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# Electron Cyclotron Instability in a Spiraling Electron Beam -Plasma System

Sunao KAWASAKI and Teiichiro YAGI

*Department of Physics, Faculty of Science, Kanazawa University*

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**Abstract** A problem of an electron cyclotron wave excitation in a bounded plasma immersed in a magnetic field by a spiraling electron beam drifting along the field is discussed. In the quasi-elctrostatic and electromagnetic approximations, relatively simple dispersion relations are obtained in the limit of small beam intensity.

## §1. Introduction

A beam-plasma interaction has been extensively studied in recent years related to the excitation of cyclotron waves in a plasma with a longitudunal homogeneous magnetic field. However, relatively less attention is paid to the problem of the beam-plasma system with a rotating electron stream, i.e. beam with a considerable portion of the transversal energy except some theoretical and experimental researches.[1),2)] This paper describes the result of the theoretical study in the range of a linear approximation of the plasma-electron layer composed of the spiraling beam in a radially finite geometry. The situation corresponds to the conditions of our experiment being done at present and the result of which will be published in near future.

Kusse and Bers[1)] treated a similar problem in an electrostatic mode taking account for a finite temperature of the plasma, although they used a simpler model in which they "unwrapped" the essential cylindrical geometry to get a planar model. Here the equations will be solved with the boundary conditions probably more appropriate to the real experimental state. The effect of the plasma temperature is neglected in this work and a cold plasma approximation is in use. In §2, the excitation of the electrostatic wave in a plasma is discussed by the dispersion relation of the beam-plasma system. The disperion relation in the electromagnetic mode is given in §3.

## §2. Electrostatic Mode

### 2・1 Fundamental equations

At present the high frequency oscillations should be studied and the plasma ions are supposed to be rest to neutralize the plasma and beam electrons as a background.

The basic equations appear in the first order after the linearization as follows (here the c.g.s. Gauss units are used throughout this work):

1) Maxwell equations

$$\mathrm{rot}\boldsymbol{E}=-\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \qquad (2.1.1)$$

$$\mathrm{div}\boldsymbol{H}=0 \qquad (2.1.2)$$

$$\mathrm{rot}\boldsymbol{H}=\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}+\frac{4\pi}{c}\boldsymbol{j} \qquad (2.1.3)$$

$$\mathrm{div}\boldsymbol{E}=4\pi\rho \qquad (2.1.4)$$

where the current density $\boldsymbol{j}$ and the charge density $\rho$ are given as

$$\boldsymbol{j}=-e(n_{b0}\boldsymbol{v}+n_b\boldsymbol{v}_0+n_{p0}\boldsymbol{u}) \qquad (2.1.5)$$

and

$$\rho=-e(n_b+n_p). \qquad (2.1.6)$$

All notations have usual meanings and $\boldsymbol{v}$ and $\boldsymbol{u}$ mean the velocity of electrons of the beam and the plasma respecitively. Suffixes of b and p are used for the quantities related to the beam and the plasma respectively and 0 means the value of the zero order.

2) Continuity equations

$$\frac{\partial n_b}{\partial t}+\mathrm{div}(n_{b0}\boldsymbol{v}+n_b\boldsymbol{v}_0)=0 \qquad (2.1.7)$$

$$\frac{\partial n_p}{\partial t}+\mathrm{div}(n_{p0}\boldsymbol{u})=0 \qquad (2.1.8)$$

3) Equations of motion

$$\frac{\partial \boldsymbol{v}}{\partial t}+\mathrm{grad}(\boldsymbol{v}\cdot\boldsymbol{v}_0)-\boldsymbol{v}_0\times\mathrm{rot}\boldsymbol{v}-\boldsymbol{v}\times\mathrm{rot}\boldsymbol{v}_0$$

$$+\frac{e}{m}\boldsymbol{E}+\frac{e}{mc}\{\boldsymbol{v}\times\boldsymbol{H}_0+\boldsymbol{v}_0\times\boldsymbol{H}\}=0 \qquad (2.1.9)$$

$$\frac{\partial \boldsymbol{u}}{\partial t}+\frac{e}{m}\boldsymbol{E}+\frac{e}{mc}\{\boldsymbol{u}\times\boldsymbol{H}_0\}=0 \qquad (2.1.10)$$

where the coulomb collisions between the particles are neglected.

## 2・2 Configuration of an electron beam and a plasma

We shall discuss the system of a cold plasma extended infinitely and an electron layer rotating around a fixed magnetic axis. The electron layer is supposed to be infinite in the direction along the magnetic field. The system is shown in Fig.1 schematically. All electrons should have the same angular velocity $\dot{\varphi}$ and the zero order component of the velocity $\boldsymbol{v}_0$ is given by

$$\boldsymbol{v}_0=r\dot{\varphi}\boldsymbol{e}_\varphi+v_{0z}\boldsymbol{e}_z \qquad (2.1.11)$$

where $\dot{\varphi}$ is represented with the magnetic field as

$$\dot{\varphi}=\frac{eH_0}{mc}=-\omega_{ce}.$$

When the magnetic field is in $-z$ direction, $\omega_{ce}$ takes a positive value.

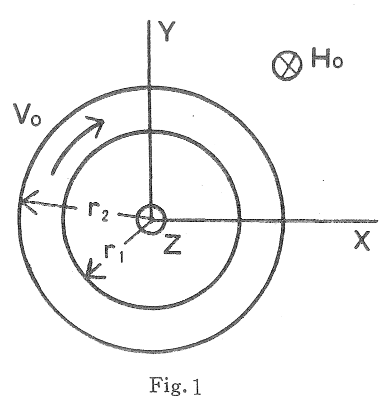

Fig. 1

## 2・3 Equation of motion

Assuming the space and time dependence of the all first order quantities in the form of $f(r)$ exp $i(\omega t-l\varphi-kz)$, representing a helical wave, the equations of motion (2.1.9) and (2.1.10) reduce to

$$i(\omega+l\omega_{ce}-kv_{0z})\begin{pmatrix} v_r \\ v_\varphi \\ v_z \end{pmatrix}+\omega_{ce}\begin{pmatrix} v_\varphi \\ -v_r \\ 0 \end{pmatrix}=\frac{e}{m}\begin{pmatrix} \frac{d\phi}{dr} \\ -il\frac{\phi}{r} \\ -ik\phi \end{pmatrix} \tag{2.3.2}$$

for the beam, and

$$i\omega\begin{pmatrix} u_r \\ u_\varphi \\ u_z \end{pmatrix}-\omega_{ce}\begin{pmatrix} u_\varphi \\ -u_r \\ 0 \end{pmatrix}=\frac{e}{m}\begin{pmatrix} \frac{d\phi}{dr} \\ -il\frac{\phi}{r} \\ -ik\phi \end{pmatrix} \tag{2.3.3}$$

for the plasma electron.

In the deduction of the two equations above, the displacement current and the perturbed magnetic field are neglected and so the electric field should be given only by an electrostatic potential $\phi$. This assumption is called "quasi-electrostatic" and requires that $\omega/k \ll c$. Thus we consider only low-frequency or short-wavelength disturbances. We are studying in this work the high frequency electronic wave so that we shall limit ourselves to the short wave oscillations.

## 2.4 Continuity equations

Equations (2.1.7) and (2.1.8) reduces to in the cylindrical co-ordinates,

$$n_b=-\frac{e}{m}N_{b0}\Big[\frac{\delta(r_1,\ r_2)}{A}\Big\{\frac{d\phi}{dr}+\frac{l\omega_{ce}}{\omega+l\omega_{ce}-kv_{0z}}\frac{\phi}{r}\Big\}$$
$$+\frac{S(r)}{A}\Big\{\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}-\frac{l^2}{r^2}\phi\Big\}+S(r)\frac{k^2\phi}{(\omega+l\omega_{ce}-kv_{0z})^2}\Big] \tag{2.4.1}$$

and

$$n_p=\frac{e}{m}\Big[N_{b0}\frac{\omega\frac{d\phi}{dr}-l\omega_{ce}\frac{\phi}{r}}{\omega}\delta(r_1,\ r_2)-\frac{N_{p0}-N_{b0}S(r)}{B\omega}\omega\Big(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}-\frac{l^2}{r^2}\phi\Big)$$
$$-\{N_{p0}-N_{b0}S(r)\}\frac{k^2\phi}{\omega^2}\Big] \tag{2.4.2}$$

where A and B are

$$A=\omega_{ce}^2-(\omega+l\omega_{ce}-kv_{0z})^2 \tag{2.4.3}$$

$$B=-\omega^2+\omega_{ce}^2 \tag{2.4.4}$$

and the density distribution of the beam is supposed to be constant $N_{b0}$ for $r_1<r<r_2$ and zero otherwise, i.e.

$$n_{b0}=N_{b0}S(r) \tag{2.4.5}$$

where $S(r)$ is defined as shown in Fig.2. The derivative of $S(r)$ produces some delta functions and $\delta(r_1,\ r_2)$ is given by

$$\delta(r_1,\ r_2)=\delta(r-r_1)-\delta(r-r_2). \tag{2.4.6}$$

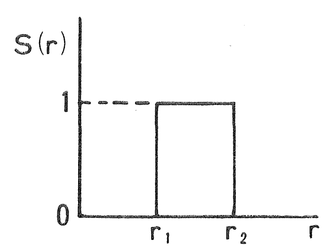

Fig. 2

## 2.5 Poisson's equation

In the approximation stated above, Maxwell's equations reduce to Poisson's equation:

$$\text{div}\ \boldsymbol{E}=-\nabla^2\phi=-4\pi e(n_b+n_p). \tag{2.5.1}$$

Insertion of (2.4.1) and (2.4.2) into (2.5.1) enables one to obtain a differential equation in $\phi$ and $r$ as

$$\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}-(\frac{l^2}{r^2}+k'^2)\phi=\delta(r_1,\ r_2)[P\frac{d\phi}{dr}+Q\frac{\phi}{r}] \tag{2.5.2}$$

where

$$k'^2=\frac{k^2\{1-\dfrac{S(r)\omega_b^2}{\omega+l\omega_{ce}-kv_{0z}}-\dfrac{\omega_p^2-\omega_o^2S(r)}{\omega^2}\}}{1+R} \tag{2.5.3}$$

$$P=\frac{-\dfrac{\omega_b^2}{A}+\dfrac{\omega_b^2}{B}}{1+R} \tag{2.5.4}$$

$$Q=-\frac{l\omega_o^2\omega_{ce}}{1+R}\{\frac{1}{A(\omega+l\omega_{ce}-kv_{0z})}+\frac{1}{B\omega}\} \tag{2.5.5}$$

and

$$R=S(r)\frac{\omega_b^2}{A}+\frac{\omega_p^2-\omega_o^2S(r)}{B} \tag{2.5.6}$$

$\omega_b$ and $\omega_p$ are the electronic plasma frequency of the beam and the plasma respectively.

2.6 Boundary conditions and dispersion relation.

The potential should be continuous at the boundaries $r=r_1,\ r_2$;

$$\phi|_{r=r_1-0}=\phi|_{r=r_1+0} \tag{2.6.1}$$

and

$$\phi|_{r=r_2-0}=\phi|_{r=r_2+0}\ . \tag{2.6.2}$$

Jump conditions in the electric field $\frac{d\phi}{dr}$ at the boundaries due to the space charge are given by integrating the equation (2.5.2) over the region $r_{1,2}-\delta r<r<r_{1,2}+\delta r$,

$$\frac{d\phi}{dr}\Big|_{r_1+0}P(+)-\frac{d\phi}{dr}\Big|_{r_1-0}P(-)=\frac{\phi(r_1)}{r_1}Q_\pm \tag{2.6.3}$$

and

$$\frac{d\phi}{dr}\Big|_{r_2+0}P(-)-\frac{d\phi}{dr}\Big|_{r_2-0}P(+)=-\frac{\phi(r_2)}{r_2}Q_\pm \tag{2.6.4}$$

where

$$\left.\begin{aligned} &P(+)=1-\frac{1}{2}P_{r_1+0}\\ &P(-)=1+\frac{1}{2}P_{r_1-0}\\ &\text{and}\\ &Q_\pm=(Q_{r_1+0}+Q_{r_1-0})/2. \end{aligned}\right\} \tag{2.6.5}$$

The solutions of the equation (2.5.2) are Bessel functions of imaginary argument, $I_l(k_1r)$ and $K_l(k_1r)$ respectively for $r<r_1$ and $r>r_2$, where $l$ is taken as zero or a positive integer. In the interior $r_1<r<r_2$, the solution is a linear combimation of Hankel functions of the first and second kind, $H_l^{(1)}(ik_2r)$ and $H_l^{(2)}(ik_2r)$. $k_1$ and $k_2$ are determined by

$$k_1 = k'_{r1-0} \qquad (2.6.6)$$
$$k_2 = k'_{r1+0} \ .$$

Equations (2.6.1), (2.6.2), (2.6.3) and (2.6.4) produce the dispersion relation with the functions

$$\begin{vmatrix} I_l(k_1r_1) & -H_l^{(1)}(ik_2r_1) & -H_l^{(2)}(ik_2r_1) & 0 \\ 0 & -H_l^{(1)}(ik_2r_2) & -H_l^{(2)}(ik_2r_2) & k_l(k_1r_2) \\ \frac{Q_\pm}{r_1}I_l(k_1r_1) + P(-)\frac{d}{dr}I_l(k_1r)|_{r_1} & -\frac{d}{dr}H_l^{(1)}(ik_2r)|_{r_1}\times P(+) & -\frac{d}{dr}H_l^{(2)}(ik_2r)|_{r_1}\times P(-) & 0 \\ 0 & \frac{d}{dr}H_l^{(1)}(ik_2r)|_{r_2}\times P(-) & \frac{d}{dr}H_l^{(2)}(ik_2r)|_{r_2}\times P(-) & -\frac{Q_\pm}{r_2}K_l(k_1r_2) - P(+)\frac{d}{dr}K_l(k_1r)|_{r_2} \end{vmatrix}$$

$$= 0 \ . \qquad (2.6.7)$$

2.7 Approximated solution

In the limit of short wave approximation which is consistent with the assumptions we adopted in (2.3), i.e., $|k_{1,\,2}|r_{1,\,2},\gg 1$, these functions can be expanded asymptoticaly to give a simple expression of the dispersion relation.

$$\tanh k_2\Delta r = \frac{-P(+)^2k_2{}^2 + \{\frac{Q_\pm}{r_1} + k_1P(-)\}P(+)k_2}{\{-\frac{Q_\pm}{r_2} + k_1P(-)\}P(+)k_2 - \{\frac{Q_\pm}{r_1} + k_1P(-)\}\{-\frac{Q_\pm}{r_2} + k_1P(-)\}} \qquad (2.7.1)$$

where

$$\Delta r = r_2 - r_1.$$

In the limit $k_1$, $k_2 \to \infty$, we can make a further simplification of (2.7.1) to

$$\tanh k_2\Delta r = -\frac{k_2}{k_1}\frac{P(-)}{P(+)}. \qquad (2.7.2)$$

For a sufficiently weak beam, these quantities reduce respectively

$$P(+) \to 1 \qquad (2.7.3)$$
$$P(-) \to 1$$

$$k_1^2 \to \frac{k^2\{1 - \frac{\omega_p^2}{(l\omega_{ce} - kv_{0z})^2}\}}{1 + \frac{\omega_p^2}{\omega_{ce}^2 - (l\omega_{ce} - kv_{0z})^2}} \qquad (2.7.4)$$

$$k_2{}^2 \to \frac{k^2\{1 - \frac{\omega_b^2}{(\omega + l\omega_{ce} - kv_{0z})^2} - \frac{\omega_p^2}{(l\omega_{ce} - kv_{0z})^2}\}}{1 + \frac{\omega_p^2}{\omega_{ce}^2 - (l\omega_{ce} - kv_{0z})^2}} \qquad (2.7.5)$$

$$Q_\pm \to 0 \ . \qquad (2.7.6)$$

The dispersion equation (2.7.2) becomes

$$\tanh k_2 \Delta r = -\frac{k_2}{k_1}. \qquad (2.7.7)$$

This form of the dispersion relation suggests that we could expect an unstable mode of the plasma oscillation at $\omega + l\omega_{ce} - kv_{0z} \simeq 0$.
More detailed investigation should be done for (2.7.7) with the numerical calculations.

## §3. Electromagnetic Mode

There may be a possibility of another type of instalility to be found in the geometry we are studing. To get the dispersion relation for the electromagentic instability,[3] in the set of Maxwell's equations, (2.1.1)~(2.1.4), no charge separation should be assumed, i.e.,

$$\mathrm{div}\ \boldsymbol{E} = 0. \qquad (3.1)$$

This condition implies physically that the electrons of the plasma could not follow with the rapidly fluctuating electric field or the wave number k would be quite small in the direction of the electric field so that it could be seen homogeneous in that direction.

Introducing the vector potential $A$,

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \qquad (3.2)$$

$$\boldsymbol{H} = \mathrm{rot}\boldsymbol{A}. \qquad (3.3)$$

Now for simplicity of the treatment of the problem, we suppose the wave propagates nearly along the static magnetic field so that the perturbing electric and magnetic fields in $z$ direction should vanish;

$$E_z = 0 \quad \text{and} \quad H_z = 0. \qquad (3.4)$$

Equations of motion corresponding to (2.3.2) and(2.3.3) are

$$i(\omega + l\omega_{ce} - kv_{0z})\begin{pmatrix} v_r \\ v_\varphi \\ v_z \end{pmatrix} + \omega_{ce}\begin{pmatrix} v_\varphi \\ -v_r \\ 0 \end{pmatrix} = \frac{e}{m}\begin{pmatrix} \frac{i\omega}{c}A_r \\ \frac{i\omega}{c}A_\varphi \\ 0 \end{pmatrix} + \frac{e}{mc}\begin{pmatrix} -ikv_{0z}A_r \\ -ikv_{0z}A_\varphi \\ -ik\omega_{ce} r A_\varphi \end{pmatrix} \qquad (3.5)$$

$$i\omega\begin{pmatrix} u_r \\ u_\varphi \\ u_z \end{pmatrix} - \omega_{ce}\begin{pmatrix} u_\varphi \\ -u_r \\ 0 \end{pmatrix} = \frac{e}{mc}\begin{pmatrix} \frac{i\omega}{c}A_r \\ \frac{i\omega}{c}A_\varphi \\ 0 \end{pmatrix}. \qquad (3.6)$$

Continuity equation corresponding to (2.4.1) is modified to give

$$n_b = -N_{b0}\delta(r_1, r_2)\frac{e}{mc}\frac{\omega'}{AB}(iBA_r - \omega_{ce}A_\varphi)$$
$$-N_{b0}S(r)\frac{e}{mc}\frac{\omega'}{AB}\left[iB\frac{\partial A_r}{\partial r} - \omega_{ce}\frac{\partial A_\varphi}{\partial r} + \frac{1}{r}\{(B - l\omega_{ce})iA_r + (lB - \omega_{ce})A_\varphi\}\right]$$
$$-N_{b0}S(r)\frac{e}{mc}\frac{k^2\omega_{ce} r}{B^2}A_\varphi \qquad (3.7)$$

where

$$A = \omega_{ce}^2 - (\omega' + l\omega_{ce})^2$$

$$\omega' = \omega - kv_{0z}$$

and $$B = \omega' + l\omega_{ce}.$$

With (3.3), the third of Maxwell's equations produces a wave equation for $\boldsymbol{A}$ as well known,

$$\nabla^2 A - \frac{\omega^2}{c^2} A = -\frac{4\pi}{c} j. \tag{3.8}$$

These two equations for $A_r$ and $A_\varphi$ can be separated conveniently using two new functions

$$A_1 \exp i(\omega t - l\phi - kz) = A_r + iA_\varphi$$

and $$A_2 \exp i(\omega t - l\phi - kz) = A_r - iA_\varphi$$

as

$$\frac{d^2 A_1}{dr^2} + \frac{1}{r}\frac{dA_1}{dr} - \{k'^2 + \frac{(l-1)^2}{r}\} A_1 = iQ \tag{3.9}$$

and $$\frac{d^2 A_2}{dr^2} + \frac{1}{r}\frac{dA_2}{dr} - \{k''^2 + \frac{(l+1)^2}{r}\} A_2 = -iQ \tag{3.10}$$

where

$$k'^2 = k^2 - \frac{\omega^2}{c^2} + \frac{\omega_{pe}^2}{c^2} \frac{-\omega(\omega - \omega_{ce})}{-\omega^2 + \omega_{ce}^2} + \frac{\omega_b^2}{c^2} S(r) \frac{\omega'}{\omega + (l-1)\omega_{ce} - kv_{0z}} \tag{3.11}$$

$$k''^2 = k^2 - \frac{\omega^2}{c^2} + \frac{\omega_{pe}^2}{c^2} \frac{-\omega(\omega + \omega_{ce})}{-\omega^2 + \omega_{ce}^2} + \frac{\omega_b^2}{c^2} S(r) \frac{\omega'}{\omega + (l+1)\omega_{ce} - kv_{0z}} \tag{3.12}$$

and

$$Q = \frac{\omega_b^2}{c^2} \delta(r_1, r_2) \frac{r\omega_{ce}\omega'(iBA_r - \omega_{ce}A\ )}{AB} + \frac{\omega_b^2}{c^2} S(r) \frac{r\omega_{ce}\omega'(iB\frac{dA_r}{dr} - \omega_{ce}\frac{dA_\varphi}{dr})}{AB}$$
$$+ \frac{\omega_b^2}{c^2} S(r) \frac{\omega_{ce}\omega'\{(B - l\omega_{ce})iA_r + (lB - \omega_{ce})A_\varphi\}}{AB} + \frac{\omega_b^2}{c^2} S(r) \frac{(k\omega_{ce}r)^2}{B^2} A_\varphi. \tag{3.13}$$

Although (3.9) and (3.10) are not separated completely due to the term of $iQ$ in the right side, in the limit $\omega_b^2 \to 0$ Q is expected to be a small quantity and could be seen as a perturbation term.

If we consider a right circularly polarized wave, i.e., $A_\varphi = -iA_r$, only (3.9) is to be solved because

$$A_1 = 2A_r$$

and

$$A_2 = 0.$$

The zeroth order solution of (3.9), $A_{(0)}{}^1$, is obtained by taking Q=0 to be Bessel's functions in the completely same form as we obtained in the previons section, substituting $l-1$ in place of $l$, where $l-1$ is taken as zero or a positive integer. $A_r$ and $A_\varphi$ in Q might be replaced by the Bessel's functions in the region $r_1 < r < r_2$ as

$$\mathrm{i}Q_{r_1<r<r_2}=a[C_1\{r\frac{dH^{(1)}_{l-1}}{dr}-(l-1)H^{(1)}_{l-1}\}+C_2\{r\frac{dH^{(2)}_{l-1}}{dr}-(l-1)H^{(2)}_{l-1}\}]$$

$$+b\{C_1(kr)^2H^{(1)}_{l-1}+C_2(kr)^2H^{(2)}_{l-1}\}=f(r) \tag{3.14}$$

where
$$a=\frac{\omega_{be}^2\omega_{ce}(\omega-kv_{0z})}{2c^2\{\omega+(l-1)\omega_{ce}-kv_{0z}\}\{\omega+l\omega_{ce}-kv_{0z}\}} \tag{3.15}$$

and
$$b=\frac{\omega_b^2}{2c^2}\ \frac{\omega_{ce}^2}{\{\omega+l\omega_{ce}-kv_{0z}\}^2}$$

$C_1$ and $C_2$ are the constants determined by the boundary conditions. The first order solution of (3.9) is given for $r_1<r<r_2$ using (3.14) in the right side in the following form

$$A_1^{(1)}=\frac{\pi}{4i}H^{(1)}_{l-1}\int rf(r)H^{(2)}_{l-1}dr-\frac{\pi}{4i}H^{(2)}_{l-1}\int rf(r)H^{(1)}_{l-1}dr \tag{3.16}$$

or

$$=a\{C_1M(r)+C_2N(r)\}+b\{C_1K(r)+C_2L(r)\}\ . \tag{3.17}$$

Taking into accoount the boundary conditions similar to (2.6.1)$\sim$(2.6.4)

$$\left.\begin{aligned} A_{1r_1-0}&=A_{1r_1+0}\\ A_{1r_2-0}&=A_{1r_2+0}\end{aligned}\right\} \tag{3.18}$$

$$\frac{dA_1}{dr}\Big|_{r_1+0}-\frac{dA_1}{dr}\Big|_{r_1-0}=\frac{a}{2}\left(rA_1^{(0)}\Big|_{r_1+0}+rA_1^{(0)}\Big|_{r_1-0}\right) \tag{3.19}$$

$$\frac{dA_1}{dr}\Big|_{r_2+0}-\frac{dA_1}{dr}\Big|_{r_2-0}=-\frac{a}{2}\left(rA_1^{(0)}\Big|_{r_2+0}+rA_1^{(0)}\Big|_{r_2-0}\right) \tag{3.20}$$

the dispersion relation for the electromagnetic mode is given by (3.21),

$$\begin{vmatrix}
I_{l-1}(k_1r_1) & \begin{matrix}-H^{(1)}_{l-1}(ik_2r_1)-aM(r_1)\\ -bK(r_1)\end{matrix} & \begin{matrix}-H^{(2)}_{l-1}(ik_2r_1)-aN(r_1)\\ -bL(r_1)\end{matrix} & 0 \\
\begin{matrix}-\frac{d}{dr}I_{l-1}(k_1r)\big|_{r_1}\\ -\frac{a}{2}r_1I_{l-1}(k_1r_1)\end{matrix} & \begin{matrix}\frac{d}{dr}H^{(1)}_{l-1}(ik_2r)\big|_{r_1}\\ +a\frac{dM(r)}{dr}\Big|_{r_1}+b\frac{dK(r)}{dr}\Big|_{r_1}\\ -\frac{a}{2}r_1H^{(1)}_{l-1}(ik_2r_1)\end{matrix} & \begin{matrix}\frac{d}{dr}H^{(2)}_{l-1}(ik_2r)\Big|_{r_1}\\ +a\frac{dN(r)}{dr}\Big|_{r_1}+b\frac{dL(r)}{dr}\Big|_{r_1}\\ -\frac{a}{2}r_1H^{(2)}_{l-1}(ik_2r_1)\end{matrix} & 0 \\
0 & \begin{matrix}\frac{d}{dr}H^{(1)}_{l-1}(ik_2r)\Big|_{r_2}\\ +a\frac{dM(r)}{dr}\Big|_{r_2}+b\frac{dK(r)}{dr}\Big|_{r_2}\\ -\frac{a}{2}r_2H^{(1)}_{l-1}(ik_2r_2)\end{matrix} & \begin{matrix}\frac{d}{dr}H^{(2)}_{l-1}(ik_2r)\Big|_{r_2}+a\frac{dN(r)}{dr}\Big|_{r_2}\\ +b\frac{dL(r)}{dr}\Big|_{r_2}\\ -\frac{a}{2}r_2H^{(2)}_{l-1}(ik_2r_2)\end{matrix} & \begin{matrix}-\frac{d}{dr}K_{l-1}(k_1r)\big|_{r_2}\\ -\frac{a}{2}r_2K_{l-1}(k_1r_2)\end{matrix} \\
0 & \begin{matrix}-H^{(1)}_{l-1}(ik_2r_2)-aM(r_2)\\ -bK(r_2)\end{matrix} & \begin{matrix}-H^{(2)}_{l-1}(ik_2r_2)-aN(r_2)\\ -bL(r_2)\end{matrix} & K_{l-1}(k_1r_2)
\end{vmatrix}$$

$$=0 \tag{3.21}$$

$$\text{where } k_1^2=k^2-\frac{\omega^2}{c^2}+\frac{\omega_p^2}{c^2}\ \frac{\omega}{\omega+\omega_{ce}} \tag{3.22}$$

and $k_2^2=k^2-\frac{\omega^2}{c^2}+\frac{\omega_p^2}{c^2}\frac{\omega}{\omega+\omega_{ce}}+\frac{\omega_b^2}{c^2}\frac{\omega-kv_{0z}}{\omega+(l-1)\omega_{ce}-kv_{0z}}$ .

The term of the lowest order representing the effect of the rotating beam appears in Q where

$$\frac{\omega_b^2}{\omega+(l-1)\omega_{ce}-kv_{0z}}$$

takes a small but finite value for $\omega_b\to 0$. To simplify (3.21) we adopt the approximation of the high frequency oscillation

$$\frac{\omega}{k}\gg c \tag{3.23}$$

which is consistent with the quasi-magnetic condition.

Because $k_1$ and $k_2$ are given by (3.22), the condition (3.23) eventually can be written in the same form as in the electrostatic case,

$$|k_1,\ k_2|r_1,\ r_2\gg 1$$

to lead a much simpler form of the dispersion relation,

$$\tanh k_2\Delta r=\frac{-2k_1k_2+\frac{a}{2}\Delta r k_2}{k_1^2+k_2^2-\frac{a}{2}\Delta r k_1-\frac{a^2}{4}r_1r_2} \tag{3.24}$$

where

$$a=\frac{\omega_b^2}{2c^2}\frac{\omega_{ce}(\omega-kv_{0z})}{\{\omega+(l-1)\omega_{ce}-kv_{0z}\}(\omega+l\omega_{ce}-kv_{0z})}\ . \tag{3.25}$$

For the detailed analysis of the equation (3.24) the numerical calculations are also required.

## §4. Conclusion

Electrostatic and electromagnetic electron wave excitations by a gyrating electron beam passing through the plasma are discussed. In the geometry of the beam-plasma treated here, in the limit of short wave and weak beam the electrostatic wave excitation can be expected for the frequency $\omega+l\omega_{ce}-kv_{0z}\sim\omega_b$. With the quasi-electromagnetic condition the wave may be excited when $\omega+(l-1)\ \omega_{ce}-kv_{0z}\sim\omega_b^2$. In the either cases to get more detailed informations about the excitation of the instabilities such as the growth rate numerical analyses would be required.

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