

A study of the semi-dynamic traffic assignment using the sensitivity analysis

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Dissertation abstract

A study of the semi-dynamic traffic
assignment using the sensitivity analysis

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Abstract: In traffic assignment problems, static traffic assignment (STA) considering the only one-time period and the average traffic volume may not be able to reflect the time-varying phenomenon in many urban areas. In contrast, the dynamic traffic assignment (DTA) aims to describe the changes in traffic conditions in detail that requires large computational cost and detailed origin-destination (OD) data. As an effective alternative, the semi-DTA is practically applicable when the available OD data is coarse, and small computational cost is considered. This study will develop a semi-DTA stochastic user equilibrium (SUE) model based on the logit model. Besides, we propose an efficient sensitivity analysis method for solving the equilibrium problem of the model and an efficient calculation procedure containing only link and node variables for saving memory and computational time. Our approach is from the link-based STOCH3 algorithm, “STOCH3-efficient route” definition, and Depth-First-Search (DFS) algorithm. Also, for solving the route overlapping and route length problems of the multinomial logit model, we extend the sensitivity analysis method and the semi-dynamic model with cross-nested logit and q -generalized logit approaches. Numerical experiments and applications to Kanazawa City road network depict that the proposed method and model are effective for real network applications.

1. Introduction

The daily STA model widely used in practice defined on a relatively long time-of-day period and the traffic condition has been described by the average or steady-state travel time on a link as a function of the volume of traffic on the link. However, STA could not adequately represent traffic flow, especially time-varying congestion phenomena, because traffic conditions change significantly with time of day. The DTA model is more preferable to represent significantly changing traffic conditions with the time of day. However, DTA models require a large calculation load and detailed OD matrix data. Considering limited computational resources and coarse OD data, a semi-DTA model has been studied in which both STA and DTA have been considered. In this approach, a day is split into several periods in which each period has the same interval of length L , static network equilibrium is reached in each period and the flow propagation between periods is considered. In one period, all vehicles departing from their origin but some of them do not reach their destinations. The flow on a link that cannot exit the link (called “residual flow”) in a given period is propagated to the next period; this effect is known as flow propagation. Depending on the formulation of residual flow, there are three different semi-DTA models including the demand modification approach (Miyagi *et al.*¹⁾), the link flow approach (Fujita *et al.*²⁾), and the queue approach (Akamatsu *et al.*³⁾). While the ordinary STA technique and algorithm can be applied, the semi-DTA model is also an alternative for describing the daily traffic dynamics of large-scale networks with a small calculation load. Once the result of the SUE is more reasonable, realistic, and general⁴⁾, developing semi-DTA models with SUE is crucial to contribute to advanced traffic assignment models. In the techniques and algorithms of the SUE traffic assignment model, a typical logit-based model has been commonly used because of its practicality and applicability. If the logit-based route choice is assumed in the semi-DTA model and the link travel time is a function of its inflow, the double-looped fixed-point problem needs to be solved for the existence and uniqueness of semi-dynamic SUE flow. Nevertheless, the huge cost of computing is a big problem, especially when applying the semi-DTA model to a large traffic network. In our study, the above problem will be solved by the approximation method using sensitivity analysis and the accuracy of the approximate results as well as the calculation efficiency will be shown.

Besides, if the residual flow is eliminated and propagated to the next period, the number of OD pairs will escalate dramatically and therefore the number of routes will increase greatly and the computer may be overloaded. Thus, we will also propose a link-based and node-based variable approach for saving storage. For avoiding route enumeration, Dial’s algorithm⁵⁾ may be used. Nevertheless, the definition of route choice set in Dial’s algorithm varies depending on iterative methods and the unique solution is not guaranteed. To deal with this problem, the definition of the STOCH3-efficient route based on fixed “reference travel costs” such as the free-flow travel cost pattern and STOCH3 algorithm⁶⁾ are essential in this study. In the process of researching and applying the STOCH3 algorithm, a tree search algorithm called the DFS algorithm⁷⁾ can be regarded as an effective integration tool with the STOCH3 algorithm.

Also, the multinomial logit-based (MNL) traffic assignment used in this model has also been criticized because of independent distribution and identical variance assumptions. The use of the MNL model brings the stable results even when we change the network structure such as merging some parts of links without changing the total travel times that causes the route overlapping problem or shortening the link travel times without changing the travel different between two links that cause the route length problem. Various models have been proposed to address those two weaknesses, of which the cross-nested logit (CNL) model⁸⁾ and the q -generalized logit model⁹⁾ are essential proposals with many advantages. Thus, this semi-DTA model needs to be extended for applying to the various extensions of the logit model, such as the CNL model and the q -generalized logit model. The use of CNL and q -generalized logit models will help to consider the effectiveness of the projects where the MNL model cannot take into account different aspects.

This dissertation consists of 6 chapters. While Chapter 1 introduces the background of the research and research purpose, Chapter 2 will locate the position of this research after organizing these past studies. Chapter 3 will propose a sensitivity analysis method for the logit-based SUE traffic assignment model. The concept and formulation of a semi-DTA will be described in Chapter 4. Besides, the sensitivity analysis method for solving the SUE of this model will be proposed in the link-based STOCH3 algorithm. The sensitivity analysis method and the semi-DTA model for the CNL model and the q -generalized logit model would be conducted in Chapter 5. Small virtual and Kanazawa city

road networks will also be considered as case studies to illustrate the efficient calculation and validity of the proposed models. Chapter 6 will summarize the research and consider future works. The following sections will show the main results of this dissertation.

2. Sensitivity analysis method for logit-based Stochastic User Equilibrium traffic assignment

In this study, we consider the traffic network with N nodes, A links, and W OD pairs, where $N = \{i, j, \dots\}$ is the set of nodes, $A = \{ij, gh, \dots\}$ is the set of links and $W = \{rs, \dots\}$ is the set of OD pairs. The solution of the SUE problem is iterative in the sense of a fixed-point problem and the method of successive averages (MSA) can be used to achieve an equilibrium state. The fixed-point problem with the route-traffic flow is considered as

$$f_k^{rs} = Q_0^{rs} \frac{\exp(-\theta c_k^{rs}(\mathbf{f}))}{\sum_{k \in K^{rs}} \exp(-\theta c_k^{rs}(\mathbf{f}))}, \quad (1)$$

where f_k^{rs} , and c_k^{rs} denote the traffic volume and the required travel time on the route k between a certain OD pair rs ; Q_0^{rs} is the fixed travel demand of an OD pair; and K^{rs} is the set of routes connecting each OD rs , θ is the logit parameter. \mathbf{f} is the vector of route flows.

The logit-based model, however, when applied to large facing difficulties is that the enumeration of possible alternatives routes is implausible. To avoid route enumeration in a logit stochastic loading, the logit-based stochastic network loading algorithms obviating route enumeration (Dial⁵) have been widely used and the well-known MSA algorithm has been used for achieving SUE solution. However, travel time may be changed from one iteration to the next because of flow-dependent assumption and these definitions are unstable (Leurent⁶). Based on Dial's algorithm, Leurent proposed the STOCH3 algorithm and the definition of the STOCH3-efficient route. He also introduced a definition of a maximum "elongation ratio" for a link concerning the origin, h_{ij}^r . If we assume that C_{rn}^0 denotes the reference shortest generalized travel time from origin node r to node n , based on the free-flow travel time $\{t_{ij}^0\}$, the "reasonable enough" of route k can be expressed through efficient links that satisfy the following condition:

$$(1 + h_{ij}^r)(C_{rj}^0 - C_{ri}^0) \geq t_{ij}^0 \quad \forall ij \in A. \quad (2)$$

Using the STOCH3-efficient route, Leurent performed the effectiveness of the STOCH3 logit model with MSA and stated that better behavioral models and computationally useful would be attained with stable route identification. In the process of applying the STOCH3 algorithm, we discovered that the DFS⁷ can be utilized to list all implicit efficient routes. DFS is an algorithm for traversing tree data structures. When the traffic network is represented as adjacent links instead of matrices and lists types, the algorithm starts at the origin node, explores along with each link, and ends at the destination node before backtracking. The algorithm is handled with a recursive technique. For example, if the following network consists of efficient links in **Figure 1** and the network is saved as adjacent links instead of matrices, a DFS is the following order: $r1, 12, 2s, 25, 56, 6s, 14, 45, 56, 6s$.

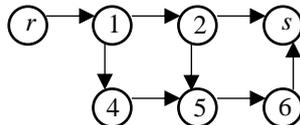


Figure 1 A virtual network with efficient links

Based on the DFS algorithm with the STOCH3-efficient route definition, a new algorithm to present stochastic network loading is proposed. Instead of relying on STOCH loading, the DFS algorithm is adopted. Instead of using well-known "forward" and "backward" techniques⁶ with the requirement of sorting $\{C_{rn}^0\}$ in order of increasing access time from r , the DFS algorithm exploited only link-based and node-based variables. The DFS algorithm with the MSA method is proposed to achieve the SUE state.

When we have a result of SUE, the problem of sensitivity analysis is the calculation of the changes of link flow x_{ij} caused by small changes in travel demand parameter (ξ^{rs}) and free-flow travel time parameter (ζ_{ij}), i.e. computing the partial derivatives. Sensitivity analysis for logit-based SUE traffic assignment is proposed by Ying and Miyagi¹⁰ based on Dial's algorithm. However, the weakness of Dial's algorithm with the instability of the reasonable route definition makes the sensitivity analysis method difficult to keep the stability. Besides, the mathematical formulations of the proposed sensitivity analysis method confused practice users with the definitions of "true" derivative and "apparent" derivative. We try to apply the STOCH3 algorithm with the proposed formulation and calculation process. Nevertheless, some equations can not be computed in the case a link has no flow. Hence, a new approach with a new formulation and calculation procedure based on the DFS algorithm and STOCH3 algorithm is proposed.

From logit-based traffic assignment, the link flow is calculated as:

$$x_{ij} = \sum_{rs \in W} Q^{rs}(\xi) \frac{\sum_{k \in K^{rs}} \delta_{ij,k}^{rs} \exp(-\theta c_k^{rs}(\mathbf{t}(\mathbf{x}, \zeta)))}{\sum_{k \in K^{rs}} \exp(-\theta c_k^{rs}(\mathbf{t}(\mathbf{x}, \zeta)))}, \quad (3)$$

here $\delta_{ij,k}^{rs}$ is the link-route incidence variable, which is 1 if the link ij exists on the route k , and 0 if it does not exist. \mathbf{x} , $\boldsymbol{\zeta}$, and $\boldsymbol{\xi}$ are the vectors of link traffic flow, free-flow travel time parameter, and travel demand parameter. $Q^{rs}(\cdot)$ is the function of the travel demand between OD pair rs . $c_k^{rs}(\cdot)$ is the function of the route travel time between OD pair rs . $\mathbf{t}(\cdot)$ is the vector-valued function of the link travel time.

Let b_{ij} denotes the right-hand side of **Equation (3)** (b_{ij} is a function of two variables \mathbf{x} and $\boldsymbol{\zeta}$). The following vector-valued equation is also defined as a function with three vector-valued variables $\boldsymbol{\xi}$, \mathbf{x} , and $\boldsymbol{\zeta}$:

$$\mathbf{d}(\boldsymbol{\xi}, \mathbf{x}, \boldsymbol{\zeta}) = \mathbf{x} - \mathbf{b}(\mathbf{Q}(\boldsymbol{\xi}), \mathbf{t}(\mathbf{x}, \boldsymbol{\zeta})). \quad (4)$$

From the general formula for derivative of the implicit function, we have:

$$\nabla_{\boldsymbol{\zeta}} \mathbf{x} = -\nabla_{\mathbf{x}} \mathbf{d}^{-1} \nabla_{\boldsymbol{\zeta}} \mathbf{d} = (\mathbf{I} - \nabla_{\mathbf{t}} \mathbf{b} \nabla_{\mathbf{x}} \mathbf{t})^{-1} (\nabla_{\mathbf{t}} \mathbf{b} \nabla_{\boldsymbol{\zeta}} \mathbf{t}), \quad (5)$$

$$\nabla_{\boldsymbol{\xi}} \mathbf{x} = -\nabla_{\mathbf{x}} \mathbf{d}^{-1} \nabla_{\boldsymbol{\xi}} \mathbf{d} = -(\mathbf{I} - \nabla_{\mathbf{t}} \mathbf{b} \nabla_{\mathbf{x}} \mathbf{t})^{-1} \nabla_{\mathbf{Q}} \mathbf{b}, \quad (6)$$

where $\nabla_{\boldsymbol{\zeta}} \mathbf{x}$ and $\nabla_{\boldsymbol{\xi}} \mathbf{x}$ are the derivative matrices of link traffic flow with respect to ζ_{ij} and ξ^{rs} , respectively. \mathbf{I} is a unit matrix. $\nabla_{\mathbf{x}} \mathbf{t}$ and $\nabla_{\boldsymbol{\zeta}} \mathbf{t}$ are the diagonal matrix of derivatives of t_{ij} with respect to x_{ij} and ζ_{ij} , respectively. Since $\nabla_{\mathbf{x}} \mathbf{t}$ and $\nabla_{\boldsymbol{\zeta}} \mathbf{t}$ are easily computed from implicit BPR function, the difficulties in here are the calculations of $\nabla_{\mathbf{t}} \mathbf{b}$ and $\nabla_{\mathbf{Q}} \mathbf{b}$ in which each element of these matrices is calculated by

$$\begin{aligned} \frac{\partial b_{ij}}{\partial t_{gh}} &= \sum_{rs \in W} \left(Q_0^{rs} \frac{\left[-\theta \sum_{k \in K^{rs}} \delta_{ij,k}^{rs} \delta_{gh,k}^{rs} \exp(-\theta c_k^{rs}) \right] \left[\sum_{k \in K^{rs}} \delta_{ij,k}^{rs} \exp(-\theta c_k^{rs}) \right] \left[-\theta \sum_{k \in K^{rs}} \delta_{gh,k}^{rs} \exp(-\theta c_k^{rs}) \right]}{\sum_{k \in K^{rs}} \exp(-\theta c_k^{rs}) \left(\sum_{k \in K^{rs}} \exp(-\theta c_k^{rs}) \right)^2} \right) \\ &= \theta \sum_{rs \in W} \left(\left[-x_{ij,gh}^{rs} + \frac{x_{ij}^{rs} x_{gh}^{rs}}{Q_0^{rs}} \right] \right), \end{aligned} \quad (7)$$

and,

$$\frac{\partial b_{ij}}{\partial Q^{rs}} = \frac{\sum_{k \in K^{rs}} \delta_{ij,k}^{rs} \exp(-\theta c_k^{rs})}{\sum_{k \in K^{rs}} \exp(-\theta c_k^{rs})} = \frac{x_{ij}^{rs}}{Q^{rs}}. \quad (8)$$

where x_{ij}^{rs} is the reference flow on link ij of OD pair rs , $x_{ij \rightarrow gh}^{rs}$ is the number of travellers from r to s choosing some routes which consist of both links ij and gh satisfying link ij is travelled prior to link gh . And $x_{ij,gh}^{rs}$ denotes the number of travellers who use both link ij and link gh without attention of arrangement in which $x_{ij,gh}^{rs} = x_{ij \rightarrow gh}^{rs} + x_{gh \rightarrow ij}^{rs}$, if $ij \neq gh$ and $x_{ij,gh}^{rs} = x_{ij}^{rs}$ otherwise.

We only need variables including link-based variables and node-based variables that reduce storage cost. At SUE state, when the travel time of all links are fixed, x_{ij}^{rs} can be calculated by using the DFS algorithm or STOCH3 algorithm once because in these, x_{ij} could be computed for each OD pair and $x_{ij} = \sum_{rs \in W} x_{ij}^{rs}$. After x_{ij}^{rs} is easily computed with the link-based approach, we need to compute $x_{ij,gh}^{rs}$ through x_{ij}^{rs} and $x_{ij \rightarrow gh}^{rs}$. We try to apply the technique of Ying and Miyagi¹⁰⁾ in which $x_{ij \rightarrow gh}^{rs} = x_{ij}^{rs} p_{gh}^{js}$ and p_{gh}^{js} is computed for all links gh by running STOCH3 algorithm once from j to s with $Q_0^{js} = 1$. However, this technique causes inaccuracy problem because the set of effective routes is different depending on the origin node r and p_{gh}^{js} will also be different in each OD pair rs . We have proved that $x_{ij \rightarrow gh}^{rs}$ could be computed by running STOCH3 algorithm once from j to s for assigning x_{ij}^{rs} to all links gh by keeping the order of reference shortest generalized travel time from origin r . Thus, STOCH3 algorithm could be used to calculate $x_{ij \rightarrow gh}^{rs}$. Consequently, we also wield the DFS method to compute $x_{ij \rightarrow gh}^{rs}$ solving the problem of storage space, accuracy as well as calculation time. As highlighted above, by running the DFS algorithm from origin node r to destination node s with recursive technique, each time we reach the destination s , we will have the information of all the links that are queried, i.e. all links belong to a route k . So, we can calculate $x_{ij \rightarrow gh}^{rs}$ from the route information brought from the DFS algorithm. After calculating x_{ij}^{rs} and $x_{ij,gh}^{rs}$, we easily calculate $\nabla_{\mathbf{t}} \mathbf{b}$, $\nabla_{\mathbf{Q}} \mathbf{b}$, $\nabla_{\boldsymbol{\zeta}} \mathbf{x}$ and $\nabla_{\boldsymbol{\xi}} \mathbf{x}$.

Consequently, Kanazawa road network is used as the case study to show the efficiency of the proposed method. This network is composed of 272 nodes and 964 links. In this study, we handle the data from the previous survey. The OD demand data with 383 OD pairs of the morning peak from 6:00 to 7:00 AM is used. The parameters of the BPR function are $\alpha = 1.0$, $\beta = 2.0$ and the logit-based parameter is $\theta = 1$. We can not use the method of Ying and Miyagi¹⁰⁾ using Dial's algorithm and STOCH3 algorithm because Dial's algorithm does not converge to the link flows equilibrium solution and some links have no flows in the equilibrium solution of STOCH3 algorithm. Thus, we compare three calculation processes for calculating the new sensitivity analysis formulation in which the first uses STOCH3 algorithm and calculate $x_{ij \rightarrow gh}^{rs} = x_{ij}^{rs} p_{gh}^{js}$ in the technique of Ying and Miyagi (A), the second uses

STOCH3 algorithm and calculate $x_{ij \rightarrow gh}^{rs}$ with the new approach of the STOCH3 algorithm (B), the third uses the DFS algorithm (C). The STOCH3-efficient-route definition is used in all three processes where h_{ij}^r is set to 1.5 for every link of the network. The programs are coded by Fortran.90 programming language and ran on a personal computer with an Intel Core i7-8700 CPU of 3.20 GHz, 16GB RAM, and Windows 10 Home. For comparison, we have assumed 4 cases: Case 1 assumes all parameters are equal to 0, case 2 sets the ζ parameter to 0.1 at all links and the ξ parameter is equal to 0, case 3 assumes that the ζ parameter is 0 and the ξ parameter is 5 in all OD pairs, case 4 assumes that the ζ and ξ parameters change simultaneously with 0.1 and 5, respectively. To achieve equilibrium state, either the algorithm based on DFS or the STOCH3 algorithm could be used to achieve actual equilibrated calculation link flows. Compared to the STOCH3 algorithm, the DFS-based algorithm brought the same link flow results and had the lower calculation time than the STOCH3 algorithm. In the approximated approach, after having the results at SUE state of case 1, the sensitivity analysis could be applied to have the approximated results in the three remaining cases. The calculation time of (B) and (C) is only half of the calculation time of (A). If not mention the inverse matrix calculation time, the difference is greater when only mentioning the time for calculating $\nabla_t \mathbf{b}$, $\nabla_Q \mathbf{b}$ in which the proposed methods will save at least 70%. This demonstrates the time-efficient calculation of the proposed method. Also, the difference in accuracy is compared. While the error of comparing between approximated link flows and actual calculation link flows of (A) is quite large, it of the proposed method shows high accuracy. The percentage error (%RMS) of (B) and (C) are the same and do not exceed 0.5% if only one parameter is changed, either ζ or ξ . When changing both parameters at the same time, the approximated results of (B) and (C) are also quite exact. Thus, the accurate calculation of (B) and (C) is equal. The results suggest that the proposed method is an accurate method to estimate the SUE STOCH3 model, robust to different parameter settings, and highly applicable to complex networks.

3. Semi-dynamic stochastic user equilibrium traffic assignment with flow propagation based on the sensitivity analysis method

To detail this model, a small virtual network in **Figure 2** with 4 nodes, 3 links, and 1 OD pair is used.

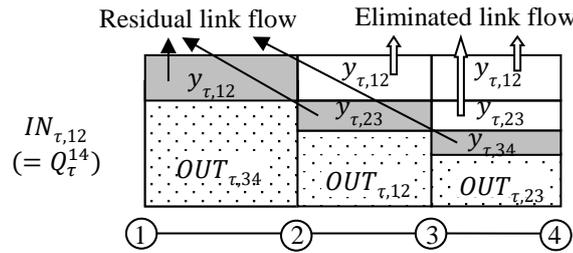


Figure 2 A semi-DTA model with flow propagation

The description of the semi-DTA model with flow propagation as follows: OD demand of the network is from node 1 to node 4 in period τ , denoted by Q_{τ}^{14} ; The inflow to link 12 in period τ is $IN_{\tau,12} = Q_{\tau}^{14}$; According to assumption 3, the travel time on link 12 in period τ is the function of its inflow $t_{\tau,12} = t_{\tau,12}(IN_{\tau,12})$; Some of the inflow to link 12 in period τ cannot exit this link and becomes the residual flow on this link in this period τ , $y_{\tau,12}$, which is eliminated from link 23 and 34 in period τ . Besides, $y_{\tau,12}$ is propagated to the next period as travel demand from node 2 to node 4 in period $\tau + 1$. The value of $y_{\tau,12}$ depends on $t_{\tau,12}$ and this part does not travel on the subsequent links, link 23 and link 34 in period τ . The outflow from link 12 in period τ is $OUT_{\tau,12} = IN_{\tau,12} - y_{\tau,12}$. This outflow exit link 12, reaches link 23 in period τ and become the inflow to link 23 in period τ . This inflow will experience travel time $t_{\tau,23} = t_{\tau,23}(IN_{\tau,12} - y_{\tau,12})$. Some of the inflow to link 23 in period τ become the residual flow on this link in this period, $y_{\tau,23}$, that is eliminated from link 34 in period τ and considered as travel demand from node 3 to node 4 in period $\tau + 1$. The outflow from link 23 in period τ is $OUT_{\tau,23} = IN_{\tau,12} - y_{\tau,12} - y_{\tau,23}$. This outflow exits link 23, becomes inflow to link 34 in period τ and experiences travel time $t_{\tau,34} = t_{\tau,34}(IN_{\tau,12} - y_{\tau,12} - y_{\tau,23})$. Similarly, the residual flow on link 34 in period τ , $y_{\tau,34}$, is calculated. Because the destination is reached, this residual flow arrives immediately at the destination without consideration in the next period. Besides, the outflow from link 34 in period τ is $OUT_{\tau,34} = IN_{\tau,12} - y_{\tau,12} - y_{\tau,23} - y_{\tau,34}$. This outflow is the flow that can reach the destination in period τ .

To formulate the model, the logit model (Sheffi⁴) is used. The reference route flow in period τ of route k between OD pair rs is calculated by:

$$f_{\tau,k}^{rs} = Q_{\tau}^{rs} p_{\tau,k}^{rs} = Q_{\tau}^{rs} \frac{\exp(-\theta c_{\tau,k}^{rs})}{\sum_{k \in K^{rs}} \exp(-\theta c_{\tau,k}^{rs})} \quad (9)$$

where all variables are calculated for each period τ .

As mentioned above, flow propagation is considered in the semi-DTA model. If the users continuously enter a link

in a stable rate and the residual flow still travels on the link at the end of the considering period, the residual flow on each link is calculated by:

$$y_{\tau,k,ij}^{rs} = \frac{f_{\tau,k}^{rs} \delta_{\tau,ij,k}^{rs}}{L} t_{\tau,ij}, \quad (10)$$

where $y_{\tau,k,ij}^{rs}$ denotes residual flow on link ij of route k between OD pair rs in period τ .

Because residual flow on a link cannot exit this link in the present period, it is also removed from the subsequent links. The total eliminated link flows is denoted by \mathbf{s}_τ . The link flow after the elimination of residual flow will be called ‘‘adjusted link flow’’ which is also the inflow to link ij in period τ . It is assumed that travel time on a link is a continuous and non-decreasing function of its flow. The problem is that the link flow is also the function of travel costs. Thus, the link travel costs themselves are needed to determine the link travel costs. The fixed-point problem of link travel cost in period τ is given by

$$\mathbf{t}_\tau = \mathbf{t}_\tau(\Delta \mathbf{f}_\tau - \mathbf{s}_\tau[\mathbf{f}_\tau, \mathbf{t}_\tau]), \quad (11)$$

where \mathbf{f}_τ is the vector of all reference route flows in period τ , Δ is the link-route incidence matrix and $\mathbf{t}_\tau(\cdot)$ denotes the vector-valued function of link travel cost in period τ .

From the above equations, the reference route flow in each time is calculated by

$$\mathbf{f}_\tau = \mathbf{Q}_\tau \mathbf{p}_\tau \left(\Delta_\tau^T \mathbf{t}_\tau(\Delta_\tau \mathbf{f}_\tau - \mathbf{s}_\tau[\mathbf{f}_\tau, \mathbf{t}_\tau]) \right), \quad (12)$$

where \mathbf{Q}_τ is the diagonal matrix of all travel demands, \mathbf{p}_τ is the vector of all route probabilities and T is the transpose.

The reference flow represents the amount of the flow which departs from the origin node in period τ . To achieve equilibrium in this model, the double-looped fixed-point problem must be solved. One is the fixed-point problem for travel costs and the other is the problem for route flows. As a consequence, a large amount of calculation time will be wasted for equilibrium solution, and applying to the real road network is very difficult.

In the route-based approach, the approximate route flow influenced by the elimination of residual flow in each period (\mathbf{f}_τ) is estimated through route flow and propagated flow at the static state (\mathbf{f}_0 and \mathbf{s}_0) according to first-order Taylor expansion. The details could be referred to as Itagaki et al.¹¹⁾. By cause of the existing inverse route-based matrix in the formulation, nevertheless, applying the proposed approach to very large-scale networks is arduous. To update this method, increase the applicability and reduce the computational cost, hence, we will propose a link-based approach with STOCH3 algorithm (Leurent⁶⁾).

From the viewpoint of practical use, we will first consider how to obtain a unique approximate solution and prevent route enumeration to achieve efficient calculation in our research, called the first link-based algorithm. At static SUE model, there is no eliminated flow ($\mathbf{s}_0 = \mathbf{0}$), we can calculate reference link flow by running a link-based STOCH3 algorithm. At semi-DTA model, when eliminated flow \mathbf{s}_τ is assumed and calculated by using the results of the static SUE model, the reference link flow could be approximated by calculating $\nabla_{\mathbf{s}} \mathbf{x}_0$ (the partial derivatives of reference link traffic flow with respect to eliminated flow at the static SUE model). Since \mathbf{s}_τ and $\nabla_{\mathbf{s}} \mathbf{x}_0$ are completely calculated, the reference link flow influenced by the elimination of residual flow is given by (according to first-order Taylor expansion)

$$\mathbf{x}_\tau = \mathbf{x}_0 + \nabla_{\mathbf{s}} \mathbf{x}_0 \mathbf{s}_\tau, \quad (13)$$

where \mathbf{x}_τ is reference link flows in period τ of the semi-DTA model; \mathbf{x}_0 is the link flows of the STA model that can be calculated by running the link-based STOCH3 algorithm. We can calculate $\nabla_{\mathbf{s}} \mathbf{x}_0$ from the fixed-point problem with the link flow at the static SUE model in the case $\mathbf{s}_0 = \mathbf{0}$

$$x_{0,ij} = \sum_{rs \in W_\tau} Q_\tau^{rs} \frac{\sum_{k \in K^{rs}} \delta_{ij,k}^{rs} \exp(-\theta c_{0,k}^{rs}(\mathbf{t}_0(\mathbf{x}_0, \mathbf{s}_0)))}{\sum_{k \in K^{rs}} \exp(-\theta c_{0,k}^{rs}(\mathbf{t}_0(\mathbf{x}_0, \mathbf{s}_0)))} \quad (14)$$

Denoting the right-hand side of the above equation by $g_{0,ij}$. With the same approach as in the previous section, it is stated that:

$$\nabla_{\mathbf{s}} \mathbf{x}_0 = (\mathbf{I} - \nabla_{\mathbf{t}} \mathbf{g}_0 \nabla_{\mathbf{x}} \mathbf{t}_0)^{-1} (\nabla_{\mathbf{t}} \mathbf{g}_0 \nabla_{\mathbf{s}} \mathbf{t}_0), \quad (15)$$

where $\nabla_{\mathbf{x}} \mathbf{t}_0$ and $\nabla_{\mathbf{s}} \mathbf{t}_0$ are calculated from the link travel time function (e.g. BPR function). And, the entries of $\nabla_{\mathbf{t}} \mathbf{g}_0$ are calculated in the same way as $\nabla_{\mathbf{t}} \mathbf{b}$ in Section 2. Also, the DFS-based algorithm and STOCH3 algorithm could be used to calculate $\nabla_{\mathbf{t}} \mathbf{g}_0$.

The next problem is how to calculate $y_{\tau,ij}^{rs}$ (the residual flow on link ij of OD pair rs in period τ that is added to the travel demand from node j to original destination node s in period $\tau + 1$) and $s_{\tau,ij}$ (the eliminated flow of link ij in period τ) with only link-based and node-based variables. Because these variables are assumed and calculated by using the results of the static SUE model, we have:

$$y_{\tau,ij}^{rs} = \sum_{k \in K^{rs}} \frac{f_{0,k}^{rs} \delta_{ij,k}^{rs}}{L} t_{0,ij} = \frac{t_{0,ij}}{L} \sum_{k \in K^{rs}} f_{0,k}^{rs} \delta_{ij,k}^{rs} = \frac{t_{0,ij}}{L} x_{0,ij}^{rs}, \quad (16)$$

$$s_{\tau,gh} = \sum_{rs \in W_\tau} \sum_{ij \in A} \frac{x_{0,ij \rightarrow gh}^{rs}}{L} t_{0,ij}. \quad (17)$$

where $x_{0,ij}^{rs}$ and $x_{0,ij \rightarrow gh}^{rs}$ could be computed by running the DFS loading procedure or the STOCH3 algorithm once for each OD pair based on the link travel times, $\{t_{0,ij}\}$ (see section 2). Since \mathbf{s}_τ and $\nabla_s \mathbf{x}_0$ are calculated at static SUE, the reference link flow can be approximated.

However, because the residual link flows and eliminated link flows in the first link-based algorithm are assumed and calculated at the static SUE, overestimation may occur and the equilibrium solution of original semi-DTA with flow propagation may deviate from the approximate model. In the second link-based algorithm, the residual link flows and eliminated link flows are not fixed and be considered as unknown variables. A fixed-point problem of the eliminated link flow needs to be resolved.

Because only \mathbf{x}_0 and $\nabla_s \mathbf{x}_0$ are fixed, the reference link flow is the function of \mathbf{s}_τ as follows:

$$\mathbf{x}_\tau = \mathbf{x}_\tau(\mathbf{x}_0 + \nabla_s \mathbf{x}_0 \mathbf{s}_\tau), \quad (18)$$

where eliminated link flow \mathbf{s}_τ is considered as an unknown variable and changes according to the change of link travel time. $\mathbf{x}_\tau(\cdot)$ is the vector-valued function of the reference link flow in period τ .

If we use the above approximation, the link travel cost are also the functions of eliminated link flows as follows:

$$\mathbf{t}_\tau = \mathbf{t}_\tau(\mathbf{z}_\tau(\mathbf{x}_\tau(\mathbf{s}_\tau) - \mathbf{s}_\tau)), \quad (19)$$

where $\mathbf{z}_\tau(\cdot)$ is the vector-valued function of the adjusted link flow in period τ .

The residual link flow ($y_{\tau,ij}^{rs}$) and the eliminated link flow ($s_{\tau,gh}$) are calculated from $\{x_{\tau,ij}^{rs}\}$ that is calculated by using STOCH3 algorithm or the DFS algorithm based \mathbf{t}_τ , thus

$$y_{\tau,ij}^{rs} = \frac{x_{\tau,ij}^{rs}(\mathbf{t}_\tau)}{L} t_{\tau,ij}, \quad (20)$$

$$s_{\tau,gh} = \sum_{rs \in W_\tau} \sum_{ij \in A} \frac{x_{\tau,ij \rightarrow gh}^{rs}(\mathbf{t}_\tau)}{L} t_{\tau,ij} \quad (21)$$

The fixed-point problem of eliminated link flow is given as

$$s_{\tau,gh} = \sum_{rs \in W_\tau} \sum_{ij \in A} \frac{x_{\tau,ij \rightarrow gh}^{rs}(\mathbf{t}_\tau(\mathbf{s}_\tau))}{L} t_{\tau,ij}(\mathbf{s}_\tau) \quad (22)$$

For solving the fixed-point problem, we can use the well-known MSA or some of the other efficient methods such as the self-regulated averaging method (SRA) (see Liu et al. ¹²).

As a result, the algorithm with link-based and node-based approach in this study includes the following four steps. This process is computed in each period.

Step 1: Calculate static SUE by using the STOCH3 algorithm or DFS loading procedure with the MSA method in the first period.

Step 2: Based on the results at SUE state, calculating $\nabla_s \mathbf{x}_0$ and the initial solution of eliminated flow \mathbf{s}_τ .

Step 3: Solving the fixed-point problem of the eliminated link flows.

Step 4: Calculate the residual link flow for flow propagation and deliver the propagated flow to the next period.. The propagated flow of a link is considered as the demand between the end node of that link and the original destination. And, the above procedure is repeated for the next period.

To examine the applicability of the proposed model to the Kanazawa road network. There are three periods including period 1 (6:00-7:00 AM), 2 (7:00-8:00 AM), 3 (8:00-9:00 AM) with the previously personal trip survey OD demand data. The OD pairs are classified according to the departure time. The proposed semi-dynamic traffic assignment model is calculated for each period so it is possible to calculate for more periods. Here we only apply to 3 periods because of limited data input.

Firstly, we will compare two approximated approaches: the proposed first-order link-based STOCH3 approach and the route-based approach. For comparison, the set of route choices is identical. The BPR function of the travel time of the car uses assumed parameters ($\alpha = 1, \beta = 2$) and $\theta = 0.2$. Because the number of OD pairs and routes in periods 2 and 3 are too big that makes the route-based approach cannot run, we will use the data in period 1 for comparison. While the error of comparing two approaches is approximately 0 that suggests the same results, the link-based approach demonstrates the time-efficient calculation. In this comparison, using the proposed method will reduce the computational time above 97%. This effect is increased as the amount of route choice set increases. The differences between the link-based approach and route-based approach are in the calculation time and applicability. Either the extremely large alternative routes connecting an OD pair in a real network or the increase of OD pairs due to the propagation of residual flow can cause overloaded calculation in a route-based approach. The link-based approach shows independence on the number of implicit route-set. The results suggest that the proposed link-based method is highly applicable.

Secondly, to compare the real effectiveness of the two link-based STOCH3 approaches when applied in practice, the next two link-based approaches will be applied and compared to the true model. Because the route-based approach of the true model must solve the double-looped fixed-point problem, the calculation time for reaching equilibrium in period 1 (with convergence error $\sigma = 10^{-3}$ that is used to assess the convergence level of the fixed-point problem) and in period 2 exceed 1 week, and the route-based approach cannot run in period 2. Thus, we only conduct tests in period 1 (with $\sigma \in [10^{-1}, 10^{-2}]$) to compare different models. The computational time of the proposed method is

very small when compared to the calculational time of the route-based approach of the original model. With σ set to 0.1, the calculation time of the original model is 11.36 minutes, while the calculation time of the proposed method is only 0.5 minutes. The proposed method will save at least 96% in this comparison. Especially, this effect will increase as the error decreases. This demonstrates the time-efficient calculation of the proposed method. The first algorithm and the second algorithm also show the same calculation time in this test. The efficiency of calculation time is very high in both approaches. When comparing the results of the two approximation models to the original model in the results of adjusted link flows, residual link flows and eliminated link flows, the approximate models provide exact results in which the second link-based algorithm creates more exact results the first one.

Consequently, we apply both link-based algorithms to the Kanazawa road network for all three time periods. The total calculation time of both approaches is quite small, the first approach is about 2 minutes 46 seconds and the second approach is about 3 minutes. The calculation time of the second algorithm is larger than the calculation time of the first approach but the difference is insignificant. The second algorithm must solve one more problem of finding a fixed point. Moreover, the comparisons between observed link flows and calculated link flows of the static SUE model and two link-based algorithms of the semi-DTA SUE model are conducted. Because the static model does not address the movement of congested traffic from one period to another, it cannot accurately reflect the level of traffic congestion at each period. For example, the static model produces underestimation results in period 3. Meanwhile, the semi-DTA model considering the flow propagation between periods solved by two algorithms gives closer results to observed link flows. Moreover, both algorithms in the semi-DTA model give guaranteed results in correlation coefficients. We also compare the calculation results of the semi-DTA model with the first link-based algorithm, the second link-based algorithm, and the static SUE model. The results indicate that the semi-DTA model can replace the static model and have a close relationship with the static model. Also, the calculation results of the second approach are better than the first one in terms of correlation coefficients in all 3 periods. In period 2 of which the congestion level is highest, the results of the first approach tend to be more underestimated because the calculation of residual flow is excluded. Meanwhile, the second approach gives results closer to reality. However, there are a tendency of slight overestimation in period 1 and the opposite trend in the remaining periods. The reason is that periods 2 and 3 are congestion periods and residual flow that is dependent on the travel time is computed a lot in these periods. Besides, the absence of a suitable OD matrix for the semi-DTA model is also considered as a reason. Future research needs to focus on solving these issues.

Because residual flow between periods plays an important role in the proposed semi-DTA model, consequently, **Figure 3** will show the residual flows on the links between periods of the second link-based algorithm. Because the residual flow from period 1 is very small in all links (less than 25 pcu), **Figure 3** only shows the residual flow from period 2 and period 3. The residual flows in period 2 are larger than those in period 3. As can be seen, the more congested the road is, the more the residual flows increase. From **Figure 3** we also see that the congestion trend is towards the city center.

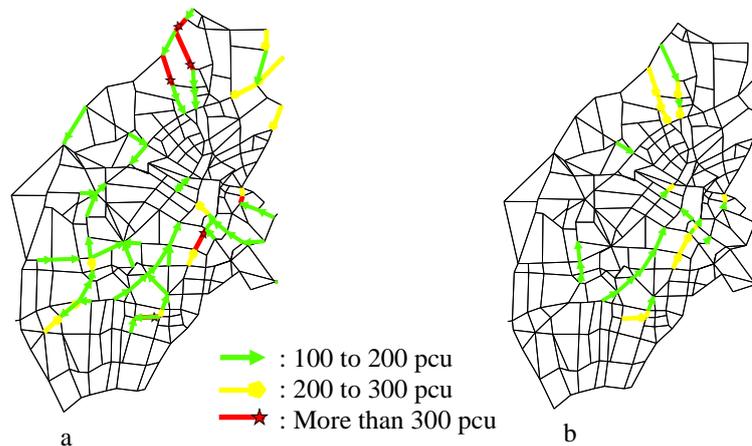


Figure 3 Residual link flows between periods of Kanazawa road network. (a) From period 2. (b) From period 3.

4. Sensitivity analysis method and semi-dynamic stochastic user equilibrium traffic assignment based on the sensitivity analysis method for cross-nested logit model and q-generalized logit model

a. Sensitivity analysis method for the cross-nested logit model

Based on the multivariate extreme value (MEV) theory of McFadden, Vovsha, and Bekhor⁸⁾ proposed the CNL model for traffic assignment in which a nest is considered as a link and each alternative (route) could belong to more than one nest depending on the relationship between the link and the route. This model can solve the overlapping problem in route choice and the SUE CNL problem could be figured out by using the MSA method to resolve the fixed-point problem of route flows. However, because of route enumeration and two-level tree structure, the calculation time is an obstacle when applying the link-nested logit model to a large-scale network. Thus, our research

uses the ‘‘STOCH3-efficient route’’ definition to create an implicit route choice set and proposes the calculation procedure using the DFS algorithm that only utilizes link and node variables saving memory. To achieve equilibrium, the self-regulated averaging (SRA) method (Liu et al. ¹²) is used.

Consequently, here we consider the sensitivity analysis method of the change of the link flow according to the change of toll pricing with π_{ij} is toll fare on link ij and φ is the value of time. Other cases are calculated similarly.

The problem of the CNL model presents the following fixed-point problem:

$$x_{ij} = \sum_{rs \in W} Q_0^{rs} \frac{\sum_{k \in K_{ij}^{rs}} \sum_{mn \in A} \left(\left\{ \alpha_{mn,k}^{rs} \exp(-\theta c_k^{rs}(\mathbf{t}(\mathbf{x}, \boldsymbol{\pi}))) \right\}^{\frac{1}{\mu}} \left[\sum_{p \in K_{mn}^{rs}} \left\{ \alpha_{mn,p}^{rs} \exp(-\theta c_p^{rs}(\mathbf{t}(\mathbf{x}, \boldsymbol{\pi}))) \right\}^{\frac{1}{\mu}} \right]^{\mu-1} \right)}{\sum_{mn \in A} \left[\sum_{k \in K_{mn}^{rs}} \left\{ \alpha_{mn,k}^{rs} \exp(-\theta c_k^{rs}(\mathbf{t}(\mathbf{x}, \boldsymbol{\pi}))) \right\}^{\frac{1}{\mu}} \right]^{\mu}}, \quad (21)$$

where K_{ij}^{rs} is the set of all routes (alternatives) included in the nest (link) ij of an OD pair rs , $\alpha_{mn,k}^{rs}$ is the degree of similarity between routes measured by the ratio of link length and route length, μ is the degree of nesting, $0 < \mu \leq 1$.

Denoting the right-hand side of the above equation by h_{ij} . Using the sensitivity analysis method proposed in Section 2, we have

$$\nabla_{\boldsymbol{\pi}} \mathbf{x} = (\mathbf{I} - \nabla_{\mathbf{t}} \mathbf{h} \nabla_{\mathbf{x}} \mathbf{t})^{-1} (\nabla_{\mathbf{t}} \mathbf{h} \nabla_{\boldsymbol{\pi}} \mathbf{t}), \quad (22)$$

where $\nabla_{\mathbf{x}} \mathbf{t}$ and $\nabla_{\boldsymbol{\pi}} \mathbf{t}$ are calculated from the link travel time function. And, entries of $\nabla_{\mathbf{t}} \mathbf{h}$ are calculated by the calculation of $\frac{\partial h_{ij}}{\partial t_{ij}}$ and $\frac{\partial h_{ij}}{\partial t_{uv}}$ for each OD pair rs .

We proposed the procedure algorithm to calculate $\nabla_{\boldsymbol{\pi}} \mathbf{x}$ by running the DFS algorithm twice. And, the proposed sensitivity analysis method is applied to a small network. We compared the results of route-based SUE CNL model and the proposed sensitivity analysis method for the CNL mode). The former is recomputed at each toll fare and the latter is calculated from the sensitivity analysis method. The results of link flow may be the same between two methods with the small value of two indicators *RMSE* and *%RMS*. However, with the sensitivity analysis method, we will save the computational time because we do not need to recompute the traffic assignment. In this case, the total saving time is up to 88.6% and we also save the memory because we do not need to explicit route choice set. In the real road network with a huge amount of OD pairs and routes connecting each OD pair, the effectiveness of the SA method will exponentially increase.

b. Sensitivity analysis method for the q -generalized logit model

For solving the route length problem, Nakayama et al. ⁹) proposed the integrated framework of q -generalization with q -analysis regarding the MEV distribution. The q -generalized exponential is given as follows:

$$\exp_q(x) = (1 + [1 - q]x)^{\frac{1}{1-q}}. \quad (23)$$

And, the solution to the SUE problem is iterative in the sense of a fixed-point problem. We also applied the STOCH3 efficient route definition and DFS algorithm to achieve efficient calculation with implicit route-set.

The fixed-point problem with the link-traffic flow can be derived as:

$$x_{ij} = \sum_{rs \in W} Q^{rs}(\boldsymbol{\xi}) \frac{\sum_{k \in K_{ij}^{rs}} \delta_{ij,k}^{rs} \exp_{2-q}(-\theta c_k^{rs}(\mathbf{t}(\mathbf{x}, \boldsymbol{\zeta})))}{\sum_{p \in K_{ij}^{rs}} \exp_{2-q}(-\theta c_p^{rs}(\mathbf{t}(\mathbf{x}, \boldsymbol{\zeta})))} \quad (24)$$

The difficulties in the calculation with the implicit route-set including c_k^{rs} and $\delta_{ij,k}^{rs}$ are solved by the DFS algorithm. With the same sensitivity analysis approach, the calculation process of $\nabla_{\boldsymbol{\zeta}} \mathbf{x}$ and $\nabla_{\boldsymbol{\xi}} \mathbf{x}$ are proposed. Because the DFS algorithm could be used to calculate route-travel time and depict the link-route relationship, it is used to calculate these matrices.

The performance of the proposed method is shown in the application to the Kanazawa road network with 383 OD pairs, $\alpha = 1.0$, $\beta = 2.0$, $\theta = 1$, $h_{ij}^r = 1.5$ and $q = 0.5$. When we change the parameter $\boldsymbol{\zeta}$ for all links in the range [1%, 5%] of the free-flow travel time \mathbf{t}^0 and the parameter $\boldsymbol{\xi}$ for all OD pairs in the range [1%, 5%] of the fixed travel demand \mathbf{Q}^0 , we need to calculate 10 the route-based approach with the MSA method for the SUE q -generalized traffic assignment times which causes computational time-consuming. Meanwhile, by using the sensitivity analysis method, we can easily calculate the approximation results with the saving time of the calculation. To assess the accuracy of the proposed sensitivity analysis method for q -generalized traffic assignment, the *RMSE* and *%RMS* indicators are utilized. The proposed method performs well with small values of both the *RMSE* and *%RMS* indicators. Also, the calculation time of the two methods is compared. In this case study, we only considered 10 scenarios and the total cumulative calculation time for 11 cases is the sum of the calculation time of 10 scenarios and the case of parameters $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ equals 0. The proposed method reduces the computational time by up to 97%. This proves the time-efficient calculation of the proposed method.

c. Semi-dynamic stochastic user equilibrium traffic assignment based on the extensions of the sensitivity analysis method for cross-nested logit model and q -generalized logit model

The semi-dynamic SUE traffic assignment model based on the logit model is proposed in Section 3. The CNL model and the q -generalized logit model are the extensions of the logit model, so they can be applied for the semi-DTA model as the fixed-point problem of the reference route flow in each period. Because solving this problem requires the computational time and memory, the method of sensitivity analysis using only link-based and node-based variables is proposed to achieve approximated results of the semi-DTA model. The two approaches that can be applied are the same as those in Section 3. As mentioned, the second approach gives better approximation results. So, the focus here will be on the second approach. In this semi-DTA approach, we calculate the equilibrium state for each period τ . In each period, we can calculate the static SUE using the CNL model and the q -generalized logit model with the DFS algorithm and SRA method. From these results and solving a fixed-point problem with eliminated link flow in each period τ , we can compute the semi-DTA model. As stated in Section 3, there are two difficulties including the calculation of $\nabla_s \mathbf{x}_0$ at the static SUE and $x_{\tau,ij}^{rs}$, $x_{\tau,ij \rightarrow gh}^{rs}$ variables. While $\nabla_s \mathbf{x}_0$ can be computed by using the sensitivity analysis method proposed in the previous sections, $x_{\tau,ij}^{rs}$ could be computed through link-based variables in both the CNL and the q -generalized logit models by using the DFS loading procedure with STOCH3-efficient-route definition. After the transformation, we also show that $x_{\tau,ij \rightarrow gh}^{rs}$ could be decomposed into link-based variables and calculated by using the DFS loading procedure with STOCH3-efficient-route definition. Therefore, the semi-DTA model for the CNL model and q -generalized logit model are completely computed with the DFS algorithm.

Consequently, three proposed semi-DTA models are applied to Kanazawa road network with the same network and input OD data shown in Section 3. The BPR parameters are assumed as $\alpha = 1$ and $\beta = 2$, the logit parameter $\theta = 0.2$, $q = 0.5$, $\mu = 0.5$ and $h_{ij}^r = 1.5$. There are three time periods: 1 (6:00-7:00 AM), 2 (7:00-8:00 AM), 3 (8:00-9:00 AM). Because the original model cannot run in periods 2 and 3, we also conduct calculational time and accuracy tests in period 1 (with convergence error $\sigma \in [10^{-1}, 10^{-2}]$). With σ set to 0.1, the calculation time is 17.6 minutes and 15 minutes in the original semi-DTA CNL model and q -generalized logit model, respectively. Meanwhile, the calculation time of the approximate models is only 48 seconds and 26 seconds. Thus, the proposed method will save at least 97% in the calculation time. Besides, the comparisons between the original models and the models using the sensitivity analysis also show that the sensitivity analysis approaches provide exact results. We also compare the results between the observed link flows, calculated link flows using the static SUE, and the semi-DTA SUE models based on the CNL and q -generalized logit. The results show that the semi-DTA models provide better results than the static models and have close relationships with the static models.

The comparison of the computational time of the three models using the same method is shown in **Figure 4**. The semi-DTA q -generalized logit model has the lowest computation time but it is not much less than the MNL model. The difference in computational time of the two semi-DTA models with q -generalized logit and MNL mainly lies in solving the fixed point problem with eliminated link flows. In all three periods, the q -generalized logit model requires fewer loops to solve this problem. Moreover, the semi-DTA CNL model takes a lot of time to calculate comparing with the other two models. The reason for this is because the CNL model requires a complex computational structure with twice runs of the DFS algorithm for each OD pair. However, it is only high when compared with the other two models. The total calculation time of the semi-DTA CNL for all three time periods is about 15.75 minutes which still reflects the computation time quite well for practical application.

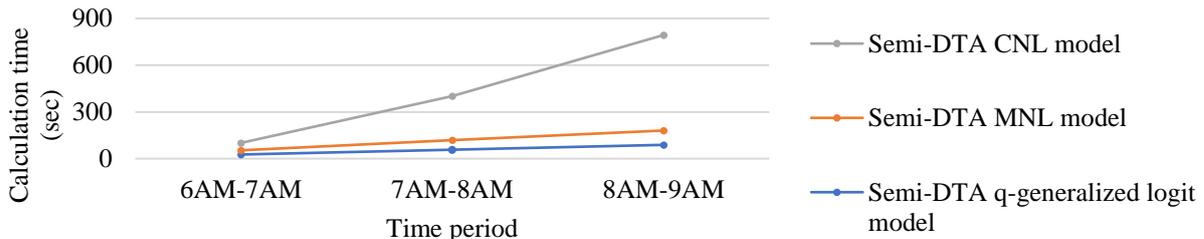


Figure 4 Calculational time comparison between three proposed models in each period of applying to Kanazawa road network

The comparisons between observed link traffic flows and calculated link traffic flows brought by three models are also performed in **Figure 5**. All three models have the results of ensuring reliability with a high correlation coefficient from 0.825 to 0.899. Interestingly, all three periods experience the same trend of the best-applied model in which the CNL model gives the best results in the correlation coefficient indicator. In these applications, all three models use the DFS algorithm with the same STOCH3-efficient routes definition. Moreover, the semi-DTA q -generalized logit model is closer to the semi-DTA MNL model when the q parameter is closer to 1 and the semi-DTA CNL model is closer to the semi-DTA MNL model when the μ is closer to 1. Based on two criteria of correlation coefficients and calculation time, all three semi-DTA models have high applicability. The semi-DTA MNL model still shows the simplicity of the calculation and the variety of applications of calculation methods such as the STOCH3 algorithm and the DFS algorithm. However, the estimation of optimal parameters for application is still a problem.

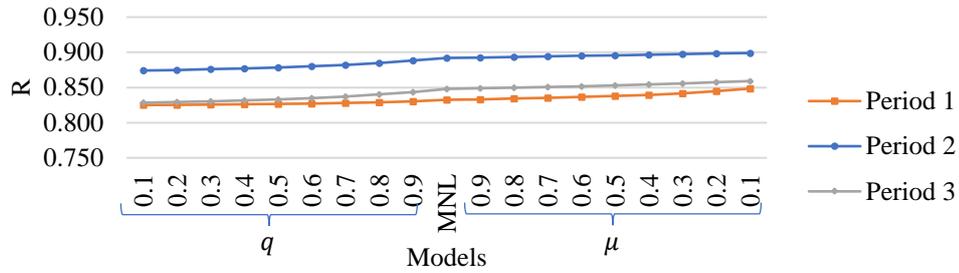


Figure 5 Correlation coefficients of different cases in the application to Kanazawa road network in three periods

5. Conclusion and future work

This study attained three main objectives: Developing sensitivity analysis method for static logit-based traffic assignment SUE model to achieve high accuracy approximation of traffic assignment results, as well as reduce the calculation time of traffic assignment model; Proposing a semi-DTA model to replace STA model in which the application of sensitivity analysis method is the main method to achieve the unique results of the proposed model and increase the applicability of the proposed model with link-based and node-based approaches; Expanding the sensitivity analysis method and the semi-DTA models for the CNL and the q -generalized logit. The proposed method and models are applied to virtual networks and Kanazawa metropolitan area network to analyze the applicability.

Besides, the results of the application also open some issues that need to be further addressed in future studies. The issues are summarized as follows: Applying the proposed method and model to more complex networks; Considering more about the assumption of residual flow; The construction of more suitable OD matrix for the semi-DTA model is another problem; Some parameters need to be estimated such as q in the q -generalized logit model and μ in the CNL model; An integrated model needs to be proposed in which both the overlapping and the route length problems are resolved and a combination of mode choice and route choice model is considered.

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学位論文審査報告書（甲）

1. 学位論文題目（外国語の場合は和訳を付けること。）

A study of the semi-dynamic traffic assignment using the sensitivity analysis

（感度分析を用いた準動的交通量配分に関する研究）

2. 論文提出者 (1) 所 属 環境デザイン学 専攻

(2) 氏 名 フイ ティエン チム
BUI TIEN THIEM

3. 審査結果の要旨（600～650字）

提出された学位論文（以下、本学位論文）に対して、審査委員の全員が口頭試問等を行うとともに、論文の内容について精査した。また、令和2年7月27日に行われた口頭発表後に、審査委員会を開催し、協議の結果、以下のように判定した。本学位論文では、実用的な利用を踏まえて、時々刻々と変化する道路ネットワークの混雑等の状態を準動的に扱う交通量配分モデル（確率的利用者均衡モデル）に関して研究を行っている。大規模な道路ネットワークでは、交通量配分の均衡計算コストは非常に大きくなるため、感度分析を適用した近似を試み、そのモデル構築および計算アルゴリズムの開発を行っている。そして、その妥当性などについて、数値計算や金沢道路ネットワークへの適用から、良好であることを示している。さらに、準動的な確率的利用者均衡における経路選択において、従前の多項ロジットモデルだけでなく、クロスネスティッドモデルや一般化ロジットモデルも適用し、より精緻な経路選択の考慮も行うことによって、準動的交通量配分の再現や予測等の精度の向上も図っている。これらの研究成果は交通ネットワークの状態を実用的に適用可能で、かつ、準動的に再現・予測する上で有用であり、工学的に価値が高いものである。以上のことから、本審査委員会は本学位論文が博士（工学）に値すると判断した。

4. 審査結果 (1) 判定（いずれかに○印） 合格 ・ 不合格

(2) 授与学位 博士（工学）