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A Theoretical Investigation of Heat Extraction from a Compost Bed by Using a Multi-Heat-Pipe Heat Exchanger

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Abstract

A buried-tube-type heat exchanger is typical and simple for extracting heat from a compost bed except the requirement of some tedious works such as removing and retubing the tubes in remixing the compost materials. In order to simplify the above tedious works, a unique heat extraction method by a multi-heat-pipe heat exchanger was proposed. The analytical solutions of the temperature in the bed and the temperature of medium at the outlet of the heat exchanger were obtained. These solutions were mathematically similar to those for the buried-tube-type heat exchanger. Then, the heat extraction ability of the multi-heat-pipe heat exchanger was calculated from the solutions.

1. Introduction

A method of extracting the heat generated in a compost bed by water flowing into tubes buried in the bed was presented by Seki and Komori [1984, 1985a]. In such a buried-tube-type heat exchanger, since the medium fluid was surrounded by the compost materials almost completely, there was no loss of heat in transmission of heat to the medium fluid through the walls of the buried tubes from the compost bed. Therefore, it was suggested that the buried-tube-type heat exchanger would be available for heat extraction from the compost bed.

Generally, the composting reactions proceed very slowly and the composting process requires a long period, moreover, the oxygen stored in the compost bed is consumed successively, so that the rate of heat generation usually decreases gradually with declining the activity of thermophilic aerobic microorganisms due to the oxygen shortage in the bed. Then, to activate the composting reaction by supplying oxygen, the compost materials must be remixed artificially in the composting process. The works, remixing and retubing the tubes for the medium fluid, for remixing are inconvenient and tedious for the case of the buried-tube-type heat exchanger. Taking account of convenience for the above works, Komori et al [1983] proposed another method for heat extraction from the compost bed with a single-heat-pipe heat extractor. For the single-heat-pipe heat extractor, the heat generated in the compost materials is pumped up to the top of the heat-pipe inserted into the bed, then, the heat is transferred to the medium fluid by convection. The heat-pipe has both functions of pumping up heat from the bed and of heat exchange.

Since the primitive heat-pipe-type heat exchanger has only a single element of heat-pipe, it would not be applicable to the practical heat extraction directly, however, the concept of the heat-pipe-type heat exchanger is unique and involves an interest in utilizing a characteristic of the heat-pipe, which has an excellent heat transfer ability.

In this investigation, for application of the heat-pipe-type heat exchanger to the practical cases, a multi-heat-pipe heat exchanger is introduced. The problem of heat extraction from the compost bed by the multi-heat-pipe heat exchanger is solved analytically with suitable boundary conditions, upon the several assumptions. The calculated results of heat extraction ability for the multi-heat-pipe heat exchanger are compared with those for the case of the buried-tube-type heat exchanger.

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2. Mechanism of heat transmission and overall heat transfer coefficient of heat-pipe

The heat-pipe has been applied to many practical engineering fields such as electronic devices, space ships, stabilization of the permafrost soil and others [Chi, 1976].

A heat-pipe transfers heat by repeating the cyclic process of vaporization and condensation of the working fluid sealed in the metalic tube and it is an excellent heat conductor.

When the bottom portion of a heat-pipe is maintained at higher temperature and the top portion is kept at lower temperature, the axial temperature profile of the heat-pipe may be schematically shown in Fig. 1 [Sun and Tien, 1975]. Heat is rapidly transferred from the bottom towards the top of the heat-pipe by vaporizing the working fluid, even though there is a relatively small temperature difference between the top and bottom portions.

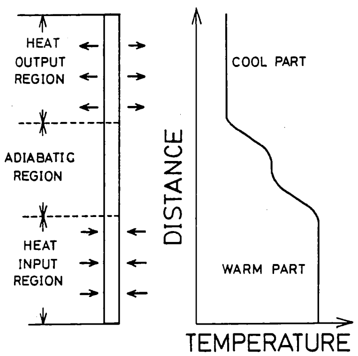


Fig. 1 Axial temperature profile of a heat-pipe

Supposing that a multi-heat-pipe heat exchanger is set in the compost bed, as shown in Fig. 2, the schematic representation of the local temperature profiles of the compost bed and of the medium fluid contacting with a heat-pipe in the multi-heat-pipe heat exchanger may be shown as Fig. 3. The heat generated in the compost bed is conducted to the bottom portion of the heat-pipe, the heat input region, through a small gap between the heat-pipe and compost materials. Therefore, a temperature at the surface of the heat input region T_e is not equal to the apparent temperature of the compost bed at the interface $T|_{r=r_1}$, exactly.

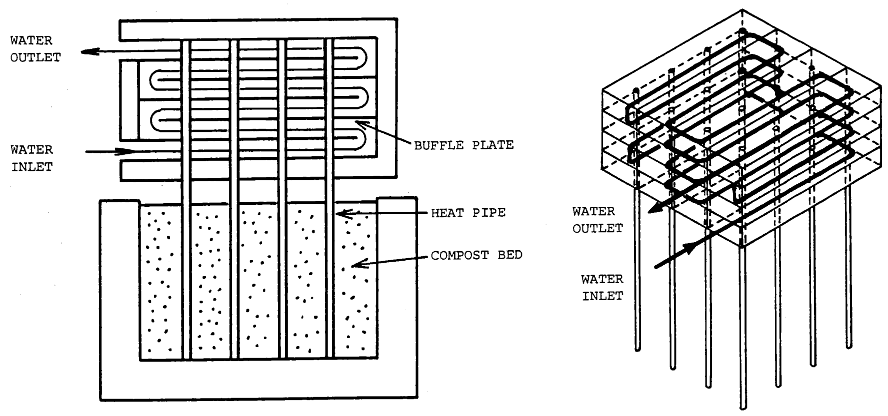
By introducing a heat transfer coefficient h_e corresponding to the heat resistance at the surface of the heat-pipe, the rate of heat transfer at the surface of the bottom portion of the heat-pipe is given by

$$Q_1=2\pi r_1 l_e h_e (T|_{r=r_1}-T_e). \qquad (1)$$

The heat conducted to the heat input region of the heat-pipe is rapidly transferred to the top portion by vaporizing the working fluid. Then, the rate of heat transfer to the axial direction of the heat-pipe is [Komori et al, 1983]

$$Q_2=\pi r_1^2 h (T_e-T_c), \qquad (2)$$

where T_e-T_c is the temperature difference be-



(a) Whole view of an apparatus for heat extraction

(b) Details of the water flow pattern in a heat exchanger

Fig. 2 Sketch of a multi-heat-pipe heat exchanger for extraction of heat from the compost bed

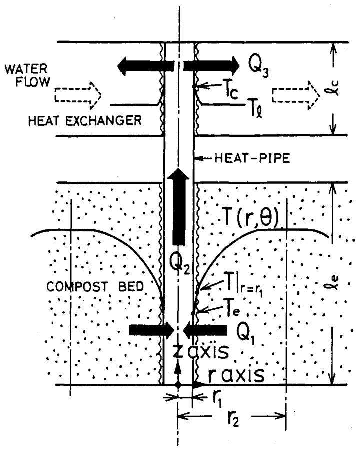


Fig. 3 Schematic representation of the local temperature profiles of the compost bed and of the medium fluid contacting with a heat-pipe

tween the bottom and the top of the heat-pipe and h is an effective heat transfer coefficient of the heat-pipe [Chi, 1976].

The rate of heat transfer to the medium fluid flowing through the heat exchanger is

$$Q_3=2\pi r_1 l_c h_c (T_c-T_l)\,, \tag{3}$$

where h_c is a liquid-film heat transfer coefficient based on the outer surface of the heat-pipe.

Supposing that the loss of heat in the heat transmission process of the system is ignored, Q_1 is related to Q_2 and Q_3 as the following equation:

$$Q_1=Q_2=Q_3\,. \tag{4}$$

Accordingly, by using the apparent driving force of temperature $T|_{r=r_1}-T_l$, and by introducing an overall heat transfer coefficient U_h based on the outer surface of the heat input region, the rate of heat transfer can be written in a form of

$$Q=2\pi r_1 l_e U_h (T|_{r=r_1}-T_e)\,, \tag{5}$$

where U_h is expressed as follows:

$$U_h=\frac{1}{1/h_e+(2l_e/r_1)/h+(l_e/l_c)/h_c}\,. \tag{6}$$

3. Mathematical treatment for the heat transfer process and analytical solutions

3.1 Differential equations, boundary and initial conditions governing the heat extraction process

In the heat extraction process by the multi-heat-pipe heat exchanger, there are two mechanisms of heat transfer. One is the conduction of heat in a compost bed and the other is the forced convective heat transfer in the heat exchanger. These mechanisms are combined through the heat-pipe, which is a carrier of heat transport. Therefore, the heat conduction equation for the compost bed and a heat balance equation for the heat exchanger must be solved simultaneously by using the suitable boundary and initial conditions.

Generally, it is difficult to obtain the exact analytical solutions for the heat extraction problem of the compost bed. Then, for simplification of the problem, the following several assumptions are made.

1) Conduction of heat in the compost bed takes place only in the radial direction around the heat-pipe (r-direction).

2) Heat-pipes are arranged in square pitches at equal spaces, and temperature profiles of the compost around any adjacent heat-pipes are the same.

3) Thermal physical properties of the compost bed are independent of temperature during the heat extraction process.

4) The heat generation term in the conduction equation is small and ignored, compared with the terms of conduction and accumulation [Seki and Komori, 1984].

5) The axial temperature profiles of the heat-pipe are uniform in the top and bottom portions, respectively.

Upon these assumptions, the conduction equation for the compost bed is

$$\frac{\partial T}{\partial \theta}=\kappa\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{7}$$

The boundary condition at the surface of the bottom portion of the heat-pipe is

$$K\frac{\partial T}{\partial r}=U_h(T-T_l) \qquad \text{at} \quad r=r_1. \tag{8}$$

According to the assumption 2), there is no flow of heat at $r=r_2$, therefore, the boundary condition at $r=r_2$ is

$$\frac{\partial T}{\partial r}=0 \qquad \text{at} \quad r=r_2\,, \tag{9}$$

and the initial condition is

$$T=T_i \qquad \text{at} \quad \theta=0. \qquad (10)$$

On the other hand, the equation of heat balance of the medium fluid in the heat exchanger is

$$C_{pl}\dot{m}(T_{lou}-T_{lin})=2\pi r_1 l_e U_h (\overline{T|_{r=r_1}-T_l})\, n, \quad (11)$$

where $\overline{T|_{r=r_1}-T_l}$ is a mean temperature difference between $T|_{r=r_1}$ and T_l, and n is the number of heat-pipes set in the heat exchanger.

Usually, for the design of a multi-tube-type heat exchanger such as a shell-and-tube and a cross-flow heat exchanger, the mean temperature difference is equal to a logarithmic mean temperature difference multiplied by a modified factor F which is determined by several parameters, i.e. the number of flow passes and the temperature efficiency. The value of F is smaller than unity. According to the practical operating conditions, since the temperature efficiency $(T_{lou}-T_{lin})/(T|_{r=r_1}-T_{lin})$ is less than 0.2 and it is comparatively small for the heat-pipe-type heat exchanger, the modified factor F may be approximated by unity. Therefore, the term $\overline{T|_{r=r_1}-T_l}$ of Eq. (11) may be expressed as

$$\overline{T|_{r=r_1}-T_l}=\frac{(T|_{r=r_1}-T_{lou})-(T|_{r=r_1}-T_{lin})}{\ln\{(T|_{r=r_1}-T_{lou})/(T|_{r=r_1}-T_{lin})\}}. \quad (12)$$

By the several experimental results for the buried-tube-type heat exchanger, the ratio $(T|_{r=r_1}-T_{lou})/(T|_{r=r_1}-T_{lin})$ is less than 2.0. Similarly, for the case of the multi-heat-pipe heat exchanger, since $(T|_{r=r_1}-T_{lou})/(T|_{r=r_1}-T_{lin})<2.0$, the term of Eq. (12) may be replaced by the arithmetic mean temperature difference (Mizushina, 1972). Therefore, Eq. (12) becomes

$$\overline{T|_{r=r_1}-T_l}=\frac{(T|_{r=r_1}-T_{lou})+(T|_{r=r_1}-T_{lin})}{2}, \quad (13)$$

and

$$T_l=\frac{T_{lou}+T_{lin}}{2}. \quad (14)$$

Substitution of Eq. (14) into Eq. (11) gives

$$T_{lou}=\frac{N_h}{1+N_h/2}T|_{r=r_1}+\frac{1-N_h/2}{1+N_h/2}T_{lin}$$
$$=N_h^*T|_{r=r_1}+N_h^*\left(\frac{1}{N_h}-\frac{1}{2}\right)T_{lin}, \quad (15)$$

where N_h is the number of transfer units, and is defined by

$$N_h=\frac{2\pi r_1 l_e U_h n}{C_{pl}\dot{m}}. \quad (16)$$

By using Eqs. (14) and (15), the boundary condition Eq. (8) becomes

$$K\frac{\partial T}{\partial r}=\frac{U_h}{1+N_h/2}(T-T_{lin}) \quad \text{at} \quad r=r_1. \quad (17)$$

3.2 Non-dimensional expression of the equations and the analytical solutions

To manipulate the mathematical treatment, the following dimensionless variables and parameters are introduced:

$$\Phi=\frac{T-T_i}{T_{li}-T_i}, \quad (18)$$

$$\Theta=\frac{\kappa\theta}{(r_2-r_1)^2}, \quad (19)$$

$$\xi=\frac{r}{r_2-r_1}, \quad (20)$$

$$B_{ih}=\frac{U_h(r_2-r_1)}{K}, \quad (21)$$

$$\eta_2=\frac{r_2}{r_1}. \quad (22)$$

By using the above dimensionless variables and parameters, Eqs. (7), (17), (9) and (10) can be rewritten in

$$\frac{\partial\Phi}{\partial\Theta}=\frac{\partial^2\Phi}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\Phi}{\partial\xi} \quad \text{in} \quad \frac{1}{\eta_2-1}<\xi<\frac{\eta_2}{\eta_2-1}, \quad (23)$$

$$\frac{\partial\Phi}{\partial\xi}=\frac{B_{ih}}{1+N_h/2}(\Phi-1)=B_h^*(\Phi-1) \quad \text{at} \quad \xi=\frac{1}{\eta_2-1}=\nu, \quad (24)$$

$$\frac{\partial\Phi}{\partial\xi}=0 \quad \text{at} \quad \xi=\frac{\eta_2}{\eta_2-1}=\nu\eta_2, \quad (25)$$

$$\Phi=0 \quad \text{at} \quad \Theta=0. \quad (26)$$

According to the analytical procedure described in the previous paper [Seki and Komori, 1985], the analytical solution of Eq. (23), which satisfies the above boundary and initial conditions Eqs. (24)–(26), is given by

$$\Phi=1+2B_h^*\sum_{n=1}^{\infty}\frac{\{J_0(\alpha_n\xi)Y_1(\alpha_n\nu\eta_2)-Y_0(\alpha_n\xi)J_1(\alpha_n\nu\eta_2)\}e^{-\alpha_n^2\theta}}{\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_n B_1(\alpha_n\nu)+B_h^*B_0(\alpha_n\nu)\}}, \tag{27}$$

and, T_{lou} is

$$\frac{T_{lou}-T_{lin}}{T_i-T_{lin}}=-2N_h^*B_h^*\sum_{n=1}^{\infty}\frac{Z_0(\alpha_n\nu)e^{-\alpha_n^2\theta}}{\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_n B_1(\alpha_n\nu)+B_h^*B_0(\alpha_n\nu)\}}, \tag{28}$$

where $Z_m(x)$ and $B_m(x)$ are expressed by Eqs. (29) and (30), respectively, and α_n is the n-th positive root of Eq. (31):

$$Z_m(x)=J_m(x)Y_1(x\eta_2)-Y_m(x)J_1(x\eta_2), \tag{29}$$

$$B_m(x)=J_m(x)Y_0(x\eta_2)-Y_m(x)J_0(x\eta_2), \tag{30}$$

$$\alpha_n Z_1(\alpha_n\nu)+B_h^*Z_0(\alpha_n\nu)=0. \tag{31}$$

4. Evaluation of heat extraction ability of multi-heat-pipe heat exchanger

To estimate the heat extraction ability for the multi-heat-pipe heat exchanger from the analytical solutions, Eqs. (27) and (28), an illustrative example is presented as shown in Fig. 4(a). The multi-heat-pipe heat exchanger has 16 elements of heat-pipes. The material of the pipe is carbon steel and the working fluid of the heat-pipe is ethanol. A single element of the heat-pipe is 1200mm length, 21.5mmϕ I.D. and 25.5mmϕ O.D., respectively. The specifications of the heat-pipe are similar to those of the heat-pipe used in the previous investigation [Komori et al, 1983]. The depth of the pipes contacting with the compost bed is 800mm, approximately.

To discuss the heat extraction ability for the multi-heat-pipe heat exchanger, a buried-tube-type heat exchanger is illustrated in Fig. 4(b). For the buried-tube-type heat exchanger shown in Fig. 4(b), the dimensions of heat exchanger are prepared to be similar to those for the multi-heat-pipe heat exchanger, that is, the buried-tube-type heat exchanger has 16 elements of carbon steel tubes with 21.5mmϕ I.D., 25.5mmϕ O.D. and 800mm length contacting with the compost bed. Each element of the tube is connected in series by the flexible tubes at the end of the tube.

In this section, heat transfer ability for the above illustrative examples are calculated by the

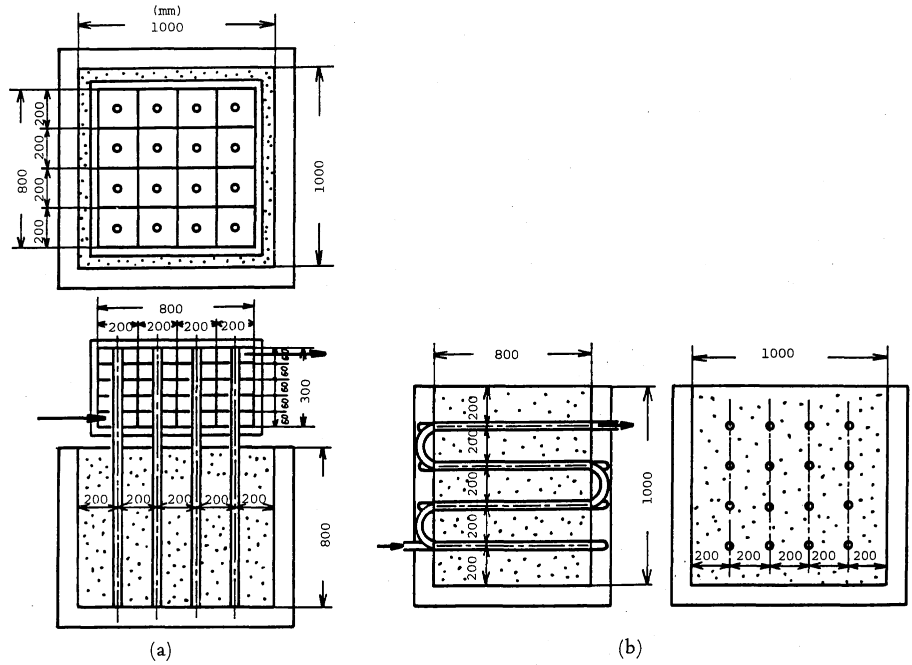


Fig. 4 Illustrative representation of the multi-heat-pipe (a) and buried-tube-type (b) heat exchangers for evaluating the heat transfer ability

analytical solutions under the several operating conditions.

4.1 Overall heat transfer coefficient

As mentioned above, the analytical solutions for the multi-heat-pipe heat exchanger are similar to the approximate analytical solutions for the buried-tube-type heat exchanger. Therefore, the heat extraction ability may be evaluated by comparing the values of the overall heat transfer coefficient for the two exchangers.

For the case of the multi-heat-pipe heat exchanger, the overall heat transfer coefficient U_h is given by Eq. (6). The overall heat transfer coefficient U_b for the buried-tube-type heat exchanger is [Seki and Komori, 1984]

$$U_b=\frac{1}{1/h_s+(r_1/K_w)\ln(r_1/r_i)+(r_1/r_i)/h_i}. \quad (32)$$

Generally, the flow rate and temperature of medium in the heat exchanger have influence on the heat transfer coefficients h_c in Eq. (6) and h_i in Eq. (32). In the practical heat extraction process from the compost bed, since the temperature of the medium rises to the maximum limit within 30°C at the outlet of heat exchanger [Seki and Komori, 1985b], consequently, h_c and h_i are dependent on the flow rate of medium fluid only. In the illustrative examples, h_c could be estimated by Hilpert's equation for the forced convective heat transfer at the outer surface of the circular tube [JSME, 1985], and h_i was evaluated from Colburn's equation [Uchida, 1972].

The values of several heat transfer coefficients and thermal conductivity of wall material of the buried tube or heat-pipe are shown in Table 1.

Table 2 summarizes the calculated results of the overall heat transfer coefficients by Eqs. (6) and (32) with the flow rate of the medium fluid. The overall heat transfer coefficients increases gradually with the flow rate of the medium fluid, and there

Table 1 Values of the several heat transfer coefficients and thermal conductivity of wall material of the buried tube or heat-pipe

Heat transfer coefficient at the interface between the buried tube and compost bed h_s [kcal/m²hr°C]	Thermal conductivity of carbon steel (Wall material of the buried tube) K_w [kcal/mhr°C]	Heat transfer coefficient at the interface between the heat pipe and compost bed h_e [kcal/m²hr°C]	Effective heat transfer coefficient of the heat pipe h [kcal/m²hr°C]
65	40	65	11570

Table 2 Calculated results of h_i, h_c, U_b, U_h and U_h/U_b for some values of the flow rate of medium fluid

Flow rate of water	Heat transfer coefficient at the inner surface of the buried tube h_i [kcal/m²hr°C]	Heat transfer coefficient at the interface between water and the heat pipe h_c [kcal/m²hr°C]	Overall heat transfer coefficient (buried tube type heat exchanger) U_b [kcal/m²hr°C]	Overall heat transfer coefficient (multi heat pipe heat exchanger) U_h [kcal/m²hr°C]	Ratio of U_b and U_h U_h/U_b [-]
2.5	530	210	56.7	25.6	0.451
5.0	921	288	59.9	28.0	0.467
10.0	1605	398	62.0	30.2	0.487

is a small variation of the values of U_b and U_h against the flow rate of water. According to the calculated results for the case of the buried-tube-type heat exchanger, the thermal resistance at the tube-compost interface is dominant, and U_b is equivalent to h_s. For the case of the multi-heat-pipe heat exchanger, the resistance of the heat-pipe $(2l_e/r_1)/h$ is almost equal to the thermal resistance at the heat-pipe-compost interface $1/h_e$, and the overall heat transfer coefficient U_h becomes $1/2U_b$ approximately.

4.2 Period for heat extraction, water temperature at the outlet of heat exchanger and rate of heat extraction

The heat extraction ability may be evaluated by the calculated results of three items, the period available for heat extraction θ_{ext}, the average temperature of water at the outlet of the heat exchanger $\widetilde{T}_{lou}$, and the average of the rate of heat extraction from the compost bed per unit volume of compost $\widetilde{q}_{ext}$.

As described in Section 3.2, the analytical solutions of the temperatures of the compost bed and the medium fluid for the case of the multi-heat-pipe heat exchanger are similar to the approximate solutions for the buried-tube-type heat exchanger, so that $\widetilde{T}_{lou}$ and $\widetilde{q}_{ext}$ may be expressed as follows:

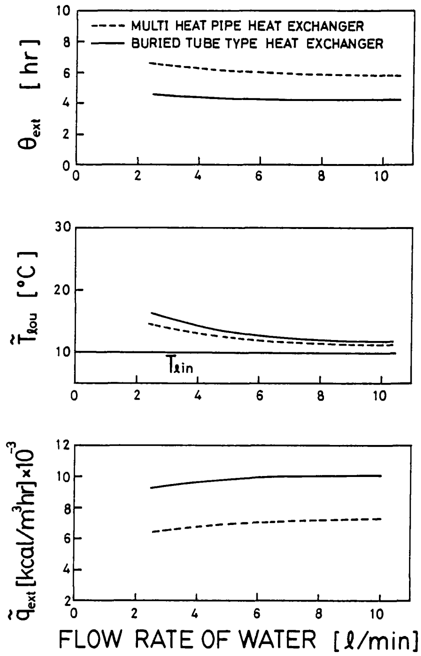


Fig. 5 Calculated results of θ_{ext}, $\widetilde{T}_{lou}$ and $\widetilde{q}_{ext}$ plotted against the flow rate of water under the operating conditions of $T_i=65$°C and $T_{lin}=10$°C, and for the physical properties of the compost bed $K=0.6$kcal/mhr°C, $C_p=0.76$kcal/kg°C and $\rho=700$kg/m^3

$$\frac{\widetilde{T}_{lou}-T_{lin}}{T_i-T_{lin}}=-\frac{2N_h^*B_h^*}{\theta_{fh}}\sum_{n=1}^{\infty}\frac{Z_0(\alpha_n\nu)\left[1-e^{-\alpha_n^2\theta_{fh}}\right]}{\alpha_n^2\left[\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_nB_1(\alpha_n\nu)+B_h^*B_0(\alpha_n\nu)\}\right]}, \tag{33}$$

$$\widetilde{q}_{ext}=\frac{2U_h}{r_1\eta_2^2N_h}(\widetilde{T}_{lou}-T_{lin}), \tag{34}$$

and θ_{ext} is a root of the following equation [Seki and Komori, 1985]:

$$\frac{T_{avf}-T_{lin}}{T_i-T_{lin}}=\frac{4B_h^*}{\nu(\eta_2^2-1)}\sum_{n=1}^{\infty}\frac{Z_1(\alpha_n\nu)\,e^{-\alpha_n^2\theta_{fh}}}{\alpha_n\left[\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_nB_1(\alpha_n\nu)+B_h^*B_0(\alpha_n\nu)\}\right]}. \tag{35}$$

In Fig. 5, the calculated results of θ_{ext}, $\widetilde{T}_{lou}$ and $\widetilde{q}_{ext}$ are plotted against the flow rate of medium fluid at $T_i=65$°C and $T_{lin}=10$°C. From Fig. 5, it is clear that $\widetilde{q}_{ext}$ and $\widetilde{T}_{lou}-T_{lin}$ for the multi-heat-pipe heat exchanger are about 30% smaller than those for the buried-tube-type heat exchanger. On the other hand, the amount of heat possible to extract from the compost bed is proportional to T_i-T_{avf}, and the following relationship between $\widetilde{q}_{ext}$ and θ_{ext} holds in the heat extraction process:

$$C_p\rho V(T_i-T_{avf})=\widetilde{q}_{ext}\,\theta_{ext}. \tag{36}$$

In this illustrative example, θ_{ext} for the multi-heat-pipe heat exchanger is about 50% larger than that for the buried-tube-type heat exchanger.

4.3 A simple calculation method for comparison of the heat extraction ability of the heat exchangers

As mentioned above, in Section 4.2, the items $\widetilde{q}_{ext}$, θ_{ext} and $\widetilde{T}_{lou}$, which are index parameters of the heat extraction ability of the heat exchanger, can be estimated theoretically by Eqs. (33), (34) and (35), however, the calculation of these items by the above equations is very complicated. Only for estimation of guide on heat extraction performance of the heat exchangers, it is advisable to simplify the calculation method for comparison of the heat extraction ability. Then, simplification of the calculation method is described here.

Now, by introducing a factor H similar to a heat conductance, the rate of heat transfer from the compost bed to the wall of the heat-pipe is expressed as follows:

$$Q=2\pi r_1 l_e K\frac{\partial T}{\partial r}\Big|_{r=r_1}=2\pi l_e r_1 H_h (T_{av}-T|_{r=r_1}). \tag{37}$$

By applying the analytical solution Eq. (27) to Eq. (37), H_h is obtained by

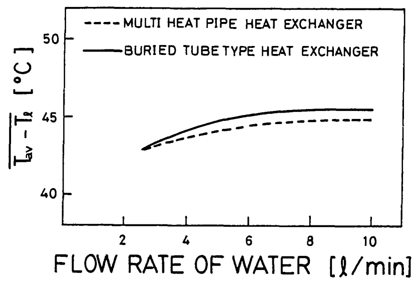


Fig. 7 Comparison of the average value of total temperature difference $\overline{T_{av}-T_l}$ for the multi-heat-pipe heat exchanger with that for the buried-tube-type heat exchanger

$$H_h=\frac{K\frac{\partial T}{\partial r}\Big|_{r=r_1}}{T_{av}-T|_{r=r_1}}=\frac{K}{r_2-r_1}\left[\frac{\sum_{n=1}^{\infty}\frac{\alpha_n Z_1(\alpha_n\nu)\,e^{-\alpha_n^2\Theta}}{\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_n B_1(\alpha_n\nu)+B_h^* B_1(\alpha_n\nu)\}}}{\sum_{n=1}^{\infty}\frac{[2Z_1(\alpha_n\nu)/\{\alpha_n\nu(\eta_2^2-1)\}+Z_0(\alpha_n\nu)]\,e^{-\alpha_n^2\Theta}}{\nu(\alpha_n^2+B_h^{*2})Z_0(\alpha_n\nu)+\nu\eta_2\alpha_n\{\alpha_n B_1(\alpha_n\nu)+B_h^* B_1(\alpha_n\nu)\}}}\right] \tag{38}$$

Similarly, a factor H_b for the buried-tube-type heat exchanger is equivalent to Eq. (38) by changing a subscript h into a subscript b. As is obvious from Eq. (38), H_h depends on time. Fig. 6 shows the changes of H_h and H_b with the dimensionless time Θ. The values of H_h and H_b significantly change during the period from $\Theta=0$ to $\Theta=0.15$, however, these factors become constant for $\Theta>0.25$. From the calculated results, it is suggested that heat penetrates rapidly toward the wall of the heat-pipe or the tube from the compost bed at the beginning of heat extraction. For $\Theta>0.25$ the change of Q with time gradually becomes small, and the heat transfer in the compost bed reaches the quasi-steady state. Therefore, assuming that heat conduction in the compost bed may be maintained at the quasi-steady state approximately, a factor U_c corresponding to a heat conductance of the compost bed may be given by the following equation (Appendix):

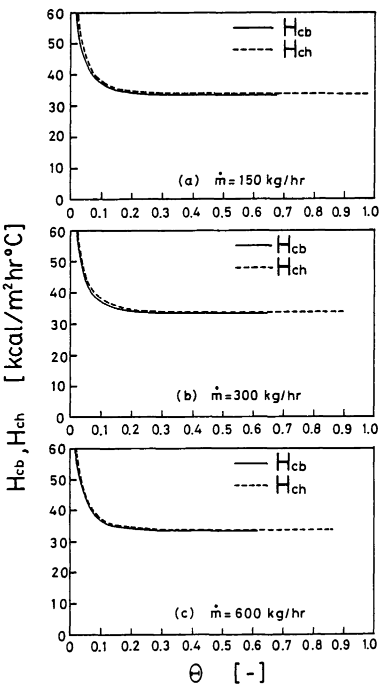


Fig. 6 Changes of factors H_h and H_b with time for the both cases of the multi-heat-pipe and buried-tube-type heat exchangers [(a) at $\dot{m}=150$kg/hr, (b) at $\dot{m}=300$kg/hr, (c) at $\dot{m}=600$kg/hr]

$$U_c=\frac{K(r_2^2-r_1^2)}{r_1\{r_2^2\ln(r_2/r_1)-(r_2^2-r_1^2)/2\}}. \tag{39}$$

The value of U_c is 29.5 kcal/m^{2}hr°C in this illustrative example. According to Fig. 7, the range of H_h or H_b is from 33.3 to 33.6 kcal/m^{2}hr°C for $\Theta>0.25$, and the values of H_h and H_b are almost equal to the value of U_c. Therefore, H_h and H_b may be replaced by U_c for $\Theta>0.25$, and the rate of heat transfer Q for both of the multi-heat-pipe and buried-tube-type heat exchangers may be written by the following simple forms, using a temperature difference $T_{av}-T_l$ as a total driving force of temperature over the heat transfer processes:

$$Q=2\pi r_1 l_e U_{Th}(T_{av}-T_l)$$
for multi-heat-pipe heat exchanger, (40)

$$Q=2\pi r_1 l_e U_{Tb}(T_{av}-T_l)$$
for buried-tube-type heat exchanger, (41)

where U_{Th} and U_{Tb} are defined by total heat conductances over the heat transfer processes, and are expressed as

$$U_{Th}=1/(1/U_h+1/U_c)\ , \quad (42)$$
$$U_{Tb}=1/(1/U_b+1/U_c)\ . \quad (43)$$

From the calculated results, since there is a slight difference in $\overline{T_{av}-T_l}$ between the two exchangers as shown in Fig. 7, the value of $\tilde{q}_{exth}/\tilde{q}_{extb}$ is almost equal to the ratio of the total heat conductance U_{Th}/U_{Tb}. Consequently, U_T is a representative term for estimation of the heat extraction ability of the heat exchanger. At the beginning of the heat extraction process, since H_h or H_b is a function of time and U_T is not available for estimating T_{av} and T_{lou} exactly.

5. Conclusions

The problem of heat extraction from the compost bed with a multi-heat-pipe heat exchanger was solved analytically. The heat extraction ability was calculated numerically by the analytical solutions, using the illustrative examples of heat extraction for the multi-heat-pipe and buried-tube-type heat exchangers. The following results were obtained.

1. For the heat extraction from the compost bed with the multi-heat-pipe heat exchanger, the solutions of the temperature profile in the compost bed and the temperature of water at the outlet of the heat exchanger were obtained, and the solutions were similar to the approximate analytical solutions for the buried-tube-type heat exchanger.

2. For the several operating conditions, the overall heat transfer coefficient U_h of the multi-heat-pipe heat exchanger was about 50% of the overall heat transfer coefficient U_b for the buried-tube-type heat exchanger, and the rate of heat extraction was approximately 30% smaller than that of the buried-tube-type heat exchanger.

3. The heat extraction ability of the two heat exchangers can be estimated by a total heat conductance U_T for $\tilde{q}_{ext}$. An evaluation of the heat transfer ability by U_T is convenient and simple.

4. For the heat extraction from the compost by the multi-heat-pipe heat exchanger, the time required to extract the heat will be larger compared with that for heat extraction by the buried-tube-type heat exchanger. However, a steady composting reaction or methabolism of thermophilic aerobic microorganisms will be maintained during the heat extraction process without a rapid change of the temperature in the compost materials around the heat-pipe.

5. The multi-heat-pipe heat exchanger does not require the troublesome works, removing and retubing the tubes for medium fluid in remixing the compost materials, so that it will be one of the available heat extractors for extraction of heat from the compost bed.

Appendix Derivation of Eq. (39)

For the case of steady state, supposing that $T=T|_{r=r_1}$ at $r=r_1$, $T=T_2$ at $r=r_2$, the temperature profile of a hollow cylinder is (Carslaw and Jaeger, 1959)

$$T=\frac{T_2-T|_{r=r_1}}{\ln(r_2/r_1)}\ln(r/r_1)+T|_{r=r_1}\ . \quad \text{(A-1)}$$

The average temperature of a hollow cylinder is

$$T_{av}=\frac{\int_{r_1}^{r_2}2\pi rT\,\mathrm{d}r}{\int_{r_1}^{r_2}2\pi r\,\mathrm{d}r}$$
$$=\frac{r_2^2\ln(r_2/r_1)-(r_2^2-r_1^2)/2}{(r_2^2-r_1^2)\ln(r_2/r_1)}\{T_2-T|_{r=r_1}\}+T|_{r=r_1}\ . \quad \text{(A-2)}$$

The heat flux at $r=r_1$ is

$$K\frac{\mathrm{d}T}{\mathrm{d}r}\bigg|_{r=r_1}=\frac{K}{r_1\ln(r_2/r_1)}(T_2-T|_{r=r_1})\ . \quad \text{(A-3)}$$

Substitution of Eq. (A-2) into Eq. (A-3) gives

$$K\frac{\mathrm{d}T}{\mathrm{d}r}\bigg|_{r=r_1}=\frac{K}{r_1}\frac{r_2^2-r_1^2}{r_2^2\ln(r_2/r_1)-(r_2^2-r_1^2)/2}(T_{av}-T|_{r=r_1})$$
$$=U_c(T_{av}-T|_{r=r_1})\ . \quad \text{(A-4)}$$

From Eq. (A-4), U_c is

$$U_c=\frac{K(r_2^2-r_1^2)}{r_1\{r_2^2\ln(r_2/r_1)-(r_2^2-r_1^2)/2\}}\ . \quad \text{(A-5)}$$

Nomenclature

B_i = Biot number [−]
C_p = heat capacity of compost bed [kcal/kg°C]
H = factor corresponding to heat conductance of compost bed [kcal/m^2 hr°C]
h = effective heat transfer coefficient of heat-pipe [kcal/m^2 hr°C]
h_c = liquid film heat transfer coefficient at water-heat-pipe interface [kcal/m^2 hr°C]
h_e = heat transfer coefficient at compost-heat-pipe interface [kcal/m^2 hr°C]
h_i = liquid film heat transfer coefficient at water-tube interface [kcal/m^2 hr°C]
h_s = heat transfer coefficient at compost-tube interface [kcal/m^2 hr°C]
K = effective thermal conductivity of compost bed [kcal/mhr°C]
K_w = thermal conductivity of material of tube for medium fluid [kcal/mhr°C]
l_c = length of heat output region of heat-pipe [m]
l_e = length of heat input region of heat-pipe [m]
$\dot{m}$ = mass velocity of medium fluid passing through heat exchanger [kg/hr]
N = number of transfer units [−]
n = number of heat-pipes set in heat exchanger [−]
Q = rate of heat transfer [kcal/hr]
Q_1 = rate of heat transfer from compost bed to heat-pipe [kcal/hr]
Q_2 = rate of heat transfer to axial direction of heat-pipe [kcal/hr]
Q_3 = rate of heat transfer from compost bed to medium fluid [kcal/hr]
$\tilde{q}_{ext}$ = average of rate of heat extraction per unit volume of compost bed [kcal/m^3 hr]
r = distance in radial direction [m]
r_1 = outer radius of heat-pipe [m]
r_2 = apparent radius of compost bed available for heat extraction [m]
T = temperature in compost bed [°C]
T_{av} = average temperature in compost bed [°C]
T_{avf} = average temperature in compost bed at the end of water supply [°C]
T_c = temperature at the wall of cooler portion of heat-pipe [°C]
T_e = temperature at the wall of warmer portion of heat-pipe [°C]
T_i = initial temperature in compost bed [°C]
T_l = temperature of water in heat exchanger [°C]
T_{lin} = temperature of water at the inlet of heat exchanger [°C]
T_{lou} = temperature of water at the outlet of heat exchanger [°C]
$\tilde{T}_{lou}$ = average of T_{lou} over the period of heat extraction [°C]
U = overall heat transfer coefficient [kcal/m^2 hr°C]
U_c = heat conductance of compost bed [kcal/m^2 hr°C]
U_T = total heat conductance for heat extraction process [kcal/m^2 hr°C]
V = volume of compost bed [m^3]
Φ = dimensionless temperature of compost bed [−]
Θ = dimensionless time [−]
θ = time [hr]
θ_{ext} = period available for heat extraction [hr]
κ = effective thermal diffusivity of compost bed [m^2/hr]
ξ = dimensionless distance in r direction [−]
η_2 = dimensionless parameter defined by Eq. (22) [−]
ρ = apparent density of compost bed [kg/m^3]

〈subscripts〉

b = buried-tube-type heat exchanger
h = multi-heat-pipe heat exchanger

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マルチ・ヒートパイプ熱交換器による堆肥発酵熱抽出に関する理論的研究

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要　約

埋設管型熱交換器を用いる方法は，堆肥そうからの熱抽出において典型的で簡単な方法であるが，混合素材の切返しを行う際に埋設管の取り外し，再配管などの煩雑な作業を必要とする。このような煩雑な作業を簡単にするためにマルチ・ヒートパイプ熱交換器を用いたユニークな熱抽出方法を提示し，熱抽出過程における堆肥そう内温度及び熱交換器出口における熱媒体の温度の解析解を導いた。これらの解は埋設管型熱交換器に対する解と数学的に同一であった。そして，解析解に基づき，マルチ・ヒートパイプ熱交換器の熱抽出能力を計算した。