

Optimum Structure of CT Probe and Spectral Components in 2-D Magnetic Field Visualization Based on the Magnetic CT Method

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The precise measurement of two-dimensional and three-dimensional magnetic flux distribution is indispensable for the development of magnetic machinery and device. The magnetic computed tomography (CT) method is proposed to visualize magnetic flux distribution. In order to determine the optimum structure of the magnetic CT probe, an evaluation model is employed to simulate the reconstruction accuracy of magnetic flux distribution. The spectrum of a distribution of magnetic field has been discussed of the components. As a result, the numerical simulation shows the optimum design of the multi layer CT probe.

Index Terms—Magnetic computed tomography (CT) method, magnetic field visualization, multilayer magnetic CT probe.

I. INTRODUCTION

VISUALIZATION of two-dimensional (2-D) and three-dimensional (3-D) magnetic flux distribution is very important to develop the precise design of magnetic machineries and devices. Basically, magnetic field is measured by using some magnetic sensors or scanning at every each point at present. But these techniques need a lot of measurement time. Projection method [1] based on the computed tomography (CT) algorithm [2] can reduce the measurement time, and achieve the visualization of magnetic field in real time. The simple magnetic CT probe is constructed by the array of one-turn coil which detects magnetic alternating current (ac) field. Moreover, by use of the portable multilayer magnetic CT probe [3], the magnetic field can be measured without mechanical movement. In future, the measurement of three-axis magnetic field will be enabled by arranging magnetic CT probes in 3-D. However, in probe design, it is important to choose the number of probe coil and probe layer (=rotation step) because these parameters affect the reconstruction accuracy directly. In this paper, we proposed a numerical magnetic field model considered with a feature of magnetic field in general, and the reconstruction simulation had been carried out to decide optimal parameters of the multilayer CT probe by comparing reconstruction error.

II. RECONSTRUCTION ALGORITHM

The principle of magnetic CT method and the structure of magnetic CT probe had been reported in [4] and [5]. In this section, the outline of magnetic CT reconstruction algorithm is described, then we discuss on the relationship between number of probe coil and reconstruction error.

Consider the 2-D magnetic field distributions; the projection p with the magnetic CT probe, as shown in Fig. 1, is defined as

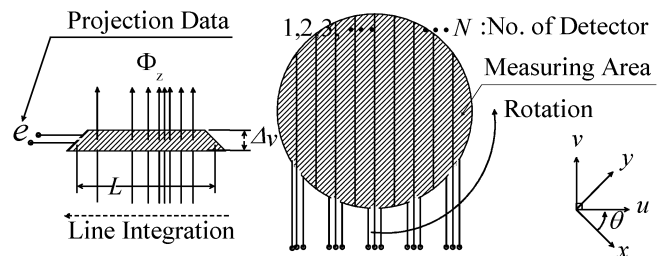


Fig. 1. 2-D magnetic CT probe and its principle.

the following phasor equation under the time varying field (refer to [3] for the proof of this expression)

$$\dot{p}_\theta(u) \equiv j \frac{\dot{e}(u)}{w\Delta u} = \int_{-\infty}^{\infty} \dot{B}(x, y) dv \quad (1)$$

where u and v are the rotation of $x-y$ coordinate at θ angle, respectively, \dot{e} is the induced ac voltage, and w is the angular frequency of objective field.

From (1), the projection data is obtained with probe rotation, and projection data is resolved by a back projection algorithm. We use the filtered back projection (FBP). To resolve with the FBP method, (2) and (3) are used with the filter function g

$$f(x, y) = \int_0^\pi (p(u, \theta) * g(u))_{u=x \cos \theta + y \sin \theta} d\theta \quad (2)$$

$$g(u) = \int_{-\infty}^{\infty} |\xi| \exp(2\pi j \xi u) d\xi \quad (3)$$

where ξ is the spatial frequency of v , $g(u)$ is the filter function. Although the projection angle is in fact discrete and it demands a mechanical rotation from (2), 2-D distribution can be reconstructed by interpolation techniques and using multilayer probe, as shown in Fig. 2. In this probe, the same probes are stacked with $\Delta\theta$ pitch, and all projection data is obtained through the multiplexer. This probe can measure the 2-D magnetic flux distribution easily without mechanical moving.

In the reconstruction process, there are two main factors that affect to the reconstruction error $\propto N \times R$ for which N is the

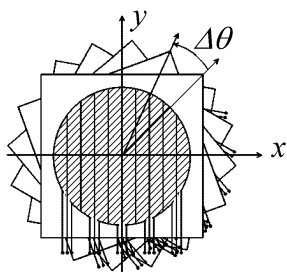


Fig. 2. Multilayer magnetic CT probe.

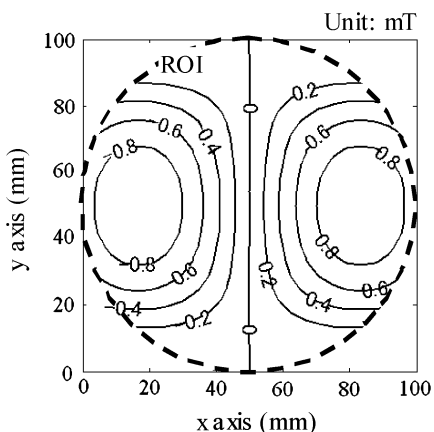


Fig. 3. Model field of 2-D magnetic flux distribution.

number of coil partition and R is the number of rotation step is the probe layer.

It is desirable that the large number of probe coil and rotation step will provide high resolution reconstruction results, however, it makes a knotty problem of the probe preparation and increases a processing time greatly. Therefore, it is important to choose the optimum number of probe coil and rotation step.

III. NUMERICAL RECONSTRUCTION SIMULATION

In order to discuss the probe design, we remark on the spatial frequency in the magnetic flux distribution. A measured data is resolved into the spatial frequency components using the FFT. On the other hand, there are many kinds of magnetic flux distribution, and most of them have the spectrum of low spatial frequency ingredient [6]–[9]. It is possible to decrease the number of coil and layer because the partition number is equal to the number of spectrum component. Hence, it is possible to determine the aspect of the number of coil and layer using a numeric criterion of the reconstruction accuracy by changing these numbers respectively. Based on the above evidence, we assume an analytical example of a normalized ($0 \leq x, y \leq 1$) 2-D magnetic flux distribution fitted with a real leakage field expressed as (4) for the discussion of probe figure. 2-D contour plot is shown in Fig. 3 and its calculated projection data is shown as Fig. 4.

$$B(x, y) = \sin\left(\frac{3\pi}{2}(x - 0.5)\right) \cos\left(\frac{11\pi}{10}(y - 0.5)\right) \times \sin c(3\pi(y - 0.5)^3) \quad (4)$$

From (4), the projection of 2-D distribution can be explicitly calculated by Radon transform, then multiplied with the

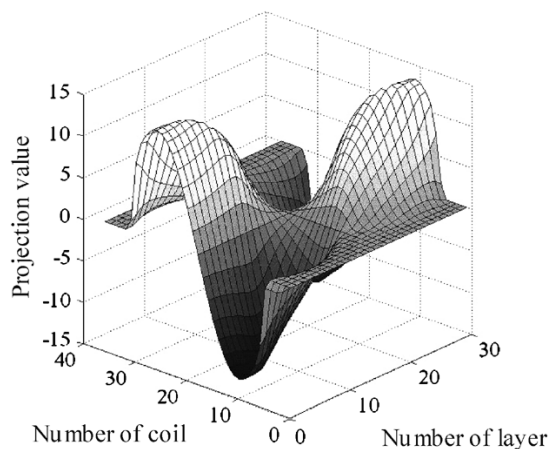


Fig. 4. Projection data of the model field.

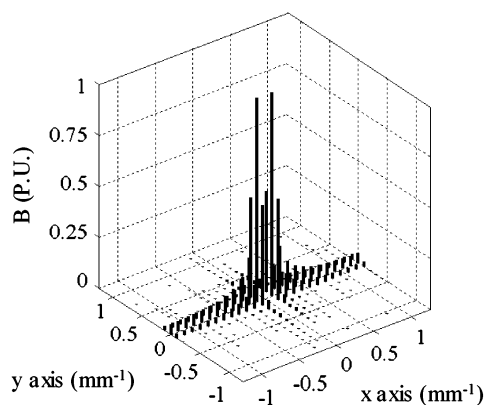


Fig. 5. 2-D Fourier power spectrum. 1 (P.U.) is equal to 0.725 (mT).

filter function in the spatial frequency domain, and finally reconstructed by the back projection with interpolation. The frequency spectrum of this model is shown in Fig. 5. With this model, the numerical reconstruction simulation is carried out to compare the original model with the reconstructed distribution by means of the relative error. Since the coil number and layer number are independent factors, the simulation results are evaluated in each parameter. In the case of each simulation, the other parameter should be taken as large number. On the error evaluation, there are some methods such as a maximum value at the peak, or comparing in a mean value and etc., but it is difficult to discuss about the accuracy of the 2-D distribution. Therefore, we propose the method for evaluation error that the relative error in each point is calculated and corrected them to make the histogram of number of relative errors within each interval. From this histogram, it is possible to determine the suitable number, i.e., almost reconstruction error within 5%.

Fig. 6 shows the result of the reconstruction error for the difference N . Note that the number of N is taken in odd number because of a data processing from the probe, and every histogram are normalized dividing by N^2 . In this figure, we can realize that almost error is within 5% at $N \geq 15$. In case of the number of $N \leq 13$, the spectrum is not separable of the main spectrum so that the shuffling is caused under $N \leq 13$.

Fig. 7 shows the result of the reconstruction error for the difference R . Note that the number of coil is fixed to $N = 25$.

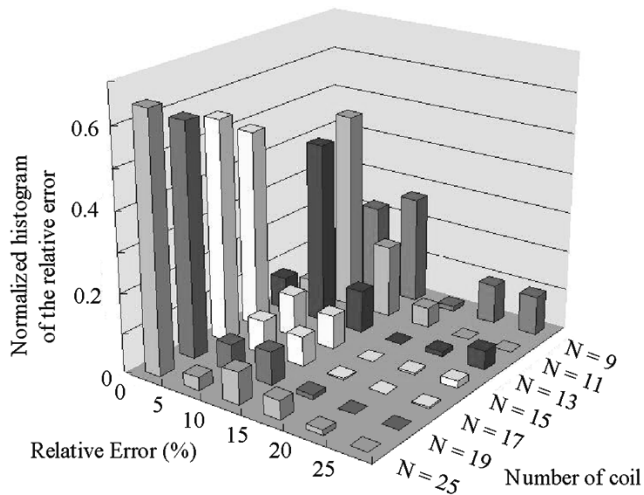


Fig. 6. Histogram of the reconstruction relative error (N : variables).

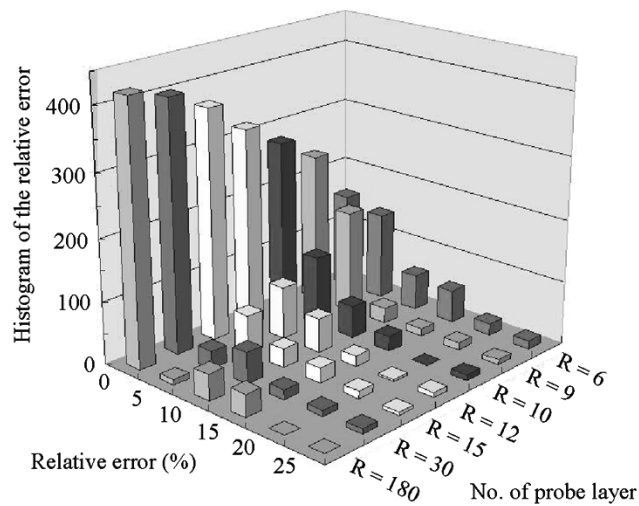


Fig. 7. Histogram of the reconstruction relative error (R : variables).

From this figure, it is concerned that the number of the probe layer should not be needed a lot of number because mainly relative error is almost same by changing R , such that $R = 10$ can be reconstructed roughly within 5% relative error.

From the above result, it is clarified that the minimum number of the coil N and layer R are $N = 15$ and $R = 10$ within 5% relative error.

IV. CONCLUSION

The visualization of a magnetic flux distribution can be realized by use of the magnetic CT method and the multilayered magnetic CT probe. This probe can be applied to any of magnetic field, and it is possible to reduce measuring and calculating time. In order to determine the optimum structure of the probe design, we have been focused on the spatial discrete frequency ingredient of a general magnetic flux distribution, and a reconstruction simulation has been carried out numerically with a sample model. The optimum number is judged by the histogram of the relative error. As a result, the optimum number of probe coil N should be $N \geq 15$ with taking into the consideration of reconstruction accuracy within 5% error, and the number of layer R should be $R \geq 10$. This result is very useful to design the actual magnetic CT probe.

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