Time-periodic magnetic field analysis with saturation and hysteresis characteristics by harmonic balance finite element method

TIME-PERIODIC MAGNETIC FIELD ANALYSIS WITH SATURATION AND HYSTERESIS CHARACTERISTICS BY HARMONIC BALANCE FINITE ELEMENT METHOD

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Abstract—In this paper we shall further discuss the harmonic balance finite element method (HBFEM) for the time-periodic magnetic field with saturation and hysteresis characteristics and its applications. The HBFEM enables us to calculate the distribution of harmonic magnetic flux at an AC magnetization, and it dose not need the intricate calculation concerned with the time variation. Comparisons between numerical analysis and experimental results are presented.

Introduction

The HBFEM is a new method which deals with time-periodic magnetic field problems. This HBFEM is combined by FEM and harmonic balance method [1], and developed for the purpose of analyzing the time-periodic problems with a saturated core. The distribution of magnetic flux for each harmonic is provided by HBFEM. Since the HBFEM works in the harmonic domain, we do not need to calculate the time variation directly.

In this paper the formulation of the HBFEM is described for the 2-dimensional time-periodic corresponding to any expression of the magnetizing characteristics including the hysteresis, and the comparisons are made between numerical analysis and experimental results. The formulation for the magnetizing characteristics expressed by low-order polynomial functions is discussed. The HBFEM is also suited to an AC magnetic field connected with voltage source and the external circuit. Several examples are given in this paper.

Formulation of HBFEM

As mentioned in the above section, the HBFEM is a new method which is combined by FEM and Harmonic Balance. For simplicity of the formulation, a magnetic field is assumed as two-dimensional in the (x,y) plane, the present problem is quasi-stationary. Therefore, the vector potential A(0,0,A) satisfies the region of interest surrounded with some boundary conditions

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J_0+\sigma\frac{\partial A}{\partial t} \quad (1)$$

where ν and σ are the magnetic reluctivity and the conductivity. The formulation is made by use of the Galerkin procedure. Its integral form is

$$\iint_\Omega\left\{\frac{\partial N_i}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial N_i}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)\right\}dx\,dy$$
$$-\iint_\Omega\left(J_0-\sigma\frac{\partial A}{\partial t}\right)N_i\,dx\,dy=0 \quad (2)$$

where $N_i(x,y)$ is the interpolating function.

We are only interested in the time-periodic solution (the harmonic problem), when an alternating magnetizing current is applied. According to the harmonic balance method all variables, such as vector potentials A, flux densities B and magnetizing current J, are approximated as harmonic solutions, the expression of function is

$$\left.\begin{aligned}
A^e&=\sum_{n=1,3,5,\ldots}\{A_{ns}^e\sin(n\omega t)+A_{nc}^e\cos(n\omega t)\}\\
Bx^e&=\sum_{n=1,3,5,\ldots}\{Bx_{ns}^e\sin(n\omega t)+Bx_{nc}^e\cos(n\omega t)\}\\
By^e&=\sum_{n=1,3,5,\ldots}\{By_{ns}^e\sin(n\omega t)+By_{nc}^e\cos(n\omega t)\}\\
J_0^e&=\sum_{n=1,3,5,\ldots}\{J_{ns}^e\sin(n\omega t)+J_{nc}^e\cos(n\omega t)\}
\end{aligned}\right\} \quad (3)$$

where ω is the fundamental angular frequency.

The magnetizing characteristic of the core can be expressed by an arbitrary function of the flux density B, that is

$$H(B)=H_{sat}(B)+H_{hys}(B)\frac{dB}{dt} \quad (4)$$

where the first term is saturation characteristic, and the second term indicates the hysteresis characteristics [2]. The magnetic reluctivity which is obtained by the Fourier expansion has the form of

$$\begin{aligned}\nu(t)&=H\{B(t)\}/B(t)\\&=\nu_0+\sum_{n=2,4,6,\ldots}\{\nu_{ns}\sin(n\omega t)+\nu_{nc}\cos(n\omega t)\}\end{aligned} \quad (5)$$

where

$$B=(B_x^2+B_y^2)^{1/2} \quad (6)$$

$$\left.\begin{aligned}
\nu_0&=1/T\int_0^T\nu(t)dt\\
\nu_{ns}&=2/T\int_0^T\nu(t)\sin(n\omega t)dt\\
\nu_{nc}&=2/T\int_0^T\nu(t)\cos(n\omega t)dt
\end{aligned}\right\} \quad (7)$$

Substituting Eqs.(3) and (5) into Eq.(2) and equating the coefficients of $\sin(n\omega t)$ and $\cos(n\omega t)$ $(n=1,3,\ldots)$ on both sides according to the harmonic balance method, as a result, the matrix for one element is obtained as follows:

$$\frac{1}{4\Delta}\begin{bmatrix}(b_1b_1+c_1c_1)D & (b_1b_2+c_1c_2)D & (b_1b_3+c_1c_3)D\\(b_2b_1+c_2c_1)D & (b_2b_2+c_2c_2)D & (b_2b_3+c_2c_3)D\\(b_3b_1+c_3c_1)D & (b_3b_2+c_3c_2)D & (b_3b_3+c_3c_3)D\end{bmatrix}\{A^e\}$$
$$+\frac{\sigma\omega\Delta}{12}\begin{bmatrix}2N & N & N\\N & 2N & N\\N & N & 2N\end{bmatrix}\{A^e\}-\{K^e\} \quad (8)$$

where

$$\left.\begin{aligned}
&b_i=y_j-y_k,\quad c_i=x_k-x_j,\quad \Delta:\text{cross section}\\
&\{A^e\}=\{A_{1s}^1\ A_{1c}^1\ A_{3s}^1\ A_{3c}^1,\cdots,\\
&\qquad A_{1s}^2\ A_{1c}^2\ A_{3s}^2\ A_{3c}^2,\cdots,\\
&\qquad A_{1s}^3\ A_{1c}^3\ A_{3s}^3\ A_{3c}^3,\cdots,\}^T
\end{aligned}\right\} \quad (9)$$

$$\{K^{*}\} = \Delta/3\,\{\, J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots,\ J_{1s}\ J_{1c}\ J_{3s}\ J_{3c},\cdots,\}^{T}$$

The coefficients of the block matrix D are determined by only the Fourier coefficients in Eq.(7). The matrix D acts as a reluctivity and is called RELUCTIVITY MATRIX. On the other hand, the matrix N is a constant concerned with harmonic orders and is called HARMONIC MATRIX. The block matrices D and N are given in (3).

The system equation for the entire region is obtained by the same procedure as the conventional FEM, and is solved by the iteration procedure for a nonlinear static magnetic field. When the harmonic components is up to (2m-1) order, the size of the block matrices D and N become 2m x 2m and the order of the system matrix is 2m times bigger than the number of nodes. But it is a sparse and band matrix.

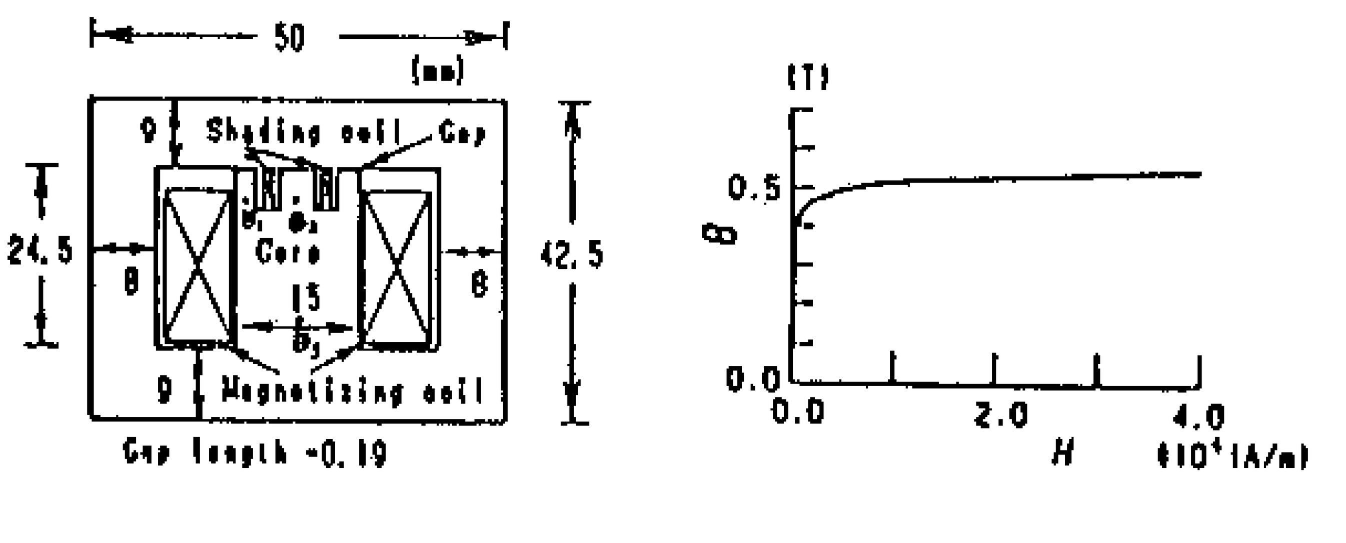
Applications of HBFEM

To illustrate the application of this method, several samples are given as the analysis models in which the eddy-current, saturation and hysteresis characteristics are considered.

Analysis with Eddy-current

Usually the HBFEM is used for the time-periodic magnetic field problems considered with non-linear magnetizing characteristic. The analysis model is given in Fig.1-a, where the shading coil and air gap are applied. Eddy-currents are induced in the shading coil. The magnetic core is Mn-Zn ferrite and its magnetizing characteristic is obtained approximately as follows:

$$\left.\begin{array}{ll} |B|<0.400 : & H=90.5B+1.58\times 10^{3}B^{5} \\ 0.400<|B|<0.435 : & H=\pm 50.0+1.43\times 10^{3}(B\mp 0.400) \\ 0.435<|B|<0.460 : & H=\pm 100.+4.00\times 10^{3}(B\mp 0.435) \\ 0.460<|B|<0.490 : & H=\pm 200.+1.00\times 10^{4}(B\mp 0.460) \\ 0.490<|B|<0.510 : & H=\pm 500.+2.00\times 10^{4}(B\mp 0.490) \\ 0.510<|B|<0.520 : & H=\pm 900.+6.00\times 10^{4}(B\mp 0.510) \\ 0.520<|B| : & H=\pm 1500.+1.00\times 10^{5}(B\mp 0.520) \end{array}\right\} \quad (10)$$



1-a 1-b

Fig.1 Analysis model and approximated magnetizing curve

The analysis region is a half of the cross section. The magnetizing characteristic is shown in Fig.1-b. The number of nodes and triangle element are 195 and 336 respectively. The parameters for the calculation are given as follows:

$$J_{1s}=1.0\times 10^{6}\,A/m^{2} \qquad \sigma=3.55\times 10^{7}\,S/m$$
$$J_{3s}=-5.4\times 10^{4}\,A/m^{2} \qquad f=180Hz. \qquad (11)$$
$$J_{1c}=J_{3c}=J_{5c}=J_{5s}=0$$

In this problem, the fundamental, third and fifth order harmonics are considered in the numerical analysis. The equi-potential lines of the harmonics are drawn at phase t=0,60,90 respectively, as shown in Fig.2, where the values of the third and fifth harmonics are enlarged by 2.5 and 10 times respectively. In this case the numerical error is appeared, because the third and fifth harmonics are too small.

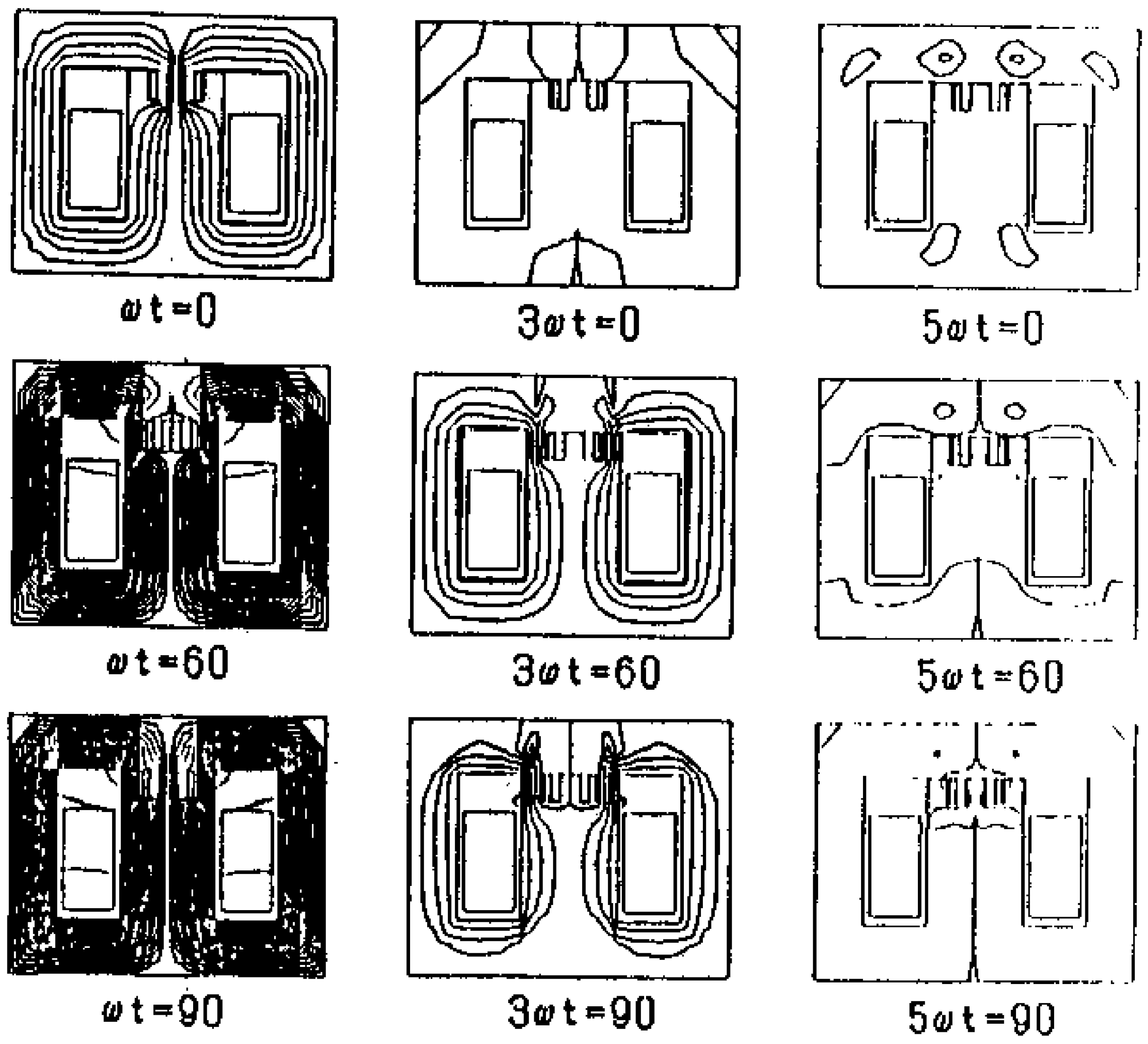
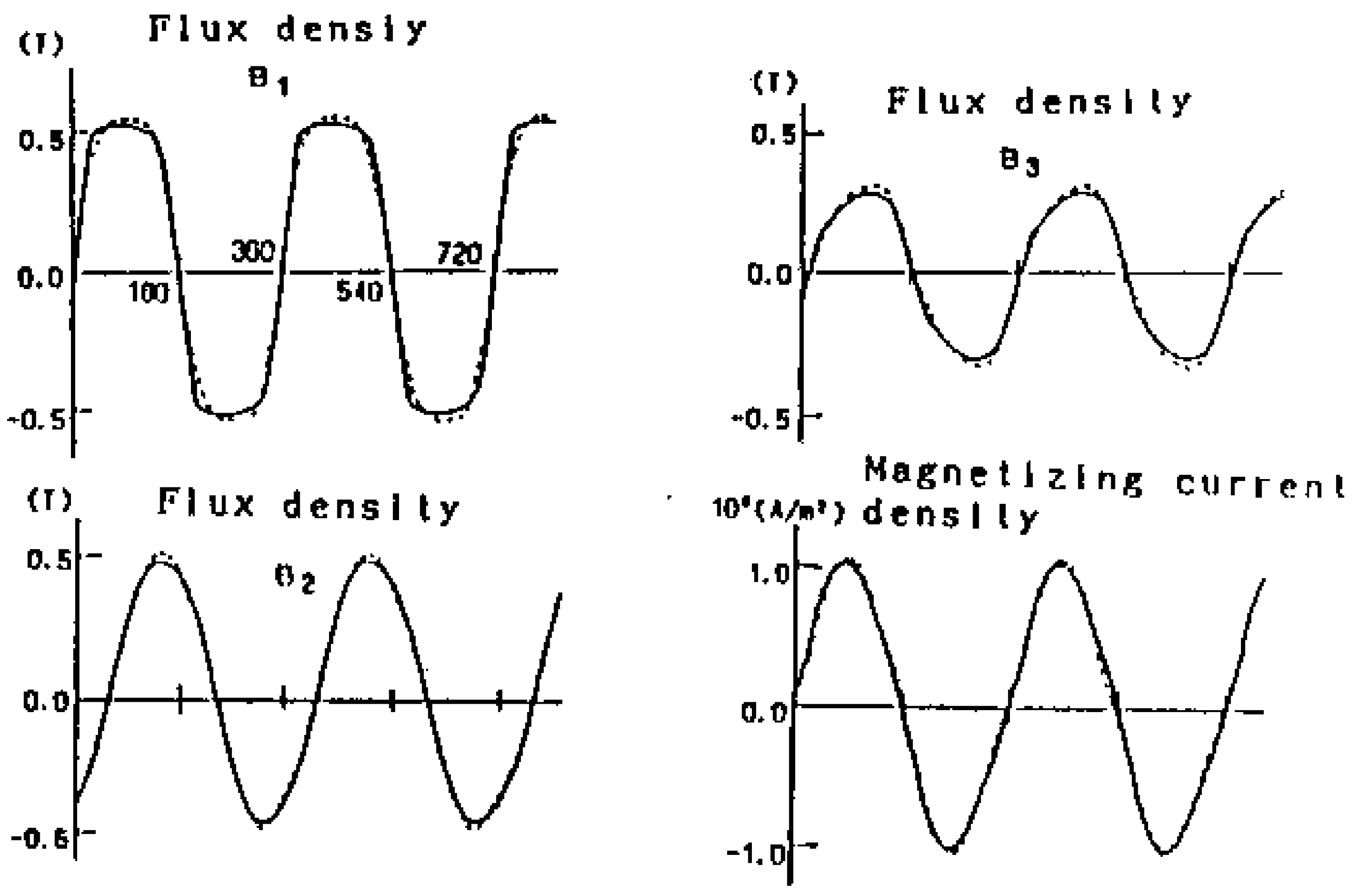


Fig.2 Distribution of Flux at 180 Hz

Fig.3 shows the waveform of the flux density and the magnetizing current, and the comparisons are made between numerical analysis and experimental results. The numerical analysis shows a reasonable agreement. In fig.1, B1 and B2 indicate the flux densities inside and outside the shading coil. B3 is the flux density in the middle of leg. The magnetic flux is delayed inside the shading coil and the waveform is sinusoidal. The flux density outside the shading coil turns to the saturating state, and then the third harmonic component is induced.



.... analysis results
—— experimental results

Fig.3 Waveform of flux density and magnetizing current

Analysis with Voltage Sources and External Circuits

Magnetic field is usually connected to the voltage sources and the external circuit which contains resistance, inductance and capacitance. The HBFEM is also suitable to solve such kind of problems. For instance, a typical magnetic frequency tripler is shown in Fig.4, which contains three-phase commercial power sources as an input, single-phase with 3 times commercial frequency as an output and external circuit connected to magnetic field.

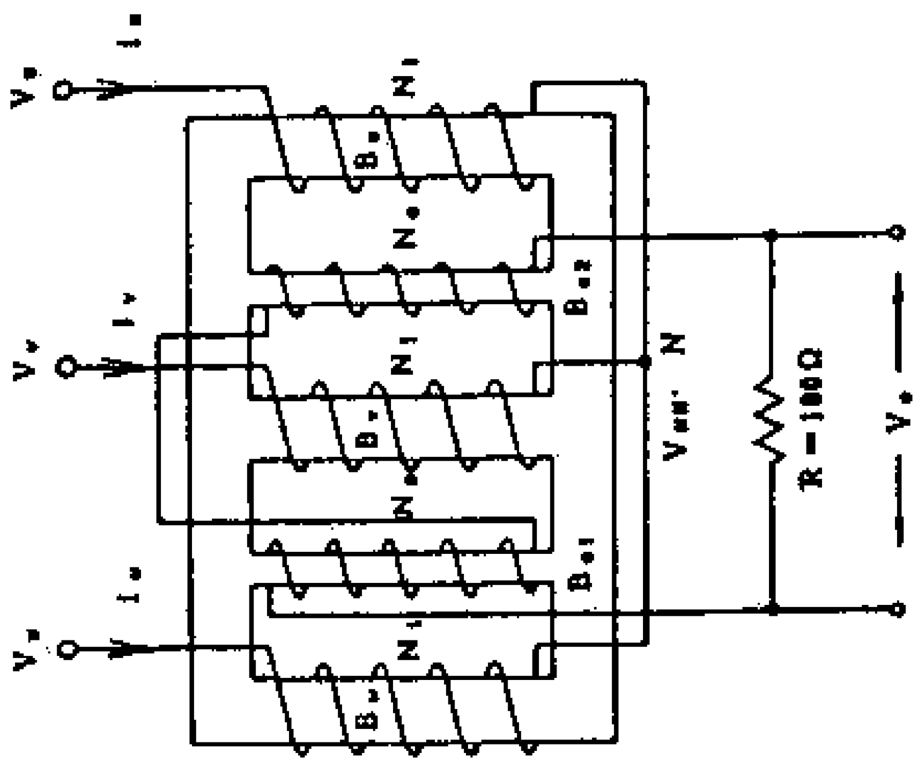


Fig.4 Magnetic frequency tripler

To this problem an approximated magnetizing characteristic is given as follows:

$$H = 100B + 40.4B^{9} \tag{12}$$

Based on Faraday's and Kirchhoff's laws, the matrix equation for this problem can be obtained as follows:

$$\begin{bmatrix} [H] & -[G_u] & -[G_v] & -[G_w] & -[G_2] & 0 \\ [C_u] & S_{cu}[Z_u] & 0 & 0 & 0 & [I] \\ [C_v] & 0 & S_{cv}[Z_v] & 0 & 0 & [I] \\ [C_w] & 0 & 0 & S_{cw}[Z_w] & 0 & [I] \\ [C_2] & 0 & 0 & 0 & S_{c2}[Z_2] & 0 \\ 0 & S_{cu}[I] & S_{cv}[I] & S_{cw}[I] & 0 & 0 \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{J_u\} \\ \{J_v\} \\ \{J_w\} \\ \{J_2\} \\ \{V_{NN'}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{V_u\} \\ \{V_v\} \\ \{V_w\} \\ \{0\} \\ \{0\} \end{Bmatrix} \tag{13}$$

where S_c, [Z] and [I] are the area of each coil, the impedance matrix of circuit and unit matrix respectively.

$$\begin{aligned} \{V_u\} &= \{V_{u1s} \quad V_{u1c} \quad V_{u3s} \quad V_{u3c} \quad \cdots\} \\ \{V_v\} &= \{V_{v1s} \quad V_{v1c} \quad V_{v3s} \quad V_{v3c} \quad \cdots\} \\ \{V_w\} &= \{V_{w1s} \quad V_{w1c} \quad V_{w3s} \quad V_{w3c} \quad \cdots\} \\ \{V_{NN'}\} &= \{V_{1sNN'} \quad V_{1cNN'} \quad V_{3sNN'} \quad V_{3cNN'} \quad \cdots\} \end{aligned} \tag{14}$$

$$[C_k] = \frac{\omega d_o \Delta}{3 S_{ck}} \begin{bmatrix} N & N & N \end{bmatrix} \tag{15}$$

where d_o is the depth of model in Z-direction.

$$[G_k] = \Delta / 3 \tag{16}$$

We use 1/2 cross section of magnetic field for analyzed region, where the number of nodes and triangle element are 340 and 594 respectively. The results of the waveform of voltage, current and flux density are given in Fig.5.

The distribution of flux in the core is shown in Fig.6, where the distribution of third harmonic flux can be clearly observed.

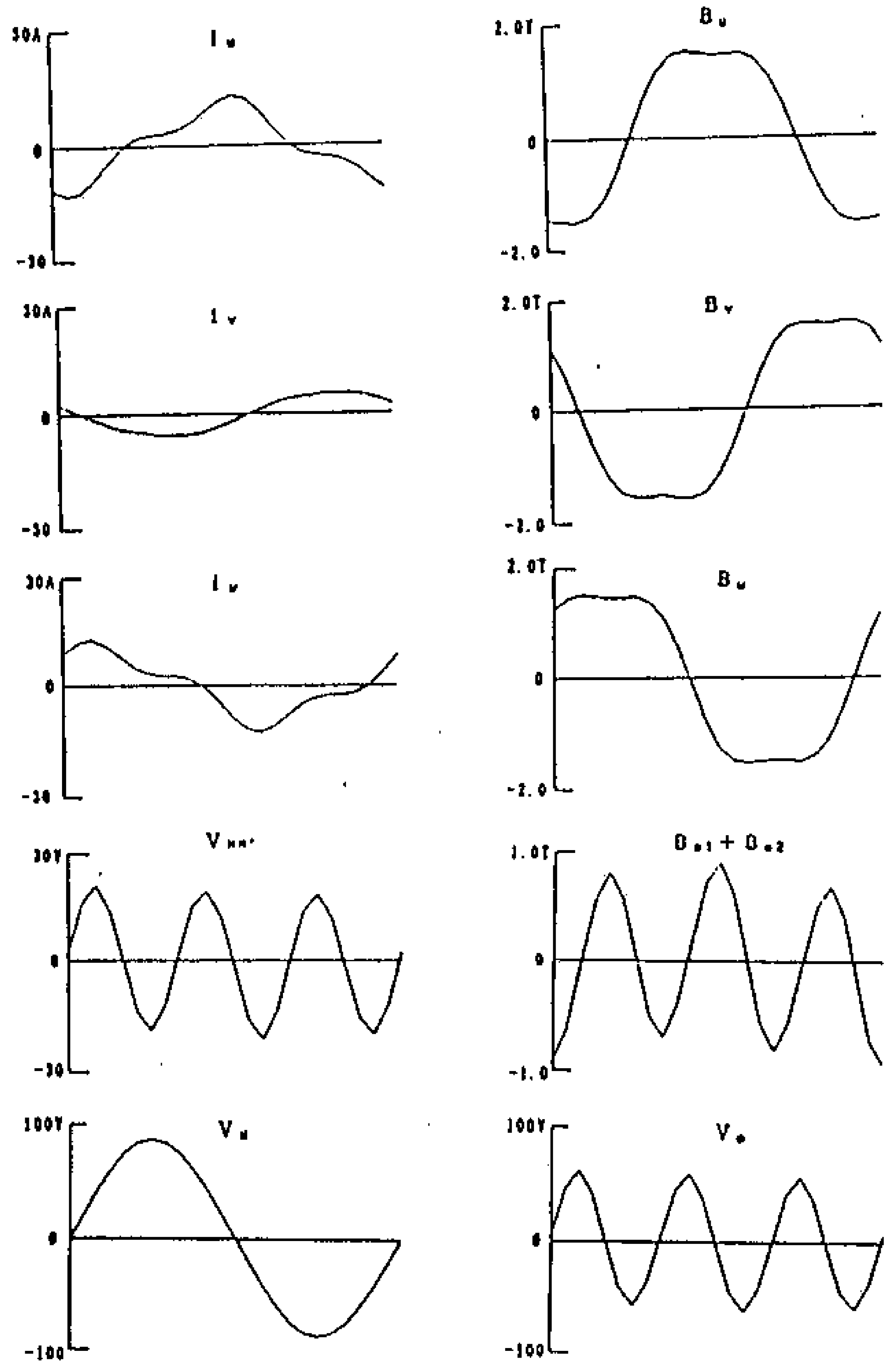


Fig.5 Results of numerical analysis

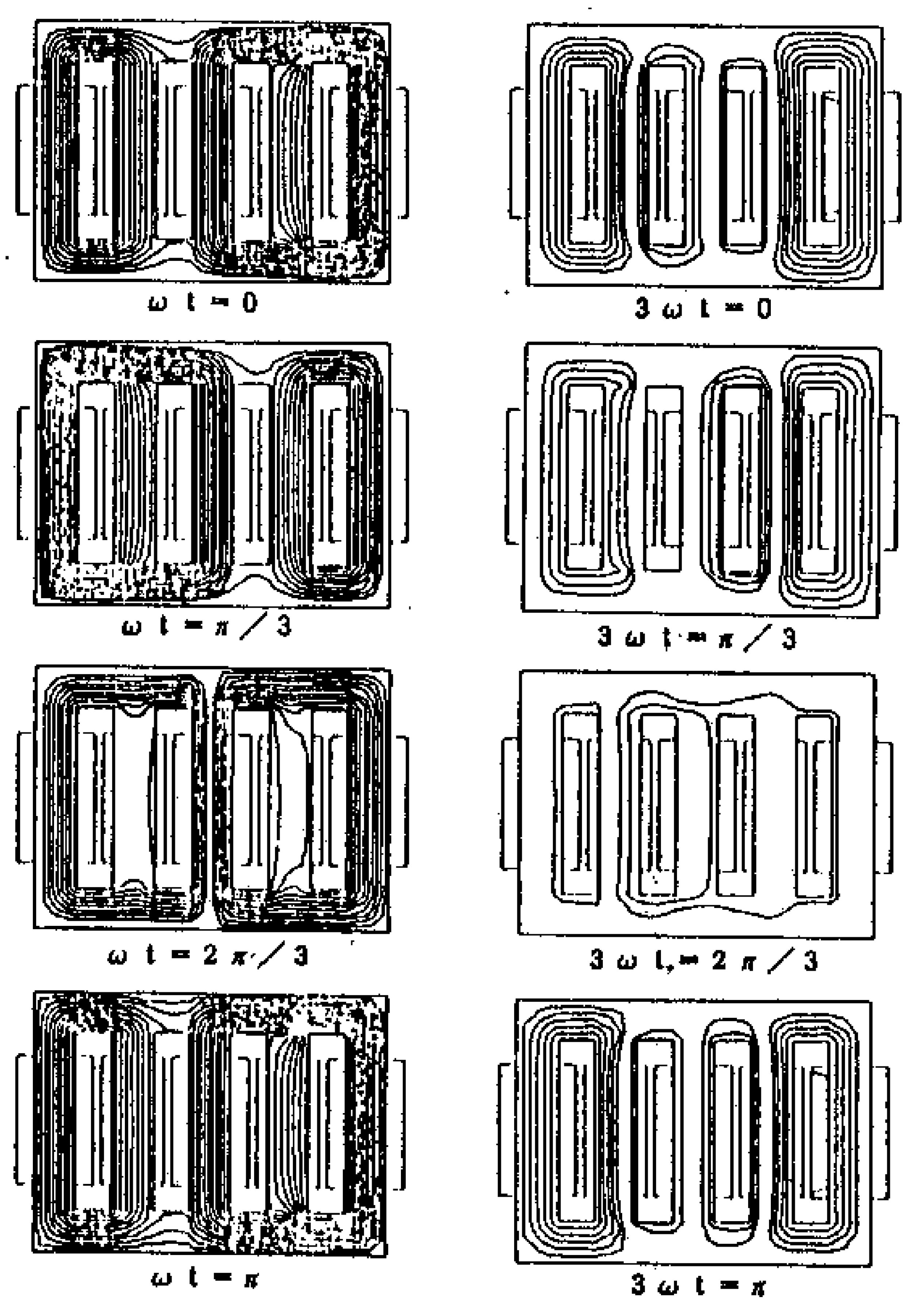


Fig.6 Distribution of flux

Analysis with Hysteresis Characteristic

A simple Magnetic reactor is used for an analysis model as shown in Fig.7. The magnetizing curve with hysteresis characteristics is expressed as shown in Fig.8. As the calculation of time component is not considered, the flux distribution for each harmonic component can be directly obtained by HBFEM. Evidently, it is convenient to solve such problems which the effect of hysteresis is considered. Fig.9 shows the flux distributions to each harmonic component. The waveform of flux density near the T-connection of the core is shown in Fig.10. The rotating fields are observed in the corner of the core, as shown in Fig.11.

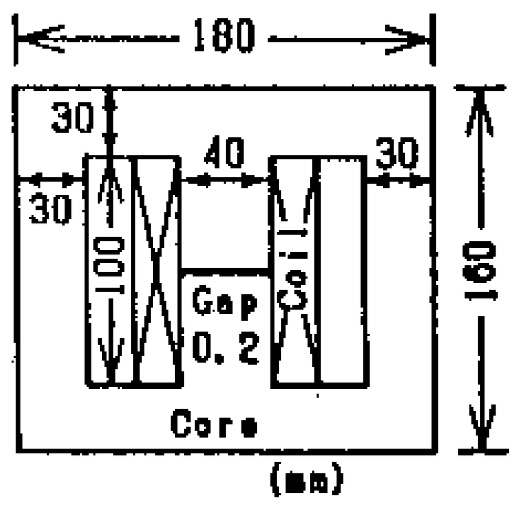


Fig.7 Analysis model

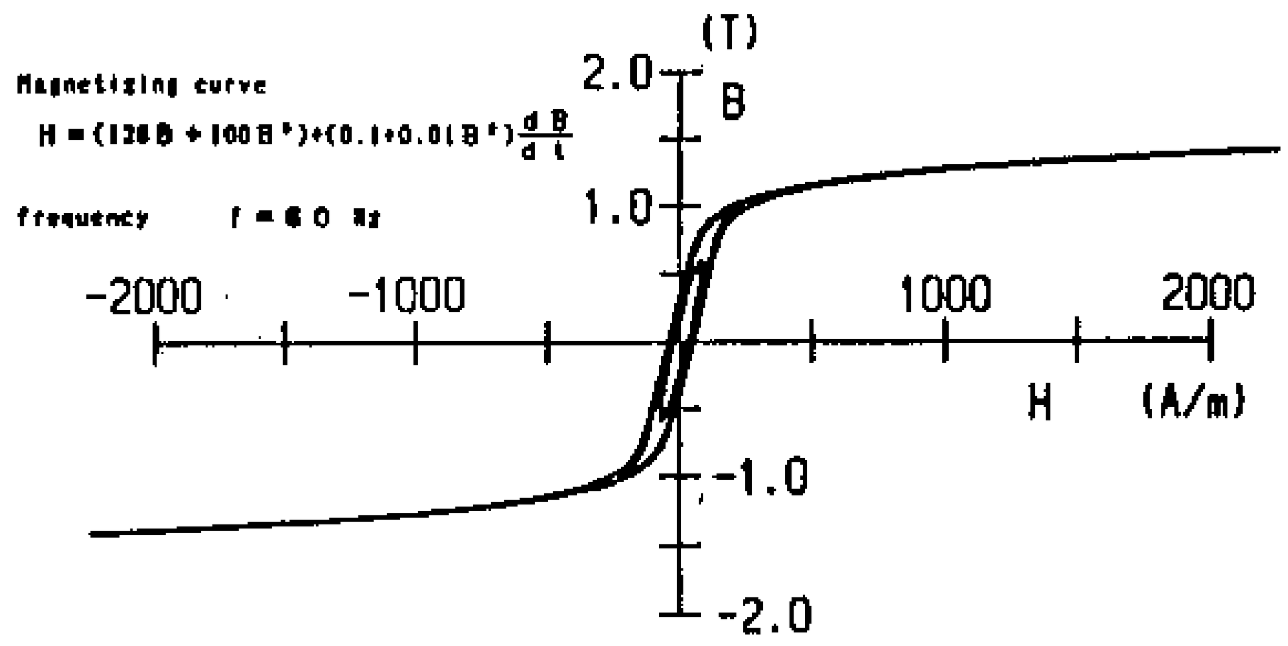


Fig.8 B-H curve of saturated core with hysteresis

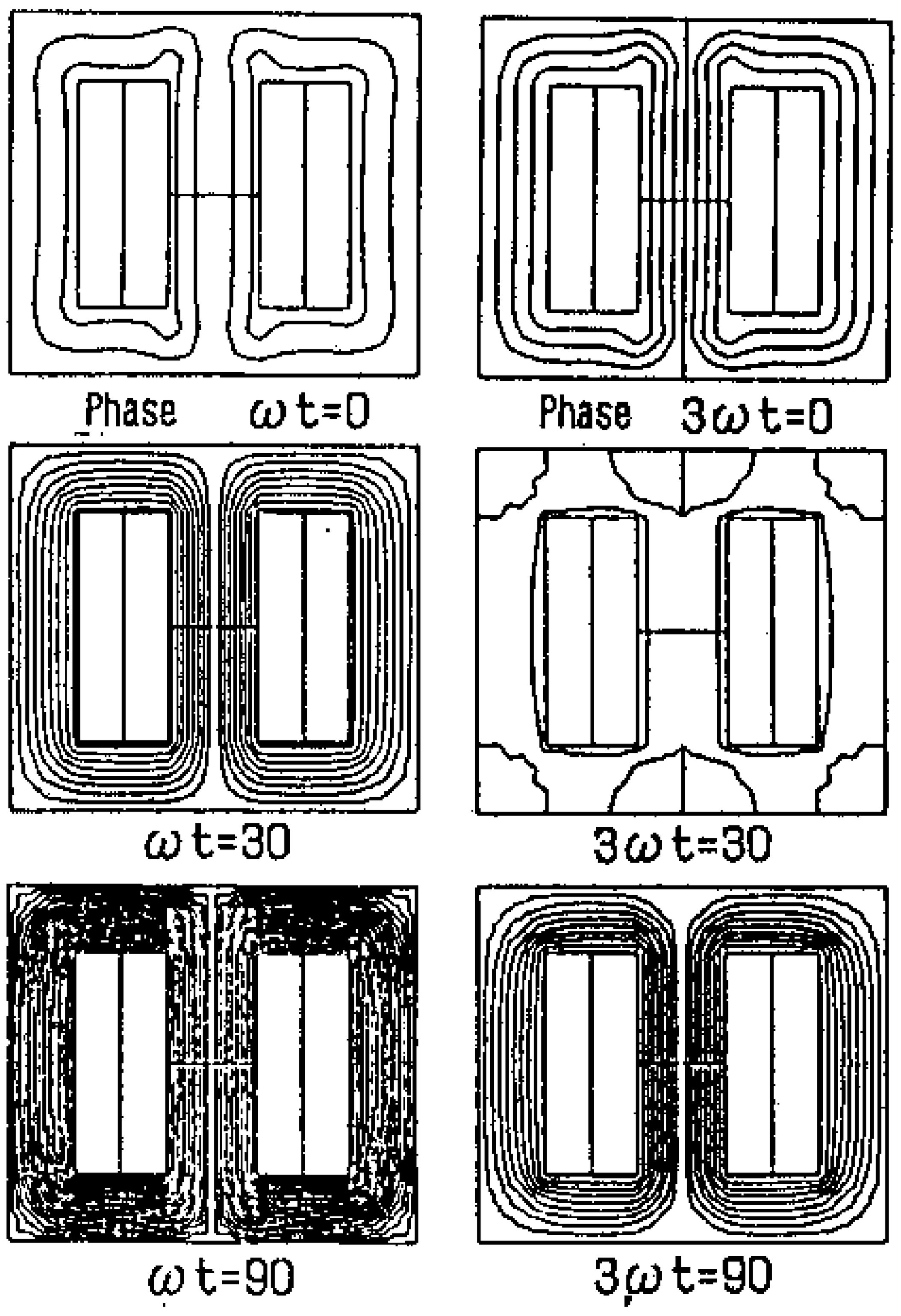


Fig.9 Flux distributions to each harmonic component

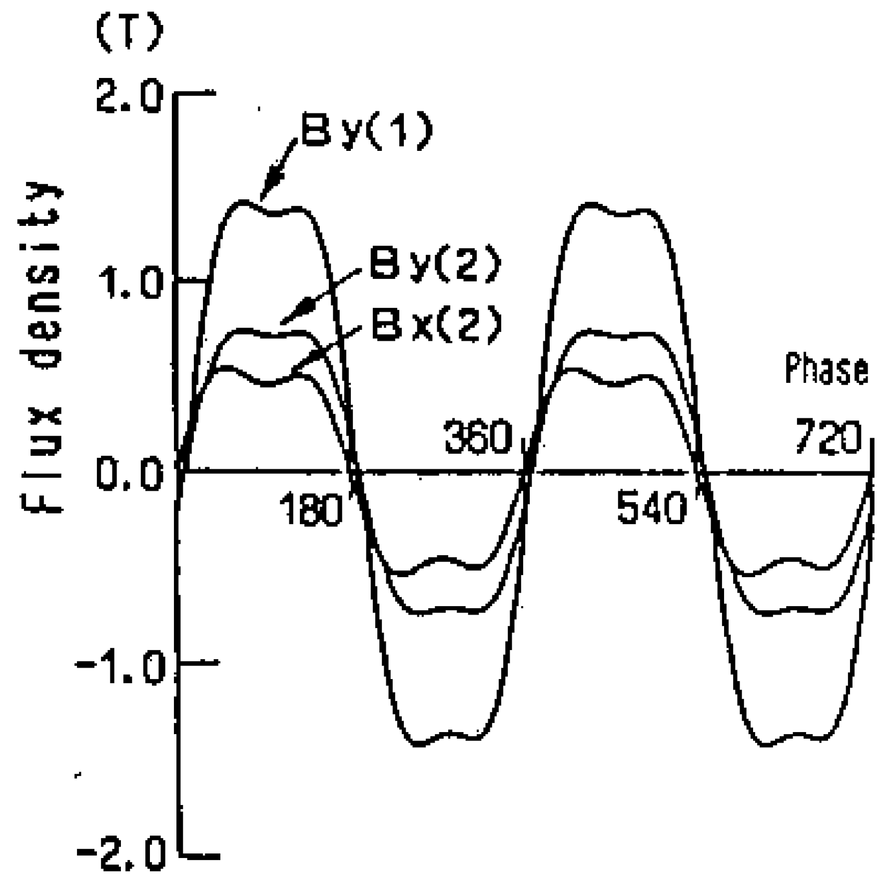


Fig.10 Waveform of flux density

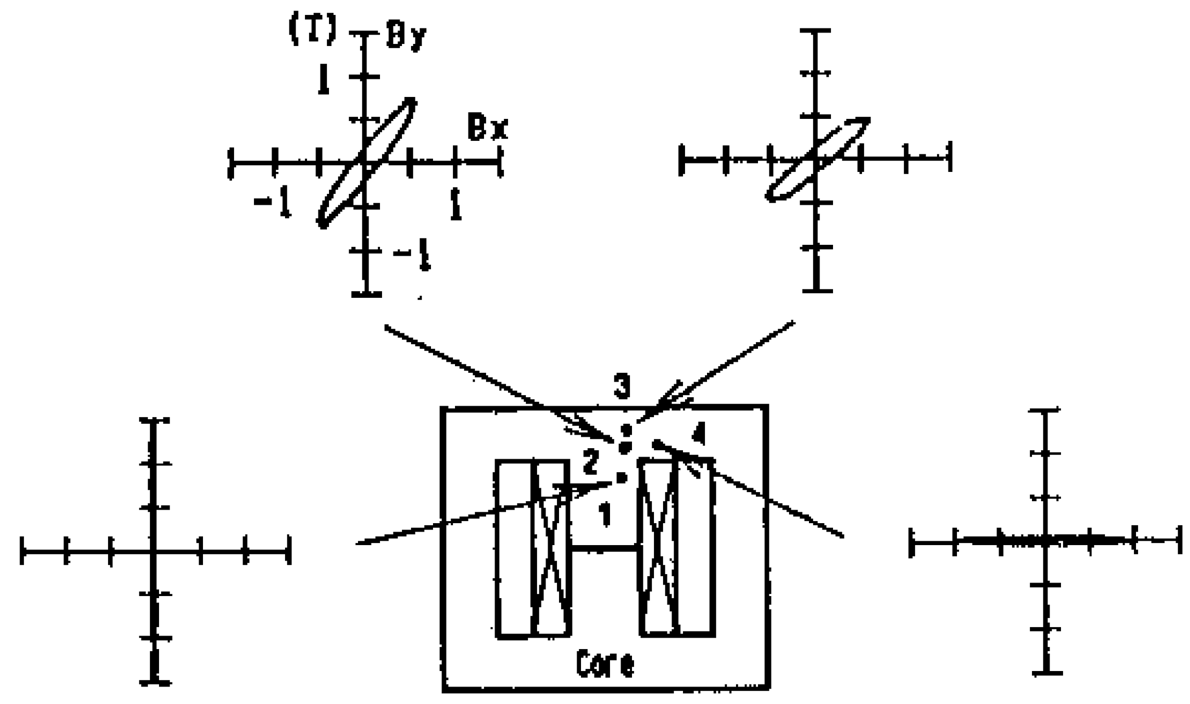


Fig.11 Vector loci of rotating field

Conclusions

From the above analysis of time-periodi magnetic field with saturation and hysteresi characteristics the conclusions can be drawn

The comparisons between numerica analysis and experimental results show reasonable agreement.

It is efficient to use HBFEM fo designing Tripler and some special hybri high speed motor connected with power sourc and external circuit, in which the thir harmonic components play an important role.

It should be pointed that the analysi considered with hysteresis characteristi needs further investigation.

References

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