

# An optimization-based model for analyzing the exertion of force vector at the human upper limb

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## ABSTRACT

The purpose of this study was to investigate muscle activation patterns for executing the force vector at the human upper limb, using optimization techniques. To predict muscle activation levels, a nonlinear optimization with the objective function as the total squared muscle activation was used. Based on the data measured during force vector regulation tasks in a human experiment, for each subject, we constructed a planer 2 degrees of freedom model with 6 muscles, and computed optimized muscle activation patterns using Lagrange multipliers methods. Comparing the data of the model and experiment, the results showed that high correlation coefficients were found in 4 of 6 muscles. In addition, in 5 of 6 muscles, preferred directions where the muscle activation level was highest were shown no significant differences. These results suggest that muscle activation levels in optimization-based model with the objective function of the total squared activation was analogous to experimental data and both patterns and directions in muscle activation levels were predicted with high aspects. However, on the other hand, 2 muscles with low coefficient correlation and 1 muscle with significant difference in the preferred direction have indicated the limitations in the modeling in the prediction of muscle activation levels. In our modeling, since we utilized muscle parameters as a constant value regardless of the subject, there is a possibility that errors in the model resulted from differences of muscle parameters for each subject. If it was estimated parameters of muscles in each subject using an appropriate manner such as magnetic resonance imaging, we may be able to predict fitted muscle activation patterns for each subject.

## KEY WORDS

Optimization, Muscle activation patterns, Modeling, Lagrange multipliers, Upper limb

## Introduction

To generate an appropriate joint moment for a given joint, the muscle activation level in muscles yield important information for a set of muscle actions. However, because a musculo-skeletal system in humans has the redundancy in which the number of the muscle around a single joint exceeds the degrees of freedom of the joint, there are infinite combinations for muscle activation patterns. For this undetermined problem, in recently, one possible solution has

been obtained by the optimization concept with minimization or maximization of the objective function. Crowninshield and Brand<sup>1)</sup> indicated that, in predicting muscle activity, muscle activation patterns were formed so as to minimize the sum of muscle stress cubed. Van Bolhuis and Gielen<sup>2)</sup> showed a model based on minimization of the sum of squares of muscle stress gave the best predictions for muscle activation patterns. Additionally, Fagg et al.<sup>3)</sup> suggested that the total squared activation criterion showed util-

ity in describing muscle recruitment patterns. Thus, the optimization with the "nice" objective function may be suitable technique to investigate muscle activation patterns.

Whereas, although a variety of objective functions have been designed to predict muscle activation patterns<sup>4)</sup>, modeling studies that used the actual human data during movement for elucidating the optimization-based model have been less likely reported. In particular, it has been often disregarded concerning the differences of characteristics in each subject for a given task. Therefore, by using the data measured in each subject, muscle activation patterns with high accuracy might be able to be examined.

The purpose of this study was to investigate muscle activation patterns to execute the force vector at the end effect in the hand, by using nonlinear optimization. We selected the objective function as the squared muscle activation in the optimization-based model in order to predict muscle activation patterns. The data used in our model, which were the length of limbs, joint angle and force vector, was utilized from force vector regulation tasks measured in human experiment<sup>5)</sup>. Based on these data, we analyzed optimized muscle activation patterns predicted from the sum of squares of muscle activation in each subject during the exertion of force vector.

## Methods

### 1. Optimization-based model

We constructed a planer 2 degrees of freedom model of the upper limb with 6 muscles using subject's anthropometric data in the actual human experiment as shown in Table 1. Six muscles modeled were the pectoralis major muscle (PMA), the posterior deltoid muscle (DEL), the brachioradialis muscle (BRD), lateral head of the triceps brachii muscle (TLA), the biceps brachii muscle (BIC) and long head of the triceps brachii muscle (TLO). Figure 1 shows the planer 2 degrees of freedom model of the upper limb. Here, assuming that the shoulder joint angle is  $\theta_s$  and the elbow joint angle is  $\theta_e$ , we can obtained that the hand position in the horizontal plane is expressed as the following equation,

$$X = \begin{bmatrix} L_1 \cos \theta_s & L_2 \cos (\theta_s + \theta_e) \\ L_1 \sin \theta_s & L_2 \sin (\theta_s + \theta_e) \end{bmatrix}, \quad (1)$$

Table 1. Anthropometric data of 11 subjects in the actual human experiment. The length of UA and LA denote the length of the upper arm and lower arm, respectively.

Subject	UA length (m)	LA length (m)
1	0.30	0.31
2	0.30	0.33
3	0.32	0.32
4	0.32	0.30
5	0.32	0.33
6	0.32	0.31
7	0.31	0.31
8	0.32	0.31
9	0.33	0.34
10	0.30	0.32
11	0.31	0.33

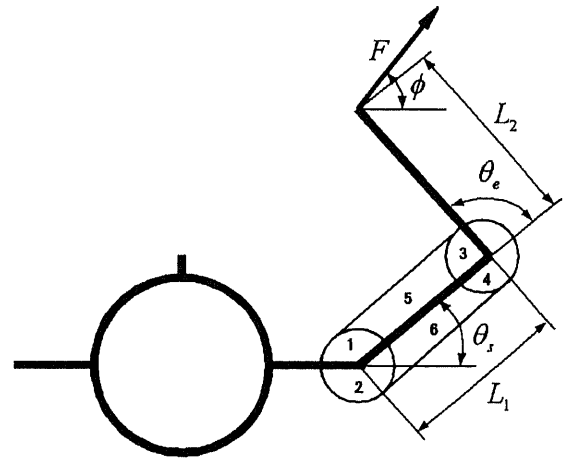


Figure 1. A planer 2 degrees of freedom human arm using the optimization-based model. The number in figure denotes the number of muscles; 1, pectoralis major (PMA); 2, posterior deltoid (DEL); 3, brachioradialis (BRD); 4, lateral head of triceps brachii (TLA); 5, biceps brachii (BIC); 6, long head of triceps brachii (TLO).  $L_1$  and  $L_2$  denote the length of the upper and lower arm, respectively.  $\theta_s$  and  $\theta_e$  denote the joint angle of the shoulder and elbow, respectively.  $F$  denotes the force vector and  $\phi$  denotes the force direction as making right direction 0 deg.

where  $X = (\chi, y)^T$  are hand position in 2 dimensional plane at which the shoulder joint is the origin of coordinates, and subscript  $T$  indicates the transpose of matrix.  $L_1$  and  $L_2$  indicate the upper and lower arm length in Table 1. In addition, by partial differentiation of Eq. (1), we can obtain matrix of the Jacobian as follows,

$$J(\theta) = \frac{\partial X}{\partial \theta} = \begin{bmatrix} -L_1 \sin \theta_s - L_2 \sin(\theta_s + \theta_e) & -L_2 \sin(\theta_s + \theta_e) \\ L_1 \cos \theta_s + L_2 \cos(\theta_s + \theta_e) & L_2 \cos(\theta_s + \theta_e) \end{bmatrix}, \quad (2)$$

where  $J$  are the Jacobian matrix. The relation between the joint moment and the force at the hand is formulated using above the Jacobian matrix,

$$M = J(\theta)^T F, \quad (3)$$

where  $M = (M_s, M_e)^T$  are the joint moment vector about the shoulder and elbow moment, that is,  $M_s$ , and  $M_e$  indicate the shoulder and elbow joint moment, respectively.  $F = (F \cos \phi, F \sin \phi)^T$  are the force vector at the hand.

The number of muscles around the shoulder and elbow joint exceeds degrees of freedom in these joints. Thus, a muscle force that is produced in each muscle is not uniquely determined from one joint moment. Concerning this undetermined problem, in order to calculate muscle activation levels in each muscle, we used a non-linear optimization technique. According to Milhorn<sup>61</sup>, the muscle activation level  $a$  is represented by using muscle force  $f$  and maximum muscle force  $f_{\max}$ ,

$$a = \frac{f}{f_{\max} - Bv}, \quad (4)$$

where  $B$  is a coefficient of viscous damping and  $v$  is muscle contraction velocity. Here,  $v$  is set to the value 0, because force vector regulation was isometric contraction tasks. Therefore, Eq. (4) can be substituted as  $a = f/f_{\max}$ . Each  $f_{\max}$  was calculated by multiplying a physiological cross-sectional area (PCSA) in Gomi<sup>71</sup> and 62 N/cm<sup>2</sup> of muscle tension per sectional

Table 2. Muscle parameters using the optimization-based model. Abbreviations for each muscle refer to Fig. 1. PCSA denotes the physiological cross-sectional area of muscles.  $f_{\max}$  denotes the maximum muscle force. The moment arm of flexion direction has negative values, and (s) and (e) denote the moment arm around the shoulder and elbow joint, respectively.

Muscle	PCSA (cm <sup>2</sup> )	$f_{\max}$ (N)	Moment arm (cm)
PMA	19.4	1202.8	-4.4
DEL	38.7	2399.4	3.5
BRD	10.3	683.6	-2.8
TLA	7.8	483.6	2.0
BIC	3.2	198.4	-2.9(s) -4.3(e)
TLO	3.9	241.8	2.5(s) 3.0(e)

area reported by Ikai et al.<sup>81</sup>. Table 2 shows PCSA and  $f_{\max}$  in each muscle. The individual muscle activation levels that produce a given joint moment can be found by solving the optimization problem in the following function to minimize,

$$U = \sum_i a_i^n, \quad (5)$$

where  $U$  is the objective function,  $a_i$  is the muscle activation level of  $i$ -th muscle, and  $n$  is order power. Since models with second-order power resulted overall in the best fits in comparing experimental muscle activation patterns<sup>21</sup>, we choose  $n=2$  as second-order power. Equality constraint functions are subjected to Eq. (5) are formulated as follows,

$$\begin{aligned} M_s &= a_1 d_1 f_{\max 1} + a_2 d_2 f_{\max 2} + a_5 d_5 f_{\max 5} + a_6 d_6 f_{\max 6} \\ M_e &= a_3 d_3 f_{\max 3} + a_4 d_4 f_{\max 4} + a_5 d_5 f_{\max 5} + a_6 d_6 f_{\max 6} \end{aligned}, \quad (6)$$

where subscripts denotes the number of muscle ; 1, PMA ; 2, DEL ; 3, BRD ; 4, TLA ; 5, BIC ; 6, TLO.  $d_i$  is moment arm of  $i$ -th muscle shown in Table 2. In addition, in order to constrain each muscle activation level to range of minimum and maximum level, we added inequality constraint functions,

$$0 \leq a_i \leq 1. \quad (7)$$

Finally, the above muscle activation level was treated as the percentage, which the maximum level is 100%.

For solving the optimization problem, the Lagrange multipliers method was employed<sup>91</sup>. Generally, for the number of unknowns  $x$  ( $x \in R^n$ ), in this case, corresponding to muscle activation levels with  $n=6$ , the Lagrange function is defined as follows,

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^m \lambda_i g_i(x) + \sum_{j=1}^l \mu_j h_j(x), \quad (8)$$

where  $\lambda$  and  $\mu$  indicate Lagrange multipliers and penalty parameters, respectively.  $m$  and  $n$  are the number of equality and inequality constraint functions, respectively. The function  $f(x)$  corresponds to Eq. (5) for which it is the objective function. Here, by imbedding equality constraint functions in Eq. (6) and inequality constraint in Eq. (7) to functions  $g(x)$  and  $h(x)$ , respectively, then we can solved Eq. (8) as the optimization without constraints under necessary conditions of  $\partial L / \partial x = 0^T$ ,  $\partial L / \partial \lambda = 0$  and  $\partial L / \partial \mu = 0$ . As requisite data to execute the optimization, that are the joint angle, arm length, and force vector at the hand, were used from the experiment data

investigated by Inumaru<sup>5)</sup>. Based on these data, we firstly computed the moment using Eq. (3), and then solved the optimization problem with regard to Eq. (8) for obtaining the muscle activation level, by using a computer program written in C++.

## 2. Experimental procedure

The data that was measured during force vector regulation in prior experiment was used for this study. Since the detailed procedure was presented before<sup>5)</sup>, therefore, we described here in briefly with regard to the experimental process.

Eleven male subjects participated in the experiment with informed consent. All subjects were right-handed and all force vector regulation tasks were executed by right upper limb. The subject performed force vector regulation in a horizontal plane monitoring a display in which the force magnitude and direction at the hand were shown. For the subject monitoring the display, we instructed to the subject exerting the force magnitude at 20 N with the force direction at one of eight directions of 45 deg intervals, on the basis of bar graph's level on the display presented values of the force vector at the hand. Simultaneously, electromyographic (EMG) activity and joint angles were recorded during force vector regulation tasks. The EMG activity that was recorded using surface electrodes were PMA as shoulder mono-flexor, DEL as mono-shoulder extensor, BRD as mono-elbow flexor, TLA as mono-elbow extensor, BIC as bi-articular flexor, and TLO as bi-articular extensor. The EMG signal was filtered at low-pass filter with 50 Hz and was normalized as percentage of EMG activity in maximum voluntary contractions according to the following equation :  $EMG = EMG_{task} / EMG_{max}$ . For elimination of the EMG activity of posture,  $EMG_{task}$  was used as the value that subtracted the background activity in posture maintained. The shoulder and elbow angle were measured by using an electrogoniometer.

## 3. Data analysis

To compare relationships of muscle activation levels between the experiment and the model, we computed the correlation coefficient, R, for all data. The number of data is 88 (11 subjects  $\times$  8 directions) for each muscle activation level of the model and

experiment. The values of R range between -1 and 1, and are close to 1 when the experimental activation level are close to the model activation level.

As in Flanders et al.<sup>10)</sup> or Hoffman et al.<sup>11)</sup>, muscle activation values were fit to a cosine-shaped function as a function of a force direction or a movement direction. In recently, Todorov<sup>12)</sup> have proved that this cosine tuning comes from the process of minimizing the variance of the end point force vector. Therefore, we also estimated the muscle activation levels in both the experiment and the model as a cosine-shaped function of the force direction. If the direction of the force is  $\phi$ , the muscle activation levels  $MA$  as a function of the force direction is expressed as

$$MA = a + b \cos(\phi - PD), \quad (9)$$

where  $PD$  is the preferred direction of muscle activation (PD) in which the activation level become maximum,  $a$  and  $b$  are coefficients. We compared PD in both the experiment and the model of the muscle activation level.

## Results

Figure 2 shows muscle activation levels for each muscle in the model computed using the optimization technique and in the experiment measured by the actual human subject. The value of the force direction along abscissa axes indicates the force direction that exerted by each subject. As a trend of all muscle, it was identified that the model activation level qualitatively agreed with the experiment activation level. In particular, we observed that there was reasonable relationship between the model and the experiment in the muscle activation level of PMA, DEL and TLO. Calculating the correlation coefficient, we found that there was the highest correlation coefficient in the muscle TLO ( $R = 0.84$ ). The lowest correlation coefficient was found in the muscle BIC ( $R = 0.33$ ) and subsequently BRD ( $R = 0.42$ ). Another muscles of PMA, DEL and TLA were also found the high correlation coefficient. Each correlation coefficient was  $R = 0.76, 0.79, 0.74$  regarding the muscle PMA, DEL, TLA, respectively. Assuming that the muscle with R above 0.7 was the muscle done modeling, we were able to demonstrate that the muscle activation level in PMA, DEL, TLA and TLO could be predicted from

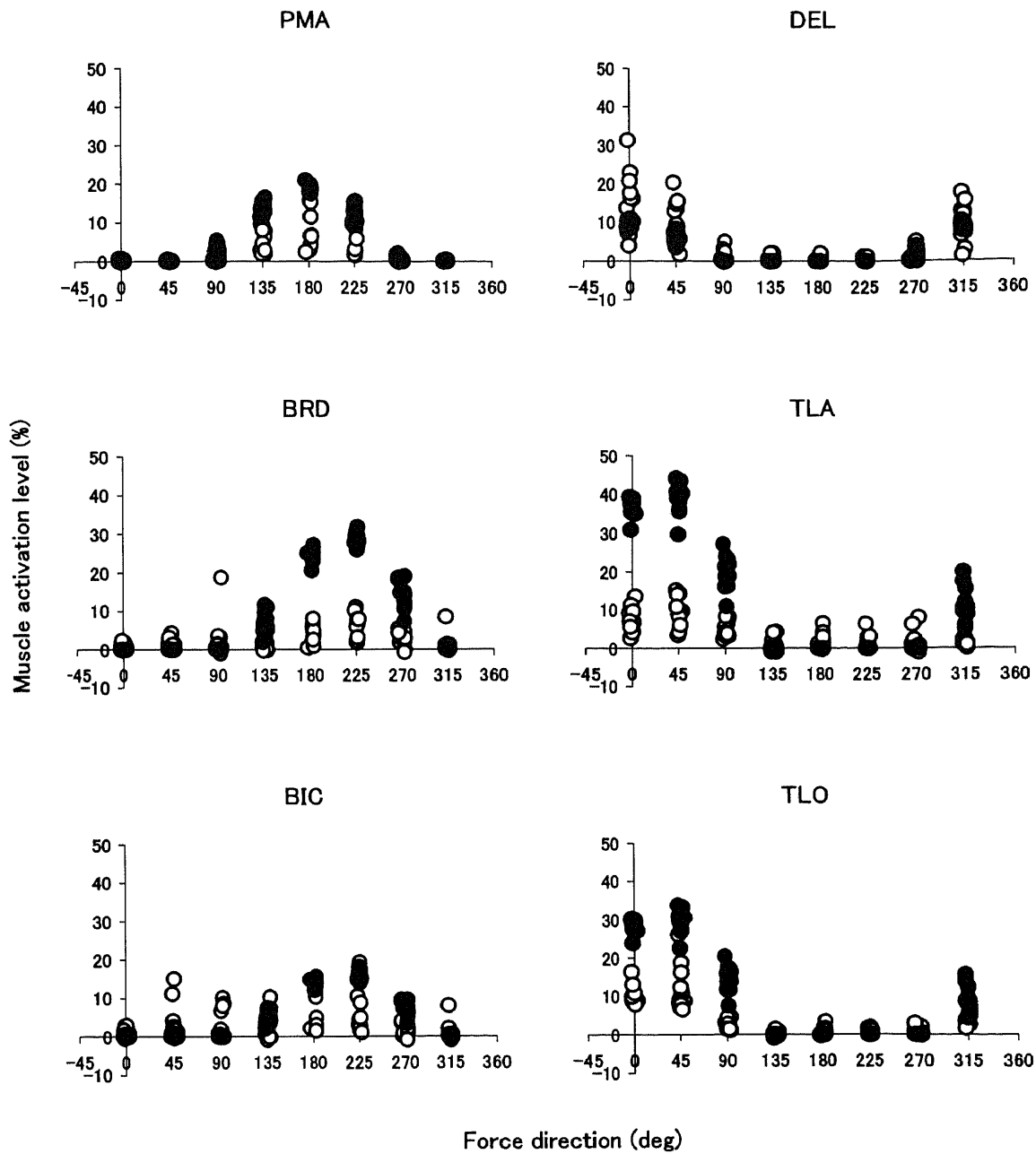


Figure 2. The muscle activation level predicted by the optimization technique (●) and measured by the actual human experiment (○). Abbreviations for each muscle refer to Fig. 1. The value of the force direction along abscissa axes denotes the force direction that exerted by human subjects. There are 88 data points (11 subjects × 8 directions) for each method.

the optimization technique.

Inspired by studies that muscle activity has been tuned by a cosine function as a function of the force direction<sup>10</sup>, we have calculated the preferred direction of muscle activation levels, which was fired maximally to a given direction. Figure 3 shows polar plots of the preferred direction of the muscle activation for

both the model and the experiment. Left plot represents the preferred direction of the model predicted by optimization method, and right plot represents that of the experiment executed by human. In this figure, the direction of polar plots indicates the force direction. All of preferred directions presented as the averaged direction in eleven subjects. In prior study, we

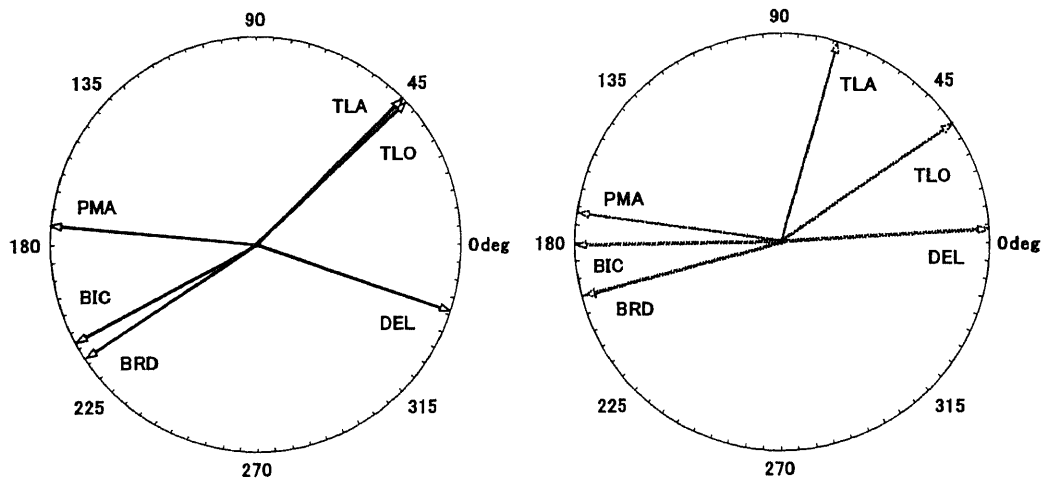


Figure 3. Polar plots of the preferred direction of muscle activation in the optimization model (bold arrows, left) and in the experiment (dotted arrows, right). Abbreviations for each muscle refer to Fig. 1. The center of a polar plot corresponds to the hand position, and the direction of that corresponds to the force direction.

Table 3. The preferred direction of muscle activation in the model and in the experiment. The value indicates by means ( $\pm$ SD) of 11 subjects, in degrees. Abbreviations for each muscle refer to Fig. 1. There were no significant differences between the model and experiment, except TLO (\* $P < 0.05$ ).

	PMA	DEL	BRD	TLA	BIC	TLO
Model	176.1 $\pm$ 6.8	341.0 $\pm$ 13.7	212.2 $\pm$ 6.7	44.2 $\pm$ 9.1	208.1 $\pm$ 6.6	43.0 $\pm$ 9.4*
Experiment	181.3 $\pm$ 6.5	363.1 $\pm$ 18.0	195.1 $\pm$ 50.9	73.9 $\pm$ 65.9	172.3 $\pm$ 52.6	34.4 $\pm$ 13.1

showed that the mechanical pulling direction of muscles have not coincided with the preferred direction of muscle activity<sup>5)</sup>. However, preferred directions of the muscle activation predicted by the optimization-based model indicated similar preferred directions that were observed in the experiment as shown in Figure 3. The deviation between the model and the experiment was 5.2 deg, 17.1 deg and 35.8 deg in PMA, BRD and BIC as the flexor, respectively. In the extensor, the deviation was 22.1 deg, 29.7 deg and 8.6 deg in DEL, TLA and TLO, respectively. The preferred direction of muscle activation in the model and experiment for all subjects shows in Table 3. The paired t-test indicated that, except for TLO, there were no significant differences between the model and the experiment, with the probability of  $P < 0.05$ .

### Discussion

Many studies have employed optimization techniques in an attempt to predict muscle activation patterns<sup>2, 4)</sup>. In our modeling, we also adapted the optimization technique with the objective function of the total squared activation criterion that might give the best prediction for muscle activation patterns, and then we computed muscle activation patterns on the basis of the data measured from human subjects. The results have shown clearly that only small differences were found between the model and experiment. Specifically, the high coefficient correlation was found in TLO, PMA, DEL and TLA of muscle activation levels between the prediction and measurement. In addition to that, except for TLO, there were no significant differences in muscle activation of the preferred direction, which represents maximum activation level. These results suggest that muscle activation

levels in optimization-based model with the objective function of the total squared activation was analogous to experimental data and both patterns and directions in muscle activation levels were predicted with high aspects. However, on the other hand, in BIC and BRD with low coefficient correlation and TLO with significant difference in the preferred direction, have indicated the limitations in the modeling in the prediction of muscle activation levels.

In other researcher's optimization-based model, Buchanan et al.<sup>4)</sup> demonstrated that, using a variety of cost functions, no particular cost function was found to adequately represent actual muscle activity at the elbow. Moreover, van Bolhuis et al.<sup>2)</sup> demonstrated that none of the existing models fitted the experimental data in all aspects. Unfortunately, our results also have indicated that it was not able to do modeling in all muscles, regardless of the total squared activation used. As the reason that this result occurred, parameters of muscles, such as PCSA, maximum force and moment arm might have been given errors in predicting muscle activation levels<sup>13,14)</sup>. If it was estimated parameters of muscles in each subject using an appropriate manner such as magnetic resonance imaging<sup>15)</sup>, we may be able to predict "best" muscle activation patterns for each subject.

In addition to relationships between the model and experiment of muscle activations, we also examined with regard to the preferred directions of muscle activation. The preferred direction indicates the direction where muscle activation level is highest in the case that the muscle activation level is fitted to the cosine regression as a function of the direction of movement or force. According to Fagg et al.<sup>3)</sup>, cosine-like recruitment behavior of muscles as a function of movement direction may result from a process of movement optimization. Additionally, this cosine-like burst also exists in the motor cortex as a neuronal population vector<sup>16)</sup>. Recently, Morrow et al.<sup>17)</sup> examined the relationship between muscle activity and motor cortex discharge, and have demonstrated that a small number of neurons recorded in the primary motor cortex contain sufficient information to reconstruct the time course of averaged muscle activity. In our optimization-based model, since cosine-like burst patterns of muscles with no significant differences in

the preferred direction was shown in both the model and the experiment, it is suggested that this pattern reflects discriminative patterns in cortical level. Therefore, producing the "best" muscle activation pattern, it may be able to investigate as aspects of the internal representation in cortical level as an optimized movement for each subject.

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## 最適化モデルから上肢でのカベクトル発揮を解析する

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### 要 旨

本研究の目的は、最適化手法を用いてヒトの上肢によるカベクトル発揮中の筋活動パターンを調査することであった。筋活動レベルを推定するために、筋活動の二乗の総和を目的関数とする非線形最適化法を使用した。ヒト実験におけるカベクトル制御課題中に測定されたデータに基づいて、各被験者に対して2自由度6筋を有する平面モデルを構築し、ラグランジュ乗数法を用いることで最適な筋活動パターンを計算した。モデルと実験のデータを比較した結果、6筋のうち4筋で高い相関係数が得られた。さらに、6筋のうち5筋では筋活動レベルが最大となる最適方向に有意差が認められなかった。これらの結果は、筋活動の二乗の総和を目的関数とする最適化ベースのモデルの筋活動レベルは実験データに類似し、筋活動レベルのパターンと最大となる方向の両方が高い様相で予測されたと示唆する。しかしながら、他方では、低い相関係数をもった2筋と最適方向に著しい差がみられた1筋は、筋活動レベルの推定におけるモデル化の限界を示している。我々は、モデル化する際に、被験者に関わらず一定の筋パラメータを利用したため、モデルにおける誤差は各被験者の筋パラメータの相違から生じた可能性がある。磁気共鳴画像診断装置などの適切な方法を使用して各被験者の筋パラメータが見積もられていたなら、我々は各被験者に対して最良の筋活動パターンを予測できたかもしれない。