Washout Control for Manual Operations

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Abstract*— It is known that limitations of human accuracy in manual manipulation hinder the quality of work performed by human operators of manual control systems. Indeed, movements of operators are apt to cause undesirable vibrations in manual control systems. In this paper, we propose a new operator-support-control scheme for suppressing harmful oscillatory motions in such systems without disturbing human operator's manipulation. The proposed scheme is based on the fact that steady-state blocking zeros of a feedback controller do not affect the steady-state control input. A finite-dimensional feedback controller with steady-state blocking zeros, called a *washout controller* in this paper, plays the central role in support for operator's manipulation. However, the dynamics of a manual control system may become different significantly from its initial model used for the design of an initial washout controller when it is applied to the manual control system. Such difference can result in poor performance of operator-support-control. In order to improve it, an iterative procedure is presented for re-design of washout controllers based on closed-loop subspace identification. Closed-loop identification is performed to brush up the model for the control design, and then a more sophisticated washout controller is obtained using the identified model. The effectiveness of the proposed scheme is demonstrated by an experiment on manual control of an inverted pendulum.

I. Introduction

This paper concerns operator-support-control for a class of manual control systems. The operator-support-control is useful to human operators for manipulation of unstable objects. In manual control systems, a crucial issue underlies undesirable vibration caused by quivering movements of human operators, due to limitations of human accuracy in manual manipulation [1]–[3]. Therefore, it is important to suppress undesirable vibration effectively during manual manipulation in order to improve control performance and to reduce operators' workload.

In operator-support-control, it is important not to hinder manual manipulation intended by human operators. There are several methods of suppressing harmful vibration in manual control systems without disturbing manual manipulation, a feedback configuration of delayed feedback control [4] or washout filters [5]–[7]. An attractive feature of these schemes is that the feedback controller has steady-state blocking zeros. It is known that a feedback controller with steady-state blocking zeros do not affect the steady-state control input. In other words, such a controller can eliminate a steady-state bias which can disturb operator's manipulation [8].

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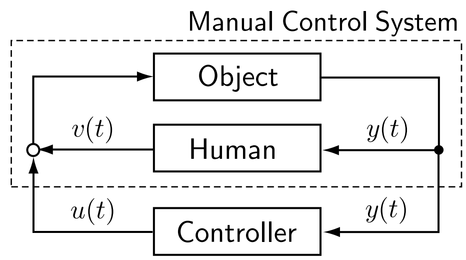

Fig. 1. Manual control system with the proposed operator-support controller.

In [8], an operator-support-control scheme has been proposed based on the technique used in delayed feedback control. Although it unveils the effectiveness of the steady-state blocking zeros, the design procedure of the operator-support-control scheme is burdensome due to infinite-dimensionality of the closed-loop system which is inherited from delayed feedback control (see [9] and [10] for the detailed design method for delayed feedback controllers).

In this paper, we propose a new control scheme which is a finite-dimensional feedback controller with steady-state blocking zeros, called a *washout controller*. The proposed washout controller is a generalization of washout filter aided controllers proposed in [5]–[7]. The proposed operator-support-control scheme is that a washout controller is added to a manual control system as an auxiliary feedback loop (see, Fig. 1). However, it is difficult to obtain a good initial model of a manual control system used for the design of a washout controller due to uncertainty and inherent nonlinearity of human operation. Therefore, the initial model of a manual control system may contains large modeling errors. To overcome it, we will adopt the iterative procedure as follows: we will carry out closed-loop identification of the manual control system compensated by the initial washout controller, in order to obtain a better model of the manual control system used for the re-design of a washout controller. Particularly, we will use the SSARX method [11] for closed-loop identification. It is known as one of closed-loop subspace identification methods. This paper is the revised version of [12], in which the manual control system is identified as a continuous-time model. In this paper, it will be identified as a discrete-time model.

To demonstrate the effectiveness of the proposed operator-support-control scheme, an experiment on manual control of an inverted pendulum is performed. The system has a pendulum attached to a moving cart whose movement is controlled by human operator's manipulation such that the pendulum actively balanced standing. The result of the experiment demonstrates that the proposed scheme successfully suppresses undesirable vibration in the manual control system.

A. Notation

Throughout this paper, we use the following notation. When a matrix $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ is divided into submatrices to form a square product with a matrix N such as $M_{22}N$, the lower linear fractional transformation of M and N is defined by

$$\mathcal{F}_l(M,N) := M_{11} + M_{12}N(I - M_{22}N)^{-1}M_{21},$$

if $|I - M_{22}N| \neq 0$.

II. Problem Statement

A. Manual Control Systems

Suppose the manual control system depicted in the dashed rectangle in Fig. 1, can be described by the nonlinear differential equation

$$\begin{array}{rcl} \dot{x}(t) & = & f(x(t), u(t)), \\ y(t) & = & g(x(t), u(t)), \end{array} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^r$ is the auxiliary input vector for operator-support-control, and $y(t) \in \Re^m$ is the measured output. f and g are assumed to be unknown smooth functions. The task of a human operator is to maintain the output $y(t)$ within a neighborhood of his/her desirable operating point $\tilde{y}$ by means of the regulation of the amount of operation $v(t) \in \Re^r$.

On the manual control system, we make several assumptions as follows. The operating point $\tilde{y}$ is assumed to be a constant which is determined by the human operator. Assume that any numerical information of $\tilde{y}$ is not available a priori for operator-support-control since anyone but the human operator cannot understand it. Moreover, it is assumed that there exist a equilibrium point of the manual control system (1) for $u(t) \equiv 0$, $v(t) \equiv \tilde{v}$, and $y(t) \equiv \tilde{y}$. That is, there exists a unique $\tilde{x}$ such that

$$\begin{array}{l} 0 = f(\tilde{x}, 0), \\ \tilde{y} = g(\tilde{x}, 0). \end{array}$$

B. Operator-Support-Control

A crucial issue on the manual control system is that the measured output $y(t)$ may vibrate due to limitations of human accuracy and the nonlinear nature of the human operator. Our goal is to suppress the vibration. To this end, we will introduce an operator-support-control scheme for manual control by means of feedback connection of an automatic controller to the manual control system (see Fig. 1). Let the automatic controller be represented as the following state space model:

$$\begin{array}{rcl} \dot{w}(t) & = & \hat{A}w(t) + \hat{B}y(t), \\ u(t) & = & \hat{C}w(t) + \hat{D}y(t), \end{array} \tag{2}$$

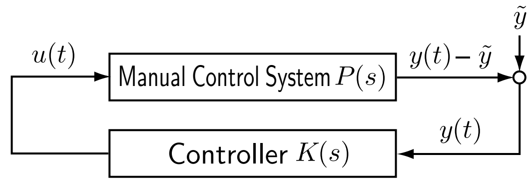

Fig. 2. Closed-loop system of the linearized system $P(s)$ around the unknown operating point $\tilde{y}$ in the manual control system and the proposed operator-support controller $K(s)$.

where $w(t) \in \Re^{\hat{n}}$ is the state vector of the controller and the control input $u(t)$ is applied to the manual operation $v(t)$.

The automatic controller is designed to locally stabilize the unknown equilibrium point $\tilde{x}$ in the manual control system (1) without changing its equilibrium point $\tilde{x}$. In other words, the stabilization should be achieved by unbiased input, i.e. $u(t) = 0$ when $y(t) - \tilde{y} = 0$. When the closed-loop system is stabilized, if the control input converges to a non-zero $\tilde{u}$, the human operator is forced to change his/her desired operation $\tilde{v}$ to $\tilde{v} - \tilde{u}$. In this way, the human operator is required to compensate the manual operation in order to eliminate the biased input $\tilde{u}$. This additional requirement imposes a burden of the human operator. To avoid it, operator-support-control should be designed such that $\lim_{t\to\infty} u(t) = 0$ and $\lim_{t\to\infty} y(t) = \tilde{y}$. From the assumption, however, the reference $\tilde{y}$ is not available for any automatic controller. Hence, the controller is required to stabilize the uncertain operating point. This may be satisfied if the coefficients of the controller satisfy the condition that there exists a $\tilde{w}$ such that

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} \begin{bmatrix} \tilde{w} \\ \tilde{y} \end{bmatrix},$$

for any constant $\tilde{y}$.

C. Stabilization of Unknown Operating Point

A salient feature of our control problem is stabilization of the unknown operating point. The feedback controller (2) is designed so that the unknown $\tilde{y}$ is locally asymptotically stabilized. The design is carried out based on the linearlized model of the manual control system. In the vicinity of the unknown $\tilde{y}$, the manual control system is linearized as

$$\begin{array}{rcl} \delta\dot{x}(t) & = & A\delta x(t) + Bu(t), \\ y(t) & = & C\delta x(t) + Du(t) + \tilde{y}, \end{array} \tag{3}$$

where $\delta x(t) = x(t) - \tilde{x}$ and

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \partial f(\tilde{x},0)/\partial x & \partial f(\tilde{x},0)/\partial u \\ \partial g(\tilde{x},0)/\partial x & \partial g(\tilde{x},0)/\partial u \end{bmatrix}.$$

(The parameters of the state-space model will be identified by methods in Section IV.) Define $P(s) = D + C(sI - A)^{-1}B$. Then, the closed-loop system of $P(s)$ and (2) is depicted in Fig. 2, where $K(s) = \hat{D} + \hat{C}(sI - \hat{A})^{-1}\hat{B}$. For the closed-loop system, the unknown operating point $\tilde{y}$ can be regarded as a step disturbance. Hence, the problem of stabilizing the unknown operating point $\tilde{y}$ can be cast into that of disturbance attenuation of the closed-loop system.

An important inherent property of controllers to stabilize unknown operating points is that their transfer functions have blocking zeros at zero frequency.

Definition 1: A transfer function matrix $K(s)$ (resp. $K(z)$ for discrete-time systems) is said to have a *steady-state blocking zero* if $K(s) = 0$ at $s = 0$ (resp. $K(z) = 0$ at $z = 1$).

If a feedback controller $K(s)$ with a steady-state blocking zero stabilizes the closed-loop system, the steady-state input always satisfies that $\lim_{t\to\infty} u(t) = 0$. Because, it follows from the final value theorem that

$$\begin{aligned}\lim_{t\to\infty} u(t) &= \lim_{s\to 0} sU(s)\\ &= \lim_{s\to 0} s\{I - K(s)P(s)\}^{-1}K(s)\tilde{Y}(s)\\ &= \{I - K(0)P(0)\}^{-1}K(0)\tilde{y}. \qquad (4)\end{aligned}$$

Therefore, the biased input can be removed by any controller with $K(0) = 0$ which stabilizes the manual control system.

Similarly, in discrete-time setting, blocking zeros play an important role in stabilization of an unknown fixed point of discrete-time systems. When we consider a linearized system around an unknown fixed point as

$$\begin{aligned}\delta x(k+1) &= A\delta x(k) + Bu(k),\\ y(k) &= C\delta x(k) + Du(k) + \tilde{y},\end{aligned} \qquad (5)$$

we stabilize it by a discrete-time controller $K(z)$ with its state-space realization

$$\begin{aligned}w(k+1) &= \hat{A}w(k) + \hat{B}y(k),\\ u(k) &= \hat{C}w(k) + \hat{D}y(k).\end{aligned} \qquad (6)$$

If $K(z)$ stabilizes (5) and has a steady-state blocking zero, then the steady-state input always satisfy $\lim_{k\to\infty} u(k) = 0$. Because,

$$\begin{aligned}\lim_{k\to\infty} u(k) &= \lim_{z\to 1}(1 - z^{-1})U(z)\\ &= \lim_{z\to 1}(1 - z^{-1})\{I - K(z)P(z)\}^{-1}K(z)\tilde{Y}(z)\\ &= \{I - K(1)P(1)\}^{-1}K(1)\tilde{y}. \qquad (7)\end{aligned}$$

D. Washout Control

It is well-known that delayed feedback control utilizes the difference between the current output and the delayed output to eliminate the biased input in the steady-state. In fact, it was applied to the operator-support-control problem in [8]. The transfer function of the delayed feedback controller can be factored $K(s) = K(s)'(1 - e^{-Ts})$ where $K(s)'$ denotes a real rational transfer function. Hence, it has a steady-state blocking zero. Contrast to the simple controller structure, the design procedure is complicated for practical use. In [5], to stabilize the continuous-time system with the unknown equilibrium point, a certainty equivalence adaptive control scheme was proposed. In [6] and [7], a design method of a washout filter aided feedback controller was discussed. The continuous-time washout filter aided feedback controller and the discrete-time one satisfy

$$\det \hat{A} \neq 0,\ \hat{B} = -\hat{A},\ \hat{D} = -\hat{C}, \qquad (8)$$

and

$$\det(I - \hat{A}) \neq 0,\ \hat{B} = I - \hat{A},\ \hat{D} = -\hat{C}, \qquad (9)$$

respectively. The washout filter is a high-pass filter which is added to a feedback controller in order to eliminate the biased input in the steady-state. However, (8) and (9) are sufficiant conditions for the finite-dimensional controller with a steady-state blocking zero. These conditions are very conservative.

In this paper, we propose less conservative conditions for finite-dimensional controllers with a steady-state blocking zero. These controllers are shown in the following.

Definition 2: A continuous-time controller with a state-space realization (2) is called a *washout controller* if it satisfies the conditions

$$\det \hat{A} \neq 0,\ \hat{D} - \hat{C}\hat{A}^{-1}\hat{B} = 0. \qquad (10)$$

Definition 3: A discrete-time controller with a state-space realization (6) is called a *discrete-time washout controller* if it satisfies the conditions

$$\det(I - \hat{A}) \neq 0,\ \hat{D} + \hat{C}(I - \hat{A})^{-1}\hat{B} = 0. \qquad (11)$$

In fact, the conditions (10) and (11) respectively satisfy (8) and (9).

Lemma 1: If the feedback controller (2) is the washout controller, then it has a steady-state blocking zero.

Proof: It is obvious that the feedback controller (2) with the condition (10) satisfies $K(s) = \hat{D} + \hat{C}(sI - \hat{A})^{-1}\hat{B} = 0$ at $s = 0$. ■

Lemma 2: If the feedback controller (6) is a discrete-time washout controller, then it has the steady-state blocking zero.

Proof: Since the controller (6) with the condition (11) satisfies $K(z) = \hat{D} + \hat{C}(zI - \hat{A})^{-1}\hat{B} = 0$ at $z = 1$, we complete the proof. ■

III. Full Order Parameterization of Washout Controllers

In this section, we will give a full order parameterization of washout controllers. The term ‘full order’ implies that the order of the controller is the same as that of the plant.

On the plants, we make an assumption of controllability and observability in this section. It can be relaxed to stabilizability and detectabiliy.

A. Continuous-time Washout Controller

Let us suppose that a state-space realization of a continuous-time linear system be given as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t) + Du(t).\end{aligned} \qquad (12)$$

Then, we have the following theorem for the full-order continuous-time washout controller [12].

Theorem 1: There exists a continuous-time washout controller stabilizing (12) if and only if A is nonsingular. Moreover, when the above condition holds, a set of stabilizing washout controllers is given by

$$\begin{bmatrix} \hat{A} & \hat{B}\\ \hat{C} & \hat{D}\end{bmatrix} = \mathcal{F}_l(J, Q),\quad Q = -FA^{-1}L, \qquad (13)$$

where

$$J = \left[\begin{array}{cc} \left[\begin{array}{cc} A+BF+LC+LDF & -L \\ F & 0 \end{array} \right] & \left[\begin{array}{c} B+LD \\ I \end{array} \right] \\ \left[\begin{array}{cc} -C-DF & I \end{array} \right] & -D \end{array} \right], \tag{14}$$

and $(F,\ L) \in \mathcal{S} :=\{(F,\ L)|\ A+BF$ and $A+LC$ are asymptotically stable, and $I+QD$ is nonsingular.$\}$.

Proof: For notational brevity, we will introduce the notations as follows: If the closed-loop system with the controller (2) is well-posed (i.e., $I-D\hat{D}$ is invertible), it can be described as

$$\left[\begin{array}{c} \dot{x}(t) \\ \dot{w}(t) \end{array} \right] = A_c \left[\begin{array}{c} x(t) \\ w(t) \end{array} \right], \tag{15}$$

where

$$\begin{aligned} A_c &= \left[\begin{array}{cc} A_{c11} & A_{c12} \\ A_{c21} & A_{c22} \end{array} \right] \\ &:= \left[\begin{array}{cc} A+B(I-\hat{D}D)^{-1}\hat{D}C & B(I-\hat{D}D)^{-1}\hat{C} \\ \hat{B}(I-D\hat{D})^{-1}C & \hat{A}+\hat{B}(I-D\hat{D})^{-1}D\hat{C} \end{array} \right]. \end{aligned}$$

(Necessity) If the controller (2) stabilizes the closed-loop system and it is a washout controller, then $\det(I-D\hat{D}) \neq 0$, $\det \hat{A} \neq 0$ and $\hat{D}-\hat{C}\hat{A}^{-1}\hat{B}=0$. Hence, we have

$$\begin{aligned} \det A_{c22} &= \det[\hat{A}\{I+\hat{A}^{-1}\hat{B}(I-D\hat{C}\hat{A}^{-1}\hat{B})^{-1}D\hat{C}\}] \\ &= \det[\hat{A}(I-\hat{A}^{-1}\hat{B}D\hat{C})^{-1}] \\ &\neq 0, \end{aligned} \tag{16}$$

and

$$\begin{aligned} A_{c12}A_{c22}^{-1}A_{c21} &= B(I-\hat{D}D)^{-1}(I-\hat{D}D)\hat{D}(I-D\hat{D})^{-1}C \\ &= B(I-\hat{D}D)^{-1}\hat{D}C. \end{aligned}$$

Therefore,

$$\begin{aligned} \det A_c &= \det A_{c22} \det[A_{c11}-A_{c12}A_{c22}^{-1}A_{c21}] \\ &= \det A_{c22} \det A. \end{aligned} \tag{17}$$

Since the closed-loop system is asymptotically stable, A_c does not have any zero eigenvalues. Hence, $\det A_c \neq 0$. Therefore, from (16) and (17), $\det A \neq 0$.

(Sufficiency) Under the condition that $\det A \neq 0$, we will show that the controller given by (13) is a washout controller which makes (15) asymptotically stable. Let us suppose that F and L be chosen from $\mathcal{S}$. Then, $I+QD$ is nonsingular, and the feedback controller is given by

$$\begin{aligned} \left[\begin{array}{cc} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{array} \right] &= \mathcal{F}_l(J,Q) \\ &= \left[\begin{array}{cc} R_1-R_2(I+DQ)^{-1}DR_3 & R_2(I+DQ)^{-1} \\ (I+QD)^{-1}R_3 & Q(I+DQ)^{-1} \end{array} \right], \end{aligned} \tag{18}$$

where $R_1=A+BF+LC-BQC$, $R_2=BQ-L$, and $R_3=F-QC$. By using $\hat{A},\ \hat{B},\ \hat{C},\ \hat{D}$ in (18), we have

$$(I-\hat{D}D)^{-1}=I+QD, \quad (I-D\hat{D})^{-1}=I+DQ,$$

and

$$\begin{aligned} A_{c11} &= A+B(I+QD)Q(I+DQ)^{-1}C = A+BQC, \\ A_{c12} &= B(I+QD)(I+QD)^{-1}R_3 = BR_3, \\ A_{c21} &= R_2(I+DQ)^{-1}(I+DQ)C = R_2C, \\ A_{c22} &= R_1-R_2(I+DQ)^{-1}DR_3 \\ &\quad +R_2(I+DQ)^{-1}(I+DQ)D(I+QD)^{-1}R_3 \\ &= R_1. \end{aligned}$$

Moreover, a similarity transformation of A_c with the matrix $T=\left[\begin{array}{cc} I & 0 \\ I & I \end{array} \right]$ is given by

$$T^{-1}A_cT = \left[\begin{array}{cc} A+BF & BF-BQC \\ 0 & A+LC \end{array} \right].$$

Since $A+BF$ and $A+LC$ are stable, so is A_c. Now, we define $R_4=A-LDF$, then it is invertible because

$$\begin{aligned} \det R_4 &= \det A \det(I-A^{-1}LDF) = \det A \det(I+QD) \\ &\neq 0. \end{aligned}$$

Hence,

$$\begin{aligned} &\left[\begin{array}{cc} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{array} \right] \\ &= \left[\begin{array}{cc} (A+BF)R_4^{-1}(A+LC) & -(A+BF)R_4^{-1}L \\ FR_4^{-1}(A+LC) & -FR_4^{-1}L \end{array} \right]. \end{aligned} \tag{19}$$

Then, from (19), we have

$$\det \hat{A} = \det[(A+BF)R_4^{-1}(A+LC)] \neq 0,$$

and

$$\hat{D}-\hat{C}\hat{A}^{-1}\hat{B} = -FR_4^{-1}L+FR_4^{-1}L=0.$$

Therefore, we conclude that (13) gives a continuous-time washout controller. ■

Since poles of the closed-loop system are included among the eigenvalues of $A+BF$ and $A+LC$ in the case of the washout controller, the pole of the closed-loop system can be arbitrarily placed by selecting matrices F and L.

B. Discrete-time Washout Controller

In this subsection, let us suppose that a state-space realization of a discrete-time linear system be given as

$$\begin{array}{rcl} x(k+1) &=& Ax(k)+Bu(k), \\ y(k) &=& Cx(k)+Du(k). \end{array} \tag{20}$$

Then, we have the following theorem for the full-order discrete-time washout controller [12].

Theorem 2: There exists a discrete-time washout controller stabilizing (20) if and only if $I-A$ is nonsingular. Moreover, when the above condition holds, a set of stabilizing discrete-time washout controller is given by

$$\left[\begin{array}{cc} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{array} \right] = \mathcal{F}_l(J,Q'), \quad Q'=F(I-A)^{-1}L, \tag{21}$$

where J is given by (14) and $(F,L) \in \mathcal{S}' := \{(F,L)|\ A+BF$ and $A+LC$ are asymptotically stable, and $I+Q'D$ is nonsingular.$\}$.

Proof: The proof of this theorem is very similar to that of Theorem 1, and hence it is omitted. ■

Remark 1: When $\det(I-A) \neq 0$ and $\det(I+Q'D) \neq 0$, $R := I - A + LDF$ is invertible, because

$$\begin{aligned}\det R &= \det(I-A)\det[I+(I-A)^{-1}LDF]\\ &= \det(I-A)\det(I+Q'D)\\ &\neq 0.\end{aligned}$$

Then, we have

$$\begin{aligned}&\hat{A} = I - (I-A-BF)R^{-1}(I-A-LC),\\ &\hat{B} = -(I-A-BF)R^{-1}L,\\ &\hat{C} = FR^{-1}(I-A-LC), \quad \hat{D} = FR^{-1}L.\end{aligned}$$

IV. Tuning of Washout Controllers for Manual Operations

In this section, we will propose an iterative design method of washout controllers for operator-support-control. Since washout controllers are implemented on digital computers, we design a discrete-time washout controller by using a discrete-time linearized model of manual control systems.

To design a washout controller assisting in manual operations, we need a discrete-time linearized model of the manual control system around the operating point. In general, however, modeling of the manual control system is difficult. Hence, using input/output data observed from the manual control system, system identification seem promising to obtain a linear model of the manual control system in the vicinity of its operating point.

First, we will obtain a linear model as an initial model via open-loop identification. Note that a relatively small number of data in the vicinity of the operating point are available for system identification since, without any operator support controller, the measured output of the manual control system oscillates significantly. Therefore, the initial model may contains large modeling errors, and such modeling errors can lead to poor performance of the initial controller.

To reduce the effect of modeling errors, closed-loop identification is implemented to brush up the model of the manual control system for the control design. Since the vibration is suppressed by the washout control in closed-loop setting, there are enough data to identify the model of the manual control system. In this paper, we will adopt a closed-loop subspace identification method, i.e., the SSARX method proposed by Jansson [11].

A. *SSARX method* [11]

The SSARX method can be categorized into the direct approach to closed loop identification. In this method, it is assumed that a linear system can be described in the innovations form by the following state space realization:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ke(k),\\ y(k) &= Cx(k) + Du(k) + e(k), \end{aligned} \tag{22}$$

where $e(k) \in \Re^m$ is the innovation. Define $\tilde{A} = A - KC$ and $\tilde{B} = B - KD$, and assume that $\tilde{A}$ is stable. From (22), we have

$$\begin{aligned} y_s(k) &= \tilde{\mathcal{O}}x(k) + \tilde{\mathcal{D}}u_s(k) + \tilde{\mathcal{K}}y_s(k) + e_s(k),\\ &= \tilde{\mathcal{O}}\tilde{\mathcal{L}}p(k) + \tilde{\mathcal{D}}u_s(k) + \tilde{\mathcal{K}}y_s(k) + v_s(k), \end{aligned} \tag{23}$$

where

$$\begin{aligned} y_s(k) &= [y(k)^{\mathrm{T}} \cdots y(k+s-1)^{\mathrm{T}}]^{\mathrm{T}},\\ u_s(k) &= [u(k)^{\mathrm{T}} \cdots u(k+s-1)^{\mathrm{T}}]^{\mathrm{T}},\\ e_s(k) &= [e(k)^{\mathrm{T}} \cdots e(k+s-1)^{\mathrm{T}}]^{\mathrm{T}},\\ p(k) &= [y(k-1)^{\mathrm{T}} \cdots y(k-s)^{\mathrm{T}} u(k-1)^{\mathrm{T}} \cdots u(k-s)^{\mathrm{T}}]^{\mathrm{T}},\\ v_s(k) &= \tilde{\mathcal{O}}\tilde{A}^s x(k-s) + e_s(k), \end{aligned}$$

$$\tilde{\mathcal{O}} = \begin{bmatrix} C \\ C\tilde{A} \\ \vdots \\ C\tilde{A}^{s-1} \end{bmatrix}, \ \tilde{\mathcal{D}} = \begin{bmatrix} D & & & \\ C\tilde{B} & D & & \\ \vdots & & \ddots & \\ C\tilde{A}^{s-2}\tilde{B} & \cdots & C\tilde{B} & D \end{bmatrix},$$

$$\tilde{\mathcal{L}} = \begin{bmatrix} K & \tilde{A}K & \cdots & \tilde{A}^{s-1}K & K & \tilde{B} & \tilde{A}\tilde{B} & \cdots & \tilde{A}^{s-1}\tilde{B} \end{bmatrix},$$

$$\tilde{\mathcal{K}} = \begin{bmatrix} 0 & & & \\ CK & 0 & & \\ \vdots & & \ddots & \\ C\tilde{A}^{s-2}K & \cdots & CK & 0 \end{bmatrix}.$$

Then, the state $x(t)$ is approximated by $\tilde{x}(k) := \tilde{\mathcal{L}}p(k)$. Moreover, when s is sufficiently large, (23) is constructed by stacking ARX models. The procedure of the SSARX method is shown as follows:

1) Estimate a high order ARX model from observed data $u,\ y$.
2) Estimate $\tilde{\mathcal{D}}$ and $\tilde{\mathcal{K}}$ from coefficients of the estimated high order ARX model. Let $\hat{\tilde{\mathcal{D}}}$ and $\hat{\tilde{\mathcal{K}}}$ denote estimates of $\tilde{\mathcal{D}}$ and $\tilde{\mathcal{K}}$, respectively.
3) From (23), we have
   $$z(k) := y_s(k) - \hat{\tilde{\mathcal{D}}}u_s(k) - \hat{\tilde{\mathcal{K}}}y_s(k) \approx \tilde{\mathcal{O}}\tilde{\mathcal{L}}p(k) + v_s(k).$$
   By performing canonical correlation analysis on $z(k)$ and $p(k)$, we obtain $\hat{\tilde{\mathcal{L}}}$ which is the estimate of $\tilde{\mathcal{L}}$. Then, the estimate of $\tilde{x}(k)$ is given by $\hat{\tilde{x}}(k) = \hat{\tilde{\mathcal{L}}}p(k)$.
4) Estimate $(A,\ B,\ C,\ D,\ K)$ and the innovation $e(k)$ from the estimated state $\hat{\tilde{x}}(k)$ and (22).

B. *Tuning of Washout Controllers*

Closed-loop identification can be iteratively performed to obtain better models of the manual control system and washout controllers. Now, we summarize a tuning procedure of the washout controller for manual operations via closed-loop identification. The procedure is shown as follows:

1) Identify the initial discrete-time linear model of the manual control system $P_0(z) = D_0 + C_0(zI - A_0)^{-1}B_0$ via system identification. $i := 0$.
2) Design the discrete-time washout controller $K_i(z) = \hat{D}_i + \hat{C}_i(zI - \hat{A}_i)^{-1}\hat{B}_i$. Then, the washout controller $K_i(z)$ are derived from (21) by using the identified model $P_i(z)$ and matrices F and L chosen from $\mathcal{S}'$.
3) Apply the discrete-time washout controller $K_i(z)$ to the manual control system. Go to 4) if necessary.
4) $i := i + 1$. Identify the discrete-time model of the manual control system under applying the discrete-time washout controller $K_{i-1}(z)$. Then, it is given by $P_i(z) = D_i + C_i(zI - A_i)^{-1}B_i$ via the SSARX method. Go to 2).

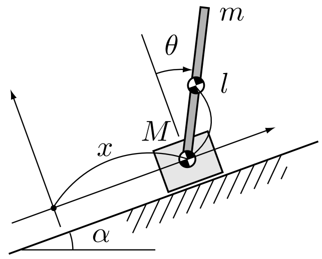

Fig. 3. Inverted pendulum system.

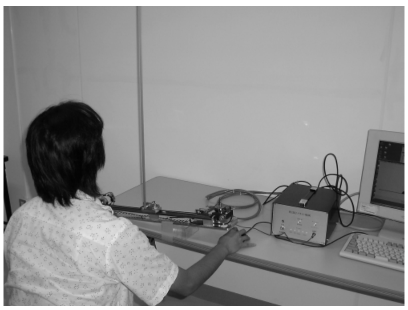

Fig. 4. View of experimental setup.

V. Experiment with Inverted Pendulum System

To demonstrate the effectiveness of the proposed technique, we applied it to manual control of the inverted pendulum system with an inclined rail shown in Fig. 3. The task of a human operator in the manual control system was to stabilize the pendulum by controlling the cart. Here are parameters of the inverted pendulum system. The length from the joint to the gravity center of the pendulum l is 0.5 m, the mass of the pendulum m is 0.056 kg, the mass of the cart M is 0.235 kg and the angle of the rail α is 0.18 rad. In the experiment, these parameters are assumed to be unknown. In this system, the observed output is the angle of the pendulum $\theta(t)$, and the operating point $\tilde{y}$ is equal to the angle of the rail α. The operator used a mouse as an input device in the experiment to move the cart right or left (see Fig. 4). Moreover, the sampling time is 20 ms, and the operator-support controller is implemented on a digital computer.

Fig. 5 shows the time responses of the angle of the pendulum $\theta(t)$, the control input by the human operator $v(t)$, and the control input by the washout controller $u(t)$. Fig. 5 (a) is a result obtained with only manual control, whereas Fig. 5 (b) has the results for manual control with the initial washout controller, and (c) has the results for manual control with the controller after tuning. Although the upright state of the pendulum is maintained, as can be seen from Fig. 5 (a), vibration occurs. Moreover, there is a great deal of vibration in the control input due to manual manipulation. On the other hand, the washout controller suppresses these vibrations (see Fig. 5 (b) and (c)). Therefore, by using our method, the control input in manual manipulation is suppressed and the load for manual operation is reduced. Moreover, the washout controller after tuning can suppress the vibration in the manual control system to be more effective than the initial controller.

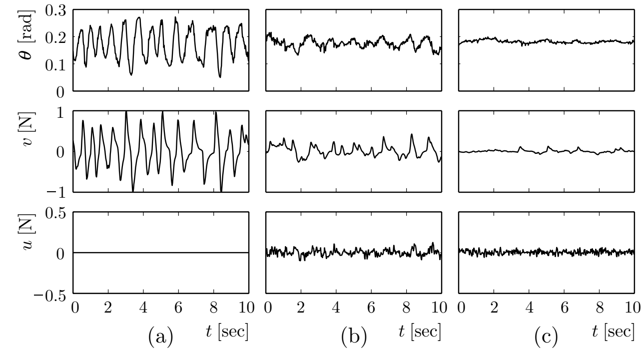

Fig. 5. Time responses for the inverted pendulum system with (a) only manual control, (b) manual control with the initial washout controller, and (c) manual control with the tuned washout controller.

VI. Conclusion

In this paper, we have proposed a new operator-support-control scheme for suppressing harmful vibration in manual control systems without disturbing human operator's manipulation. As support control for human operator, we have proposed a washout controller which is a finite-dimensional feedback control method with steady-state blocking zeros, and showed that such controller can suppress undesirable vibration in a manual control system. Moreover, we have proposed tuning technique of washout control for manual operations via closed-loop identification. The proposed washout controller is effective to construct the support system for the human operator.

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