Measurement of Electron Densities of Plasma by Microwave Free Space Method

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1. Introduction

Measurements of electron densities of plasma by microwave free space method namely by interferometer have been carried out by several workers[1] [2] [3] [4] [5]. Especially, U. Kubo and Y. Inuishi[3] measured the electron densities of plasma in the gas discharge tube and obtained the results that the electron densities obtained when electric field vector of microwave was perpendicular to the central axis of discharge tube were larger than those obtained when it was parallel to the axis. And as one idea they tried the theoretical explanation by that in the case of perpendicular polarization in the wall of discharge tube may influence the result.

In order to remove this effect of polarization in the wall of discharge tube, we make microwave beam narrow by arranging microwave window in front of and behind discharge tube and carried out measurements. Even if the windows were arranged, it is thought that the diffraction wave will radiate out the line passing through two windows and cause the polarization there. But except that electron density have distribution in the direction of radius of discharge tube, conditions become same and polarization influence the results in the same order both in the case of perpendicular and in the case of parallel, if the aperture of window is small enough.

2. Experimental Procedure

(i) **Interferometer** Fig. 1 shows the block diagram of interferometer used. Frequency of oscillation of reflex klystron 24V 10 (made in Oki Electric Company in Japan) is 24Gc (wave length 1.25cm). A part of its output power is divided into

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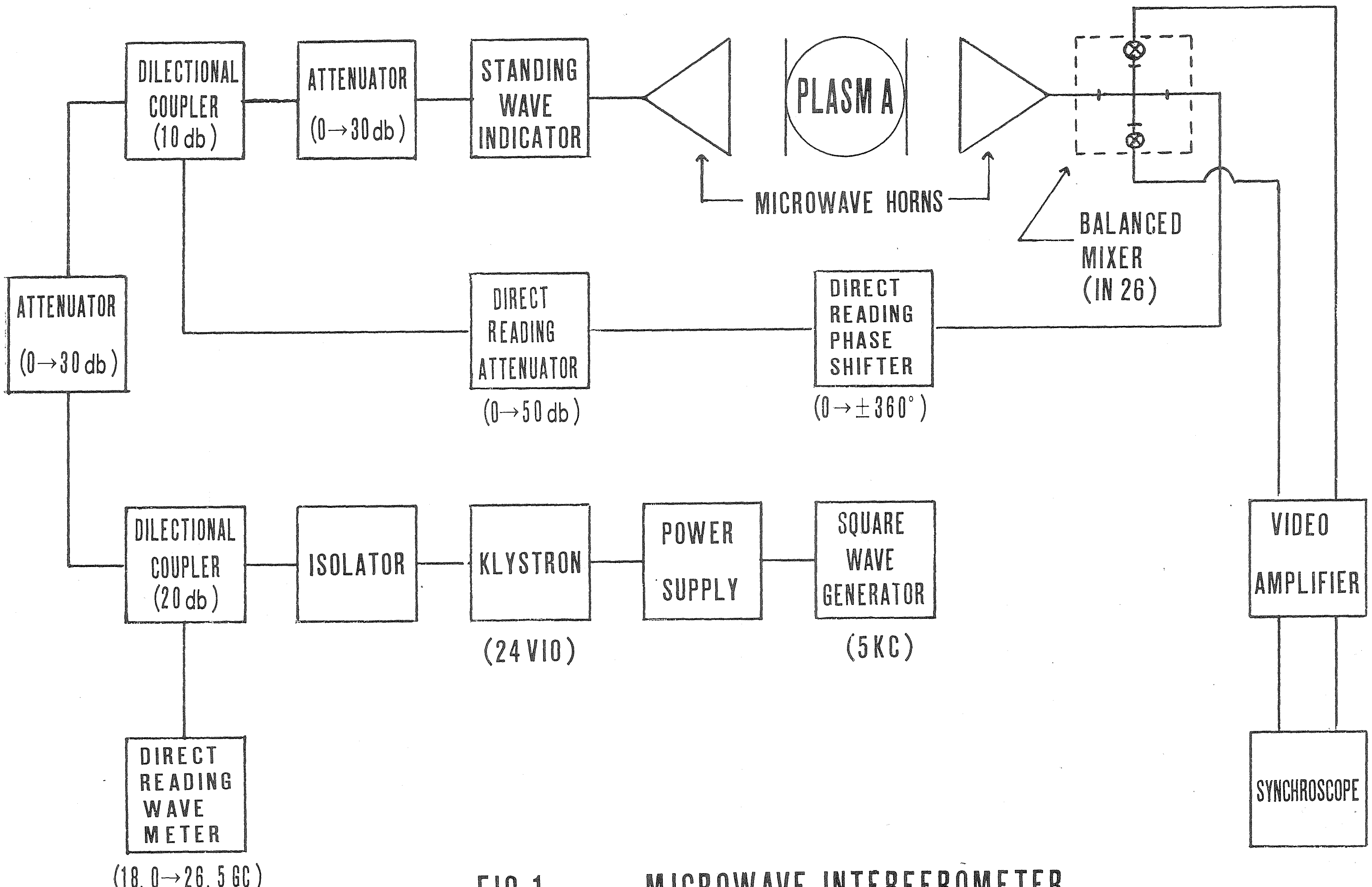

FIG 1. MICROWAVE INTERFEROMETER

reference path by 10 db directional coupler. The output powers of both transmission path including plasma and reference path enter into balanced mixer and its output is observed on synchroscope. Electromagnetic horns placed in transmission path are pyramidal horn with 20 db gain and their apertures are 48mm × 58mm and their lengths are 70mm. Distance between horns is fixed and 560mm. At the first time, when discharge does not occur in the discharge tube, we adjust phase shifter in reference path so that line shows maximum (or minimum) deflection on the synchroscope. At the next time, when discharge occur, we adjust again the phase shifter so that line make maximum (or minimum) deflection. Difference of readings of phase shifter before and after give the phase shift by transmission of microwave through plasma in the discharge tube. As the ouput of balanced mixer is small when windows of aperture 20mm × 20mm are used, we make amplitude modulation by adding square wave voltage of frequency 5kc on the repeller of klystron and amplify the output of balanced mixer by video amplifier of maximum gain 54db and put its output into synchroscope.

(ii) Discharge tube Fig. 2 shows the shape of discharge tubes used. Their lengths are 270mm and their diameters are 50 mm ϕ and 30mmϕ. Cathode is direct heating oxide cathode including five filaments of double coils. Anode is made by Tantalum. A probe is inserted for probe measurment. Probe consits of tungsten wire whose length is 4mm and diameter is 0.5mmϕ. We used the plasma of positive column as test plasma which is produced by d. c. gas discharge. Of course, we made microwave transmit in the nearest neighborhood of probe.

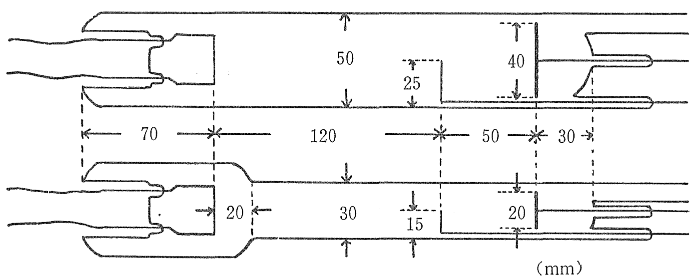

FIG. 2 DISCHARGE TUBE

(iii) Window Shape of windows and their relative position with transmitting horn, receiving horn and discharge tube are shown in Fig. 3. Window is made by copper plate of thickness 0.5mm and has pyramidal shape having inclination of 20 degrees so that reflecting wave do not come back to transmitting horn. Both windows in

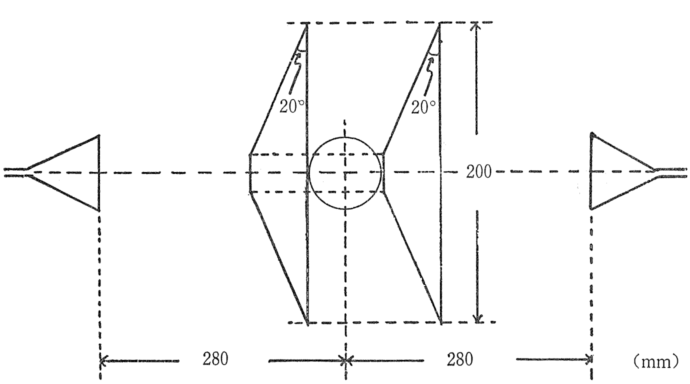

FIG. 3 MICROWAVE WINDOWS

front of and behind discharge tube are arranged in the nearest neighborhood of discharge tube. Aperture of window is changed from 50mm × 50mm to 20mm × 20mm.

(iv) Principle of mesurement When plane electromagnetic wave traverses plasma of electron density n_e, it is subjected to shift $\Delta\theta$ of its phase and its attenuation compared to the case of propagation in vacuum by its interaction with plasma. Shift $\Delta\theta$ of phase and attenuation α per wave length are expressed by following equations(6).

$$\Delta\theta = 2\pi\left[1 - \frac{1}{\sqrt{2}}\left\{\left(\frac{(1-\eta)^2+\beta^2}{1+\beta^2}\right)^{\frac{1}{2}} + \left(1 - \frac{\eta}{1+\beta^2}\right)\right\}^{\frac{1}{2}}\right]$$

$$\fallingdotseq \pi\eta\beta = \frac{4\pi^2 e^2}{m\omega^2} n_e \frac{radians}{wave\ length} \quad \beta \ll 1,\ \eta < 1 \tag{1}$$

$$\alpha = \frac{2\pi}{\sqrt{2}}\left[\left\{\frac{(1-\eta)^2+\beta^2}{1+\beta^2}\right\}^{\frac{1}{2}} - \left(1 - \frac{\eta}{1+\beta^2}\right)\right]^{\frac{1}{2}}$$

$$\fallingdotseq \pi\eta\beta = \frac{4\pi^2 e^2 \nu}{m\omega^3} n_e \frac{nepers}{wave\ length} \quad \beta \ll 1,\ \eta < 1 \tag{2}$$

$$\eta = \frac{\omega_p^2}{\omega^2} \quad \beta = \frac{\nu}{\omega} \quad \omega_p^2 = \frac{4\pi e^2 n_e}{m} \tag{3}$$

ω_p : plasma angular frequency
ω : angular frequency of electromagnetic wave
ν : collision frequency of electron with neutral molecule
n_e : electron density
e, m : charge and mass of electron

In our case, electron densities of plasma column have distribution in the radial direction. When change of electron densities along radius is so small that W. K. B. approximation(4) can be applied, mean phase shift of traversing wave is given as follows from Fig. 4.

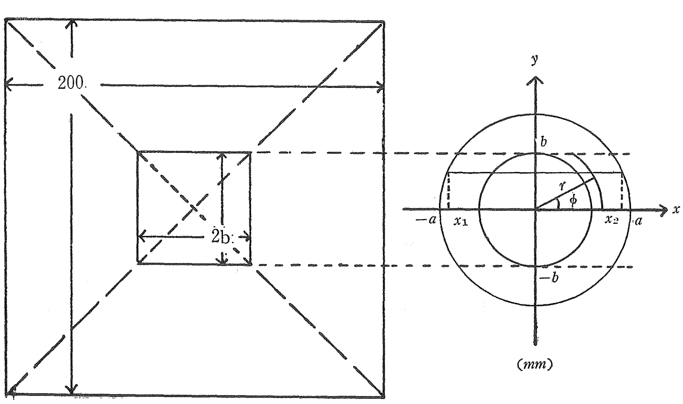

Fig. 4

$$\bar{\Gamma} = \frac{\int_{-b}^{b} dy \int_{x_1}^{x_2} \Delta\theta(x,y) \frac{dx}{\lambda}}{2b}$$

from equation (1)

$$= \frac{4\pi^2 e^2}{m\omega^2} \frac{\sqrt{a^2-b^2} + \frac{a^2}{b}\sin^{-1}\frac{b}{a}}{\lambda} \bar{n}_e \tag{4}$$

$\bar{n}_e$: mean electron density

a : radius of plasma column

b : $2b \times 2b$ is apperture of window.

We can obtain $\bar{n}_e$ from $\bar{\Gamma}$ measured by this equation.

On the other hand, electron densities obtained from probe measurements are maximum one in the center of discharge tube. Owing to comparison of them with mean electron densities obtained by microwave interferometer we must calculate mean electron densities from maximum electron densities $n_{e.\,max.}$. For this purpose, the distribution of electron density in the radial direction is assumed to be parobolic as follows.

$$n_e(r) = n_{e.\,max.}\left\{1-\left(\frac{r}{a}\right)^2\right\} \tag{5}$$

Then from Fig. 4,

$$S = 2\left(b\sqrt{a^2-b^2} + a^2\sin^{-1}\frac{b}{a}\right) \tag{6}$$

$$\bar{n}_e = \frac{1}{S}\int_0^{2\pi} d\varphi \int_0^{b} n_{e.\,max.}\left\{1-\left(\frac{r}{a}\right)^2\right\} rdr + 4\int_b^a n_{e.\,max.}\left\{1-\left(\frac{r}{a}\right)^2\right\} rdr \int_0^{\sin^{-1}\frac{b}{r}} d\varphi$$

$$= \frac{1}{S}\left[2b^2\left\{\frac{\sin^{-1}\frac{b}{a}}{\left(\frac{b}{a}\right)^2} + \cot\left(\sin^{-1}\frac{b}{a}\right)\right\} - b^2\left(\frac{b}{a}\right)^2\left\{\frac{\sin^{-1}\frac{b}{a}}{\left(\frac{b}{a}\right)^4}\right.\right.$$

$$\left.\left. + \frac{1}{3}\frac{\cos\left(\sin^{-1}\frac{b}{a}\right)}{\left(\frac{b}{a}\right)^3} + \frac{2}{3}\cot\left(\sin^{-1}\frac{b}{a}\right)\right\}\right] n_{e.\,max.} \tag{7}$$

We calculate $\bar{n}_e$ by equation (7) from $n_{e.\,max.}$ obtained by probe measurement and compare it with $\bar{n}_e$ obtained from measurent by microwave interferometer.

3. Results and discussion

About the discharge tube of 50mm ϕ, we plotted the mean electron densities $\bar{n}_{em}$ obtained from measurements by microwave interferometer and the mean electron densities $\bar{n}_{ep}$ obtained from probe measurements versus discharge currents in the cases of various apertures of windows. Their results are shown in Fig. 5, Fig. 6, Fig. 7, Fig. 8 and Fig. 9.

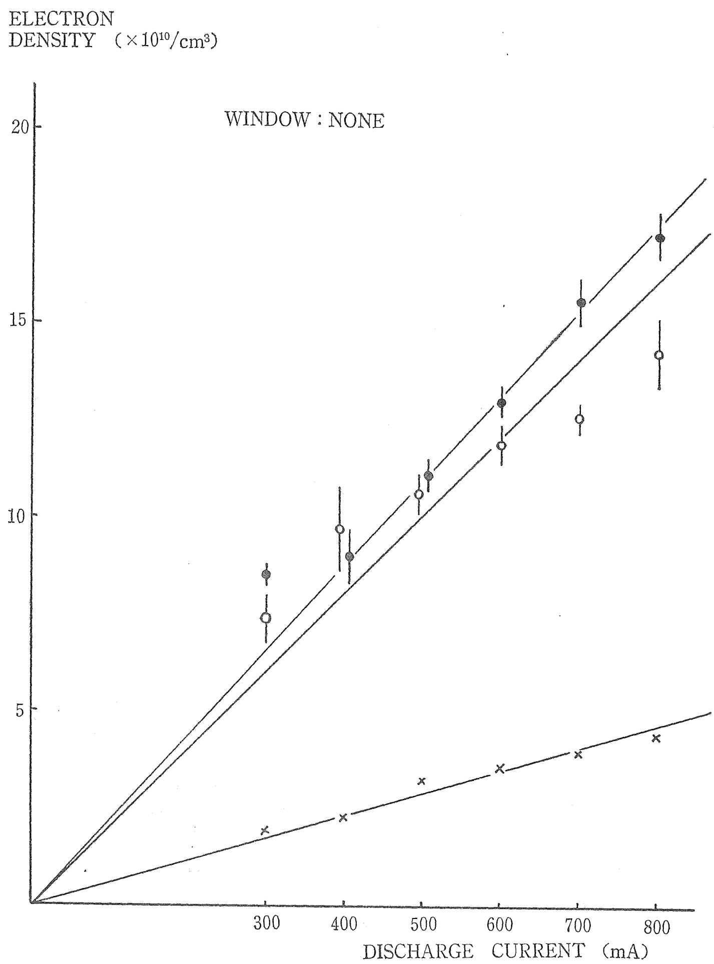

Fig. 5 Mean electron densities versus discharge current about discharge tube of 50mmϕ when windows are not used. ⏀ : mean electron densities obtained by microwave interferometer in the case when electric field vector E of microwave is parallel to the direction z of tube axis. ϕ : mean electron densities obtained by microwave interferometer in the case when E is perpendicular to z. × : mean electron densities obtained by probe measurement.

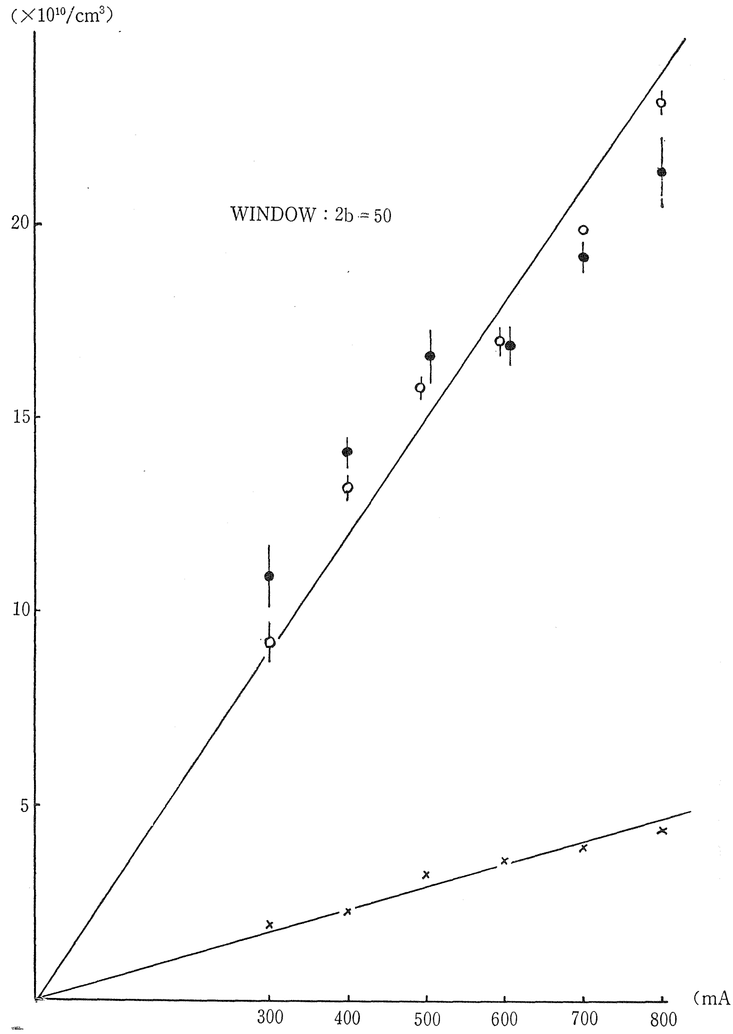

Fig. 6 Mean electron densities versus discharge tube of 50mmφ when aperture *2b* of window is 50mm. ●: mean electron densities obtained by microwave interferometer in the case when E//z. ○: mean electron densities obtained by microwave interferometer in the case when E⊥z. ×: mean electron densities obtained by probe measurement.

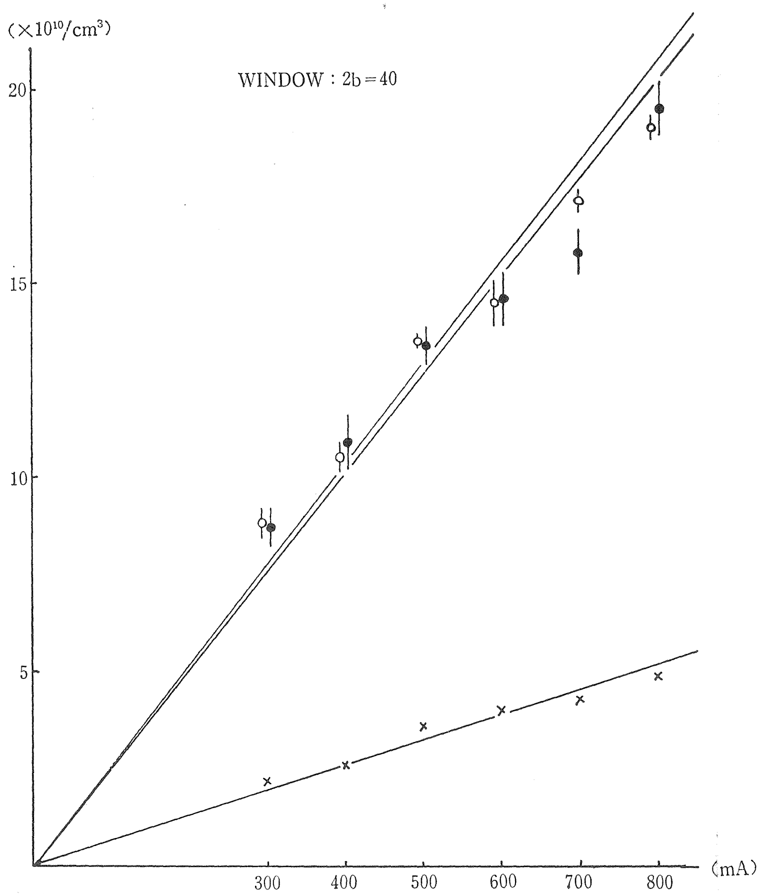

Fig. 7 Mean electron densities versus discharge current about discharge tube of 50mmϕ when aperture $2b$ of window is 40mm. ● : mean electron densities obtained by microwave interferometer in the case when E//z. ○ : mean electron densities obtained by microwave interferometer in the case when E⊥z. × : mean electron densities obtained by probe measurement.

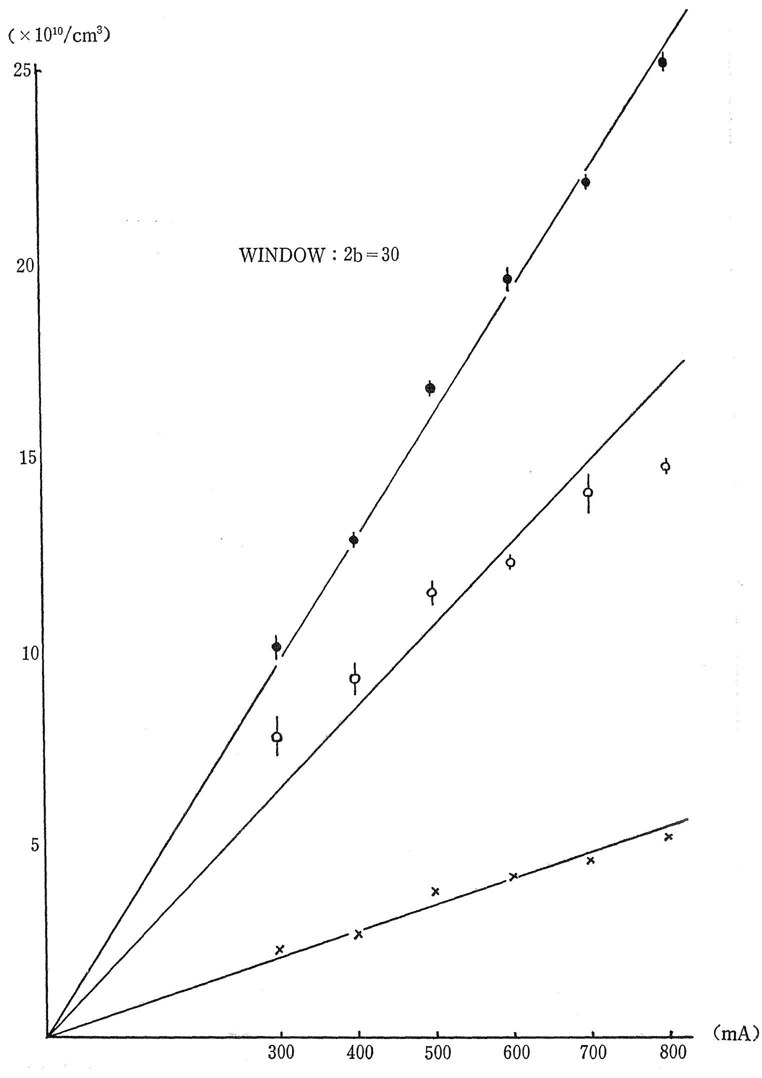

Fig. 8 Mean electron densities versus discharge current about discharge tube of 50mmϕ when aperture $2b$ of window is 30mm. ⏀: mean electron densities obtained by microwave interferometer in the case when E//z. ϕ: mean electron densities obtained by microwave interferometer in the case when E⊥z. ×: mean electron densities obtained by probe measurement.

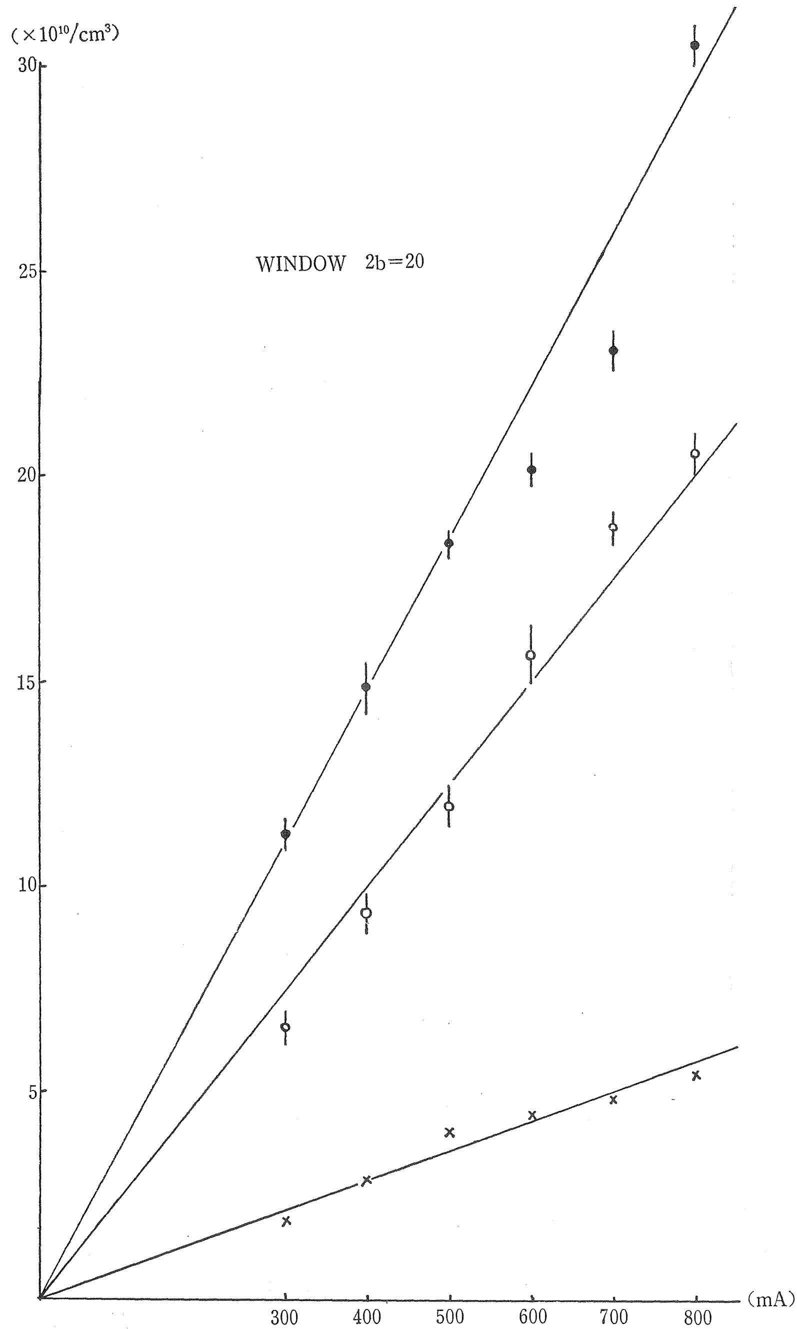

Fig. 9 Mean electron densities versus discharge current about discharge tube of 50mmϕ when aperture $2b$ of window is 20mm. ⏀: mean electron densities obtained by microwave interferometer in the case when $E//z$. ϕ: mean electron densities obtained by microwave interferometer in the case when $E \perp z$. $\times$: mean electron densities obtained by probe measurement.

(i) As seen from figures, $\bar{n}_{em}$ are larger than $\bar{n}_{ep}$. But, either $\bar{n}_{em}$ or $\bar{n}_{ep}$ increases linearly with discharge current. From this, those are thought to be quantities related with true electron densities.

At the next time, we approximate these graphs with straight lines and caculate the ratios of $\bar{n}_{em}$ to $\bar{n}_{ep}$ from them. When we plot these ratios versus the apertures of windows, Fig. 10 is obtained. This figure shows the mean value of measurements of two times.

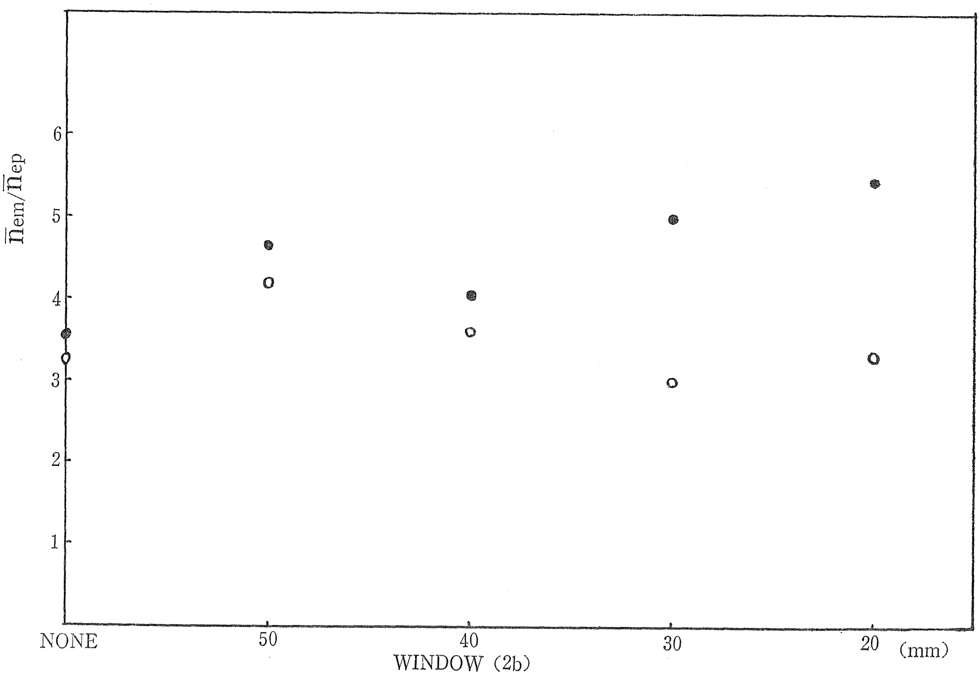

Fig. 10 Ratio of $\bar{n}_{em}$ to $\bar{n}_{ep}$ versus aperture $2b$ of window about discharge tube of 50mmϕ. $\bar{n}_{em}$ is mean electron densities obtained by microwave interferometer and $\bar{n}_{ep}$ is mean electron densities obtained by probe measurement. ●: the case when E//z ○: the case ehen E⊥z.

(ii) As seen from the figure, $\bar{n}_{em}$ amount to from 3 times to 5.5 times of $\bar{n}_{ep}$. Following causes to this result are thought.

(a) In probe measurement, owing to the great slope of straight line in the region of electron saturated current, knee in plasma potential sometimes does not appear clearly. So, errors enter into $\bar{n}_{ep}$.

(b) As S. Shiobara[4] pointed out, the change of phase shift Γ owing to the multiple reflection of microwave between horns is thought to influence the result. But, as this effect is in so order that maximum change of Γ amounts to 60 per cent, the result can not be explainad only by this effect.

(c) As U. Kubo and Y. Inuishi[3] pointed out, scattering of microwave by the divergence action of plasma is thought to influence the result. Namely, the microwave traversing the central part of section of discharge tube where electron densities are larger compared to the end part is principally measured and $\bar{n}_{em}$ becomes large.

(d) Electron densities are wholly higher in the case when windows are used than in the case when windows do not be used. From this, going of microwave around discharge tube is thought to be removed by window.

(iii) Measurements are carried out both for the case when electric field vector E of microwave is perpendicular to the axis z of discharge tube and electron densities obtained are designated as $\bar{n}_{e\perp}$ and for the case when the former is parallel to the latter and electron densities obtained are designated as $\bar{n}_{e//}$. As seen from Fig. 10, $\bar{n}_{e//}$ are larger than $\bar{n}_{e\perp}$. Especially, their differences are remarkable when the apertures $2b$ of windows are 30mm and 20mm. On the contrary to the results expected when the effect of polarization influences, namely, the results that $\bar{n}_{e//}$ and $\bar{n}_{e\perp}$ will become to coincide more and more, the narrower we make microwave beam, their differences become remarkable when we make microwave beam narrow. Therefore, the difference between $\bar{n}_{e//}$ and $\bar{n}_{e\perp}$ can not be explained by polarization. This result is thought to be caused by the circumstance that anisotropy of plasma in z direction may be caused owing to the discharge voltage adding to plasma and the relation between shift $\Delta\theta$ of phase of microwave and electron density n_e may deviate from theory.

In order to investigate whether this result is due to proper nature of plasma or not we measured the dielectric constant of polystyrene column by microwave interferometer whose diameter is 50mm and length is 260mm. If we denote the dielectric constant of matter as $\varepsilon = \varepsilon_{re} - j\varepsilon_{im}$, then phase shift $\Delta\theta$ and attenuation α of microwave per wave length are given as follows when $\varepsilon_{im} \ll \varepsilon_{re}$.

$$\Delta\theta = 2\pi(1-\sqrt{\varepsilon_{re}})$$

$$\alpha = \pi\frac{\varepsilon_{im}}{\sqrt{\varepsilon_{re}}} \qquad (8)$$

In Fig. 4, mean phase shift $\bar{\Gamma}$ is given as follows.

$$\bar{\Gamma} = \frac{\int_{-b}^{b} dy \int_{x_1}^{x_2} \Delta\theta \frac{dx}{\lambda}}{2b} = \frac{\sqrt{a^2-b^2} + \frac{a^2}{b}\sin^{-1}\frac{b}{a}}{\lambda} 2\pi(1-\sqrt{\varepsilon_{re}}) \qquad (9)$$

a : radius of column of matter

b : 2b × 2b is aperture of window.

ε_{re} is obtained from measurement of $\bar{\Gamma}$. Fig. 11 shows the result measured. As seen from this, the difference does not exist almost between the values obtained in the case when electric field vector is perpendicular to the central axis of polystyrene column and the values obtained in the case when the former is parallel to the latter.

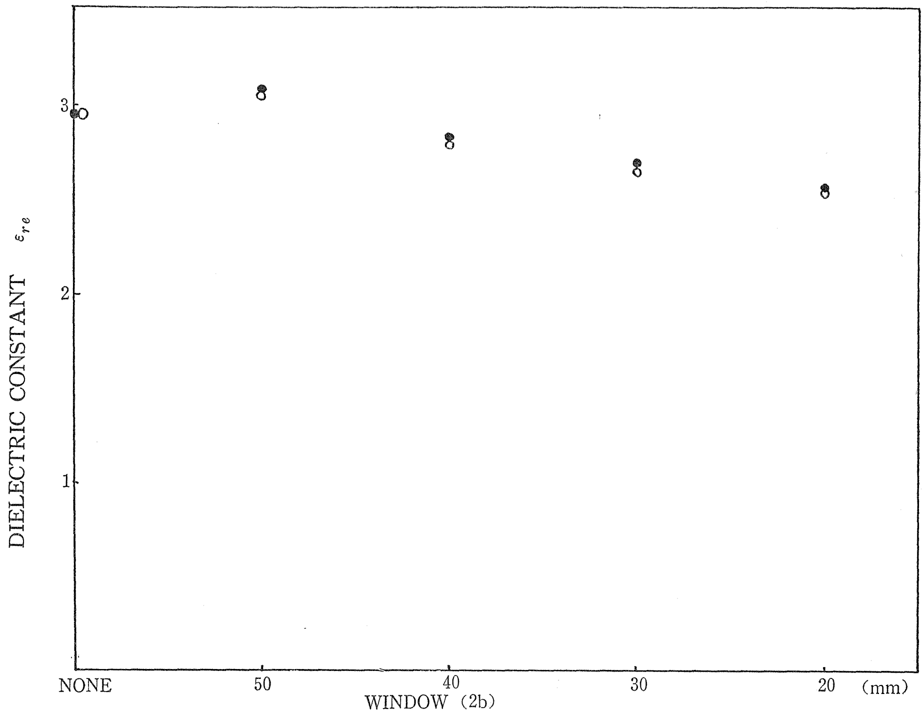

Fig. 11 Dielectric constant ε_{re} of polystyrene column versus aperture $2b$ of winodow. ●: the case when electric field vector E is parallel to the direction z of column axis. ○: the case when $E \perp z$.

Therefore, the result described above is proper to plasma.

Also we arranged standing wave indicator in front of transmitting horn and investigated the reflecting microwave measuring voltage standing wave ratios. Fig .12 shows the result measured. As seen from this, the difference does not exist almost for the various apertures of window and between the case when discharge occurs and the case when discharge does not occur. But, V. S. W. R. and thus reflections are larger in the case when electric field vector is perpendicular to the tube axis than in the case when the former is parallel to the latter.

About the discharge tube of 30mmϕ, the ratios of $\bar{n}_{em}$ to $\bar{n}_{ep}$ versus apertures of windows are plotted in Fig.13. The result shows as almost similar tendency as that about the discharge tube of 50mmϕ.

Lastly, if diffraction waves from window in front of discharge tube spread and cover the cross section of discharge tube, the influence of polarization of plasma by the boundary of tube wall may appear in the case when electric field vector E of microwave is perpendicular to the direction z of tube axis. But, as the U. Kubo and Y. Inuishi's(3) theoretical consideration, $\bar{n}_{e\perp} > \bar{n}_{e//}$ turns out as the result in that case which is contrary to our result. Therefore, above result may not be caused by diffraction wave.

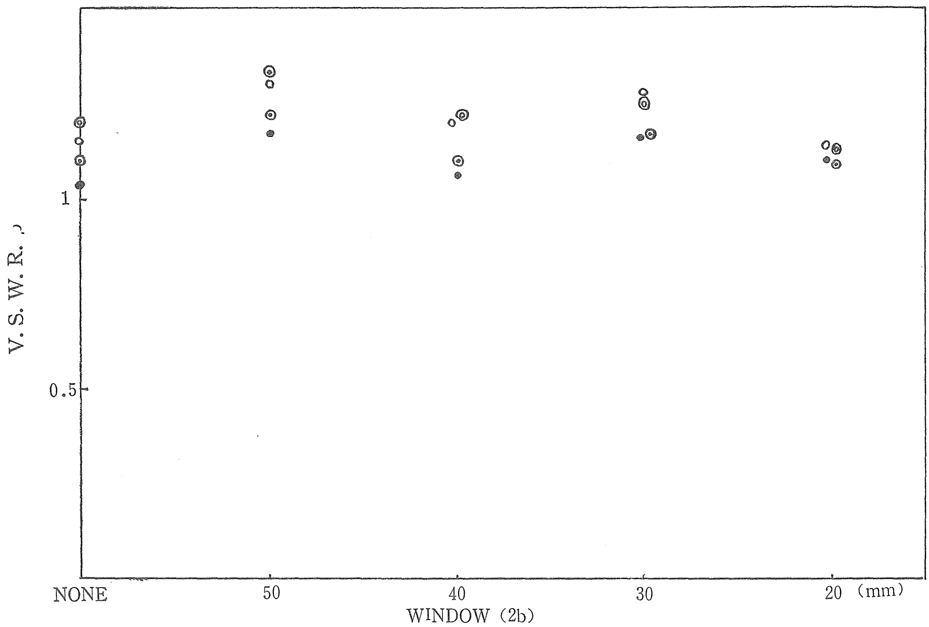

Fig. 12 Voltage standing wave ratio ρ versus aperture $2b$ of window about discharge tube of 50mmϕ. ●: the case when E//z and discharge occur. ⊙: the case when E//z and discharge does not occur. ○: the case when E⊥z and discharge occur. ◎: the case when E⊥z and discharge does not occur.

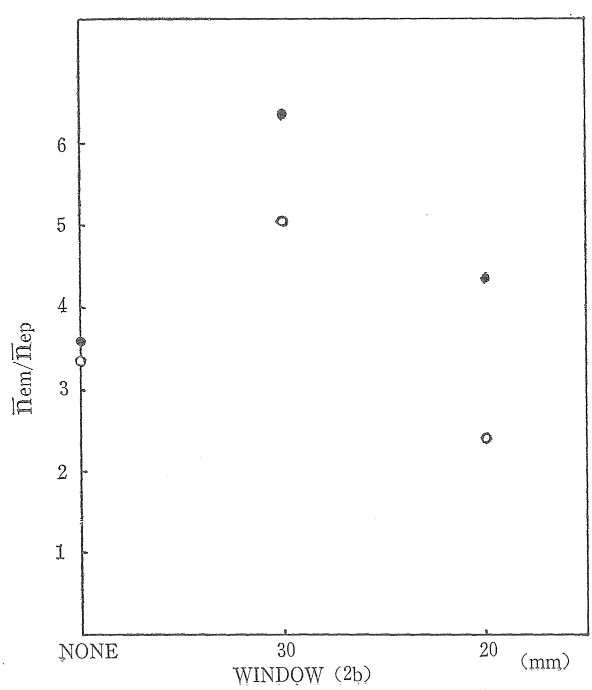

Fig. 13

Ratio of $\overline{n}_{em}$ to $\overline{n}_{ep}$ versus aperture $2b$ of window about discharge tube of 30mmϕ.
●: the case when E//z
○: the case when E⊥z.

4. Conclusions

(i) Mean electron density $\bar{n}_{em}$ measured by microwave free space method amounts to from 3 times to 5.5 times of mean electron densitiy $\bar{n}_{ep}$ measured by probe. Either of them increases linearly with discharge current and thus is thought to be quantity related with true electron density.

(ii) About mean electron density measured by microwave free space method, that obtained in the case when electric field vector E is parallel to axis z of discharge tube is larger than that obtained in the case when the former is perpendicular to the latter. Especially, when we make microwave beam narrow by windows in front of and behind discharge tube, its difference become remarkable. It is thought that this result may be caused by anisotropy of plasma of positive column owing to the addition of discharge voltage.

5. Acknowledgment

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