Simulation of A Soap Film Catenoid

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Abstract. *There are many interesting phenomena concerning soap film. One of them is the soap film catenoid. The catenoid is the equilibrium shape of the soap film that is stretched between two circular rings. When the two rings move farther apart, the radius of the neck of the soap film will decrease until it reaches zero and the soap film is split. In our simulation, we show the evolution of the soap film when the rings move apart before the film splits. We use the BMO algorithm for the evolution of a surface accelerated by the mean curvature.*

Keywords: soap film catenoid, minimal surface, hyperbolic mean curvature flow, BMO algorithm

1. Introduction

The phenomena that concern soap bubble and soap films are very interesting. For example when soap bubbles are blown with any shape of bubble blowers, the soap bubbles will be round to be a minimal surface that is the minimized surface area. One of them that we are interested in is a soap film catenoid. The catenoid is the minimal surface and the equilibrium shape of the soap film stretched between two circular rings.

In the observation of the behaviour of the soap bubble catenoid [3], if two rings move farther apart, the radius of the neck of the soap film will decrease until it reaches zero. Then the soap film is split and a small bubble will appear.

In this simulation, the soap film catenoid was simulated when the rings move apart before the film split by adapting BMO algorithm for the evolution of a surface accelerated by the mean curvature.

2. Derivation of the equation

When the force act to the interface in the normal direction, the equation of interfacial dynamics [2] is given by

$$A = -\kappa n, \tag{1}$$

where A is the interfacial normal acceleration, κ is the mean curvature, and n is the unit normal vector to the interface.

The differential equation for the evolution (1) is

$$\begin{cases} \alpha_{tt}(t,x) \;=\; -\kappa(x) & \text{in } (0,T) \times \Omega \\ \alpha_t(t=0,x) \;=\; v_0(x) & \text{in } \Omega \\ \alpha(t=0,x) \;=\; \gamma(x) & \text{in } \Omega, \end{cases}$$

where α is a position function, v_0 is an initial velocity and γ is a parametrized curve.

3. BMO algorithm for the evolution of a surface accelerated by the mean curvature

The original BMO was proposed by Bence, Merriman, and Osher in [1]. The BMO is useful for computing motion of a soap film by mean curvature. In this paper, we use the idea of BMO for the evolution of a surface accelerated by the mean curvature [2] because the the soap film catenoid has the force that act on the interface to reduce its area.

At a given time T, $h = T/M$, where $0 < h \ll 1$ and M is a positive number. For $k = 0, 1, ..., M$

Let Γ_k be a smooth curve and E_k be region enclosed by Γ_k and boundary at time k.

We use two signed distance functions for initial condition.

$$d_0(x) = \begin{cases} \inf\limits_{y\in\Gamma_0} \quad \|x-y\| & \text{if } x \in E_0 \\ \text{-}\inf\limits_{y\in\Gamma_0} \quad \|x-y\| & \text{otherwise,} \end{cases}$$

$$d_{-1}(x) = \begin{cases} \inf\limits_{y\in\Gamma_0 - v_0 h} \quad \|x-y\| & \text{if } x \in E_{-1} \\ \text{-}\inf\limits_{y\in\Gamma_0 - v_0 h} \quad \|x-y\| & \text{otherwise.} \end{cases}$$

1) Set $u_0(x) \;=\; 2d_k - d_{k-1}$, where

$$d_k(x) = \begin{cases} \inf\limits_{y\in\Gamma_k} \quad \|x-y\| & \text{if } x \in E_k \\ \text{-}\inf\limits_{y\in\Gamma_k} \quad \|x-y\| & \text{otherwise.} \end{cases}$$

2) Solve the wave equation with zero initial velocity and initial condition u_0 at a time h

$$\begin{cases} u_{tt} \;=\; \triangle u & \text{in } (0,\, h) \,\times\, \Omega \\ \frac{\partial u}{\partial \nu} \;=\; 0 & \text{on } (0,\, h) \,\times\, \partial\Omega \\ u(t \;=\; 0,\, x) \;=\; u_0 & \text{in } \Omega \\ u_t(t \;=\; 0,\, x) \;=\; 0 & \text{in } \Omega. \end{cases}$$

3) Update the interface and the region.

4) Calculate d_{k+1}.

4. Model of a soap bubble catenoid

4.1 Derivation of the equation concerning a soap bubble catenoid

For the soap bubble catenoid, there is the force that act on the interface to reduce its area because the catenoid is the minimal surface. The force act on the interface that the curvature is $1/r$, where r is the radius of the circle in the catenoid for each grid point. Adding $1/r$ in the equation (1), we get

$$A \;=\; (-\kappa + 1/r)n. \tag{2}$$

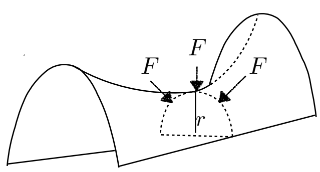


Figure 1. The half of catenoid

The differential equation concerning the soap bubble catenoid (2) is

$$\begin{cases} \alpha''(t,x) \;=\; -\kappa(x) \,+\, 1/r(x) & \text{in } (0,T) \,\times\, \Omega \\ \alpha'(t=0,x) \;=\; 0 & \text{in } \Omega \\ \alpha(t=0,x) \;=\; \gamma(x) & \text{in } \Omega, \end{cases}$$

where $\gamma(x)$ is a closed curve that be parametrised and initial velocity is zero.

4.2 BMO algorithm for a soap film catenoid

At a given time T, $h = T/M$, where $0 < h \ll 1$ and M is a positive number. For $k = 0, 1, ..., M$

1) Set $u_0(x) = d_k$.

2) Compute the radius in the catenoid for each grid point.

3) Solve the equation with zero initial velocity and initial condition u_0 at a time h

$$\begin{cases} u_{tt} = \triangle u + 1/r & \text{in } (0,\ h) \times \Omega \\ u = u_0 & \text{on } (0,\ h) \times \partial\Omega \\ u(t = 0,\ x) = u_0 & \text{in } \Omega \\ u_t(t = 0,\ x) = 0 & \text{in } \Omega. \end{cases} \tag{5}$$

4) Update the interface and the region.

5) Calculate d_{k+1}.

5. The equation solving

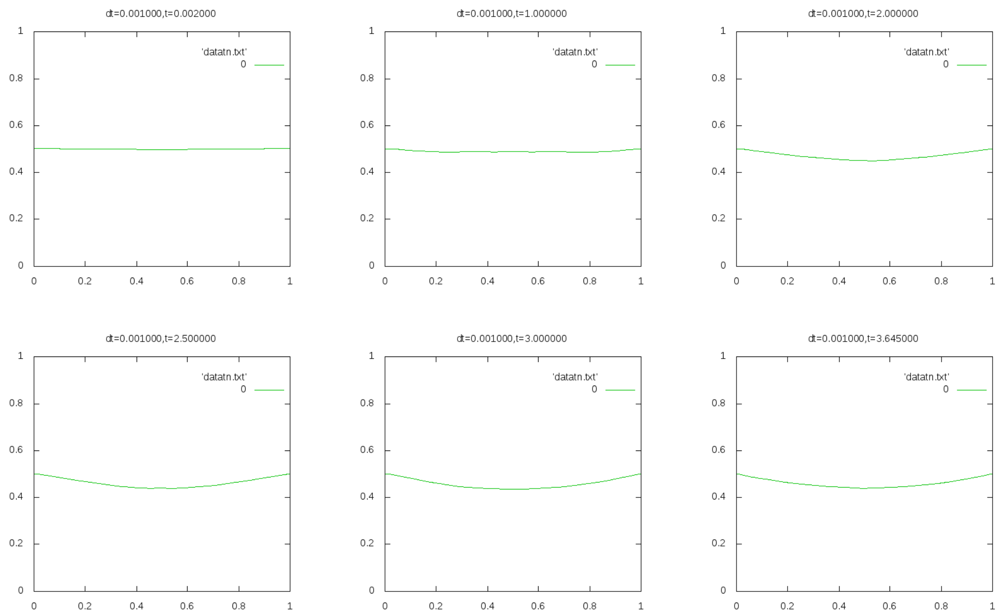
We solve the equation (5) by using finite difference method(central difference method for the second derivative). Let $u_{i,j}^k$ be an approximate solution at (t_k, x_i, y_j), $\Delta t = t_k - t_{k-1}$, $\Delta x = x_i - x_{i-1}$, $\Delta y = y_j - y_{j-1}$ and $\Delta x = \Delta y$. Then the approximate solution is

$$u_{i,j}^{k+1} = 2u_{i,j}^k - u_{i,j}^{k-1} + \left(\frac{\Delta t}{\Delta x}\right)^2 \left(u_{i+1,j}^k + u_{i-1,j}^k + u_{i,j+1}^k + u_{i,j-1}^k - 4u_{i,j}^k\right) + \Delta t^2 / r_{i,j}^k .$$

6. Numerical results

This simulation that we show is only some part of soap film catenoid.

400×400 grids, time discretisation is 0.001/10.



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