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| journal or<br>publication title | ICCAS-SICE 2009 - ICROS-SICE International<br>Joint Conference 2009, Proceedings |
| number                          | 5333846  |
| page range                      | 246-250  |
| year                            | 2009-01-01   |
| URL                             | <a href="http://hdl.handle.net/2297/24287">http://hdl.handle.net/2297/24287</a>  |

# Driver Centric Decentralized Controller Design in Traffic Flow

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**Abstract:** In this paper, we consider a decentralized control problem for suppressing the traffic jam phenomenon in traffic flow. To analyze the phenomenon, we use the so-called optimal velocity model. In the model, the optimal velocity function which is a nonlinear function of the headway of the preceding vehicle describes driver's characteristics. Without affecting the characteristics, all vehicles in traffic flow should be stabilized in a decentralized fashion. In this paper, we apply washout control which is a high pass filter based control method. We derive a stability condition and illustrate the effectiveness with several simulations.

**Keywords:** traffic flow, congestion control, human centered control

## 1. INTRODUCTION

It is shown that a decentralized delayed feedback control method can suppress the traffic jam phenomenon in the optimal velocity traffic model [1]. In the optimal velocity traffic model, driver's intention is expressed by an optimal velocity function. In this paper, we apply highpass-filter-based control, called washout control, which can stabilize the traffic flow behavior without disturbing the driver's intention.

This paper is organized as follows. Section 2 explains the optimal velocity traffic model and its dynamics. In addition, derive the linearized model for checking we analyze the stability of the optimal velocity traffic model. In Section 3, washout control is applied to the suppression of traffic jam and stability conditions are derived for parameters of washout controller. Section 4 shows numerical simulations for optimal velocity traffic model to illustrate the effectiveness of washout control. Finally, conclusions are presented in Section 5.

## 2. OPTIMAL VELOCITY TRAFFIC MODEL

We consider traffic flow which is described by the optimal velocity model [2] (Fig. 1). We denote the position and velocity of the  $i$ th vehicle by  $x_i(t)$  and  $v_i(t)$ , respectively. We assume that the lead vehicle runs at a positive velocity  $v_0(t) > 0$ , which described as

$$\dot{x}_0(t) = v_0(t). \quad (1)$$

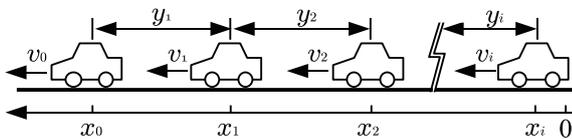


Fig. 1 Traffic flow model.

We also assume that the lead vehicle is not influenced by others. The following vehicles are modelled as

$$\begin{cases} \dot{x}_i(t) = a\{F(y_i(t)) - \dot{x}_i(t)\} + u_i(t), \\ y_i(t) = x_{i-1}(t) - x_i(t), \end{cases} \quad (2)$$

where  $y_i(t)$  is the head distance between  $(i-1)$ th and  $i$ th vehicles,  $a$  is the sensitivity of a driver,  $u_i(t)$  is the control input. The optimal velocity function  $F(y_i(t))$  is assumed to be described as

$$F(y_i(t)) = \tanh(y_i(t) - y_c) + \tanh(y_c) \quad (3)$$

where  $y_c$  is the desired forward distance which the  $i$ th driver wishes to keep. Here, denoting the velocity of the  $i$ th vehicle  $\dot{x}_i$ ,  $v_i(t)$ , we can rewrite (2) as

$$\begin{cases} \dot{v}_i(t) = a\{F(y_i(t)) - v_i(t)\} + u_i(t), \\ \dot{y}_i(t) = v_{i-1}(t) - v_i(t). \end{cases} \quad (4)$$

It is also assumed that when the leading vehicle runs with constant velocity  $v_0$  and  $u \equiv 0$ , (4) has a steady state

$$\begin{bmatrix} v_i^* & y_i^* \end{bmatrix} = \begin{bmatrix} v_0 & F^{-1}(v_0) \end{bmatrix}. \quad (5)$$

Defining

$$\begin{aligned} \bar{v}_i(t) &:= v_i(t) - v_i^*, \\ \bar{y}_i(t) &:= y_i(t) - y_i^*, \end{aligned}$$

the linearized dynamics of vehicle system (4) around the steady state (5) as

$$\begin{cases} \dot{\bar{v}}_i(t) = a\{\Lambda \bar{y}_i(t) - \bar{v}_i(t)\} + u_i(t), \\ \dot{\bar{y}}_i(t) = \bar{v}_{i-1}(t) - \bar{v}_i(t), \end{cases} \quad (6)$$

where

$$\Lambda := \left. \frac{\partial F(y)}{\partial y} \right|_{y=F^{-1}(v_0)} \quad (7)$$

is the first derivative of the optimal velocity function at  $y = F^{-1}(v_0)$ . Then, by choosing the state vector as

$$\bar{z}_i(t) = \begin{bmatrix} \bar{v}_i(t) \\ \bar{y}_i(t) \end{bmatrix}, \quad (8)$$

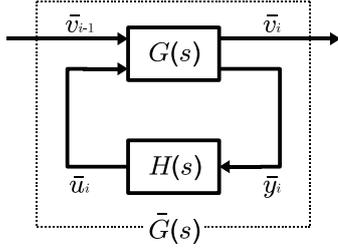


Fig. 2 Block diagram of the  $i$ th controlled vehicle.

a state space realization is given by

$$\begin{cases} \dot{\bar{z}}_i &= \begin{bmatrix} -a & a\Lambda \\ -1 & 0 \end{bmatrix} \bar{z}_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bar{v}_{i-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{u}_i, \\ \bar{v}_i &= \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{z}_i, \\ \bar{y}_i &= \begin{bmatrix} 0 & 1 \end{bmatrix} \bar{z}_i. \end{cases} \quad (9)$$

From this state space realization, we can derive the transfer function  $G(s)$  from  $\begin{bmatrix} \bar{v}_{i-1}(s) \\ \bar{u}_i(s) \end{bmatrix}$  to  $\begin{bmatrix} \bar{v}_i(s) \\ \bar{y}_i(s) \end{bmatrix}$  as

$$\begin{aligned} G(s) &= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \\ &= \begin{bmatrix} s+a & -a\Lambda \\ 1 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} a\Lambda/d(s) & s/d(s) \\ (s+a)/d(s) & -1/d(s) \end{bmatrix} \end{aligned} \quad (10)$$

where

$$d(s) := s^2 + as + a\Lambda. \quad (11)$$

### 3. WASHOUT CONTROL

In the optimal velocity traffic model, driver's intention is expressed by  $F$ . In general, it is difficult to exactly describe it. Hence, there exists uncertainty in  $F$  and  $y_c$ . Furthermore,  $v_i^*$  and  $y_i^*$  may be different from driver's intention. If we use  $v_i^*$  and  $y_i^*$  as a reference input to stabilize the system, it would be inconsistent with driver's intention. It is known that washout control can stabilize the equilibrium point without using it as a reference input [3].

A washout controller for the  $i$ th vehicle is given by

$$\begin{cases} \dot{\xi}_i(t) &= \alpha \xi_i(t) + \beta y_i(t), \\ u_i(t) &= \alpha \xi_i(t) + \beta y_i(t), \end{cases} \quad (12)$$

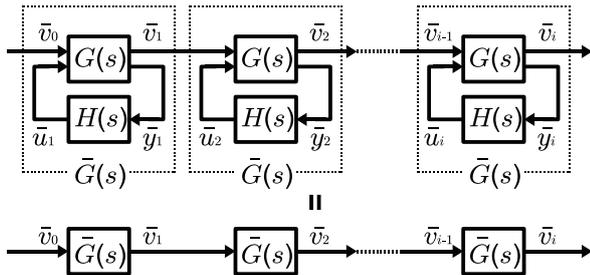


Fig. 3 Series connection of controlled vehicles.

where parameters  $\alpha$  and  $\beta$  are chosen to suppress the traffic jam. Then, we can derive the transfer function  $H(s)$  from  $y_i(t)$  to  $u_i(t)$  as

$$\begin{aligned} H(s) &= \alpha(s - \alpha)^{-1}\beta + \beta \\ &= \frac{\beta s}{s - \alpha}. \end{aligned} \quad (13)$$

When we use (12) for (10), we have the transfer function  $\bar{G}(s)$  from  $\bar{v}_{i-1}$  to  $\bar{v}_i$  as

$$\begin{aligned} \bar{G}(s) &= G_{11}(s) + G_{12}(s)H(s) \\ &\quad \times (1 - G_{22}(s)H(s))^{-1}G_{12}(s) \\ &= \frac{n_2s + n_3}{s^3 + d_1s^2 + d_2s + d_3}, \end{aligned} \quad (14)$$

where

$$\begin{aligned} d_1 &= a - \alpha, \\ d_2 &= a\Lambda + \beta - a\alpha, \\ d_3 &= -a\Lambda\alpha, \\ n_2 &= a\Lambda + \beta, \\ n_3 &= d_3. \end{aligned}$$

Here, Fig. 3 shows that the series connection of controlled vehicles. The velocity  $\bar{v}_i$  is described as

$$\bar{v}_i = \{\bar{G}(s)\}^i \bar{v}_0. \quad (15)$$

Hence, if  $\|\bar{G}(s)\|_\infty \leq 1$ , the velocity  $\bar{v}_i$  cannot diverge from the velocity of equilibrium state even if the number of connected vehicles increases.

Then, the region  $\Omega_1$  for parameters  $\alpha$  and  $\beta$  such that  $\bar{G}(s)$  is stable can be derived as

$$\Omega_1 := \{(\alpha, \beta) \mid \alpha < 0, d_2 > 0, d_1d_2 - d_3 > 0\}$$

by using the Routh stability criterion. Additionally, the region  $\Omega_2$  for parameters  $\alpha$  and  $\beta$  such that  $\|\bar{G}(s)\|_\infty \leq 1$  is derived as

$$\Omega_2 := A \cap (B \cup C \cup D),$$

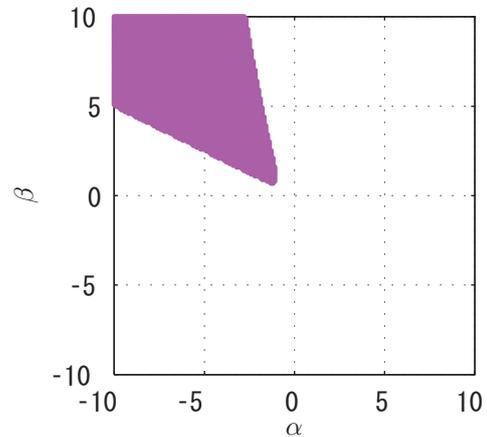
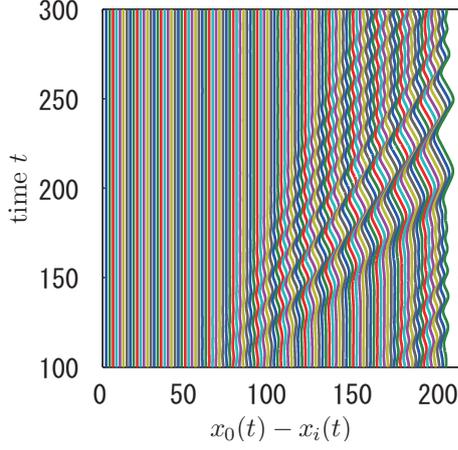
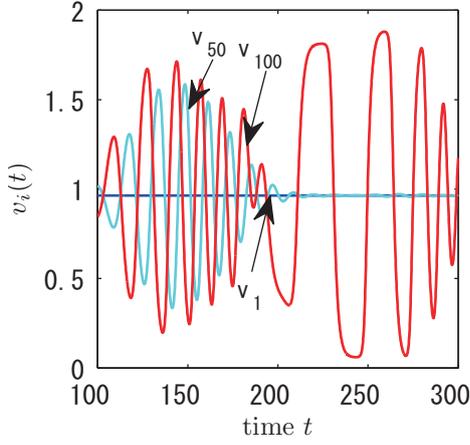


Fig. 4 The region  $\Omega_1 \cap \Omega_2$  of the controller parameters  $\alpha$  and  $\beta$  satisfying  $\|\bar{G}(s)\|_\infty \leq 1$ .



(a) Space-time plot.



(b) Clipped out snapshot of velocity behavior of three vehicles.

Fig. 5 Numerical simulation of the uncontrolled traffic flow.

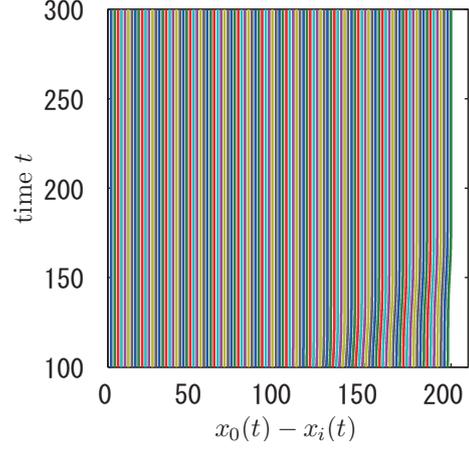
where

$$\begin{aligned}
 A &= \{(\alpha, \beta) \mid \zeta > 0\}, \\
 B &= \{(\alpha, \beta) \mid \eta > 0\}, \\
 C &= \{(\alpha, \beta) \mid \eta^2 - 4\zeta < 0\}, \\
 D &= \{(\alpha, \beta) \mid \eta^2 - 3\zeta < 0\}, \\
 \zeta &= d_2^2 - 2d_1d_3 - n_2^2, \\
 \eta &= d_1^2 - 2d_2.
 \end{aligned}$$

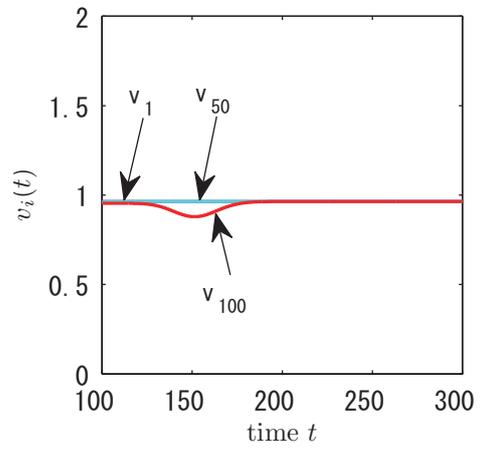
Hence, the intersection  $\Omega_1 \cap \Omega_2$  where gives parameters suppressing traffic jam is illustrated as in Fig. 4. We used parameters  $a = 1.0$  and  $\Lambda = 1.0$  to draw Fig. 4.

#### 4. SIMULATIONS

We have simulated 100 vehicles dynamics. In the simulations, we use the parameters  $a = 1.0$ ,  $y_c = 2.0$  and  $v_0 = 0.964$ . For all simulation results, we used Runge-Kutta algorithm for numerical integration with time step



(a) Space-time plot.

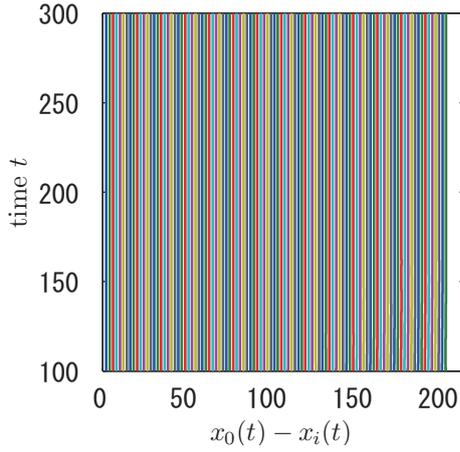


(b) Clipped out snapshot of velocity behavior of three vehicles.

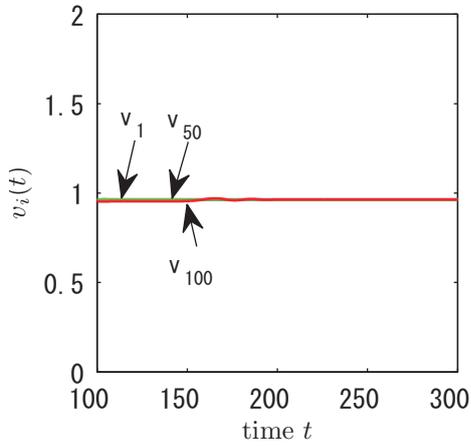
Fig. 6 Numerical simulation of the controlled traffic flow by washout control.

$\Delta t = 0.01$ . The uniform random noise with maximum amplitude  $10^{-3}$  is added to the first equation of (4) for all vehicles.

We simulated the uncontrolled optimal velocity traffic model (i.e.,  $\alpha = 0$ ,  $\beta = 0$ ). Figure 5(a) shows the space-time plot of the distance  $x_0(t) - x_i(t)$  from  $t = 100$  to 300 for all vehicles. The horizontal axis represents a distance between the leading vehicle and each following vehicles. The vertical axis is the time evolution. It can be seen from Fig. 5(a) that the upper vehicle group (the left part of Fig. 5(a)) run constantly with the lead vehicle velocity  $v_0$ , however, we observe the oscillating headway distances in the lower vehicle group (the left part of Fig. 5(a)). In Fig. 5(a), we also find a congestion moving backward. Figure 5(b) shows the velocity behavior of the 1st, 50th, and 100th vehicles. They are denoted by  $v_1$ ,  $v_{50}$  and  $v_{100}$ , respectively. The 1st vehicle runs constantly with velocity  $v_0$ . The 50th vehicle velocity



(a) Space-time plot.



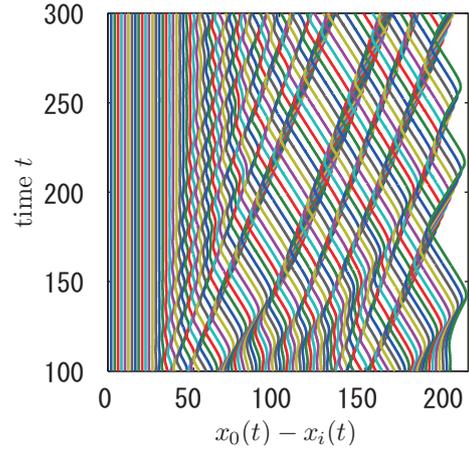
(b) Clipped out snapshot of velocity behavior of three vehicles.

Fig. 7 Numerical simulation of the controlled traffic flow by delayed feedback control.

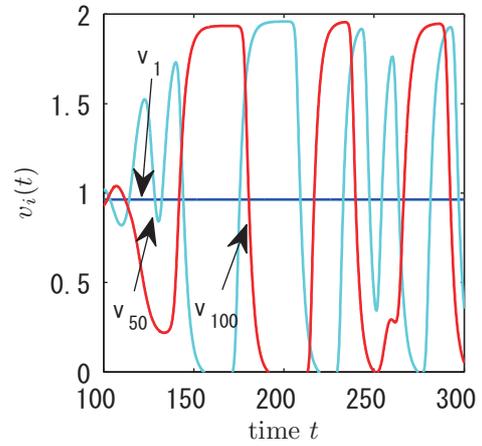
oscillates with accelerate-decelerate actions. The 100th vehicle velocity also oscillates, and the amplitude of this oscillation is larger than that of 50th. These numerical simulations mean that the traffic jam occurs in the optimal velocity traffic model.

#### 4.1 Washout control

We simulated the controlled traffic model. Figure 6(a) shows the space-time plot of the distance  $x_0(t) - x_i(t)$  from  $t = 100$  to 300 when all vehicles are controlled by washout control. The parameters of the washout controller (12) we used are  $\alpha = -5.0$  and  $\beta = 4.0$ . We find no traffic jam in the simulation. Figure 6(b) indicates the velocities of the 1st, 50th, and 100th vehicles. Although the velocity of the 100th vehicle is fluctuated, it can be seen that all vehicles run constantly. This numerical simulation substantiates that washout control is useful to suppress the traffic jam in the optimal velocity



(a) Space-time plot.



(b) Clipped out snapshot of velocity behavior of three vehicles.

Fig. 8 Numerical simulation of the uncontrolled traffic flow with driver's characteristics.

model.

#### 4.2 Delayed feedback

To compare the proposed method with another one, we used a delayed feedback controller [1]. Figure 7(a) shows the space-time plot of the distance  $x_0(t) - x_i(t)$  from  $t = 100$  to 300 when all vehicles are controlled by delayed feedback control. There is also no congestion in the traffic flow. Figure 7(b) indicates the velocities of the 1st, 50th, and 100th vehicles. It can be seen that all the vehicles run constantly.

A washout control and delayed feedback control can suppress the congestion in the traffic flow, even if they have different controller structure.

#### 4.3 Heterogeneous traffic flow

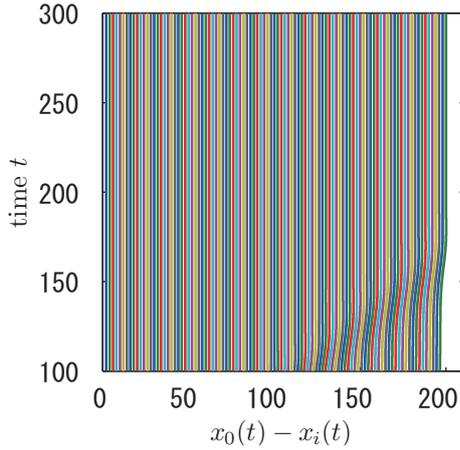
We consider the case where each vehicle has different characteristic to reflect the fact that each driver's feeling is different from others in practice. Instead of

## 5. CONCLUSION

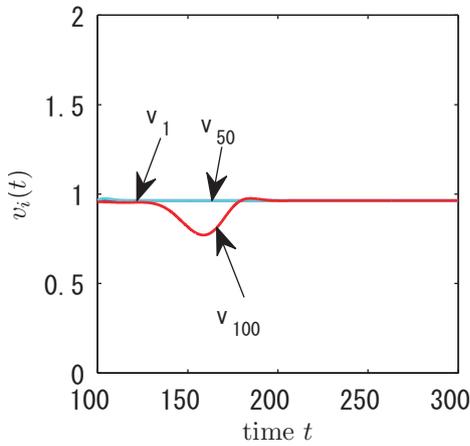
In this paper, we applied a highpass-filter-based controller, called a washout controller, to suppress the traffic jam phenomenon in the optimal velocity traffic model. A salient feature of washout control is to preserve the driver's optimal velocity function. Additionally, washout control which is implemented in each vehicle is independent of the number of vehicles in the traffic flow and does not require other vehicle information (e.g., other vehicle velocity, position, parameters, and so on). We derived a condition for parameters of a washout controller such that the controlled traffic flow is stable. By the condition, parameters of a washout controller can be easily selected. In addition, we showed that the numerical simulations which illustrate the effectiveness of the proposed method. The simulation results contain not only homogeneous traffic flow with all the same vehicle's dynamics, but also heterogeneous traffic flow with different driver's dynamics and characteristics.

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(a) Space-time plot.



(b) Clipped out snapshot of velocity behavior of three vehicles.

Fig. 9 Numerical simulation of the controlled traffic flow with driver's characteristics.

$a = [a_1, a_2, \dots, a_{100}] = [1, 1, \dots, 1]$  where  $a_i$  is the  $i$ th driver's sensitivity, we set  $a = [a_1, a_2, \dots, a_{100}]$  by using a sequence of uniform random numbers which is generated by a MATLAB command "rand" as

$$\begin{aligned} a &= \text{rand}(100, 1) \\ &= [0.67 \quad 0.45 \quad \dots \quad 0.73 \quad 0.56]. \end{aligned} \quad (16)$$

The space-time plot of the distance  $x_0(t) - x_i(t)$  from  $t = 100$  to  $300$  is shown in Fig. 8(a) for uncontrolled heterogeneous traffic flow and in Fig. 9(a) for heterogeneous traffic flow in which all vehicles are controlled by washout control. The velocity behavior of the 1st, 50th, and 100th vehicles are shown in Fig. 8(b) and Fig. 9(b). We see that the traffic jam is suppressed by washout control although there is a transient response of stabilization.