Minimum Congestion Embedding of Complete Binary Trees into Tori

PAPER

Minimum Congestion Embedding of Complete Binary Trees into Tori

Akira MATSUBAYASHI[†*], ***Member and*** **Ryo TAKASU**[†], ***Nonmember***

SUMMARY We consider the problem of embedding complete binary trees into 2-dimensional tori with minimum (edge) congestion. It is known that for a positive integer n, a 2^n-1-vertex complete binary tree can be embedded in a $(2^{\lceil n/2\rceil}+1)\times(2^{\lfloor n/2\rfloor}+1)$-grid and a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-grid with congestion 1 and 2, respectively. However, it is not known if 2^n-1-vertex complete binary tree is embeddable in a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-grid with unit congestion. In this paper, we show that a positive answer can be obtained by adding wrap-around edges to grids, i.e., a 2^n-1-vertex complete binary tree can be embedded with unit congestion in a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-torus. The embedding proposed here achieves the minimum congestion and an almost minimum size of a torus (up to the constant term of 1). In particular, the embedding is optimal for the problem of embedding a 2^n-1-vertex complete binary tree with an even integer n into a square torus with unit congestion.

key words: *graph embedding, congestion, complete binary tree, torus*

1. Introduction

The problem of efficiently implementing parallel algorithms on parallel machines has been studied as the graph embedding problem, which is to embed the communication graph underlying a parallel algorithm within the processor interconnection graph for a parallel machine with minimal communication overhead. It is well known that the dilation and/or congestion of the embedding are lower bounds on the communication delay, and the problem of embedding a guest graph within a host graph with minimal dilation and/or congestion has been extensively studied. In particular, it was pointed out by Kim and Lai [2] that minimal congestion embeddings are very important for a parallel machine that uses circuit switching for node-to-node communication.

In this paper, we consider minimal congestion embeddings of complete binary trees in tori. Complete binary trees are well known as one of the most fundamental communication graphs for divide-and-conquer algorithms. Also, tori are well known as one of the most popular processor interconnection graphs for parallel machines.

Gordon [1] showed that for a positive integer n, a 2^n-1-vertex complete binary tree denoted by $C(n)$ can be embedded into a $(2^{\lceil n/2\rceil}+1)\times(2^{\lfloor n/2\rfloor}+1)$-grid with unit congestion. Zienicke [4] showed that $C(n)$ can be embedded into a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-grid with congestion 2. Lee and Choi [3] showed that the latter result still holds under a constraint of row-column routing.

Although it is an interesting question to ask if $C(n)$ is embeddable in a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-grid with unit congestion, we have no answer for the problem. Lee and Choi [3] mentioned that this would be negative.

Since a torus contains the grid of the same side lengths as a subgraph, we can immediately obtain from the results of [1], [4], and [3] that $C(n)$ can be embedded in a $(2^{\lceil n/2\rceil}+1)\times(2^{\lfloor n/2\rfloor}+1)$-torus and a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-torus with congestion 1 and 2, respectively. However, it is not known whether $C(n)$ is embeddable in a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-torus with unit congestion. In this paper, we give a positive answer for the question by proving the following theorem:

Theorem 1: For a positive integer n, $C(n)$ can be embedded into a $2^{\lceil n/2\rceil}\times 2^{\lfloor n/2\rfloor}$-torus with unit congestion.

We construct an embedding satisfying the condition of Theorem 1 by using Gordon's embeddings [1]. The embedding proposed here achieves the minimum congestion and an almost minimum size of a torus (up to the constant term of 1). In particular, the embedding is optimal for the problem of embedding $C(n)$ with an even integer n into a square torus with unit congestion.

The paper is organized as follows: Some definitions are given in Sect. 2. In Sect. 3, we review the Gordon's embeddings. Based on the results, we prove Theorem 1 in Sect. 4.

2. Preliminaries

Let G be a graph and let $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively.

The *(two dimensional)* $m_1\times m_2$*-grid* denoted by $M(m_1,m_2)$ is the graph with vertex set $\{(i,j)\mid 0\le i<m_1, 0\le j<m_2\}$ and edge set $\{((i,j),(i+1,j))\mid 0\le i<m_1-1, 0\le j<m_2\}\cup\{((i,j),(i,j+1))\mid 0\le i<m_1, 0\le j<m_2-1\}$. The *(two dimensional)* $m_1\times m_2$*-torus* denoted by $D(m_1,m_2)$ is the graph obtained from $M(m_1,m_2)$ by adding *wrap-around edges* $((i,0),(i,m_2-1))$ $(0\le i<m_1)$ and $((0,j),(m_1-1,j))$ $(0\le j<m_2)$. We denote $M(m,m)$ and $D(m,m)$ by

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[†]The authors are with the Faculty of Engineering, Utsunomiya University, Utsunomiya, 321-8585 Japan.

[*]Presently, with the Faculty of Engineering, Kanazawa University, Kanazawa, 920-8667 Japan.

$M^2(m)$ and $D^2(m)$, respectively.

An *embedding* $\langle\phi,\rho\rangle$ of a graph G into a graph H is defined by a one-to-one mapping $\phi : V(G) \to V(H)$, together with a mapping ρ that maps each edge $(u,v) \in E(G)$ onto a set of edges of H which induces a path connecting $\phi(u)$ and $\phi(v)$. The *dilation* of $\langle\phi,\rho\rangle$ is $\max_{e_G\in E(G)} |\rho(e_G)|$. The *(edge) congestion* of $\langle\phi,\rho\rangle$ is $\max_{e_H\in E(H)} |\{e_G \in E(G) \mid e_H \in \rho(e_G)\}|$.

For an embedding $\varepsilon = \langle\phi,\rho\rangle$ of a graph G into a graph H, let ϕ^ε and ρ^ε denote ϕ and ρ, respectively. For $U \subseteq V(G)$, let $\phi^\varepsilon(U) = \{\phi^\varepsilon(v) \mid v \in U\}$. Moreover, let $\rho^\varepsilon(S) = \bigcup_{e\in S} \rho^\varepsilon(e)$ for $S \subseteq E(G)$.

3. Gordon's Embeddings

In this section, we review the embeddings given in [1] which embed complete binary trees into grids with unit congestion.

Let G_1, G_2, and G_3 be graphs. For an embedding $\varepsilon_1 = \langle\phi_1,\rho_1\rangle$ of G_1 into G_2 and a dilation-1 embedding $\varepsilon_2 = \langle\phi_2,\rho_2\rangle$ of G_2 into G_3, we denote by $\varepsilon_2 \circ \varepsilon_1$ the embedding $\langle\phi_3,\rho_3\rangle$ of G_1 into G_3 defined by $\phi_3 : u \in V(G_1) \mapsto \phi_2(\phi_1(u))$ and $\rho_3 : e \in E(G_1) \mapsto \rho_2(\rho_1(e))$. It should be noted that since $\rho_1(e)$ is a set of edges which induces a path of G_2 and the dilation of ε_2 is one, $\rho_2(\rho_1(e))$ is a set of edges which induces a path of G_3.

For an embedding ε of a graph into $M^2(m)$, we denote $\psi_m \circ \varepsilon$ by $\overline{\varepsilon}$, where ψ_m is the autoisomorphism of $M^2(m)$, or the dilation-1 embedding of $M^2(m)$ in itself which maps $(i,j) \in V(M^2(m))$ $(0 \le i \le m-1, 0 \le j \le m-1)$ to $(m-1-i, m-1-j)$. We define that w_m, x_m, y_m, and z_m are the dilation-1 embeddings of $M^2(m)$ into $M^2(2m-1)$ such that $(i,j) \in V(M^2(m))$ $(0 \le i \le m-1, 0 \le j \le m-1)$ is mapped to vertices (i,j), $(i,j+m-1)$, $(i+m-1,j)$, and $(i+m-1,j+m-1)$, respectively, of $M^2(2m-1)$.

For embeddings ε and ε' of a graph G into $M(m_1,m_2)$, we write $\varepsilon|\varepsilon'$ if ε and ε' satisfy the following conditions:

- $(i, m_2-1) \notin \phi^\varepsilon(V(G))$ or $(i,0) \notin \phi^{\varepsilon'}(V(G))$ for $0 \le i \le m_1-1$.
- $((i,m_2-1),(i+1,m_2-1)) \notin \rho^\varepsilon(E(G))$ or $((i,0),(i+1,0)) \notin \rho^{\varepsilon'}(E(G))$ for $0 \le i \le m_1-2$.

We write ε/ε' if ε and ε' satisfy the following conditions:

- $(m_1-1, j) \notin \phi^\varepsilon(V(G))$ or $(0,j) \notin \phi^{\varepsilon'}(V(G))$ for $0 \le j \le m_2-1$.
- $((m_1-1,j),(m_1-1,j+1)) \notin \rho^\varepsilon(E(G))$ or $((0,j),(0,j+1)) \notin \rho^{\varepsilon'}(E(G))$ for $0 \le j \le m_2-2$.

Lemma A (Gordon[1]): For an even integer n, there exists an embedding of $C(n+2)$ into $M^2(2m+1)$ $(m = 2^{n/2})$ with unit congestion if there exist embeddings W, X, Y, and Z satisfying the following condition:

Condition 1:

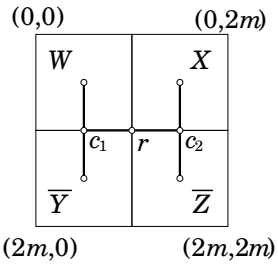


Fig. 1 Embedding of $C(n+2)$ into $M^2(2m+1)$.

(a) W, X, Y, and Z are embeddings of $C(n)$ into $M^2(m+1)$ with unit congestion.
(b) $W|X$, $Z|Y$, $W/\overline{Y}$, $X/\overline{Z}$.
(c) $(m,m) \notin \phi^\varepsilon(V(C(n)))$ for $\varepsilon \in \{W,Z\}$.
(d) $(m,0) \notin \phi^\varepsilon(V(C(n)))$ for $\varepsilon \in \{X,Y\}$.
(e) $(m,m/2) \notin \phi^\varepsilon(V(C(n)))$ for $\varepsilon \in \{W,X,Y,Z\}$.
(f) $\{((m,j),(m,j+1)) \mid m/2 \le j < m\} \cap \rho^\varepsilon(E(C(n))) = \emptyset$ for $\varepsilon \in \{W,Z\}$.
(g) $\{((m,j),(m,j+1)) \mid 0 \le j < m/2\} \cap \rho^\varepsilon(E(C(n))) = \emptyset$ for $\varepsilon \in \{X,Y\}$.
(h) $\{((i,m/2),(i+1,m/2)) \mid m/2 \le i < m\} \cap \rho^\varepsilon(E(C(n))) = \emptyset$ for $\varepsilon \in \{W,X,Y,Z\}$.
(i) ϕ^ε maps the root of $C(n)$ to $(m/2,m/2)$ for $\varepsilon \in \{W,X,Y,Z\}$.

□

This lemma can be proved by constructing a desired embedding, which is obtained by (i) embedding four $C(n)$'s with $w_{m+1} \circ W$, $x_{m+1} \circ X$, $y_{m+1} \circ \overline{Y}$, and $z_{m+1} \circ \overline{Z}$, (ii) mapping the root r of $C(n+2)$, r's child c_1, and the other child c_2 to (m,m), $(m,m/2)$, and $(m,3m/2)$, respectively, (iii) and connecting r, c_i $(i = 1,2)$, and c_i's children with the shortest paths as shown in Fig. 1. It is easy to see that this is an embedding of $C(n+2)$ into $M^2(2m+1)$ with unit congestion. We denote by $F_n(W,X,Y,Z)$ the embedding of $C(n+2)$ into $M^2(2m+1)$ which is constructed as described above from four embeddings W, X, Y, and Z satisfying Condition 1 for an even integer n and $m = 2^{n/2}$.

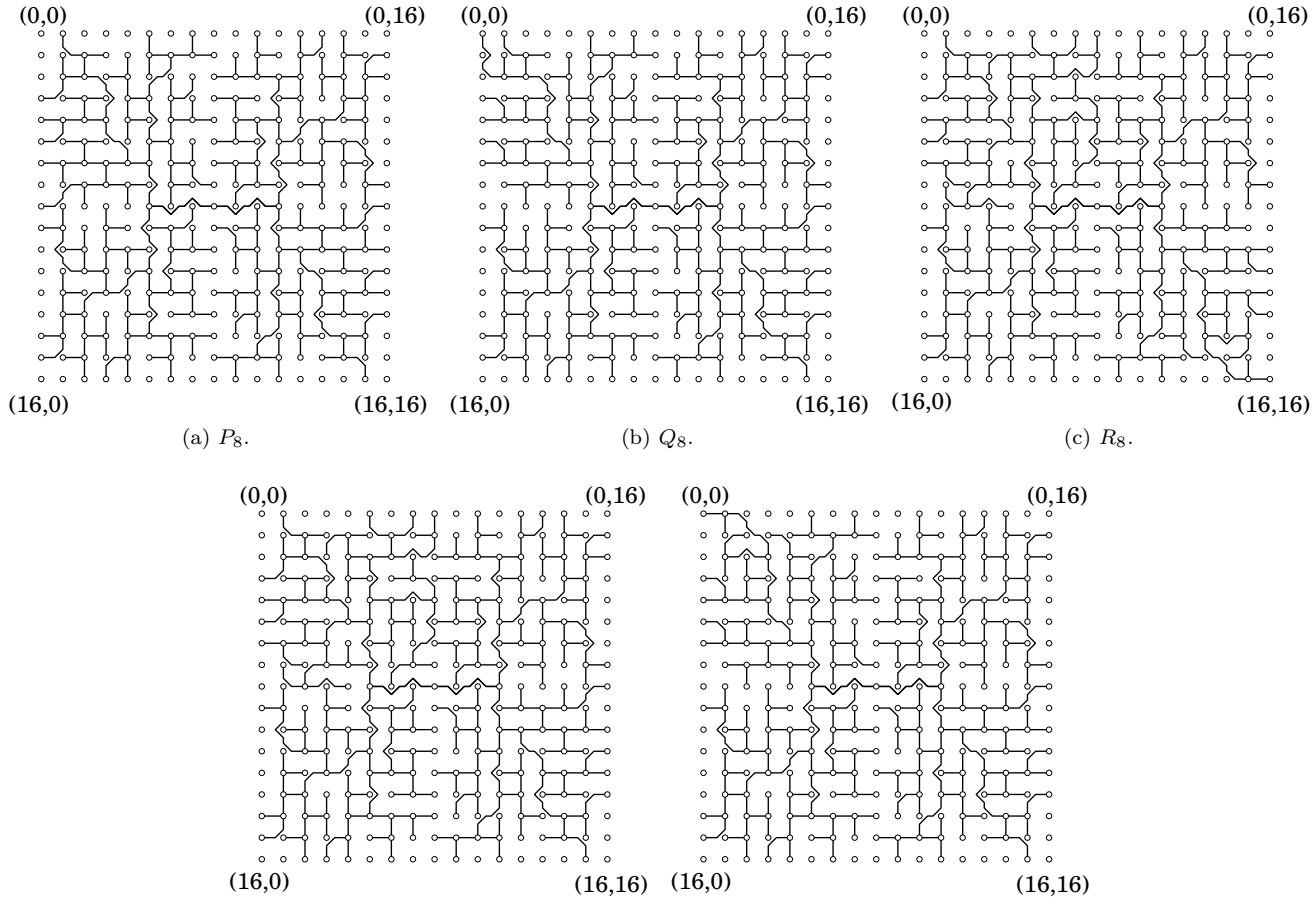
Theorem B (Gordon[1]): For an even integer $n \ge 8$, there exist embeddings P_n, Q_n, R_n, S_n, and T_n of $C(n)$ into $M^2(m+1)$ $(m = 2^{n/2})$ with unit congestion such that the following conditions are satisfied:

Condition 2:

(a) $(0,0) \notin \phi^\varepsilon(V(C(n)))$ for $\varepsilon \in \{P_n,R_n,S_n\}$.
(b) $\{(0,m),(m,0),(m,m/2)\} \cap \phi^\varepsilon(V(C(n))) = \emptyset$ for $\varepsilon \in \{P_n,Q_n,R_n,S_n,T_n\}$.
(c) $(m,m) \notin \phi^\varepsilon(V(C(n)))$ for $\varepsilon \in \{P_n,Q_n,S_n,T_n\}$.

Condition 3:

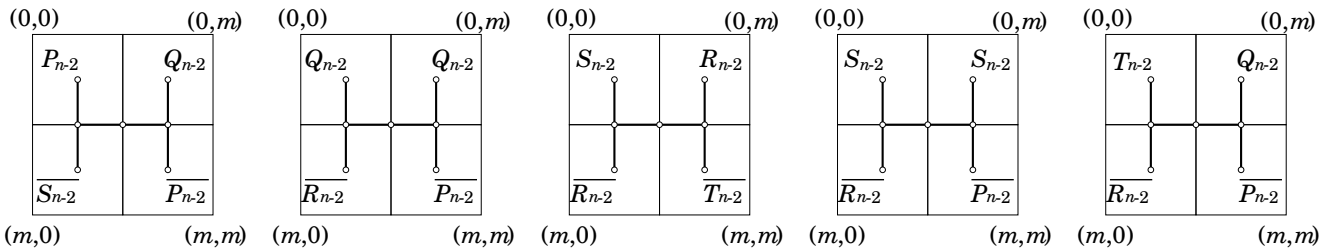
(a) $\{((0,j),(0,j+1)) \mid 0 \le j < m\} \cap \rho^\varepsilon(E(C(n))) = \emptyset$ for $\varepsilon \in \{P_n,Q_n,R_n,S_n\}$.
(b) $\{((m,j),(m,j+1)) \mid 0 \le j < m\} \cap \rho^\varepsilon(E(C(n))) = \emptyset$ for $\varepsilon \in \{P_n,Q_n,S_n,T_n\}$.



(a) P_8. (b) Q_8. (c) R_8.

(d) S_8. (e) T_8.

Fig. 2 Embeddings P_8, Q_8, R_8, S_8, and T_8.



(a) P_n. (b) Q_n. (c) R_n. (d) S_n. (e) T_n.

Fig. 3 Recursive constructions of P_n, Q_n, R_n, S_n, and T_n for $n \geq 10$ ($m = 2^{n/2}$).

(c) $\{((i,0),(i+1,0)) \mid 0 \leq i < m\} \cap \rho^{\varepsilon}(E(C(n))) = \emptyset$ for $\varepsilon \in \{P_n, R_n, S_n, T_n\}$.

(d) $\{((i,m),(i+1,m)) \mid 0 \leq i < m\} \cap \rho^{\varepsilon}(E(C(n))) = \emptyset$ for $\varepsilon \in \{P_n, Q_n, R_n, S_n, T_n\}$.

(e) $\{((m,j),(m,j+1)) \mid 0 \leq j < m/2\} \cap \rho^{R_n}(E(C(n))) = \emptyset$.

(f) $\{((i,m/2),(i+1,m/2)) \mid m/2 \leq i < m\} \cap \rho^{\varepsilon}(E(C(n))) = \emptyset$ for $\varepsilon \in \{P_n, Q_n, R_n, S_n, T_n\}$.

Condition 4: $P_n|Q_n$, $P_n|S_n$, $Q_n|Q_n$, $P_n|R_n$, $S_n|R_n$, $T_n|R_n$, $S_n|S_n$, $T_n|Q_n$, $R_n|P_n$, $Q_n|S_n$, $S_n|T_n$.

Condition 5: $P_n/\overline{S_n}$, $Q_n/\overline{P_n}$, $Q_n/\overline{R_n}$, $S_n/\overline{R_n}$, $R_n/\overline{T_n}$, $\overline{S_n}/P_n$, $\overline{P_n}/R_n$, $\overline{R_n}/T_n$, $\overline{S_n}/Q_n$.

Condition 6: ϕ^{ε} maps the root of $C(n)$ to $(m/2, m/2)$ for $\varepsilon \in \{P_n, Q_n, R_n, S_n, T_n\}$.

□

We describe here the constructions given in [1] for Q_n and S_n ($n \geq 8$), which are used to construct our embedding. Q_n and S_n, together with P_n, R_n, and T_n are recursively defined as shown in Fig. 2 for $n = 8$ and as $P_n = F_{n-2}(P_{n-2}, Q_{n-2}, S_{n-2}, P_{n-2})$, $Q_n = F_{n-2}(Q_{n-2}, Q_{n-2}, R_{n-2}, P_{n-2})$, $R_n = F_{n-2}(S_{n-2}, R_{n-2}, R_{n-2}, T_{n-2})$, $S_n = F_{n-2}(S_{n-2}, S_{n-2}, R_{n-2}, P_{n-2})$, and

$T_n = F_{n-2}(T_{n-2}, Q_{n-2}, R_{n-2}, P_{n-2})$ for $n \geq 10$ (Fig. 3).

4. Proof of Theorem 1

In this section, we prove Theorem 1 by a sequence of lemmas.

Lemma 2: For an even integer $n \geq 8$, $Q_n/\overline{S_n}$.

Proof It is easy from Fig. 2 to see that $Q_8/\overline{S_8}$. Thus, it suffices to show that $Q_n/\overline{S_n}$ for an even integer $n \geq 10$. It follows from Theorem B (Condition 5) that $\overline{P_{n-2}}/R_{n-2}$ and hence $\overline{R_{n-2}}/P_{n-2}$. Moreover, it follows from Theorem B ((a) and (b) in Condition 2) that $(0,0) \notin \phi^{R_{n-2}}(V(C(n-2)))$ and $(0, 2^{n/2-1}) \notin \phi^{P_{n-2}}(V(C(n-2)))$. From these facts and the definitions of Q_n and S_n (Fig. 3), we have that $Q_n/\overline{S_n}$. □

Lemma 3: For an even integer $n \geq 8$, Q_n, Q_n, S_n, and S_n satisfy Condition 1 for W, X, Y, and Z, respectively.

Proof First, (a) in Condition 1 is immediate. By Condition 4 of Theorem B and Lemma 2, Q_n and S_n ($n \geq 8$) satisfy (b) in Condition 1. Then, (c), (d), and (e) in Condition 1 are satisfied since Q_n and S_n satisfy (b) and (c) in Condition 2. (f) and (g) in Condition 1 are satisfied since Q_n and S_n satisfy (b) in Condition 3. Finally, (h) and (i) in Condition 1 are satisfied since Q_n and S_n satisfy (f) in Condition 3 and Condition 6, respectively. □

Lemma 4: For an even integer $n \geq 10$, there exists an embedding U_n of $C(n)$ into $M^2(2^{n/2}+1)$ with unit congestion such that the following condition is satisfied:

Condition 7: $U_n|U_n$, U_n/U_n, and $\{(2^{n/2}, 0), (0, 2^{n/2}), (2^{n/2}, 2^{n/2})\} \cap \phi^{U_n}(V(C(n))) = \emptyset$.

Proof Let $n \geq 10$ be an even integer and $m = 2^{n/2}$. From Theorem B, there exist embeddings Q_{n-2} and S_{n-2} with unit congestion such that Conditions 2 through 6 are satisfied. We define as shown in Fig. 4 that $U_n = F_{n-2}(Q_{n-2}, Q_{n-2}, S_{n-2}, S_{n-2})$, which is an embedding of $C(n)$ into $M^2(m+1)$ with unit congestion by Lemmas A and 3.

The following claims show that U_n satisfies Condition 7.

Claim 5: $U_n|U_n$.

Proof Immediate from the definition of U_n and Lemma 3. *End of proof of Claim 5*

Claim 6: U_n/U_n.

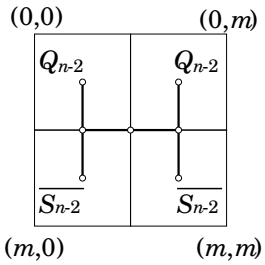


Fig. 4 Embedding U_n ($n \geq 10$, $m = 2^{n/2}$).

Proof It follows from Theorem B (Condition 5) that $\overline{S_n}/Q_n$. Moreover, it follows from Theorem B ((a) and (b) in Condition 2) that $\{(0, m/2), (0,0)\} \cap \phi^{Q_{n-2}}(V(C(n-2))) \subseteq \{(0,0)\}$ and $\{(m/2, 0), (m/2, m/2)\} \cap \phi^{\overline{S_{n-2}}}(V(C(n-2))) = \emptyset$. From these facts and the definition of U_n, we have that U_n/U_n.
End of proof of Claim 6

Claim 7: $\{(m,0), (0,m), (m,m)\} \cap \phi^{U_n}(V(C(n))) = \emptyset$.

Proof As shown in the proof of Claim 6, it follows that $(0, m/2) \notin \phi^{Q_{n-2}}(V(C(n-2)))$ and $\{(m/2, 0), (m/2, m/2)\} \cap \phi^{\overline{S_{n-2}}}(V(C(n-2))) = \emptyset$. Thus, the claim holds by the definition of U_n.
End of proof of Claim 7

Thus, U_n satisfies Condition 7. Therefore, the proof of Lemma 4 is completed. □

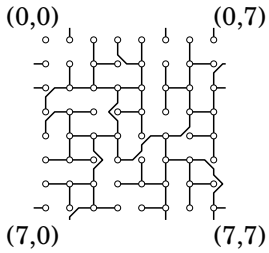
Lemma 8: For an even integer $n \geq 10$, there exists an embedding of $C(n)$ into $D^2(2^{n/2})$ with unit congestion.

Proof Let $m = 2^{n/2}$, $C = C(n)$, $M = M^2(m+1)$, and $D = D^2(m)$. We define that $\theta_m : (i,j) \in V(M) \mapsto (i \bmod m, j \bmod m) \in V(D)$ and $\lambda_m : (u,v) \in E(M) \mapsto (\theta_m(u), \theta_m(v)) \in E(D)$.

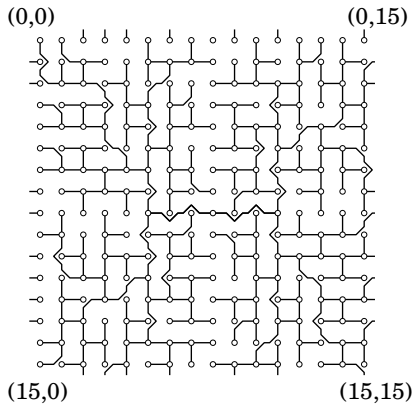
By Lemma 4, there exists an embedding U_n of C into M such that Condition 7 is satisfied. We construct from U_n a desired embedding $\langle \phi, \rho \rangle$ of C into D. Let $\phi : v \in V(C) \mapsto \theta_m(\phi^{U_n}(v))$, and for an edge $(u,v) \in E(C)$, let $\tau((u,v)) = \{\lambda_m(e) \mid e \in \rho^{U_n}((u,v))\}$. Since λ_m maps two adjacent edges of M to two adjacent ones of D by definition, $\tau((u,v))$ induces a connected subgraph of D which contains $\phi(u)$ and $\phi(v)$. Thus, there exists a subset of $\tau((u,v))$ which induces a path connecting $\phi(u)$ and $\phi(v)$[†]. We define that $\rho((u,v))$ is the subset of $\tau((u,v))$.

Since U_n satisfies Condition 7, it follows that ϕ is a one-to-one mapping of $V(C)$ to $V(D)$ and that for distinct edges e and e' of C, $\tau(e)$ and $\tau(e')$ are disjoint. Thus, $\langle \phi, \rho \rangle$ is an embedding of C into D with unit congestion. □

[†]Indeed, $\tau((u,v))$ itself induces a path for U_n constructed in the proof of Lemma 4.



(a) Embedding of $C(6)$ into $D^2(8)$.



(b) Embedding of $C(8)$ into $D^2(16)$.

Fig. 5 Congestion-1 Embeddings of $C(n)$ into $D^2(2^{n/2})$ for $n = 6$ and $n = 8$. Wrap-around edges are represented by half lines

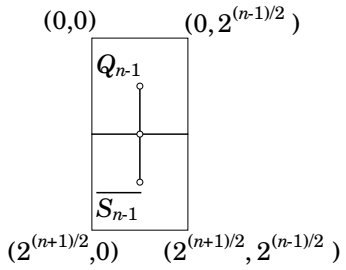


Fig. 6 Embedding U'_n $(n \geq 9)$.

It is not difficult to see that there exists an embedding of $C(n)$ into $D^2(2^{n/2})$ for an even integer $n \leq 8$. Figure 5 shows examples of such embeddings for $n = 6$ and $n = 8$. Thus, we have the following lemma, which proves Theorem 1 for the case that n is even:

Lemma 9: For an even integer n, there exists an embedding of $C(n)$ into $D^2(2^{n/2})$ with unit congestion. □

It remains to show that Theorem 1 holds for the case that n is odd. For an odd integer $n \geq 9$, we can obtain from the definition of U_{n+1} constructed in the proof of Lemma 4 an embedding U'_n of $C(n)$ into $M(2^{(n+1)/2}+1, 2^{(n-1)/2}+1)$ as shown in Fig. 6. From Theorem B (Conditions 2 and 3) and Lemma 4 (Condition 7), it is not difficult to see that U'_n satisfies the following condition:

Condition 8: $U'_n|U'_n$, U'_n/U'_n, and $\{(2^{(n+1)/2}, 0), (0, 2^{(n-1)/2}), (2^{(n+1)/2}, 2^{(n-1)/2})\} \cap \phi^{U'_n}(V(C(n))) = \emptyset$.

Thus, we can construct an embedding of $C(n)$ into $D(2^{(n+1)/2}, 2^{(n-1)/2})$ with unit congestion by a similar argument of the proof of Lemma 8. Therefore, although the details are omitted here, we have the following lemma:

Lemma 10: For an odd integer n, $C(n)$ can be embedded into $D(2^{(n+1)/2}, 2^{(n-1)/2})$ with unit congestion. □

Lemmas 9 and 10 complete the proof of Theorem 1.

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Akira Matsubayashi received the B.E. degree in electrical and electronic engineering in 1991, M.E. degree in intelligence science in 1993, and D.E. degree in electrical and electronic engineering in 1996 all from Tokyo Institute of Technology, Tokyo, Japan. From 1996 to 2000, he was a research associate in the Department of Information Science at Utsunomiya University, Utsunomiya, Japan. Currently, he is a research associate in the Department of Information and Systems Engineering at Kanazawa University, Kanazawa, Japan. His research interests are in parallel and VLSI computation. He is a member of the ACM, SIAM, and the Information Processing Society of Japan.

Ryo Takasu received the B.E. and M.E. degrees in information science from Utsunomiya University, Utsunomiya, Japan, in 1997 and 1999, respectively. He joined NEC Engineering, Ltd. in 1999.