Failure Analysis of Geomaterials under Dynamic Loading Conditions

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Introduction

The analysis of the failure of mechanical, structural and geotechnical systems is one of the main challenging tasks in the engineering computations. The challenge arises especially in the modeling of the localized failures in these systems. The failure is often characterized by the concentration of strains in narrow bands, usually referred to as the shear band. And the propagation of these bands can lead to the failure of the soil and structural foundation. In saturated porous media, the localized failure is strongly influenced by the coupling between the solid skeleton deformation and the pore fluid flow.

The material point method (MPM) is one of the latest developments in particle–in–cell (PIC) methods. MPM takes the combination of Lagrangian and Eulerian methods and has the advantages such as no need to search for contact surfaces, no distorted grid in the simulation of large strain problems. In this work, MPM is extended into the analysis of dynamic problems in saturated porous media in the framework of Biot's theory, especially the strain localization phenomenon under dynamic loading conditions.

Governing equations and their discretization

For the simulation of failure phenomena of geomaterials like saturated porous media, the u-U form governing equations are adopted:

Solid momentum equilibrium equations

$$\underbrace{\left(\sigma''_{ij,j}-(\alpha-n)p\delta_{ij}\right)_{,j}}_{\text{internal force}}+\underbrace{\rho'_s g_i+F_i}_{\text{externnal force}}\underbrace{+n^2K^{-1}\gamma_f(\dot{U}_i-\dot{u}_i)}_{\text{damping force}}=\underbrace{\rho'_s\ddot{u}_i}_{\text{inertial force}} \tag{1a}$$

Fluid momentum equilibrium equations

$$\underbrace{-np_{,i}}_{\text{internal force}}+\underbrace{\rho'_f g_i}_{\text{external force}}\underbrace{-n^2K^{-1}\gamma_f(\dot{U}_i-\dot{u}_i)}_{\text{damping force}}=\underbrace{\rho'_f\ddot{U}_i}_{\text{inertial force}} \tag{1b}$$

Mass conservation equation

$$Q((\alpha-n)\dot{u}_{i,i}+n\dot{U}_{i,i})+\dot{p}=0 \tag{1c}$$

where σ''_{ij} is the effective stress, p is the pore fluid pressure, u_i and U_i are the displacements of solid skeleton and pore fluid respectively, α is Biot constant, n is porosity, K is the permeability, K_s and K_f are the bulk moduli of solid skeleton and pore fluid, $Q=[n/K_s+(\alpha-n)/K_f]^{-1}$. γ_f is the bulk density of the fluid, $\rho'_f=n\rho_f$ and $\rho'_s=(1-n)\rho_s$, in which ρ_s and ρ_f are the density of the solid phase and the fluid phase. g_i and F_i are the body and external forces, respectively.

Because saturated porous medium is composed of solid skeleton and pore fluid, two sets of material points are invoked to characterize solid skeleton and pore fluid respectively (see Figure 1(a)). In each subdomain Ω_P, there is a pair of material points, which denote solid and fluid phase respectively. Each pair of material points is defined by the position vectors $\mathbf{X}^s_P$, $\mathbf{X}^f_P$, which are the function of time t and $\mathbf{X}^s_P(t^0)=\mathbf{X}^f_P(t^0)$ at initial time t^0, the fixed mass M^s_P, M^f_P, where $M^s_P=\rho'_sV_P$, $M^f_P=\rho'_fV_P$, V_P is the volume of subdomain P. With time passing, subdomain Ω^0_P deforms into subdomain Ω_P, and the pair of material points locates in the different position, $\mathbf{X}^s_P(t^k)$ and $\mathbf{X}^f_P(t^k)$, as shown in Figure 1(b). In Figure 1, the background element is the typical computational grid. The scripts s and f mean that the variables are relative to solid skeleton and pore fluid.

Based on the virtual work principle and considering the particular form of density, it has

$$\rho'_s(\boldsymbol{X},t^k)=\sum_{P=1}^{N_P}M^s_P\delta(\boldsymbol{X}-\boldsymbol{X}^s_P(t^k)),\quad \rho'_f(\boldsymbol{X},t^k)=\sum_{P=1}^{N_P}M^f_P\delta(\boldsymbol{X}-\boldsymbol{X}^f_P(t^k)) \tag{2}$$

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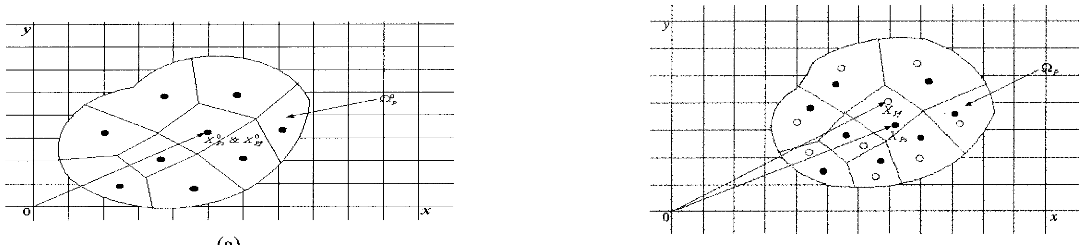
where N_P is the number of subdomains or the number of material point pairs. It is easy to obtain the discrete form of governing equations (1) as following

$$M_I^s \ddot{u}_{iI} = (F_{iI})^s_{int} + (F_{iI})^s_{ext} + (F_{iI})^s_{damp} \qquad I = 1,\ldots,N_n \tag{3a}$$

$$M_I^f \ddot{U}_{iI} = (F_{iI})^f_{int} + (F_{iI})^f_{ext} + (F_{iI})^f_{damp} \qquad I = 1,\ldots,N_n \tag{3b}$$

where M_I^s, M_I^f are the mass matrices, $\boldsymbol{F}_{int}$, $\boldsymbol{F}_{ext}$, $\boldsymbol{F}_{damp}$ represent the vectors of internal force, external force and artificial damping force respectively. Scripts P and I mean that the variables are defined on the material points and the computational grid nodes respectively. Owing to the limitation of the paper length, the derivative process is ignored.



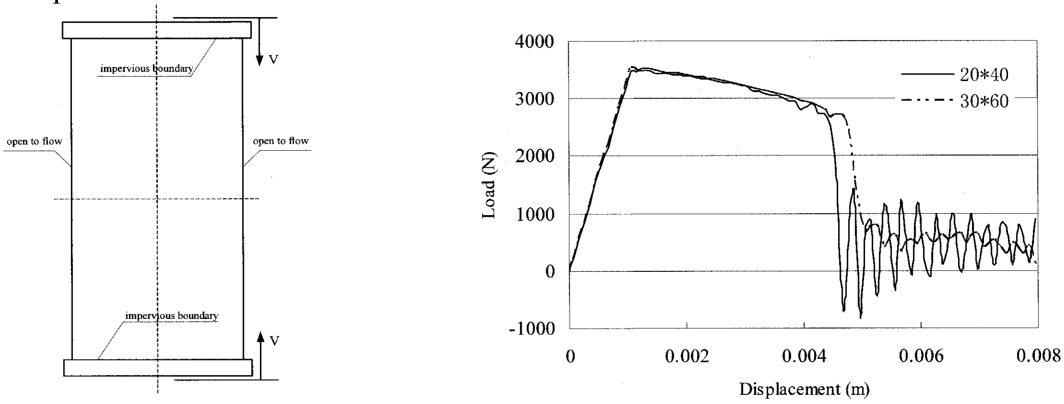
(a) (b)

Figure 1. Sketch of the typical computational grid and material element (a) original configuration (b) configuration at time t^k (solid dot denotes material points of solid skeleton, hollow one denotes pore fluid)

An explicit algorithm as mentioned in [1] for MPM is applied. A Druck-Prager model and a strong decohesion model [2] of the solid skeleton are introduced to simulate the elastoplastic behaviour and predict the evolution of localization. The two models above are coupled through the bifurcation analysis. To get the pore pressure, a linear relation is established as indicated in Equation (1c).

Numerical Example

The numerical example concerns a homogeneous rectangle panel of saturated porous media in plane strain problem. The panel is subjected to uniaxial compression between two rigid plates applied by displacement control as shown in Figure 2(a). Owing to the symmetric condition, as the gravity is neglected, the computation is only carried out a quarter of the panel. The load-displacement curve in Figure 2(b) shows the load history applied on the top of specimen and the softening behavior versus the vertical displacement.



(a) (b)

Figure 2. (a) Compression problem of a rectangle panel in the plane strain (b) Load-displacement curves obtained from different meshes

References

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