Step wandering induced by the drift of adatoms in a conserved system

Step wandering induced by the drift of adatoms in a conserved system

Masahide Sato,[1,*] Makio Uwaha,[2] Yukio Saito,[3] and Yukio Hirose[1]
[1]*Department of Computational Science, Kanazawa University, Kakuma-machi, Kanazawa 920-1192, Japan*
[2]*Department of Physics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8602, Japan*
[3]*Department of Physics, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan*
(Received 22 February 2002; published 24 June 2002)

We study step wandering induced by the drift of adatoms in a conserved system. When steps are impermeable, in-phase wandering occurs with the step-down drift. The steps are unstable for long-wavelength fluctuations and the wavelength of the most unstable mode is determined by the competition between the drift and the step stiffness. When nonlinear effects are taken into account, the steps obey the same type of equation as that of the step wandering due to the Ehrlich-Schwoebel effect in growth without evaporation. We carry out Monte Carlo simulation and compare the results with the nonlinear evolution equation.

DOI: 10.1103/PhysRevB.65.245427 PACS number(s): 81.10.Aj, 05.70.Ln, 47.20.Hw, 68.35.Fx

I. INTRODUCTION

In a vicinal face two types of step instabilities occur in many kinds of crystals. One is step wandering, which is an instability for a fluctuation along the step and the other is step bunching, which is an instability for a fluctuation of the step distance. In case of Si(111), the instabilities occur when a specimen is heated by direct electric current.[1–8] The cause of the instabilities is considered to be the drift of adatoms induced by the current.[9]

The step wandering induced by the drift has been studied with a continuum step model and by means of Monte Calro simulations.[10–14] When there is evaporation of adatoms, the step wandering occurs with the step-down drift if the drift velocity exceeds a critical value determined by the step stiffness. When a step is isolated in a large terrace, the step obeys the Kuramoto-Sivashinsky (KS) equation[15,16] whose solution shows spatiotemporal chaos. In a vicinal face, two-dimensional analysis[12] shows that grooves perpendicular to the steps appear, but the lateral fluctuation is so wild that they sometimes pinch out. Thus the unstable surface shows a chaotic pattern.

The observation of step wandering has been made in Si(111) surfaces.[4–8] The step wandering occurs when the current is in the step-down direction. Since the drift is in the same direction of the current,[5] the drift direction to induce the step wandering agrees with the linear stability analysis.[10,11] However, the surface pattern is different from the chaotic pattern expected from the theory:[12] all steps wander in phase and straight grooves parallel to the current are produced.

Recently, step wandering induced by the Ehrlich-Schwoebel (ES) effect[17–19] has been studied theoretically.[20–26] With evaporation of adatoms, steps obey the KS equation and grooves fluctuate much in a vicinal face.[20–23] Without evaporation, on the contrary, the fluctuation of grooves is suppressed and equidistant parallel straight grooves appear.[24–26]

For step wandering induced by the drift of adatoms, the surface pattern may also be affected by the presence of evaporation. In this paper we study the step wandering induced by the drift of adatoms in a conserved system. We first perform the linear stability analysis of steps in Sec. III. The time evolution of the step pattern at late stages is derived by taking the nonlinear effect into account in Sec. IV. We carry out Monte Carlo simulation and test the theory in Sec. V. We give a short summary and discussion in Sec. VI.

II. MODEL

We use a continuum step model to study the linear stability of a vicinal face. Our coordinate system is such that x axis is parallel to the steps and y axis is in the step-down direction. We consider a conserved system such that impingement and evaporation of adatoms are neglected. When the drift is in the y direction, the diffusion equation of adatom density is given by[27]

$$\frac{\partial c}{\partial t} = D_s \nabla^2 c - v \frac{\partial c}{\partial y}, \tag{1}$$

where D_s is the diffusion coefficient and v is the drift velocity. Boundary conditions at the mth step are given by[13,28]

$$\pm D_s \hat{\boldsymbol{n}} \cdot \boldsymbol{\nabla} c|_{\pm} \mp \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{y}} v c|_{\pm} = K_{\pm}(c|_{\pm} - c_m) + P(c|_{\pm} - c|_{\mp}), \tag{2}$$

where $K_{\pm}$ are kinetic coefficients, c_m is the equilibrium adatom density at the mth step, and P represents the step permeability.[29] $\hat{\boldsymbol{n}}$ is the unit vector normal to the step toward the step-down direction, $\hat{y}$ is the unit vector in the y direction, and $+(-)$ indicates the lower (upper) side terrace of the step. The first term in the right-hand side of Eq. (2) represents the number of solidified adatoms, which is proportional to the difference between the adatom density at the step and that at equilibrium. The difference of the kinetic coefficients, K_+ and K_-, represents the ES effect,[17–19] which also causes step instabilities in sublimation and in growth even if there is no drift. To focus on the effect of drift on the step instabilities, we neglect the ES effect and set $K_{\pm} = K$. The second term is the effect of step permeability and represents the adatom current between the neighboring terraces bypassing solidification. When the parameter $P = 0$, the adatom diffusion across the step vanishes and the step is called impermeable. When $P \neq 0$, the adatom diffusion

across the step occurs if $c|_+$ and $c|_-$ are different. If $P \to \infty$, then $c|_+ = c|_-$, and the step is called perfectly permeable.

If we neglect step-step interaction, the equilibrium adatom density is given by

$$c_m = c_{\rm eq}^0 + \frac{\Omega c_{\rm eq}^0 \tilde{\beta}}{k_B T} \kappa, \tag{3}$$

where $c_{\rm eq}^0$ is the equilibrium adatom density of an isolated step, Ω is the atomic area, $\tilde{\beta}$ is the step stiffness, and κ is the curvature of the step. The second term is the Gibbs-Thomson effect, which stabilizes the straight step.

By solving the diffusion equation (1) with the boundary conditions (2), the adatom density on the terraces are determined. The normal step velocity is given by

$$V_m = \Omega K(c|_- - c_m) + \Omega K(c|_+ - c_m)$$
$$= \Omega \hat{\boldsymbol{n}}[D_s(\nabla c|_+ - \nabla c|_-) - v\hat{\boldsymbol{y}}(c|_+ - c|_-)]. \tag{4}$$

The position of the mth step, $y = \zeta_m(x)$, is related to the normal velocity

$$\frac{\partial \zeta_m}{\partial t} = V_m \sqrt{1 + \left(\frac{\partial \zeta_m}{\partial x} \right)^2}. \tag{5}$$

III. LINEAR ANALYSIS

We study linear stability of an equidistant train of straight steps with the distance l for a fluctuation along the steps. We assume that the steps fluctuate in phase. With the wave number of the fluctuation q and the amplification rate ω_q, the position of the mth step is given by $\zeta_m = ml + \delta y e^{iqx + \omega_q t}$ and the linear dispersion relation is calculated as

$$\omega_q = \frac{2\Omega K v}{G_1} B_1 e^{vl/2D_s} D_s q^2 \sinh \alpha l \cosh \frac{vl}{2D_s} - \frac{\Omega K^2 v^2}{D_s G_1} B_1 e^{vl/2D_s} \sinh \alpha l \sinh \frac{vl}{2D_s} + \frac{2\Omega K^2 v}{G_1} B_1 e^{vl/2D_s} \alpha \left(\cosh \frac{vl}{2D_s} \cosh \alpha l - 1 \right)$$
$$- \frac{2D_s \Omega K \tilde{\Gamma}}{G_1} q^2 \left(D_s q^2 \sinh \alpha l - K\alpha \cosh \frac{vl}{2D_s} \right) - \frac{2\Omega K^2 \tilde{\Gamma}}{G_1} q^2 D_s \alpha \cosh \alpha l$$
$$+ P \frac{4\Omega K v}{G_1} B_1 e^{vl/2D_s} \alpha \cosh \frac{vl}{2D_s} \left(\cosh \alpha l - \cosh \frac{vl}{2D_s} \right) - P \frac{4\Omega K \tilde{\Gamma}}{G_1} q^2 D_s \alpha \left(\cosh \alpha l - \cosh \frac{vl}{2D_s} \right), \tag{6}$$

where

$$\alpha = \frac{1}{2} \sqrt{\frac{v^2}{D_s^2} + 4q^2}, \tag{7}$$

$$G_1 = 2D_s \alpha K \cosh \alpha l + D_s^2 q^2 \sinh \alpha l + K^2 \sinh \alpha l$$
$$+ 2P[D_s \alpha (\cosh \alpha l - \cosh vl/2D_s) + K \sinh \alpha l], \tag{8}$$

$$B_1 = \frac{2v c_{\rm eq}^0}{(K+2P)(e^{vl/D_s} - 1) + v(e^{vl/D_s} + 1)}, \tag{9}$$

$$\tilde{\Gamma} = c_{eq}^0 \frac{\Omega \tilde{\beta}}{k_B T}. \tag{10}$$

When the steps are perfectly permeable, $P \to \infty$, the step wandering does not occur. When the steps are not perfectly permeable $P \neq \infty$ and the step distance is small $vl/D_s \ll 1$, $Kl/D_s \ll 1$, and $Pl/D_s \ll 1$, for the long-wavelength fluctuation $\alpha l \ll 1$, Eq. (6) is expanded as

$$\frac{\omega_q}{\Omega c_{eq}^0} = vlq^2 - \frac{\Omega \tilde{\beta}}{k_B T} D_s l q^4 \tag{11}$$

up to the first order of l. The stability for the long-wavelength fluctuation is determined by the first term in Eq. (11). The critical drift velocity to induce the instability vanishes, in contrast to the case with evaporation.[12,13] With the step-down drift, $v > 0$, the step wandering always occurs for a long-wavelength fluctuation. The second term is the effect of the step stiffness, which is always negative and stabilizes the straight step. As a result of the competition between the first and the second terms, the wavelength of the most unstable mode is given by

$$\lambda_{\rm max} = 2\pi \sqrt{\frac{\Omega \tilde{\beta} D_s}{k_B T v}}. \tag{12}$$

The fluctuation with the wavelength $\lambda_{\rm max}$ appears dominantly in the initial stage of the wandering. In Ref. 8 Minoda and co-workers observed off-angle dependence of the wavelength of in-phase wandering. At 1100 °C, where steps are considered to be permeable,[29] the wavelength of in-phase wandering is independent of the step distance, in agreement with Eq. (12)

IV. NONLINEAR EVOLUTION EQUATION

When step wandering occurs, the amplitude of fluctuation increases rapidly. To predict the evolution of step position, we must take into account the nonlinear effects.

When the step wandering is caused by the ES effect in growth without evaporation, a nonlinear evolution equation is derived systematically by the multiscale expansion.[24,25] The behavior of its solution is different from a chaotic behavior represented by the KS equation.[15,16] When numerical simulation of the nonlinear evolution equation is carried out with a random initial fluctuation,[24] a cellular pattern with the wavelength of the most unstable mode appears and the amplitude of the step wandering increases with time as $t^{1/2}$.

Though an evolution equation, which the drift-induced step wandering obeys, may be derived systematically by the multiscale expansion, here we give a heuristic derivation of the nonlinear equation. For simplicity we assume that the steps are impermeable. We consider an equidistant train of steps whose normal direction is tilted from the y axis at an angle θ. We set the ξ axis along the step and the ζ axis in the step-down normal direction. In the ζ direction the step distance is given by $l_\perp = l\cos\theta$ and the drift velocity is $v_\perp = v\cos\theta$. The adatom density is given by $c(\zeta) = A_\zeta + B_\zeta \exp(v_\perp \zeta / D_s)$, where

$$A_\zeta = \frac{K(e^{v_\perp l_\perp / D_s} - 1)c_{\rm eq}^0}{K(e^{v_\perp l_\perp / D_s} - 1) + v_\perp (e^{v_\perp l_\perp / D_s} + 1)}, \tag{13}$$

$$B_\zeta = \frac{2v_\perp c_{\rm eq}^0}{K(e^{v_\perp l_\perp / D_s} - 1) + v_\perp (e^{v_\perp l_\perp / D_s} + 1)}. \tag{14}$$

The adatom current in the ζ direction $j_\zeta(\zeta)$ and in the ξ direction $j_\xi(\zeta)$ are given by

$$j_\zeta(\zeta) = -D_s \frac{dc}{d\zeta} + v_\perp c = v_\perp A_\zeta, \tag{15}$$

$$j_\xi(\zeta) = c(\zeta) v_\parallel, \tag{16}$$

where the drift velocity in the ξ direction is $v_\parallel = v\sin\theta$. The adatom current on the terrace in the x direction $J_x^{(1)}$ is given by

$$J_x^{(1)} = \int_0^l dy[-j_\zeta(\zeta)\sin\theta + j_\xi(\zeta)\cos\theta] = 2\frac{D_s v \sin\theta}{K} A_\zeta$$
$$\approx c_{\rm eq}^0 l v \sin\theta\cos\theta, \tag{17}$$

where we have assumed that the step distance is small $v_\perp l / D_s \ll 1$. Since there is no evaporation of adatoms, the change of adatom current is accompanied by solidification or melting of atoms at the steps. $J_x^{(1)}$ is the adatom current due to the tilt of the step. There is other type of adatom current $J_x^{(2)}$. It comes from the difference of the chemical potential μ along the step and given by

$$J_x^{(2)} = l\cos\theta D_s c_{\rm eq}^0 \frac{\partial}{\partial s}\left(\frac{\mu}{k_B T}\right), \tag{18}$$

where s is the arc length of the step and $\partial x/\partial s = \cos\theta$. When the step is curved, the Gibbs-Thomson effect determines the chemical potential $\mu = \Omega\tilde{\beta}\kappa$. Total adatom current in the x direction is given by $J_x = J_x^{(1)} + J_x^{(2)}$. By considering the mass conservation and the geometrical condition, the step velocity in the y direction $\partial\zeta/\partial t$ is given by

$$\frac{\partial\zeta}{\partial t} = -\Omega\frac{\partial J_x}{\partial x}$$
$$= -\frac{\partial}{\partial x}\left[\frac{\Omega c_{\rm eq}^0 l v \zeta_x}{1+\zeta_x^2} + \frac{\Omega^2 l\tilde{\Gamma}}{1+\zeta_x^2}\frac{\partial}{\partial x}\left(\frac{\zeta_{xx}}{(1+\zeta_x^2)^{3/2}}\right)\right], \tag{19}$$

where the subscript x of ζ represents the partial derivative with respect to x. The first term in Eq. (19) comes from Eq. (17), the effect of the step tilting, and the second term comes from Eq. (18). The linear dispersion obtained from Eq. (19) coincides with Eq. (11). Since we have the same equation as that of Ref. 24, we expect the same wandering behavior.

V. MONTE CARLO SIMULATION

A. Simulation model

To test the above analysis we carry out Monte Carlo simulation. The algorithm is the same as that in Ref. 13. We consider a (001) vicinal face of a cubic lattice with the lattice constant $a = 1$. x axis is parallel to the steps and y axis is in the step-down direction. The boundary conditions are periodic in the x direction and helical in the y direction. The steps are solid-on-solid steps, i.e., each step position is a single-valued function of x. Therefore, we denote the surface configuration by the y coordinate $y_m(i)$ of the mth step on the ith lattice site in the x direction. We forbid two-dimensional nucleation and formation of multiple hight steps. Then, solidification and melting occurs only at the steps.

We choose the time increment for each diffusion trial in such a way to make the surface diffusion coefficient $D_s = 1$. When the drift of adatoms is weak $va/D_s \ll 1$, the drift motion is taken into account as a biased diffusion. If the drift is in the y direction, the probability for hopping of an adatom from the site (i,j) to the site $(i,j\pm1)$ is $(1 \pm va/2k_BT)/4$ and to the site $(i\pm1,j)$ is 1/4, where $v(-v)$ corresponds to the step-down (step-up) drift.

In Sec. IV we assumed that steps are impermeable in deriving the nonlinear evolution equation, Eq. (19). Hereafter we also assume that steps are impermeable: adatoms cannot go to the neighboring terraces by diffusion. Adatoms can go to the neighboring terraces through successive solidification and melting. Solidification occurs when an adatom comes to an edge of a terrace. When solidifying adatom is at the lower side edge, the atom solidifies on site. When the adatom is at the upper side edge, it moves down to the lower side edge to solidify. However, it cannot solidify if the site is occupied by another adatom.

The probability for the solidification is given by

$$p_s = \left[1 + \exp\left(\frac{\Delta E_s + \Delta U - \phi}{k_B T}\right)\right]^{-1}. \tag{20}$$

The increment of step energy is given by $\Delta E_s = \epsilon \times$ (increment of the step perimeter length), where ϵ

is the nearest-neighbor bond energy. The bond energy ϵ is related to the step stiffness $\tilde{\beta}$ as

$$\frac{\tilde{\beta}}{k_B T} = \frac{(1-e^{-\epsilon/k_B T})^2}{2e^{-\epsilon/k_B T}}. \tag{21}$$

ΔU is the change of step interaction energy. For simplicity, the interaction between neighboring steps is approximated by an interaction within the same x coordinate in our simulation. We consider the elastic repulsive interaction, and we use $U = \Sigma_i A/|y_m(i) - y_n(i)|^2$ as the interaction potential between the mth step and $n(=m\pm1)$th step, where A is a parameter representing the strength of the interaction. ϕ is the chemical potential gain by solidification, which is related to the equilibrium adatom density $c_{\rm eq}^0$ as[13]

$$c_{\rm eq}^0 = \frac{1}{1+e^{\phi/k_B T}}. \tag{22}$$

Melting of solid atom occurs when there is no adatom on top of the atom. The probability of melting is given by

$$p_m = \left[1+\exp\left(\frac{\Delta E_s + \Delta U + \phi}{k_B T}\right)\right]^{-1}. \tag{23}$$

The melted atom stays there with probability 1/2 and moves onto the upper terrace with probability 1/2 if the site is not occupied by another adatom (otherwise, it cannot melt). The weak ES effect appears in the algorithm, but it does not give serious effects in our simulation (see Ref. 13 for details).

B. Simulation result

Figure 1 represents a vicinal face with drift of adatoms in a conserved system. We start simulation with equidistant train of straight steps. When the step is impermeable, both step wandering and step bunching occur with step-down drift.[12,13] Then we use strong step repulsion and suppress the step bunching. Figures 1(a) and 1(b) represent the vicinal face with step-down drift. In-phase step wandering occurs and a train of equidistant grooves appears. In the initial stage, short small grooves appear like nucleation [Fig. 1(a)]. With increasing time, the groove structure spreads the whole surface and the amplitude of grooves grows [Fig. 1(b)]. The pattern is similar to that of Ref. 8. When the amplitude of step wandering becomes comparable to the system size, the groove structure is broken and the step bunching starts. This extreme situation is unphysical because we neglected the interaction between steps in the x direction. With step-up drift, the vicinal face is stable and wandering does not occur [Fig. 1(c)]. The drift direction to induce the step wandering agrees with the linear analysis, and the formation of straight grooves agrees with the solution of the nonlinear evolution equation.[24,25]

Figure 2 represents the time evolution of the step width w defined by

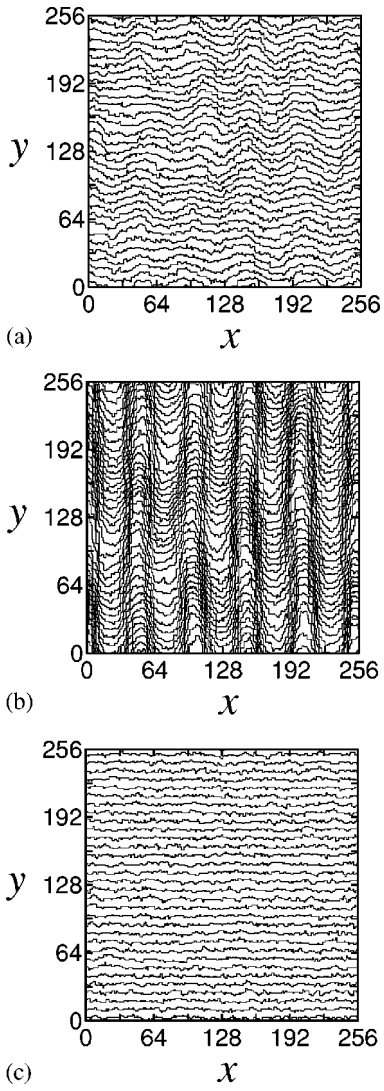

FIG. 1. Snapshots of surface without evaporation of adatoms (a) with the drift velocity $va/2k_BT = 0.2$ at $t \approx 1.3\times10^4$, (b) with the drift velocity $va/2k_BT = 0.2$ at $t \approx 3.6\times10^4$, and (c) with $va/2k_BT = -0.2$ at $t \approx 4.1\times10^5$. The system size is $L\times H = 256\times256$ and the step number is $N=32$. Other parameters are $\epsilon = 1.0$, $\phi = 1.5$, $A = 64.0$.

$$w(t) = \frac{1}{N}\sum_n \sqrt{\frac{1}{L}\sum_i \left(y_n(i,t) - \frac{1}{L}\sum_i y_n(i,t)\right)^2}, \tag{24}$$

where N is the number of steps, L is the system size in the x direction. To suppress the step bunching we use the strong step repulsion $A=64$. In the initial stage ($t \leqslant 2.0\times10^4$), the local wandering hardly occurs and the step width is small. Once the in-phase step wandering occurs, the width grows rapidly ($2.0\times10^4 \leqslant t \leqslant 2.0\times10^5$). With inceasing the width, the growth becomes slow ($t \geqslant 2.0\times10^5$) and obeys $w \sim t^\beta$ with $\beta \leqslant 1/2$. In the nonlinear analysis,[24,25] the step width increases with time as $t^{1/2}$, which agrees with our simulation. The power law growth continues until the step width becomes unphysically large and the validity of the model breaks down ($t > 6.0\times10^5$).

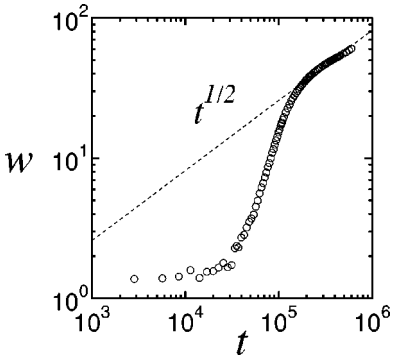

FIG. 2. Time evolution of the step width. The system size is $L \times H = 128 \times 128$ with step number $N = 16$. The drift velocity is $va/2k_BT = 0.1$. Other parameters are the same as in Fig. 1.

Figure 3 represents snapshots of the step wandering with a weaker step repulsion. Since the repulsion is weak, the step bunching, which is suppressed in Fig. 1, occurs. In the initial stage [Fig. 3(a)], local wandering occurs easily and many narrow grooves appear. The initiation of groove formation is inhomogeneous and looks like nucleation. This feature agrees with experiment.[8] The fluctuation of grooves induce the local step bunching, in contrast to the regular array of grooves with strong step repulsion (Fig. 1).

Figure 4 represents a snapshot of step wandering with evaporation of adatoms. Except the evaporation, the parameters are the same as those in Fig. 1. With the evaporation, correlation between steps becomes weak and fluctuation of grooves, which is suppressed in in Fig. 1, is large and break of grooves occurs. In Ref. 12 we studied time evolution of a vicinal face with impermeable steps under evaporation. When step bunching is suppressed by the strong step repulsion, in the initial stage, in-phase wandering occurs and parallel straight grooves appear. However, with increasing amplitude of wandering, the grooves fluctuate wildly and pinching out of grooves occurs. The result of Monte Carlo simulation (Fig. 4) exhibits this tendency.[12] In our simulation, the fluctuation of grooves is small because of weak evaporation. If the evaporation is strong, the fluctuation probably becomes large. However, since the wavelength of step wandering is longer, large scale simulation is necessary.

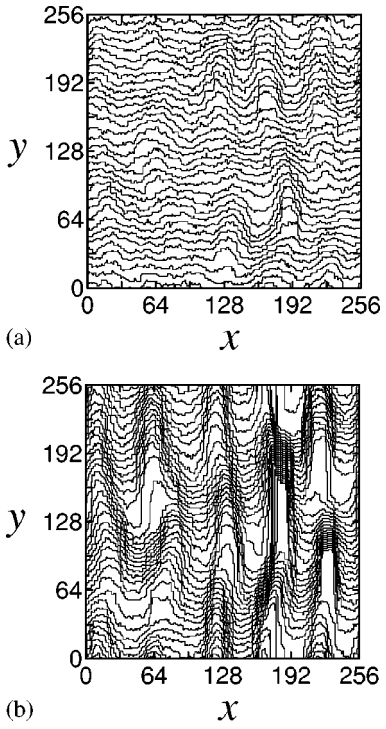

FIG. 3. Snapshots of step wandering with weak step repulsion $A = 40.0$ (a) in an early stage ($t \approx 1.8 \times 10^4$) and (b) in a late stage ($t \approx 3.2 \times 10^4$) Other parameters are the same as in Fig. 1.

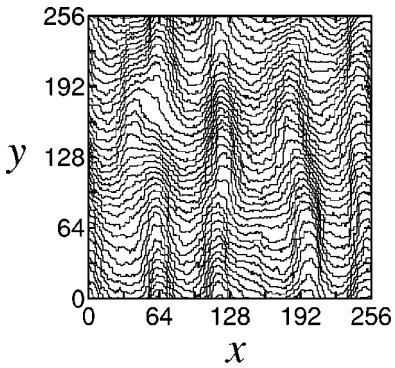

FIG. 4. Snapshots of step wandering with evaporation of adatoms at $t \approx 3.6 \times 10^4$. The adatom lifetime is $\tau = 1024$. The other parameters are the same as in Fig. 1.

VI. SUMMARY AND DISCUSSION

We studied step wandering induced by the drift of adatoms in a conserved system. In the linear analysis the in-phase step wandering occurs with the step-down drift if the step is not perfectly permeable, $P \neq \infty$. We derived the nonlinear evolution equation of in-phase wandering and showed that the type of the nonlinear equation is the same as that in Refs. 24 and 25. We carried out Monte Carlo simulation with impermeable steps. The results of simulation qualitatively agree with the theoretical analysis.

In the step instabilities of Si(111) vicinal faces, temperature is separated into at least three ranges: range I (830 °C–1000 °C), range II (1000 °C–1180 °C) and range III (1180 °C–1300 °C) according to the current direction to induce the step bunching.[1–8,30] Since the step bunching is observed in range I with step-down current and in range II with step-up current, the steps are considered to be impermeable in range I and permeable in range II.[1–8,30]

Recently, in-phase step wandering has been observed in range I and II when a crystal is sublimated with step-down current.[6] Since the drift of adatoms is parallel to the current,[5] the step wandering occurs irrespective of step permeability. Then our analysis agrees with the experiment although we have neglected the evaporation of adatoms in our analysis. In Si(111) the surface diffusion length is much larger than the step distance and the effect of evaporation is not crucial.

In a recent experiment[8] the time dependence of the amplitude of step wandering was measured in Si(111) vicinal face at 1100 °C. When the step wandering occurs, the amplitude of wandering increases rapidly in the initial stage. In the

late stage the growth rate decreases and saturation of the amplitude occurs. In the initial stage of the step wandering, the growth of amplitude is rapid in our simulation and the exponential growth is expected from the linear analysis. Both the linear analysis and the simulation agree with the experiment. In a late stage the amplitude increases as t^{β} with $\beta \sim 1/2$ in our simulation and in the nonlinear analysis. In the experiment, slow growth is observed before the saturation, though it is not clear whether the growth law is $t^{1/2}$. The saturation of amplitude observed in the experiment does not agree with the nonlinear analysis. Even if the x component of the step repulsion, which we neglected in our simulation, is taken into account, the saturation of the amplitude does not occur.[26] There are many discussions on this discrepancy of step fluctuation amplitude at the very late stages in the experiment and in the theoretical and numerical analyses, but the problem is not yet settled.

In our analysis, both the step wandering and the step bunching occur with the step-down drift if the steps are impermeable. In experiment,[31] in range I, where the steps are considered to be impermeable, the in-phase step wandering occurs on the surface with large off angles and the step bunching occurs with small off angles. We may interpret the result as follows. When the off angle is large, the step distance is small and the repulsion between steps is strong. Then the step bunching is suppressed and only the step wandering is observed. When the off angle is small, the step distance is large and the step repulsion is weak. Then the step bunching occurs.

When the steps are perfectly permeable, neither the step bunching nor the step wandering occurs in a conserved system, In range II, where the step is considered to be permeable, both the step wandering and the step bunching are observed.[4,6–8] To interpret the result, two scenarios are possible. One is that the steps are almost perfectly permeable and the evaporation of adatoms is not negligible. Then both instabilities are possible but evaporation is not strong enough to destroy the straight grooves produced by the step wandering. The other is the instabilities of the partially permeable steps, and evaporation is negligible. When the steps are partially permeable, as expected by Eq. (11), the in-phase step wandering occurs with the step-down drift despite the absence of evaporation. Considering the result for perfectly permeable steps,[13] we think that the step bunching may occur with step-up drift if the step permeability is large enough. To clarify the scenario, we are investigating the instabilities with a finite permeability.

ACKNOWLEDGMENTS

This work was performed as a part of the program ''Research for the Future'' of the Japanese Society for the Promotion of Science (JSPS) and supported by a Grant-in-Aid from JSPS. Y. S. and M. V. benefited from the interuniversity cooperative research program of the Institute for Materials Research, Tohoku University. Y.S. acknowledges the grant from JSPS.

*Electronic address: sato@cs.s.kanazawa-u.ac.jp

[1] A.V. Latyshev, A.L. Aseev, A.B. Krasilnikov, and S.I. Stenin, Surf. Sci. **213**, 157 (1989).
[2] Y. Homma, R. McCland, and H. Hibino, Jpn. J. Appl. Phys., Part 2 **29**, L2254 (1990).
[3] Y.-N. Yang, E.S. Fu, and E.D. Williams, Surf. Sci. **356**, 101 (1996).
[4] M. Degawa, H. Nishimura, Y. Tanishiro, H. Minoda, and K. Yagi, Jpn. J. Appl. Phys., Part 2 **38**, L308 (2000).
[5] M. Degawa, H. Minoda, Y. Tanishiro, and K. Yagi, Surf. Sci. **461**, L528 (2000).
[6] M. Degawa, H. Minoda, Y. Tanishiro, and K. Yagi, J. Phys. Soc. Jpn. **70**, 1026 (2001).
[7] M. Degawa, K. Thürmer, I. Morishima, H. Minoda, K. Yagi, and E.D. Williams, Surf. Sci. **487**, 171 (2001).
[8] H. Minoda, I. Morishima, M. Degawa, Y. Tanishiro, and K. Yagi, Surf. Sci. **493**, 487 (2001).
[9] S. Stoyanov, Jpn. J. Appl. Phys., Part 1 **30**, 1 (1991).
[10] M. Sato and M. Uwaha, J. Phys. Soc. Jpn. **65**, 2146 (1996).
[11] M. Sato, M. Uwaha, and Y. Saito, Phys. Rev. Lett. **80**, 4233 (1998).
[12] M. Sato and M. Uwaha, Phys. Rev. E **60**, 7120 (1999).
[13] M. Sato, M. Uwaha, and Y. Saito, Phys. Rev. B **62**, 8452 (2000).
[14] N. Suga, J. Kimpara, N.-J. Wu, H. Yasunaga, and A. Natori, Jpn. J. Appl. Phys., Part 1 **39**, 4412 (2000).
[15] Y. Kuramoto and T. Tsuzuki, Prog. Theor. Phys. **55**, 356 (1976).
[16] G.I. Sivashinsky, Acta Astronaut. **4**, 1177 (1977).
[17] R.L. Schwoebel and E.J. Shipsey, J. Appl. Phys. **37**, 3682 (1966).
[18] R.L. Schwoebel, J. Appl. Phys. **40**, 614 (1969).
[19] G. Ehrlich and F.G. Hudda, J. Chem. Phys. **44**, 1039 (1966).
[20] G.S. Bales and A. Zangwill, Phys. Rev. B **41**, 5500 (1990).
[21] I. Bena, C. Misbah, and A. Valance, Phys. Rev. B **47**, 7408 (1993).
[22] Y. Saito and M. Uwaha, Phys. Rev. B **49**, 10 677 (1994).
[23] O. Pierre-Louis and C. Misbah, Phys. Rev. B **58**, 2276 (1998).
[24] O. Pierre-Louis, C. Misbah, Y. Saito, J. Krug, and P. Politi, Phys. Rev. Lett. **80**, 4221 (1998).
[25] F. Gillet, O. Pierre-Louis, and C. Misbah, Eur. Phys. J. B **18**, 519 (2000).
[26] S. Paulin, F. Gillet, and O. Pierre-Louis, and C. Misbah, Phys. Rev. Lett. **86**, 5538 (2001)
[27] S. Stoyanov, Jpn. J. Appl. Phys., Part 1 **30**, 1 (1991).
[28] In Ref. 13, the boundary conditions Eqs. (2.4), (2.5), (4,1), and (4.2) are with misprints. The correct boundary conditions are given by Eq. (2) with $P=0$.
[29] S. Stoyanov, Surf. Sci. **416**, 200 (1998).
[30] Existence of range IV is also reported in Ref. 1 and Y. Homma and N. Aizawa, Phys. Rev. B **62**, 8323 (2000).
[31] In Ref. 6 the interpretation is different from ours. The authors interpreted their experiment by an off-angle dependence of the step permeability.