indicators, usually of the order of tens, which is a very small number comparing to the number of financial instruments associated with the portfolio. Subsequently, the computation can be significantly accelerated and the time constraint overcome. The method can also provide a clearer view of the otherwise complex system because it reduces the study of the system to the analysis of a few representative indicators. These, if chosen correctly, can serve as efficient gauges for expected reactions of the whole portfolio to specific changes in the market.

The paper is organized as follows: Section 2 introduces the new method using economic indicators and section 3 explains the method of choosing the economic indicators. In section 4 we continue with an empirical analysis of the proposed method on the example of a portfolio consisting of one stock, Toyota Motor Corporation, using a basis of 18 indicators.

2 Methodology

The main idea of the proposed method is to approximate given financial instrument(s) by a linear combination of time-series data of selected economic indicators. There are three equally important steps in building the method. The first step is to give general guidelines for the selection of economic indicators, while the second step is to develop an asset pricing model using the selected economic indicators. The third step is to calibrate the method for a given portfolio.

Let a be the vector of time-series data of the considered instrument's closing prices from the first day up to the m-th day of the considered period :

$$a = (a_1, a_2, \cdots, a_m).$$

Further, let r^i denote the vector of time-series data of i-th indicator values from the first day up to m-th day of the considered period:

$$r^i = (r^i_1, r^i_2, \cdots, r^i_m).$$

Then we consider the following linear approximation

$$a \simeq \sum_{i=1}^{n} \alpha_i r^i, \tag{2.1}$$

where $\alpha_i \in \mathbb{R}$ denotes the coefficients of the linear expansion, and n is the total number of indicators. The values of the coefficients α_i are obtained from the least-squares approach as the minimizers of the following functional:

$$I(\alpha_1, \alpha_2, \cdots, \alpha_n) = |a - \sum_{i=1}^{n} \alpha_i r^i|^2 \to \min,$$

where $|.|$ is an m-dimensional Euclidean norm.

2.1 Data management

According to the Dow theory, at any time the stock market has three kinds of trends. The primary trends last from less than one year to several years, the secondary trends last from ten days to three months and the minor trends last from one day to three weeks or more. The thirdly classified trends are subject to some degree of manipulation and thus these trends are not important for our analysis, or even can be misleading. Therefore, instead of the raw data we consider the means of the daily closing prices over certain short periods of several days. This smoothes out local fluctuations while preserving the global tendency of the data. We calculate the averaged value x'_J at J-th day by the following expression:

$$x'_J = \frac{1}{2d+1} \sum_{j=J-d}^{J+d} x_j, \tag{2.2}$$

where d is a suitably chosen natural number expressing the time-span over which averages are taken.

In order to be able to observe the influence of individual indicators on the value of the portfolio without any bias, the time-series data of both the portfolio and the economic indicators are scaled from the actual time-series data by the following expression:

$$x = \frac{x' - \bar{x}'}{\bar{x}'}, \tag{2.3}$$

where x represents the scaled time-series data, x' is the smoothed time-series data from (2.2) and $\bar{x}'$ is the mean of x'.

In what follows, we will use the notion of correlation coefficient between two time-series data a and b defined by

$$\rho(a,b) \quad = \quad \frac{\mathrm{cov}(a,b)}{\hat{a}\hat{b}},$$

where $\hat{a}$, $\hat{b}$ are the standard deviations of a, b, respectively, and the covariance of a and b is given by the formula

$$\mathrm{cov}(a,b) \quad = \quad \frac{1}{m}\sum_{j=1}^{m}(a_j - \bar{a})(b_j - \bar{b}).$$

The value of the correlation coefficient determines the degree of relationship between two data sets.

As a measure of quality of approximation of data set a by another set b, we will use the following cummulative relative error

$$e(a,b) = \frac{1}{m}\sum_{j=1}^{m}\frac{|a_j - b_j|}{|a_j|}. \tag{2.4}$$

3 Principles of selection of economic indicators

Concepts for evaluation of the risk associated with a portfolio are based on models describing changes in the factors that influence the value of individual assets and the correlation between these factors. In this section, we discuss the basic guidelines for the selection of a representative set of such factors, called economic indicators in the sequel. There are various factors that may influence the portfolio prices such as basic market factors, market indices that can judge the health of economy, and currency exchange, a factor related to trading.

3.1 Basic market factors

Market factors are all the market participants selling or taking delivery of the products that include any type of goods, services and information. As an example, in crude oil market, the market factors are the oil companies, refiners and the physical trading companies. Financial derivatives, such as forwards, futures and options are also important market factors in the financial industry. Derivatives are settled against the spot price based on the daily transaction of company's stocks. From the viewpoint of indicator selection, our method considers derivatives only in the sense of futures (see section 3.2).

Indicators falling into the category of basic market factors are selected in the following steps:

1. *Selection of region and market*: regions are considered based on where the components of a given portfolio get their earnings such as revenues, return on investment, etc., and how they pay their costs such as production cost, operating cost, labor cost, overhead cost, etc. On the basis of the important regions the financial markets are specified depending on where the portfolio is traded.

2. *Selection of candidates for basic market factors*: a relatively large number of market factors holding top positions in the financial markets obtained in step 1.

3. *Selection of related industry categories*: In order to select the market factors we need to assess the performance of individual factor not only by region or country but also by industry. We adopt the Industry Classification Benchmark (ICB) system developed by Dow Jones. This system divides the market into increasingly specific categories according to similar functions and related products. According to the composition of the portfolio, suitable categories from ICB system are considered in order to select the market factors.

4. *Computation of correlation coefficients*: determine the degree of correlation of market factors from step 2 and the portfolio.

5. *Final selection*: in general, a market factor is selected as an indicator if it is a member of the candidates from step 2, falls under an industry category selected in step 3 and the correlation coefficient is comparatively high.

The selection of market factors outlined in the above five steps can be modified according to the focused product and business area of the portfolio components. For a specific portfolio, a detailed analysis of its structure becomes an important ingredient of the selection of indicators from the group of basic market factors. In this case, the correlation coefficients may not be high but there exists strong relation with some of the portfolio components. On the other hand, a highly correlated market factor can be excluded, if there is no reasonable relation with any of the portfolio components or, more probably, if its connection with the portfolio is too strong.

3.2 Market indices

A market index is a value computed from a group of data to measure a certain sector of the market. These data may be derived from any sources, including company performance, prices, productivity, and employment. We select this area in order to identify the impact of country's economic health on the portfolio. Moreover, market indices are capable of involving other factors, for example, domestic and foreign political events such as war and terrorism, as well as natural disasters, while the physical market factors may not be able to reflect these events. The financial indices measure the performance of selected companies in order to evaluate the economic trends.

To give a few examples, the *Dow Jones Industrial Average* detects the USA market trend, the *consumer price index* is an economic index detecting the variation in prices for different consumer goods and services over time, the *real GDP* measures the level of prices of all new, domestically produced, final goods and services in an economy and the *commodity index* tracks the overall performance of commodity markets. Commodity indices often use the prices of future contracts for commodities that are listed on commodity exchanges, and can be grouped into subcategories such as energy, metals, agriculture, livestock, etc.

3.3 Currency exchange

Currency exchange rates can affect the returns on portfolio, especially when the portfolio consists of components from foreign markets. A mutual fund is an example of such kind of portfolio that holds international stocks. Fluctuations in the value of the base currency relatively to any foreign currency can affect the value of the portfolio. For example, when the base currency decreases relative to a foreign currency, it results in a higher valuation of the portfolio as measured in base currency.

Moreover, there may exist a nonlinear relationship between the currency exchange rates and the gaining of a portfolio component. For example, a portfolio component can use foreign currency to accumulate its input while the revenue is calculated in local currency. In order to maintain the stability and consistency of the approximation method, the exchange rates should not be considered only as components of the linear expansion but should also be used, e.g., as weights for certain indicatros, such as commodity indices.

4 Empirical analysis

According to the methodology discussed in section 2, we now provide a detailed empirical analysis and model testing. We discuss the optimal choice of economic indicators, approximate the given portfolio by a linear expansion and examine the properties of the expansion on the example of a portfolio consisting of one component, Toyota Motor Corporation (TMC) stock. Although our method is primarily designed for the analysis of complex multi-component portfolios, the analysis of a simple one-component portfolio is the first indispensable step in the validation of the model. Toyota Motor was chosen because it is one of the Japan's biggest employers and the backbone of the country's manufacturing sector. Moreover, the changes in the entire stock market are related to the stock price of TMC.

4.1 Data Description

We use daily closing prices of TMC stock. The data were gained from Google finance and Forex database, ranging from January, 2006 to December, 2010. Collected data are refined according to the formulae (2.2) and (2.3) with d=2 (see Fig. 1).

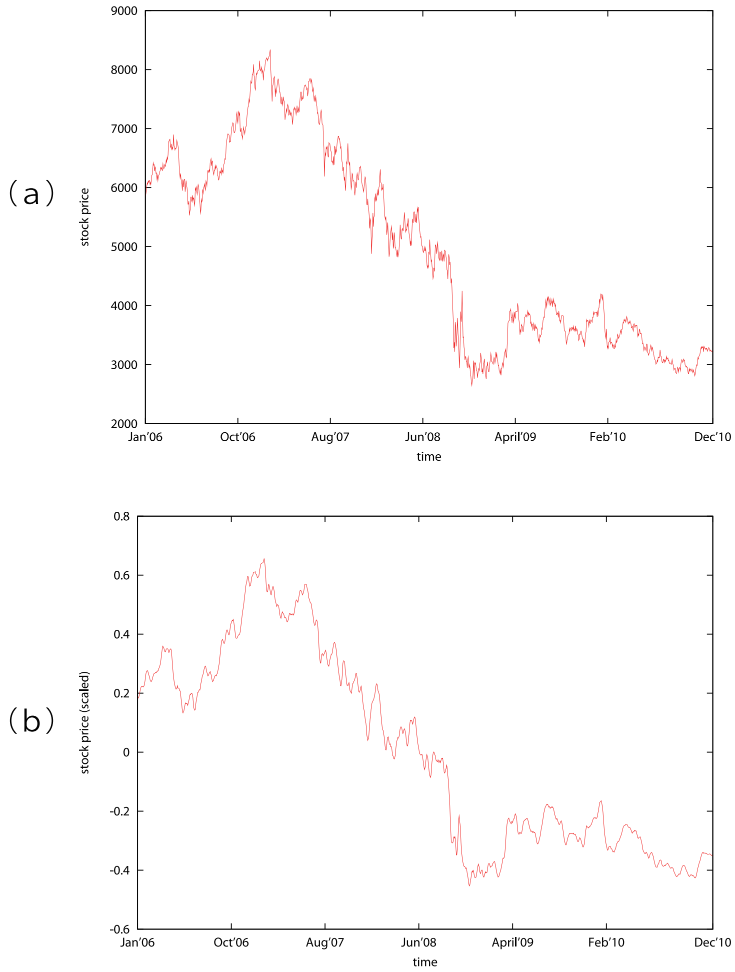

Figure 1: (a) closing prices of TMC stock from January, 2006 to December, 2010 and (b) the corresponding averaged and scaled data.

4.2 Economic indicators

In this subsection, we perform the selection of specific indicators for our empirical portfolio according to the procedure discussed in section 3. First, we study the activities of TMC from the geographical viewpoint. We would like to observe the activities of TMC related to its earnings and expenditures throughout the world and accumulate relevant information, as suggested in step 1 of section 3.1.

As of end of March, 2012, Toyota conducts its business worldwide with 50 overseas manufacturing companies in 27 countries. Toyota's vehicles are sold in more than 160 countries.

Region	**Production** (%)	**Sales** (%)	**Export from Japan** (%)
Japan	48	23	
North America	18	36	45
Europe	8	15	16
Asia	20	22	7
Others	6	4	32

From the above table (obtained from the Japan Automobile Manufacturers Association, Inc.), it is found that the most important regions for TMC are the domestic region, Japan, and the overseas region, North America. Therefore, we have selected two financial markets: Tokyo Stock Exchange (TSE) and the New York Stock Exchange (NYSE). It is also found from the databook of TMC that the highest percentage of Toyota's revenue is sales revenue and the sales revenue mostly consists of the income from export to overseas from Japan. On the other hand, the expenses associated with administering the business on a day to day basis include the fixed cost, such as overhead cost that remains the same regardless of the number of products produced, and variable costs, such as materials, which can vary according to the quantity to be produced. Moreover, an expense incurred by activities not related to the core operations of the business can be denoted by indirect costs, such as advertising the products locally or internationally.

As in step 2 in subsection 3.1, we have considered the top 100 companies that are most actively traded on the Tokyo Stock Exchange as the candidates to select indicators under the basic market factors group. Since TMC is keeping more manufacturing companies in order to maintain the level of quality, we choose the basic market factors from the domestic market only.

According to the step 3 of subsection 3.1, we adopt the Nikkei sectors and components for the selection of related industry categories. Nikkei is the most widely quoted average of Japanese equities, similar to the Dow Jones Industrial Average. The Nikkei represents the performance of the market and components have been rebalanced so that it can represent the Japan's industrial structure. Nikkei component stocks are balanced based on six sectors. For each sector we have analyzed the structure of the portfolio and its relationship to the corresponding market factors. Brief discussion on indicator selection based on six sectors of Nikkei is given below:

- *Financial sector*: This sector is one of the important sectors for TMC. Toyota's expertise extends beyond its automobile manufacturing business into a variety of other fields, including housing, financial services, communications, marine technology, biotechnology and afforestation. On the basis of company analysis, the financial services section is much correlated with automobile manufacturing business while the others are not much important to it. We have selected the Mizuho Financial Group because it is highly correlated with TMC stock comparing to other market factors in financial sector. One of the operating subsidiaries, Mizuho Bank, is the only bank, other than Japan Post Bank, to have branches in every prefecture in Japan. Moreover, the Mizuho Financial Group offers financial services including banking, securities, trust and asset management services in 30 countries throughout the world.

- *Capital goods*: JTEKT is selected from the subsector machinery under capital goods sector. The main activities of JTEKT are manufacturing and sale of machine tools, such as metal cutting applications that are used in electric devices or automotive engines and chassis. It also manufactures and sales auto parts, such as steering systems, driveline components etc. Moreover, it is a member of Toyota group and the correlation is high. Since TMC needs different types of machinery for its business operations, it is assumed that it collects most of its machineries from the group member, JTEKT.

- *Technology*: Electric machinery is the most important subsector related to our empirical portfolio under the technology sector. Hoya and Aisin are two important factors under this sector from the viewpoint of TMC stock. Hoya is a global technology company and the leading supplier of innovative and necessary high-tech products and services. The Aisin Group offers a wide lineup of products covering almost every automobile-related field and it is also a member of Toyota Group.

- *Materials*: Raw materials in automobile industry may differ from a manufacturer to another, direct or by-products. Steel and petroleum-based products represent an increasingly large percentage of automobile components. Besides, aluminium, glass, rubber and plastics are the main materials used in automobile manufacturing sector.

  In cars, steel is used to create the underlying chassis or cage beneath the body that forms the skeleton of the vehicle. It is also used in a variety of areas throughout the body to accommodate the engine or other parts, such as door beams, roofs, etc., and exhausts are often made from stainless steel. Steel manufacturing has evolved greatly, so carmakers can make different types of steel for different areas of the vehicle. In January, 1934, the Aichi Steel Corporation was founded by the TMC as ”Steel Production Department” for the research and creation of specialty steel for car manufacture.

  Nippon Electric Glass Co. Ltd manufactures and sales a wide range of glass related products, such as glass for electronic devices, heat-resistant glass etc. On the other hand, Toyota door glasses aren't just an ordinary part of the vehicle, but rather considered as one of the most essential auto part. These two companies are highly correlated to each other.

In 1949, Toyota Motor founded Toyota Gosei Co. Ltd, a rubber research operation and continues to own nearly 45% of the company. 95% of sales revenue of this company is composed from manufacture of auto components and Toyota Motor is one of the major customers of its products. This company is key partner in the globalization activities of Toyota Motor Corporation in North America, Europe and Southeast Asia.

- *Consumer goods*: Toyota Tsusho is the trading unit for the Toyota Group, which includes Toyota Motor and auto parts maker Denso. The company deals with a wide range of goods, including metals such as steel and aluminium, machinery and electronics, energy and chemicals, and various consumer products. Toyota Tsusho exports Toyota vehicles around the world, but plenty of non-automotive consumer products also contribute to its business. Moreover, Toyota Motor Corporation owns 22% of the company.

- *Transportation & Utilities*: Toyota started research and development of fuel cell vehicles in 1992 with the aim of developing zero-emission ecology cars so that these cars will reduce the amount of CO_2 emitted by the transportation sectors. For example, as a result of hybrid technology advancement, it is developing the fuel cell hybrid vehicle (FCHV) based on its own proprietary technology. Japanese automakers are working to reduce the cost of manufacturing a fuel-cell system for successful adoption of this vehicle. In this regard, more than 20 Japanese companies including TMC and Tokyo Gas Co. Ltd-Japan's number one gas company, are working together for the development of hydrogen supply infrastructure. In 1980's the Tokyo Gas Co. Ltd started to develop fuel cells, which constitute potential applications for natural gas. In the field of transportation fuel, this company is trying to spread the use of natural gas vehicles. Also it is trying to develop the technology for organizing a hydrogen supply infrastructure based on natural gas. This technology is intended to disseminate the use of fuel cell vehicles.

Next, we deliberate on the selection of the remaining indicators related to market indices and currency exchange. The economic impacts on automobile industry associated with movements in crude oil prices is one of the important points. Crude oil price movements can be driven by several factors, including fundamental changes in supply and demand, speculation and geopolitical concerns [1]. Japan imports about 80% of its oil and natural gas since its natural resources are extremely low. The overall demand for automobiles declined during 2008-2010 due to a global financial crisis. The financial downturn also affected auto manufacturers all around the globe. Despite the decline in demand Toyota had a competitive edge enhanced. It had also created hybrid engine designs that consumed less fuel and produced less environmental pollution and thus maintaining their overall sales at a higher level even the recession period was worsened by the rapid increase in the prices of fuel.

The West Texas Intermediate (WTI) crude oil price, quoted in U.S. dollar (USD) and acquired from the U.S. Energy Information Administration, is chosen as a representative of the world oil price. Oil price increased in the first six months of 2008 and then collapsed. Peak oil theory, originated by M. King Hubbert in 1956, would imply that the sharp fall in oil prices in 2008 was due to the sudden discovery of new oil reserves [7]. On the other hand, our portfolio prices also experienced a downward trend in the same period (see Fig. 2).

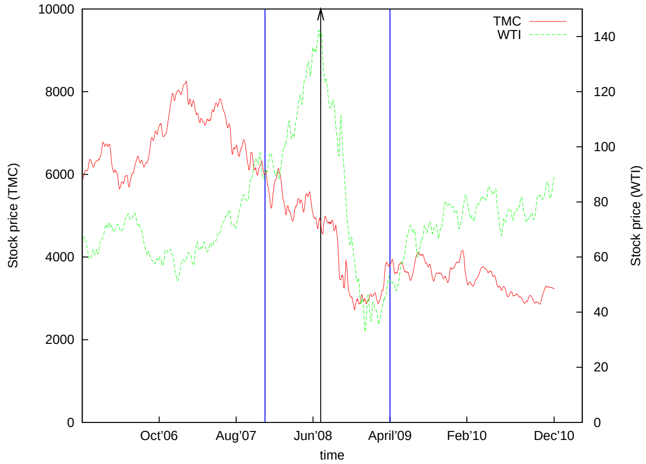

Figure 2: Data of the TMC stock price and the WTI oil price.

In addition, Speir (2004) concludes that oil price volatility alone explains not more than a half of gasoline price movements. This result is supported by Ashton and Upton (2004). The U.S. Gulf coast area is one of the most important regions for energy resources and infrastructure. Over 40 percent of total U.S. petroleum refining capacity is located along the Gulf coast. The Gulf coast has plentiful access to gasoline from its own refineries and it supplies a large proportion of the gasoline. Therefore, we have also considered U.S. Gulf coast conventional gasoline spot price as one of the indicators.

In principle, building linear expansion only through commodity producing companies is inadequate for getting exposure to the given portfolio. Portfolio returns are not only linked to company specific risk but also closely related to the prices of commodities. Thus, the diversified portfolio holders carefully watch the benchmark of commodity indices. Protection against inflation is also one of the main considerations to include commodity indices as indicators. Moreover, the commodities are an important part of the production process of many goods. We have selected the commodity indices Gasoline index, Rubber index, and Crude oil index from the Tokyo Commodity Exchange Inc. In this section, we have also chosen the Dow Jones Industrial Average (DJIA) as an indicator in order to gauge the performance of industrial sector within the American economy. Dow is one of the most closely watched U.S. benchmark indices, tracking targeted stock market activity.

The majority of Toyota vehicles are sold in the U.S., Japan, Europe and Australia and hence, it is exposed to fluctuating economic and political conditions of those markets. The Japanese currency, one of the major currencies in the world, strengthened 45 percent over five years. The stronger yen makes it harder for companies, such as TMC to make vehicles in Japan and sell them profitably in the overseas, since Toyota is keeping more manufacturing companies in Japan in order to maintain the level of quality. Therefore, we consider three of major exchange rates: U.S. dollar, Euro and Australian dollar (respective to the Japanese yen), which have significant impact on the earnings and expenditure of our empirical portfolio. We also convert the WTI crude oil prices and gasoline spot prices from USD to Japanese yen to reflect the effect of the exchange rates.

According to the above discussion, we have selected the following 18 economic indicators (see Table 1).

	Nikkei Sectors	**Subsectors**	**Market Factors**	**Correlation coefficient**
Basic Market Factors	Financial	Banking	Mizuho Financial Group	0.91
	Capital Goods	Machinery	JTEKT Corporation	0.95
	Technology	Electric Mechinery	Hoya Corporation	0.91
			Aisin Seiki Co. Ltd.	0.88
	Materials	Glass & Ceramics	Nippon Electric Glass Co. Ltd.	0.87
		Rubber	Toyota Gosei Co. Ltd.	0.63
		Steel Products	Aichi Steel Corporation	0.78
		Crude Oil	West Texas Intermediate Crude Oil	0.25
		Gasoline	U.S. Gulf Coast Conventional Gasoline	0.46
	Consumer Goods	Retail	Toyota Tsusho Corporation	0.96
	Transportation & Utilities	Gas	Tokyo Gas Company Limited	0.91
Market Indices			Gasoline index	0.71
			Rubber index	0.53
			Crude Oil index	0.43
			Dow Jones Industrial Average	0.78
Currency			USD/JPY	0.93
			EUR/JPY	0.85
			AUD/JPY	0.78

Table 1: Selected 18 economic indicators

4.3 Approximation of the portfolio

In order to approximate the value of the portfolio, we have divided the total period (January, 2006 to December, 2010) into two smaller subperiods [2]. The first period is used to determine the coefficients of expansion (2.1), and will be called "historical period". In practice, the data of economic indicators and portfolio prices are known for this period, which makes the calculation of the least-squares approximation possible. The second period, called "expansion period" is where the portfolio is actually evaluated and in practice corresponds to the future. For experimentation and analysis purposes, we can choose the starting point of expansion period arbitrarily. Considering the financial crisis of 2007-2008, a downturn in global economic activity leading to the "Great Recession", we decided to start the expansion from August, 2007.

First, we need to determine a suitable length for the historical period, so that it yields a sufficiently precise approximation in the expansion period. The dependence of the relative error of the approximation on the length of historical period was investigated for several lengths of expansion period.

The results are shown in Fig. 3 and suggest a monotone decrease in the error with increasing length of the historical period and with decreasing length of the expansion period. A suitable length of the historical period can then be set based on prescribed values for the error tolerance. In our case, for example, based on the requirement that the relative errors should be below 4% for expansion periods up to 50 days, the historical period was set to 400 days. The behaviour of the expansion may vary according to the starting point - we address this topic in the following subsection.

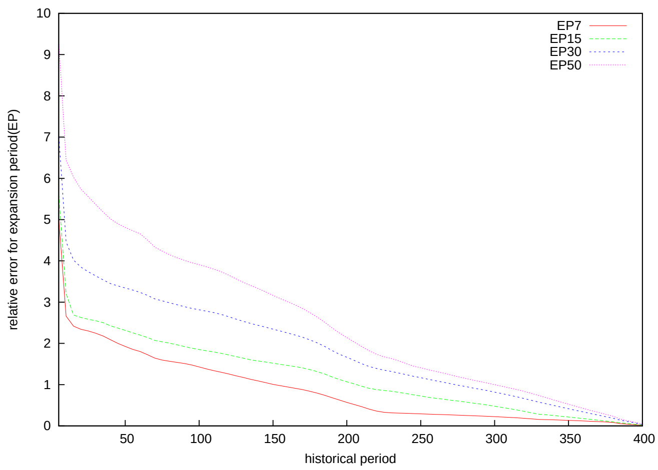

Figure 3: Dependence of relative error on the length of historical period for several choices of expansion period.

4.4 Analysis of the approximation

4.4.1 Error analysis

The resulting approximation of the portfolio with starting point August, 2007 and historical period 400 days is depicted in Fig. 4 (a). Here we used the linear approximation for the TMC stock price obtained from the historical period and extended it by substituting the known data for economic indicators during the expansion period. The figure thus gives the comparison of evolution of the extrapolated linear expansion with the motion of the real portfolio prices. Fig. 4 (b) shows the relative error evolution from the starting point up to the end of available data in December, 2010. The portfolio prices are scaled according to the equation (2.3) but relative errors are computed by (2.4) using the actual prices. The obtained values of expansion coefficients α_i from (2.1) for each indicator are shown in Table 2.

(a)

(b)

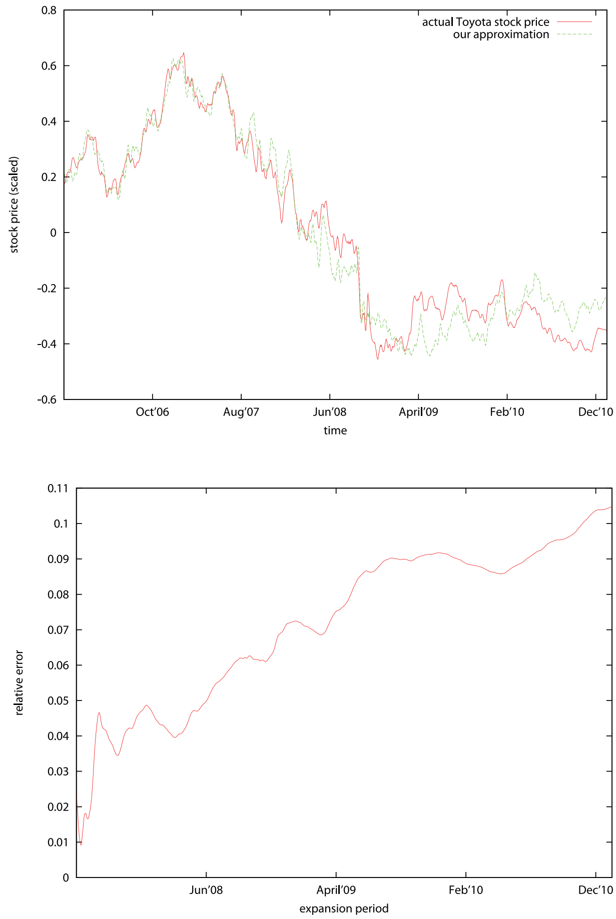

Figure 4: (a) real portfolio value (red line) and its approximation (green line), and (b) the corresponding relative error.

Economic indicator	**Coefficient**
Aisin Seiki Co. Ltd.	0.76
Tokyo Gas Company Limited	0.39
JTEKT Corporation	0.38
Dow Jones Industrial Average	0.32
EUR/JPY	0.31
U.S. Gulf Coast Conventional Gasoline	0.21
Mizuho Financial Group	0.17
Nippon Electric Glass Co. Ltd.	0.11
Crude Oil index	0.009
Rubber index	-0.02
USD/JPY	-0.07
Toyota Tsusho Corporation	-0.08
AUD/JPY	-0.12
West Texas Intermediate Crude Oil	-0.14
Hoya Corporation	-0.18
Aichi Steel Corporation	-0.25
Toyota Gosei Co. Ltd.	-0.27
Gasoline index	-0.48

Table 2: Indicator coefficients of TMC stock for historical period January 4, 2006 to August 17, 2007

One can observe that the error is kept below 5% for almost one year. Moreover, the correlation coefficient of the actual prices and the approximated prices within the period from starting point to December, 2010, is as high as 0.916272. These facts support the applicability of the approximation method.

Focusing on the economic event of 2008 and 2009, known as “Great Recession 2008-2009” we examine the behaviour of our expansion from January, 2008 to December, 2009. The purpose is to test the robustness of the expansion method in situations with rapid and significant changes in the economy. The recession began by the end of 2007 and the recovery phase tentatively began in mid-2009 [3]. Japan went collectively into the recession at the middle of 2008.

It is found that the error of the expansion during the whole recession period is less than 10% (see Fig. 5), which suggests that the expansion has a relatively stable behaviour even in extreme situations, such as economic recession.

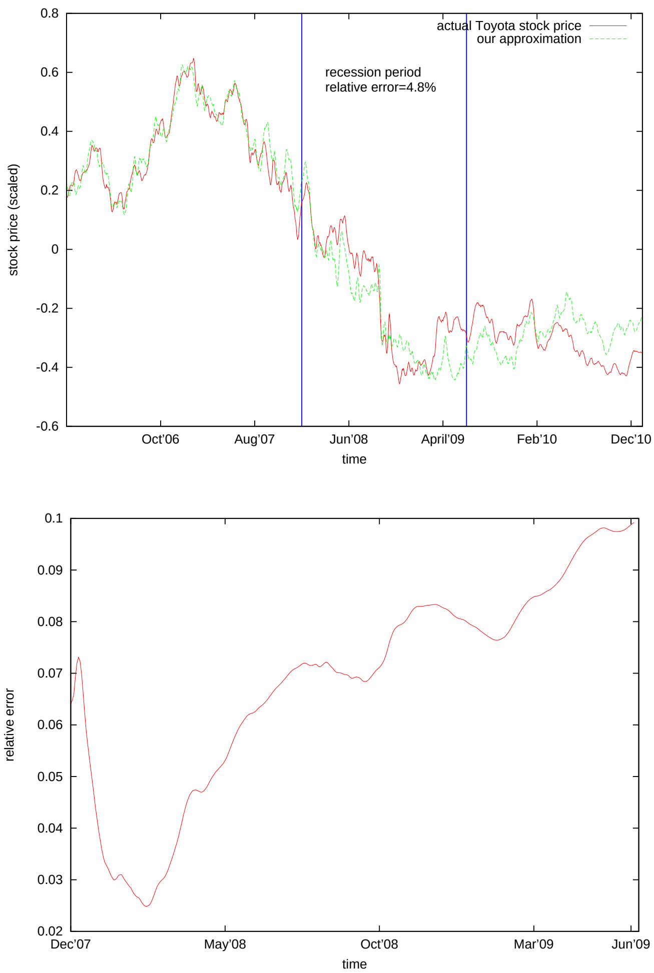

Figure 5: Graphical representation of different periods and the relative error for recession period.

4.4.2 Feasible expansion period

In order to find the expansion period, for which the approximation gives reasonable results, we have calculated the evolution of relative errors for a large number of different starting points. The results for five selected starting points are shown in Fig. 6.

We observe that the error for starting points falling into the recession period cross the 5% borderline within approximately one month. Since the error continues to rise after 4 weeks for all starting points, we conclude that the safe expansion period (in the sense of allowed error of 5%) is not more than 3-4 weeks. If we allow errors only up to 3%, the expansion method works for circa 2 weeks.

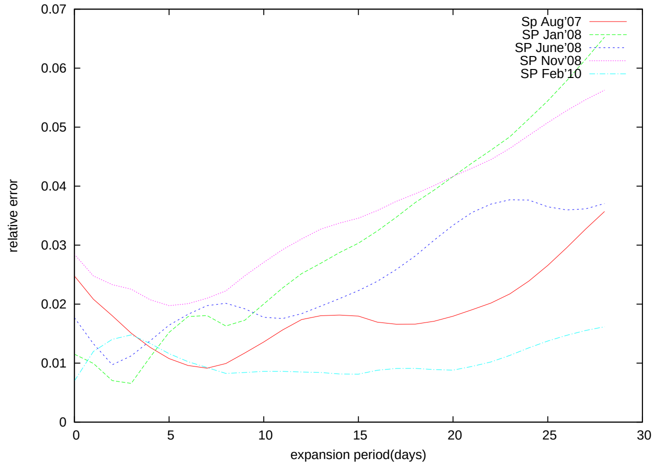

Figure 6: Evolution of relative error for different starting points.

4.4.3 Ability to follow changes in indicators

In this section, we verify the robustness of the expansion method by investigating its ability to accurately pursue rapid changes in certain aspects of the market. If the method is robust in this sense, it can become a convenient tool for the evaluation of impacts on the portfolio of various expected or forseen changes in the market, such as abrupt changes in oil prices and exchange rates, decline or growth of economy in some countries, etc.

The robustness is tested on past data by picking an indicator and a test period during which the indicator displays a significant monotone change. The value of this reference indicator within this period is replaced by a simple linear interpolation and the remaining indicators are generated with random changes correlated to the reference indicator. Values of indicators obtained in this way are substituted into the approximate expansion for the portfolio, the predicted value at the end of the testing period is calculated and this experiment is repeated for a large number of times, giving a probability distribution for the value of the portfolio at the end of the test period.

In particular, we chose the DJIA since its value decreases almost monotonely from August, 2007 to August, 2008 (see Fig. 7). The setting for the approximate expansion is as in section 4.4.1. The value of DJIA is prescribed as a linear function interpolating between its values on 19th of August, 2007 and 22nd of August, 2008. The historical period for the approximation is then from 4th of January, 2006 to 18th of August, 2007. The remaining 17 indicators are random values correlated with the above prescribed DJIA.

The results of 10,000 experiments yielded a probability distribution for TMC stock price on 22nd of August, 2008 (see Fig. 8). The mean of this distribution was found to be 0.0063, which corresponds the value of 5031.50, and is close to the actual value

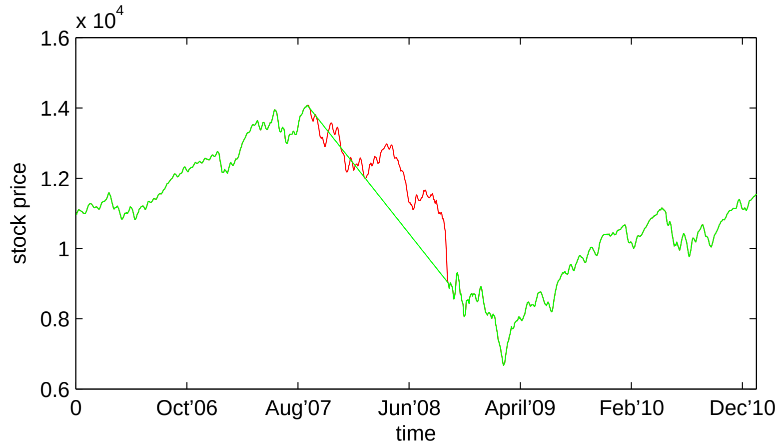

Figure 7: Original and interpolated DJIA data.

of 4896.50. The stock prices move within the interval $(4770.00, 4920.00)$ (red mark in the figure) from 19th to 27th of August, 2008. Several different test periods within the monotone decrease of the DJIA were examined with similar agreement of the predicted and actual value of TMC stock price. Therefore, we conclude that the proposed method has a good potential to accurately respond to changes in market factors.

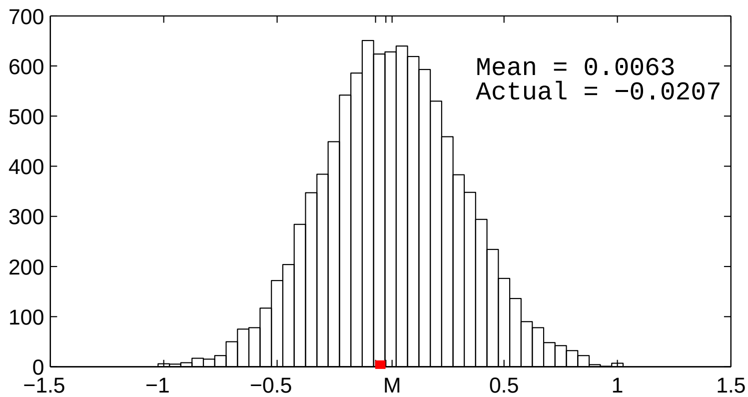

Figure 8: Probability distribution for TMC stock on Aug 22, 2008 obtained from 10,000 simulations.

5 Conclusion

In this paper, we introduced a method for portfolio evaluation based on the approximation of the portfolio by a linear combination of a relatively small number of market indicators.

In this way, it is expected that the analysis of a huge number of financial instruments (often including complex derivatives) and their complicated correlations can be reduced to the investigation of several carefully picked simple gauges, which will provide a clear view of the tendencies and properties of the portfolio.

The method was tested on a one-component portfolios, and favorable results were obtained. Namely, it was found that the suggested approximation gives a good agreement with real data for certain feasible extrapolation periods, whose lengths can be estimated. Moreover, the approximation is robust with respect to large-scale changes in the market situation and therefore can be used as a decision tool for investment policies.

It is assumed that the full strength of the developed technique can be brought into play by its application to complex portfolio. Thus, the next step of the research will include the extension of the ideas contained in the present work for such complicated sets of financial instruments, and refinement of the statistical methods used for the analysis of the method.

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