Higgs potential in a minimal S3 invariant extension of the standard model

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Higgs potential in a minimal S_3 invariant extension of the standard model

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Minimal S_3 invariant Higgs potential with real soft S_3 breaking masses is investigated. It is required that without having a problem with triviality, all physical Higgs bosons, except one neutral one, become heavy $\gtrsim 10$ TeV in order to sufficiently suppress flavor-changing neutral currents. There exist three nonequivalent soft mass terms that can be characterized according to their discrete symmetries, and the one that breaks S_3 completely. The S_2' invariant vacuum expectation values (VEVs) of the Higgs fields are the most economic VEVs in the sense that the freedom of VEVs can be completely absorbed into the Yukawa couplings so that it is possible to derive, without referring to the details of the VEVs, the most general form for the fermion mass matrices in minimal S_3 extension of the standard model. We find that except for the completely broken case of the soft terms, the S_2' invariant VEVs are unique VEVs that satisfy the requirement of heavy Higgs bosons. It is found that they also correspond to a local minimum in the completely broken case.

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I. INTRODUCTION

A non-Abelian flavor symmetry is certainly a powerful tool to understand flavor physics. In the case of the standard model (SM), where only one Higgs $SU(2)_L$ doublet is present, any non-Abelian flavor symmetry has to be explicitly broken to describe experimental data. However, if the Higgs sector is extended, and Higgs fields belong to a nontrivial representation of a flavor group [1,2], phenomenologically viable possibilities may arise. The smallest non-Abelian discrete group is S_3.[1] It is a permutation group of three objects, and offers a possible explanation why there are three generations of the quarks and leptons [8,9]. An S_3 invariant Yukawa sector of the SM has exactly five independent couplings [8,9]:

$$1:\ L_aR_aH_a+L_bR_bH_b+L_cR_cH_c,$$

$$2:\ L_a(R_b+R_c)H_a+L_b(R_a+R_c)H_b+L_c(R_a+R_b)H_c,$$

$$3:\ (L_b+L_c)R_aH_a+(L_a+L_c)R_bH_b+(L_a+L_b)R_cH_c,$$

$$4:\ (L_bR_b+L_cR_c)H_a+(L_aR_a+L_cR_c)H_b+(L_aR_a+L_bR_b)H_c,$$

$$5:\ (L_bR_c+L_cR_b)H_a+(L_aR_c+L_cR_a)H_b+(L_aR_b+L_bR_a)H_c, \tag{1}$$

where L_a, R_a, and H_a correspond to three left-handed leptons, right-handed leptons and Higgs bosons, which are subject to permutations. The three dimensional representation **3** of S_3 is not an irreducible representation; **3** can be decomposed into **1** and **2** as

$$\mathbf{1}:\ H_S=\frac{1}{\sqrt{3}}(H_a+H_b+H_c), \tag{2}$$

$$\mathbf{2}:\ (H_1,H_2)=\left(\frac{1}{\sqrt{2}}(H_a-H_b),\frac{1}{\sqrt{6}}(H_a+H_b-2H_c)\right), \tag{3}$$

and similarly for L's and R's. In terms of the fields in the irreducible basis, the five independent Yukawa couplings are [8,9]:

$$L_iR_iH_S,\quad f_{ijk}L_iR_jH_k,\ L_SR_SH_S,\ L_SR_iH_i,\ L_iR_SH_i, \tag{4}$$

where i,j,k run from 1 to 2, and

$$f_{112}=f_{121}=f_{211}=-f_{222}=1. \tag{5}$$

It has been found in [8,9] that these Yukawa couplings are sufficient to reproduce the masses of the quarks and their mixing, and that they are not only consistent with the known observations in the leptonic sector, but also can make testable predictions in the neutrino sector if one assumes an additional discrete symmetry in this sector. In deriving the fermion mass matrices, it has been assumed in [8,9] that the vacuum expectation values (VEVs) of the Higgs fields are S_2' invariant, i.e.,

$$\langle H_S\rangle\neq 0,\quad \langle H_1\rangle=\langle H_2\rangle\neq 0. \tag{6}$$

[1]Flavor symmetries based on a permutation symmetry have been considered by many authors in the past. One of the first papers on permutation symmetries are [1–3,5,6]. See [7] for a review. Phenomenologically viable models based on non-Abelian discrete flavor symmetries S_3,D_4 and A_4 and also on a product of Abelian discrete symmetries have been recently constructed in [8–12,14–18] and [19,20], respectively. (See also [21–25].) However, it is difficult to understand bilarge mixing of neutrinos in terms of Abelian discrete symmetries alone [13].

By the S_2' invariance we mean an invariance under the interchange of H_1 and H_2, i.e.

$$H_1 \leftrightarrow H_2. \tag{7}$$

Note that this permutation symmetry is not a subgroup of the original S_3. Although the Yukawa couplings (4) do not respect this symmetry, each term in the S_3 invariant Higgs potential [given in (9)], except for one term, respects this discrete symmetry. Moreover, as we can see from (4), the S_2' invariant VEVs (6) are the most economic VEVs in the sense that the freedom of VEVs can be completely absorbed into the Yukawa couplings so that we can derive the most general form for the fermion mass matrices

$$\mathbf{M} = \begin{pmatrix} m_1+m_2 & m_2 & m_5 \\ m_2 & m_1-m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix} \tag{8}$$

without referring to the details of VEVs. In other words, if $\langle H_1 \rangle \neq \langle H_2 \rangle$, the mass matrices would have one more independent parameter that should be determined in the Higgs sector.

In the present paper we investigate how different the S_2' invariant vacuum is under the requirement that except for one neutral physical Higgs boson, all the physical Higgs bosons can become heavy $\gtrsim 10$ TeV without having a problem with triviality [26]. This bound results in order to suppress three-level flavor-changing neutral currents (FCNCs) that contribute, for instance, to the mass difference Δm_K of K^0 and $\overline{K}^0$ in S_3 invariant extension of the SM [27,28]. (See also Refs. [3] and [4].) The investigations are presented in Secs. III and IV, and the conclusions are summarized in the last section. In Sec. V we discuss the Pakvasa-Sugawara vacuum [1], and a supersymmetric case is treated in Sec. VI.

II. S_3 INVARIANT HIGGS POTENTIAL AND SOFT S_3 BREAKING

A. S_3 invariant Higgs potential and its problem

The most general, S_3 invariant, renormalizable potential is given by [1]

$$V_H = V_{2H} + V_{4H}, \tag{9}$$

$$V_{2H} = -\mu_1^2 (H_1^\dagger H_1 + H_2^\dagger H_2) - \mu_3^2 H_S^\dagger H_S,$$

$$\begin{aligned} V_{4H} = &+ \lambda_1 (H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2 (H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\ &+ \lambda_3 [(H_1^\dagger H_2 + H_2^\dagger H_1)^2 + (H_1^\dagger H_1 - H_2^\dagger H_2)^2] \\ &+ [\lambda_4 f_{ijk} (H_S^\dagger H_i)(H_j^\dagger H_k) + \text{H.c.}] + \lambda_5 (H_S^\dagger H_S)(H_1^\dagger H_1 \\ &+ H_2^\dagger H_2) + \lambda_6 \{(H_S^\dagger H_1)(H_1^\dagger H_S) + (H_S^\dagger H_2)(H_2^\dagger H_S)\} \\ &+ \{\lambda_7 [(H_S^\dagger H_1)(H_S^\dagger H_1) + (H_S^\dagger H_2)(H_S^\dagger H_2)] + \text{H.c.}\} \\ &+ \lambda_8 (H_S^\dagger H_S)^2, \end{aligned} \tag{10}$$

where λ_4 and λ_7 can be complex.[2] We first redefine H_i as

$$H_\pm = \frac{1}{\sqrt{2}} (H_1 \pm H_2), \tag{11}$$

and write the $SU(2)_L$ Higgs doublets in components:

$$H_\pm = \begin{pmatrix} h_\pm + i\chi_\pm \\ \frac{1}{\sqrt{2}} (h_\pm^0 + i\chi_\pm^0) \end{pmatrix}, \quad H_S = \begin{pmatrix} h_S + i\chi_S \\ \frac{1}{\sqrt{2}} (h_S^0 + i\chi_S^0) \end{pmatrix}. \tag{12}$$

The down components of the Higgs doublets have zero electric charge, and therefore, we assume that only the down components can acquire a VEV. Further, because of $U(1)_Y$ gauge invariance, it is always possible to make a phase rotation for H_S so that only the real part h_S^0 can get VEV. We denote the VEVs as follows:

$$\langle h_\pm^0 \rangle = v_\pm, \quad \langle h_S^0 \rangle = v_S, \quad \langle \chi_\pm \rangle = c_\pm, \tag{13}$$

which should satisfy the constraint

$$(v_+^2 + v_-^2 + v_S^2 + c_+^2 + c_-^2)^{1/2} = v \simeq 246 \text{ GeV}. \tag{14}$$

In order to reproduce realistic fermion masses and their mixings [8], we also require that

$$v_S \neq 0, \quad \text{and at least one of} \quad v_\pm \quad \text{and} \quad c_\pm \neq 0 \tag{15}$$

is satisfied, and do not allow a large hierarchy among the nonvanishing VEVs, unless it is noticed. (In Secs. II and III, however, we allow such hierarchy.)

There are five minimization conditions:

$$0 = -v_S \mu_3^2 + \partial V_{4H} / \partial h_S^0, \tag{16}$$

$$0 = -v_+ \mu_1^2 + \partial V_{4H} / \partial h_+^0, \tag{17}$$

$$0 = -v_- \mu_1^2 + \partial V_{4H} / \partial h_-^0, \tag{18}$$

$$0 = -c_+ \mu_1^2 + \partial V_{4H} / \partial \chi_+^0, \tag{19}$$

$$0 = -c_- \mu_1^2 + \partial V_{4H} / \partial \chi_-^0. \tag{20}$$

We regard VEVs as independent parameters and express the parameters of the potential (9), especially the mass parameters μ_1^2 and μ_3^2, in terms of the VEVs. To make all the physical Higgs bosons except one neutral Higgs boson without having large values of the Higgs quartic couplings λ's, we have to have either $-\mu_3^2, -\mu_1^2 \gg v^2$ or $-\mu_1^2 \gg v^2$, where v is defined in (14). For the first case, none of the VEVs can be $O(v)$ because the derivative terms, i.e., $\partial V_{4H}/\partial h_+^0$ etc., are of $O(\text{VEV}^3)$. Therefore, this case cannot satisfy the con-

[2] The S_3 invariant potential has been studied in [1,21], for instance. Similar potentials with non-Abelian discrete symmetries have been also studied in [2,3,14,29].

straint (14). For the second case, μ_3 and v_S can be $O(v)$, but none of v_+, v_-, c_+, c_- can be $O(v)$. That is, the hierarchy $|v_+/v_S|, |v_-/v_S|, |c_+/v_S|, |c_-/v_S| \ll 1$ has to be satisfied. This hierarchy is consistent with the minimization conditions (17)–(20), only if at least one of the derivative terms, i.e., $\partial V_{4H}/\partial h_+^0$ etc., contains at least a term proportional to v_S^3. However, this is not the case, as we can see from the potential V_{4H} (10). Moreover, (15) does not allow $v_+ = v_- = c_+ = c_- = 0$.

It is thus clear, if the two conditions (14) and (15) are satisfied, that $\mu_1^2, \mu_3^2 \sim O(\mathrm{VEV}^2)$, which means that all the masses of the physical Higgs bosons are of $O(\mathrm{VEV})$. That is, to have a large Higgs mass, the value of certain Higgs couplings λ's have to be large. Then we run into the problem with triviality; the Higgs mass cannot be larger than the cutoff. As we see from (9), the model has many Higgs couplings, so that the known triviality bound on the Higgs mass, ~ 700 GeV [26], cannot be directly applied. But we may assume that the bound for the present case does not differ very much from that of the SM. However, this upper bound is too low to suppress three-level flavor changing neutral currents (FCNCs) that contribute, for instance, to the mass difference Δm_K of K^0 and $\overline{K}^0$; certain Higgs masses in S_3 invariant extension of the SM have to be larger than $\sim O(10)$ TeV [3,27,28]. Therefore, in a phenomenologically viable S_3 extension of the SM, S_3 symmetry should be broken, unless there is some cancellation mechanism of FCNCs.

B. Soft S_3 breaking terms and their characterization

As we have seen above, we have to modify the Higgs potential (9) to make it possible that the Higgs masses can become lager than 10 TeV. How should we break S_3? We would like to maintain the consistency and predictions of S_3 in the Yukawa sector, while simultaneously satisfying the experimental constraints from the FCNC phenomena. Therefore, we break S_3 as softly as possible. The softest operators in the case at hand are those of dimension two; that is, mass terms. There are four soft-breaking mass terms

$$V_{SB} = -\mu_2^2(H_+^\dagger H_+ - H_-^\dagger H_-) - \sqrt{2}(\mu_4^2 H_S^\dagger H_+ + \mathrm{H.c.}) - (\mu_5^2 H_+^\dagger H_- + \mathrm{H.c.}) - \sqrt{2}(\mu_6^2 H_S^\dagger H_- + \mathrm{H.c.}). \quad (21)$$

μ_4^2, μ_5^2, and μ_6^2 can be complex parameters.[3] However, we assume that they are real parameters in following discussions except in Sec. V. We would like to characterize these four mass terms according to discrete symmetries:

$$R: H_S \to -H_S, \quad (22)$$

$$S_2': H_- \to -H_-, \quad (23)$$

$$S_2'': H_+ \to -H_+, \quad (24)$$

$$R \times S_2': H_S \to -H_S \ \text{and} \ H_- \to -H_-, \quad (25)$$

$$R \times S_2'': H_S \to -H_S \ \text{and} \ H_+ \to -H_+, \quad (26)$$

$$S_2' \times S_2'': H_- \to -H_- \ \text{and} \ H_+ \to -H_+, \quad (27)$$

where S_2' and S_2'' are not a subgroup of the original S_3. Accordingly, we characterize the soft mass terms (21) as

$$R: \mu_4 = \mu_6 = 0, \quad (28)$$

$$S_2': \mu_5 = \mu_6 = 0, \quad (29)$$

$$S_2'': \mu_4 = \mu_5 = 0, \quad (30)$$

$$R \times S_2': \mu_4 = \mu_5 = \mu_6 = 0, \quad (31)$$

$$R \times S_2'': \mu_4 = \mu_5 = \mu_6 = 0, \quad (32)$$

$$S_2' \times S_2'': \mu_4 = \mu_5 = \mu_6 = 0. \quad (33)$$

Actually, there are only four nonequivalent soft-breaking mass terms, including one without any discrete symmetry. This is because S_2' and S_2'' are not independent: The Higgs potential (9) and the soft terms (21) are invariant under the interchange of H_+ and H_- if one appropriately redefines the coupling constants and mass parameters. In the next section we will discuss the three cases, i.e., R, S_2' and $R \times S_2'$ invariant cases, and in Sec. IV we will treat the completely broken case, in which all the soft mass terms (21) are present. Each possibility is renormalizable because all the other interactions are S_3 invariant and cannot induce infinite S_3 violating breaking terms (21). In principle, μ_4^2, μ_5^2, and μ_6 can be complex. As announced, however, we assume that they are real, except for Sec. V. This is consistent with renormalizability from the same reason above.

Before we go to the next sections, it may be worthwhile to write down explicitly the λ_4 and λ_7 terms of the potential V_{4H} (10):

$$\begin{aligned}2\sqrt{2}V_{\lambda_4 H} = {} & [\mathrm{Re}(\lambda_4)\chi_S^0 - \mathrm{Im}(\lambda_4)h_S^0][(\chi_+^0)^3 + 3(\chi_+^0)^2\chi_-^0 \\ & - 3\chi_+^0(\chi_-^0)^2 - (\chi_-^0)^3 + \chi_+^0(h_+^0)^2 + \chi_-^0(h_+^0)^2 \\ & + 2\chi_+^0 h_+^0 h_-^0 - 2\chi_-^0 h_+^0 h_-^0 - \chi_+^0(h_-^0)^2 \\ & - \chi_-^0(h_-^0)^2] + [\mathrm{Re}(\lambda_4)h_S^0 + \mathrm{Im}(\lambda_4)\chi_S^0] \\ & \times[(\chi_+^0)^2 h_+^0 - (\chi_-^0)^2 h_+^0 + 2\chi_+^0\chi_-^0 h_+^0 \\ & - 2\chi_+^0\chi_-^0 h_-^0 + (h_+^0)^3 - (h_-^0)^3 + (\chi_+^0)^2 h_-^0 \\ & - (\chi_-^0)^2 h_-^0 + 3(h_+^0)^2 h_-^0 - 3h_+^0(h_-^0)^2], \end{aligned} \quad (34)$$

[3]The soft mass terms (21) may be generated from a S_3 invariant Higgs potential by introducing certain S_3 singlet Higgs fields [4].

$$V_{\lambda_7 H}=\frac{\mathrm{Re}(\lambda_7)}{2}\{[(\chi_+^0)^2+(\chi_-^0)^2-(h_+^0)^2-(h_-^0)^2]$$
$$\times[(\chi_S^0)^2-(h_S^0)^2]+4[\chi_+^0 h_+^0+\chi_-^0 h_-^0]\chi_S^0 h_S^0\}$$
$$+\mathrm{Im}(\lambda_7)\{(\chi_+^0 h_+^0+\chi_-^0 h_-^0)[(\chi_S^0)^2-(h_S^0)^2]$$
$$-[(\chi_+^0)^2+(\chi_-^0)^2-(h_+^0)^2-(h_-^0)^2]\chi_S^0 h_S^0\}, \quad (35)$$

where only those terms containing the neutral components are written above. The rest of the terms in V_{4H} have the form

$$(h_S^0)^{2n_1}(h_+^0)^{2n_2}(h_-^0)^{2n_3}(\chi_S^0)^{2n_4}(\chi_+^0)^{2n_5}(\chi_-^0)^{2n_6}$$

with

$$\sum_{i=1}^{6} n_i=2 \quad \text{and} \quad n_i=0,1,2. \quad (36)$$

III. MINIMIZATION CONDITIONS AND HIGGS MASSES

Below we will analyze the total potential $V_T=V_H+V_{SB}$ for the three nonequivalent cases (28), (29), and (31). We consider only phenomenologically viable cases (15). But we do allow, if necessary, a large hierarchy among the nonvanishing VEVs. In all the cases, $\lambda_4=0$ follows from the discrete symmetry in question.

$R\times S_2'$ ($\mu_4=\mu_5=\mu_6=0;\lambda_4=0$): The five minimization conditions in this case are given by

$$0=-v_S\mu_3^2+\partial V_{4H}/\partial h_S^0, \quad (37)$$

$$0=-v_+(\mu_1^2+\mu_2^2)+\partial V_{4H}/\partial h_+^0, \quad (38)$$

$$0=-v_-(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial h_-^0, \quad (39)$$

$$0=-c_+(\mu_1^2+\mu_2^2)+\partial V_{4H}/\partial \chi_+^0, \quad (40)$$

$$0=-c_-(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial \chi_-^0, \quad (41)$$

where the second derivative terms, i.e., $\partial V_{4H}/\partial h^0$ and $\partial V_{4H}/\partial \chi^0$, are $\sim O(\mathrm{VEV}^3)$. We first observe that, because of the absence of λ_4, the condition (37) requires $\mu_3 \sim O(\mathrm{VEV})$. If $|\mu_1^2\pm\mu_2^2|\gg v^2$ should be satisfied, then none of v_+,v_-,c_+,c_- can be $O(v)$. But this is not consistent with (38)–(41) because of the absence of v_S^3 terms in the derivative terms of (38)–(41). Therefore, taking into account the condition (15), at least one of v_+,v_-,c_+,c_- has to be $O(v)$. Assume that $v_+\sim O(v)$, which means that $\mu_1^2=-\mu_2^2+O(\mathrm{VEV}^2)$. Consequently, the total Higgs potential in this case can be written as

$$V_T=-2\mu_1^2 H_-^\dagger H_-+\cdots, \quad (42)$$

where the terms indicated by$\cdots$are those that are proportional to VEV^n ($n=1,\ldots,4$). Therefore, only H_- can obtain a large mass, if $-2\mu_1^2$ is positive and large. So, this case does not satisfy the phenomenological requirement that all the physical Higgs bosons, except one, can be made heavy without running into the problem with triviality.

One can perform similar analyses for other cases such as $c_-\sim 0(v)$. [$v_S\neq 0$ is always assumed.] As before, one finds that only one $SU(2)_L$ doublet can become heavy. So, the soft masses with the discrete symmetry $R\times S_2'$ cannot be used for a phenomenologically viable model.

R ($\mu_4=\mu_6=0;\lambda_4=0$): The five minimization conditions in this case are given by

$$0=-v_S\mu_3^2+\partial V_{4H}/\partial h_S^0 \quad (43)$$

$$0=-v_+(\mu_1^2+\mu_2^2)-v_-\mu_5^2+\partial V_{4H}/\partial h_+^0, \quad (44)$$

$$0=-v_+\mu_5^2-v_-(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial h_-^0, \quad (45)$$

$$0=-c_+(\mu_1^2+\mu_2^2)-c_-\mu_5^2+\partial V_{4H}/\partial \chi_+^0, \quad (46)$$

$$0=-c_+\mu_5^2-c_-(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial \chi_-^0. \quad (47)$$

Again, because of (43), $\mu_3\sim O(\mathrm{VEV})$. $|\mu_5|$ has to be large, otherwise the situation is the same as in the previous case. Equations (44) and (45) have a nontrivial solution

$$\mu_1^2=-\frac{\mu_5^2(v_+^2+v_-^2)+O(\mathrm{VEV}^4)}{2v_+v_-},$$

$$\mu_2^2=\frac{\mu_5^2(v_+^2-v_-^2)+O(\mathrm{VEV}^4)}{2v_+v_-}, \quad (48)$$

if $v_+\neq 0, v_-\neq 0$. Then the total potential becomes

$$V_T=m_H^2 H_H^\dagger H_H+\cdots, \quad (49)$$

where, as before, the terms indicated by $\cdots$ are those that are proportional to VEV^n ($n=1,\ldots,4$), and

$$H_H=\frac{v_-H_+-v_+H_-}{(v_+^2+v_-^2)^{1/2}}, \quad m_H^2=\frac{v_+^2+v_-^2}{v_+v_-}\mu_5^2. \quad (50)$$

Therefore, only H_H can become heavy.

If $v_-=0$, Eq. (45) requires $|v_+/v|\ll 1$ because $|\mu_5|\gg v$ has to be satisfied. To satisfy Eq. (45), on one hand, at least one of c_+, and c_- has to be $O(v)$ because of the absence of v_S^3 terms in the derivative term. On the other hand, we obtain Eq. (48) with $v_\pm\to c_\pm$. [$c_+\sim O(\mathrm{VEV}), c_-=0$ and $c_+=0, c_-=O(\mathrm{VEV})$ cannot satisfy (46) and (47).]

The case $v_+=0$ is equivalent to the case $v_-=0$. If $v_+=v_-=0$, the situation does not change. From these considerations, we conclude that the case at hand does not satisfy the phenomenological requirement.

S_2' ($\mu_5=\mu_6=0;\lambda_4=0$): The five minimization conditions in this case are given by

$$0=-v_S\mu_3^2-\sqrt{2}v_+\mu_4^2+\partial V_{4H}/\partial h_S^0, \quad (51)$$

$$0=-v_+(\mu_1^2+\mu_2^2)-\sqrt{2}v_S\mu_4^2+\partial V_{4H}/\partial h_+^0, \quad (52)$$

$$0=-v_-(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial h_-^0, \quad (53)$$

$$0=-c_{+}(\mu_1^2+\mu_2^2)+\partial V_{4H}/\partial\chi_{+}^0, \tag{54}$$

$$0=-c_{-}(\mu_1^2-\mu_2^2)+\partial V_{4H}/\partial\chi_{-}^0. \tag{55}$$

Note that the derivative terms in (53)–(55) contain at least of one of v_-, c_+, and c_-. Therefore, large values for μ_1 and μ_2 can be consistent with (53)–(55), only if (i) $v_-=c_+=c_-=0$ and (ii) $\mu_1^2=\mu_2^2+O(\mathrm{VEV}^2)$ or (iii) $\mu_1^2=-\mu_2^2+O(\mathrm{VEV}^2)$. Keeping this in mind, we next solve (51) and (52) to obtain

$$\mu_3^2=\frac{v_+^2(\mu_1^2+\mu_2^2)+O(\mathrm{VEV}^4)}{v_S^2},$$

$$\mu_4^2=-\frac{v_+(\mu_1^2+\mu_2^2)+O(\mathrm{VEV}^3)}{\sqrt{2}v_S}. \tag{56}$$

Inserting (56) into the total Higgs potential V_T, we obtain

$$V_T=-(\mu_1^2-\mu_2^2)H_-^\dagger H_- -\frac{\mu_1^2+\mu_2^2}{2v_S^2}[(v_S H_+^\dagger - v_+ H_S^\dagger)\times(v_S H_+ - v_+ H_S)+\mathrm{H.c.}]+\cdots. \tag{57}$$

We see from (57) that case (ii) can be ruled out, because in this case H_- cannot obtain a large mass. We can also see from (57) that case (iii) allows large values of the Higgs masses if $|v_+/v_S|\gtrsim 40$. However, (53) and (55) require that $|v_-/v|,|c_-/v|\ll 1$. Note that the derivative terms of (53) and (55) contain at least one of v_-, c_-, which implies that $v_-=c_-=0$ to satisfy (53) and (55). c_+ is nonvanishing in case (iii). For case (i) we obtain the same form of the leading potential V_T, but no restriction on the ratio v_+/v_S. In terms of VEVs, we have $v_-=c_+=c_-=0$ for case (i), and $v_-=c_-=0$ for case (iii). These two types of VEVs are S_2' invariant VEVs (6). Both types of VEVs give rise to the general form of the fermion mass matrix (8).

Below we would like to consider only the case (i) ($v_-=c_+=c_-=0$), and give the mass matrix $\mathbf{m}_\mathrm{h}^2$ of the neutral scalar Higgs bosons

$$h_-^0, h_L^0=\sin\gamma h_+^0+\cos\gamma h_S^0, h_H^0=\cos\gamma h_+^0-\sin\gamma h_S^0, \tag{58}$$

and the mass matrix $\mathbf{m}_\chi^2$ the neutral pseudoscalar Higgs bosons

$$\chi_-^0,\quad \chi_L^0=\sin\gamma\chi_+^0+\cos\gamma\chi_S^0, \chi_H^0=\cos\gamma\chi_+^0-\sin\gamma\chi_S^0, \tag{59}$$

are, respectively, given by

$$\mathbf{m}_\mathrm{h}^2=\begin{pmatrix} m_{h_-^0}^2 & 0 & 0\\ 0 & m_{h22}^2 & m_{h23}^2\\ 0 & m_{h23}^2 & m_{h33}^2\end{pmatrix},\quad \mathbf{m}_\chi^2=\begin{pmatrix} m_{\chi_-^0}^2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & m_{\chi_H^0}^2\end{pmatrix}, \tag{60}$$

where

$$m_{h_-^0}^2=2\mu_2^2+\sqrt{2}\cot\gamma\mu_4^2\simeq-(\mu_1^2-\mu_2^2), \tag{61}$$

$$m_{h22}^2=v^2\left[2(\lambda_1+\lambda_3)\sin^4\gamma+\frac{1}{2}(\lambda_5+\lambda_6+2\lambda_7)\sin^2 2\gamma+2\lambda_8\cos^4\gamma\right], \tag{62}$$

$$m_{h23}^2=\frac{v^2}{2}\sin 2\gamma[(\lambda_1+\lambda_3)(1-\cos 2\gamma)+(\lambda_5+\lambda_6+2\lambda_7)\cos 2\gamma-2\lambda_8\cos^2\gamma], \tag{63}$$

$$m_{h33}^2=2\sqrt{2}\mu_4^2/\sin 2\gamma+\frac{v^2}{2}(\lambda_1+\lambda_3-\lambda_5-\lambda_6-2\lambda_7+\lambda_8)\sin^2 2\gamma, \tag{64}$$

$$m_{\chi_-^0}^2=2\mu_2^2\sqrt{2}+\mu_4^2\cot\gamma-2(\lambda_2+\lambda_3)v_+^2-2\lambda_7 v_S^2, \tag{65}$$

$$m_{\chi_H^0}^2=2\sqrt{2}\mu_4^2/\sin 2\gamma-2\lambda_7 v^2, \tag{66}$$

and we have introduced $[v=(v_+^2+v_S^2)^{1/2}]$

$$\tan\gamma=\frac{v_+}{v_S}. \tag{67}$$

In (61)–(65), we have taken into account the higher order terms of (57) with $\lambda_4=\mathrm{Im}(\lambda_7)=0$. ($\lambda_4=0$ follows from the S_2' symmetry): If λ_4 and $\mathrm{Im}(\lambda_7)$ do not vanish, there is no local minimum for case (i). As we can see from the mass matrices (60) with (61)–(65), the pseudoscalar boson (60), χ_L, is the would-be Goldstone boson, and that except for h_L^0 all the physical Higgs bosons can become heavy without large Higgs couplings λ's. We also find from (58) and (67) that only h_L^0 acquires VEV. Since only h_L^0 acquires VEV, its coupling to the fermions is flavor diagonal, while the other physical neutral Higgs bosons have FCNC couplings. However, h_L^0 still mixes with h_H^0 because of the nonvanishing entry m_{h23}^2. Therefore, we have to fine tune so that m_{h23}^2 vanishes. (Of course, the mixing is suppressed by $v^2/\mu_4^2\sim 6\times 10^{-4}$ for $\mu_4\sim 10$ TeV.) In this limit, m_{h33} and m_{h22} are the masses of h_H^0 and the lightest Higgs h_L^0, respectively.

IV. SOFT BREAKING WITHOUT SYMMETRY

Here we would like to investigate the full potential $V_T=V_H+V_{SB}$ without any assumption on Abelian discrete symmetries. The reason is that S_2' is not a symmetry of the theory; it can be a symmetry only in the Higgs potential. So, radiative corrections can induce finite non-S_2'-invariant terms in the Higgs potential, for instance. Here we assume that all the soft masses (21) are present, and that they are still real. We, however, do not allow an unnatural large hierarchy of the VEVs, in contrast to the previous sections. There are exactly nine nonequivalent possibilities that satisfy the phenomenological requirement (15):

$$A_1:\ v_- = c_+ = c_- = 0;\ \ A_2: v_+ = v_- = c_- = 0; \tag{68}$$

$$B_1: c_+ = c_- = 0;\ \ B_2: v_- = c_- = 0;\ \ B_3: v_- = c_+ = 0;$$

$$B_4:\ v_+ = v_- = 0; \tag{69}$$

$$C_1:\ c_- = 0;\ \ C_2: v_- = 0; \tag{70}$$

$$D:\ \text{none of them } = 0. \tag{71}$$

It will turn out that among these nine possibilities only two cases, A_1 and B_1, satisfy the phenomenological constraint that all the Higgs bosons except one can be made heavy without running into the problem with triviality. Note that A_1 and also B_2 exhibit the S_2' invariant VEVs (6).

A_1 ($v_- = c_+ = c_- = 0$): We start with the case A_1. The first case A_1 corresponds to the S_2' invariant VEVs (6). The nontrivial minimization conditions at $v_- = c_+ = c_- = 0$ are given by

$$0 = -v_S\mu_3^2 - \sqrt{2}v_+\mu_4^2 + \partial V_{4H}/\partial h_S^0, \tag{72}$$

$$0 = -v_+(\mu_1^2 + \mu_2^2) - \sqrt{2}v_S\mu_4^2 + \partial V_{4H}/\partial h_+^0, \tag{73}$$

$$0 = -\sqrt{2}v_S\mu_6^2 - v_+\mu_5^2 + \partial V_{4H}/\partial h_-^0, \tag{74}$$

$$0 = \partial V_{4H}/\partial \chi_+^0 = v_+ v_S[v_+ \mathrm{Im}(\lambda_4)/2\sqrt{2} + v_S \mathrm{Im}(\lambda_7)], \tag{75}$$

$$0 = \partial V_{4H}/\partial \chi_-^0 = -v_+^2 v_S \mathrm{Im}(\lambda_4)/2\sqrt{2}. \tag{76}$$

Equation (75) requires $0 = \mathrm{Im}(\lambda_7) + (v_+/2\sqrt{2}v_S)\mathrm{Im}(\lambda_4)$, and (76) requires $\mathrm{Im}(\lambda_4) = 0$. So, we assume that λ_7 and λ_4 are real. (In the case of the S_2' invariant soft term (29), λ_4 has to vanish for the S_2' VEVs (6) to correspond to a local minimum.) We then use (72)–(74) to express μ_1^2, μ_3^2, and μ_5^2 in terms of VEVs:

$$\begin{aligned}
\mu_1^2 &= -\mu_2^2 - \sqrt{2}\mu_4^2 \cot\gamma + O(\mathrm{VEV}^2),\\
\mu_3^2 &= -\sqrt{2}\mu_4^2 \tan\gamma + O(\mathrm{VEV}^2),\\
\mu_5^2 &= -\sqrt{2}\mu_6^2 \cot\gamma + O(\mathrm{VEV}^2),
\end{aligned} \tag{77}$$

where γ is defined in (67). Inserting μ's of (77) into the total potential, we can compute the mass matrices and find

$$\mathbf{m}_\mathrm{h}^2 \simeq \begin{pmatrix} 2\mu_2^2 + \sqrt{2}\mu_4^2\cot\gamma & 0 & \sqrt{2}\mu_6^2/\sin\gamma \\ 0 & 0 & 0 \\ \sqrt{2}\mu_6^2/\sin\gamma & 0 & 2\sqrt{2}\mu_4^2/\sin 2\gamma \end{pmatrix} + O(\mathrm{VEV}^2) \simeq \mathbf{m}_\chi^2 \tag{78}$$

for the basis (58) and (59). Comparing these results with (60), we find that apart from the $O(\mathrm{VEV}^2)$ terms, the masses (78) reduce to those of the S_2' invariant case (60) as μ_6^2 [and hence μ_5^2 because of (77)] goes to zero. Therefore, the S_2' invariant local minimum exists in the full Higgs potential, if all the mass parameters are real.

A_2 ($v_+ = v_- = 0$): The five minimization conditions at $v_+ = v_- = c_- = 0$ [which is of the S_2' invariant type (6)] are given by

$$0 = -v_S\mu_3^2 + \partial V_{4H}/\partial h_S^0, \tag{79}$$

$$0 = -\sqrt{2}v_S\mu_4^2 + \partial V_{4H}/\partial h_+^0, \tag{80}$$

$$0 = -\sqrt{2}v_S\mu_6^2 + \partial V_{4H}/\partial h_-^0, \tag{81}$$

$$0 = -c_+(\mu_1^2 + \mu_2^2) + \partial V_{4H}/\partial \chi_+^0, \tag{82}$$

$$0 = -c_+\mu_5^2 + \partial V_{4H}/\partial \chi_-^0. \tag{83}$$

Equations (79)–(83) imply[4] that $\mu_3^2, (\mu_1^2 + \mu_2^2), \mu_4^2, \mu_5^2, \mu_6^2 \sim O(\mathrm{VEV}^2)$. Inserting μ's above into the total potential, we find

$$V_T = 2\mu_2^2 H_-^\dagger H_- + O(\mathrm{VEV}^4). \tag{84}$$

So, only H_- can become heavy.

B_1 ($c_+ = c_- = 0$): The five minimization conditions at $c_+ = c_- = 0$ are given by

$$0 = -v_S\mu_3^2 - \sqrt{2}v_+\mu_4^2 - \sqrt{2}v_-\mu_6^2 + \partial V_{4H}/\partial h_S^0, \tag{85}$$

$$0 = -v_+(\mu_1^2 + \mu_2^2) - \sqrt{2}v_S\mu_4^2 - v_-\mu_5^2 + \partial V_{4H}/\partial h_+^0, \tag{86}$$

$$0 = -v_+\mu_5^2 - \sqrt{2}v_S\mu_6^2 - v_-(\mu_1^2 - \mu_2^2) + \partial V_{4H}/\partial h_-^0, \tag{87}$$

$$0 = -(v_+^2 + 2v_+v_- - v_-^2)v_S \mathrm{Im}(\lambda_4)/2\sqrt{2} - v_+ v_S^2 \mathrm{Im}(\lambda_7), \tag{88}$$

$$0 = -(v_+^2 - 2v_+v_- - v_-^2)v_S \mathrm{Im}(\lambda_4)/2\sqrt{2} - v_- v_S^2 \mathrm{Im}(\lambda_7). \tag{89}$$

Equations (88) and (89) require $\mathrm{Im}(\lambda_7) = \mathrm{Im}(\lambda_4) = 0$. Solving (85)–(87) to express μ_1^2, μ_3^2 and μ_6^2 in terms of v_S, v_+, and v_-, and inserting them into the total potential, we obtain

[4]As announced, we do not allow an unnatural large hierarchy among the VEVs. If, for instance, $|v_S/v| \ll 1$, then μ_3^2 can be large thanks to the nonvanishing λ_4. In this case, H_S can become heavy.

$$\begin{aligned}V_T=&\{[2\mu_2^2 v_-^2+\mu_5^2(v_-^3/v_+-v_+v_-)]/v_S^2)\\&+\sqrt{2}\mu_4^2(v_++v_-^2/v_+)/v_S\}H_S^\dagger H_S\\&+[2\mu_4^2(v_S/v_+)+\mu_5^2(v_-/v_+)]H_+^\dagger H_+\\&+[2\mu_2^2+\mu_5^2(v_-/v_+)+\sqrt{2}\mu_4^2(v_S/v_+)]H_-^\dagger H_-\\&+\{-\sqrt{2}\mu_4^2(v_-/v_+)-2\mu_2^2(v_-/v_S)+\mu_5^2[(v_+/v_S)\\&-(v_-^2/v_-v_S)]\}(H_S^\dagger H_-+\text{H.c.})-\mu_5^2(H_+^\dagger H_-+\text{H.c.})\\&-\sqrt{2}\mu_4^2(H_S^\dagger H_++\text{H.c.})+O(\text{VEV}^4).\end{aligned}\tag{90}$$

One can show that except for $h_L=(v_S h_S^0+v_+h_+^0+v_-h_-^0)/(v_+^2+v_-^2+v_S^2)^{1/2}$ all the physical Higgs bosons can become heavy. So, this case satisfies the phenomenological requirements.

$B_{2,3,4},C_{1,2},D$: We have performed similar analyses for the rest of the cases and found that none of $B_{2,3,4}$, $C_{1,2}$, and D cases satisfy our requirement (if we do not allow a large hierarchy among the VEVs).

V. THE PAKVASA-SUGAWARA VACUUM

The Pakvasa-Sugawara (PS) VEVs [1] are given by

$$v_-=c_+=0,\tag{91}$$

which is nothing but the case B_3 given in (69). As we mentioned, the S_3 invariant potential (9) does meet the requirement that except for one neutral physical Higgs boson, all the physical bosons can become heavy. On the other hand, the PS VEVs (91) are the most economic VEVs in the case of a spontaneous *CP* violation; only one phase, which should be determined in the Higgs sector, enters into the Yukawa sector. Here we would like to analyze the most general case with complex soft masses in contrast to the previous sections. The minimization conditions are

$$0=-v_S\mu_3^2-\sqrt{2}v_+\text{Re}(\mu_4^2)+\sqrt{2}c_-\text{Im}(\mu_6^2)+\partial V_{4H}/\partial h_S^0,\tag{92}$$

$$\begin{aligned}0=&-v_+(\mu_1^2+\mu_2^2)-\sqrt{2}v_S\text{Re}(\mu_4^2)+c_-\text{Im}(\mu_5^2)\\&+\partial V_{4H}/\partial h_+^0,\end{aligned}\tag{93}$$

$$0=-\sqrt{2}v_S\text{Re}(\mu_6^2)-v_+\text{Re}(\mu_5^2)+\partial V_{4H}/\partial h_-^0,\tag{94}$$

$$0=\sqrt{2}v_S\text{Im}(\mu_4^2)-c_-\text{Re}(\mu_5^2)+\partial V_{4H}/\partial\chi_+^0,\tag{95}$$

$$\begin{aligned}0=&\sqrt{2}v_S\text{Im}(\mu_6^2)+v_+\text{Im}(\mu_5^2)-c_-(\mu_1^2-\mu_2^2)\\&+\partial V_{4H}/\partial\chi_-^0.\end{aligned}\tag{96}$$

We solve (92)–(96) to express $\mu_1^2,\mu_3^2,\text{Im}(\mu_4^2),\text{Re}\,\mu_5^2$, and $\text{Im}(\mu_5^2)$ in terms of VEVs. We find that in the leading order, they are given by

$$\begin{aligned}\mu_1^2=&[\mu_2^2v_+^2+\sqrt{2}\,\text{Re}(\mu_4^2)v_+v_S+\sqrt{2}\,\text{Im}(\mu_6^2)v_Sc_-\\&+\mu_2^2c_-^2]/(c_-^2-v_+^2)+O(\text{VEV}^2),\end{aligned}$$

$$\mu_3^2=\sqrt{2}(-\text{Re}(\mu_4^2)(v_+/v_S)+\text{Im}(\mu_6^2)(c_-/v_S))+O(\text{VEV}^2),$$

$$\text{Im}(\mu_4^2)=-\text{Re}(\mu_6^2)(c_-/v_+)+O(\text{VEV}^2),$$

$$\text{Re}(\mu_5^2)=-\sqrt{2}\,\text{Re}(\mu_6^2)(v_S/v_+)+O(\text{VEV}^2),$$

$$\begin{aligned}\text{Im}(\mu_5^2)=&[\sqrt{2}\,\text{Re}(\mu_4^2)v_Sc_-+\sqrt{2}\,\text{Im}(\mu_6^2)v_+v_S\\&+2\mu_2^2v_+c_-]/(c_-^2-v_+^2)+O(\text{VEV}^2).\end{aligned}\tag{97}$$

Inserting these mass parameters into the full potential, we have verified numerically that, except for one neutral physical Higgs boson, all the physical bosons can become heavy. In the limit, in which the imaginary parts of $\mu_4^2,\mu_5^2,\mu_6^2,\lambda_4$, and λ_7 vanish, the Pakvasa-Sugawara VEVs reduce to the S_2' invariant VEVs (6), as we can see also from

$$\begin{aligned}c_-\to&[-4\,\text{Im}(\mu_4^2)+\text{Im}(\lambda_4)v_+^2+2\sqrt{2}\,\text{Im}(\lambda_7)v_+v_S]\\&\times[v_+/4\,\text{Re}(\mu_6^2)]+\cdots,\end{aligned}\tag{98}$$

where $\cdots$ stands for higher orders in the limit.

VI. SUPERSYMMETRIC EXTENSION

As in the case of the minimal supersymmetric standard model (MSSM), we introduce two S_3 doublet Higgs superfields, $H_i^U,H_i^D(i=1,2)$, and two S_3 singlet Higgs superfields, H_S^U,H_S^D [10,11]. The same *R*-parity is assigned to these fields as in the MSSM. Then the most general renormalizable S_3 invariant superpotential is given by

$$W_H=\mu_1H_i^UH_i^D+\mu_3H_S^UH_S^D.\tag{99}$$

The S_3 invariant soft scalar mass terms are [10,11],

$$\begin{aligned}\mathcal{L}_S=&-m_{H_1^U}^2(|\hat{H}_1^U|^2+|\hat{H}_2^U|^2)-m_{H_1^D}^2(|\hat{H}_1^D|^2+|\hat{H}_2^D|^2)\\&-m_{H_S^U}^2(|\hat{H}_S^U|^2)-m_{H_S^D}^2(|\hat{H}_S^D|^2),\end{aligned}\tag{100}$$

and the S_3 invariant B terms are,

$$\mathcal{L}_B=B_1(\hat{H}_1^U\hat{H}_1^D+\hat{H}_2^U\hat{H}_2^D)+B_3(\hat{H}_S^U\hat{H}_S^D)+\text{H.c.},\tag{101}$$

where hatted fields are scalar components. Given the superpotential (99) along with the S_3 invariant soft supersymmetry breaking (SSB) sector (100) and (101), we can now write down the scalar potential. For simplicity we assume that only the neutral scalar components of the Higgs supermultiplets acquire VEVs. The relevant part of the scalar potential is then given by

$$V=(|\mu_1|^2+m^2_{H_1^U})(|\hat{H}_1^{0U}|^2+|\hat{H}_2^{0U}|^2)+(|\mu_1|^2+m^2_{H_1^D})(|\hat{H}_1^{0D}|^2+|\hat{H}_2^{0D}|^2)+(|\mu_3|^2+m^2_{H_S^U})(|\hat{H}_S^{0U}|^2)+(|\mu_3|^2+m^2_{H_S^D})(|\hat{H}_S^{0D}|^2)+\frac{1}{8}\left(\frac{3}{5}g_1^2+g_2^2\right)(|\hat{H}_1^{0U}|^2+|\hat{H}_2^{0U}|^2+|\hat{H}_S^{0U}|^2-|\hat{H}_1^{0D}|^2-|\hat{H}_2^{0D}|^2-|\hat{H}_S^{0D}|^2)^2-[B_1(\hat{H}_1^{0U}\hat{H}_1^{0D}+\hat{H}_2^{0U}\hat{H}_2^{0D})+B_3(\hat{H}_S^{0U}\hat{H}_S^{0D})+\text{H.c.}], \quad (102)$$

where $g_{1,2}$ are the gauge-coupling constants for the $U(1)_Y$ and $SU(2)_L$ gauge groups. As one can easily see, the scalar potential V (102) has a continues global symmetry $SU(2)\times U(1)$ in addition to the local $SU(2)_L\times U(1)_Y$. As a result, there will be a number of pseudo-Goldstone bosons that are phenomenologically unacceptable. This is a consequence of S_3 symmetry. Therefore, we would like to break S_3 symmetry explicitly. As in the nonsupersymmetric case, we would like to break it as softly as possible to preserve predictions from S_3 symmetry, while breaking the global $SU(2)\times U(1)$ symmetry completely. There is a unique choice for that: Since the softest terms have the canonical dimension two, the soft S_3 breaking should be in the SSB sector. As for the soft scalar masses, we have an important consequence (100) from S_3 symmetry that they are diagonal in generations. Since we would like to preserve this, the only choice is to introduce the soft S_3 breaking terms in the B sector [11]. Moreover, looking at the S_3 invariant scalar potential V (102), we observe that it has again an Abelian discrete symmetry

$$S_2':H_1^{U,D}\leftrightarrow H_2^{U,D}, \quad (103)$$

which is the same as (7). We assume that the soft S_3 breaking terms respect this discrete symmetry (103), and add the following soft S_3 breaking Lagrangian:

$$\mathcal{L}_{S_3B}=B_4(\hat{H}_1^U\hat{H}_2^D+\hat{H}_2^U\hat{H}_1^D)+B_5\hat{H}_S^U(\hat{H}_1^D+\hat{H}_2^D)+B_6\hat{H}_S^D(\hat{H}_1^U+\hat{H}_2^U)+\text{H.c.} \quad (104)$$

In the following discussions, we assume that all the B parameters are real. The resulting scalar potential can be analyzed, and one finds that a local minimum respecting S_2' symmetry, i.e.,

$$\langle\hat{H}_1^{0U}\rangle=\langle\hat{H}_2^{0U}\rangle=v_U/2\neq 0,\ \langle\hat{H}_1^{0D}\rangle=\langle\hat{H}_2^{0D}\rangle=v_D/2\neq 0,$$
$$\langle\hat{H}_S^{0U}\rangle=v_{SU}/\sqrt{2}\neq 0,\ \langle\hat{H}_S^{0D}\rangle=v_{SD}/\sqrt{2}\neq 0, \quad (105)$$

can occur. To see this, we write down the minimization conditions in this case, which can be uniquely solved:

$$(|\mu_1^2|^2+m^2_{H_1^U})=(B_1v_{SD}+B_4v_{SD}+\sqrt{2}B_6v_{SD})/v_U+O(\text{VEV}^2), \quad (106)$$

$$(|\mu_3^2|^2+m^2_{H_S^U})=(B_3v_{SD}+\sqrt{2}B_5v_D)/v_{SU}+O(\text{VEV}^2), \quad (107)$$

$$(|\mu_1^2|^2+m^2_{H_1^D})=(B_1v_U+B_4v_U+\sqrt{2}B_5v_{SU})/v_D+O(\text{VEV}^2), \quad (108)$$

$$(|\mu_3^2|^2+m^2_{H_S^D})=(B_3v_{SU}+\sqrt{2}B_6v_U)/v_{SD}+O(\text{VEV}^2). \quad (109)$$

Inserting these solutions into the scalar potential (102) with (104), we obtain the mass matrices for the Higgs fields. As in the nonsupersymmetric case, we redefine the Higgs fields as

$$\hat{H}_{\pm}^{D,U}=\frac{1}{\sqrt{2}}(\hat{H}_1^{D,U}\pm\hat{H}_2^{D,U}). \quad (110)$$

Then the mass matrices can be written as

$$\mathbf{M}_-^2=\begin{pmatrix}[(B_1+B_4)v_D+\sqrt{2}B_6v_{SD}]/v_U & -B_1+B_4\\ -B_1+B_4 & [(B_1+B_4)v_U+\sqrt{2}B_5v_{SU}]/v_D\end{pmatrix} \quad (111)$$

for the $[\hat{H}_-^U,(\hat{H}_-^D)^\dagger]$ basis, and

$$\mathbf{M}^2=\begin{pmatrix}M_{US} & 0 & -B_3 & -\sqrt{2}B_5\\ 0 & M_{U+} & -\sqrt{2}B_6 & -B_1-B_4\\ -B_3 & -\sqrt{2}B_6 & M_{DS} & 0\\ -\sqrt{2}B_5 & -B_1-B_4 & 0 & M_{D+}\end{pmatrix}+O(\text{VEV}^2) \quad (112)$$

for the $[\hat{H}_S^U,\hat{H}_+^U,(\hat{H}_S^D)^\dagger,(\hat{H}_+^D)^\dagger]$ basis, where

$$M_{US}=(B_3v_{SD}+\sqrt{2}B_5v_D)/v_{SU},$$
$$M_{U+}=[(B_1+B_4)v_D+\sqrt{2}B_6v_{SD}]/v_U, \quad (113)$$

$$M_{DS}=[B_3v_{SU}+\sqrt{2}B_6v_U]/v_{SD},$$
$$M_{D+}=[(B_1+B_4)v_U+\sqrt{2}B_5v_{SU}]/v_D. \quad (114)$$

From the mass matrices (111) and (112), we find that the lightest physical Higgs boson, the MSSM Higgs boson, can be written as a linear combination

$$h_{\rm MSSM}=(v_D\hat{H}_+^{0D}+v_{SD}\hat{H}_S^{0D}+v_U\hat{H}_+^{0U}+v_{SU}\hat{H}_S^{0U})/v, \tag{115}$$

where $v=(v_U^2+v_{SU}^2+v_D^2+v_{SD}^2)^{1/2}\simeq 246$ GeV, and its mass is approximately given by

$$m_h^2\simeq\frac{1}{2}[(3/5)g_1^2+g_2^2](v_U^2+v_{SU}^2-v_D^2-v_{SD}^2)^2/v^2 \tag{116}$$

for $\mu^{2\prime}s,B's\gg v^2$. It can be shown that the masses of the other physical Higgs bosons can be made arbitrarily heavy. From (116), we see that the tree-level upper bound for m_h is exactly the same as in the MSSM.

Because of the very nature of the SSB terms, the explicit breaking of S_3 in the B sector (104) does not propagate to the other sector. Moreover, although the superpotential (99) and the corresponding trilinear couplings do not respect S_2' symmetry (103), they cannot generate S_2' violating infinite B terms because they can generate only S_3 invariant terms, which are, however, automatically S_2' invariant.

VII. CONCLUSIONS

We recall that our investigations have been carried out under the two phenomenological conditions (14) and (15). Below we would like to summarize our conclusions:

(i) The S_3 invariant Higgs potential (9) does not satisfy the phenomenological requirement that except one neutral physical Higgs boson all the physical Higgs bosons can become heavy $\gtrsim 10$ TeV without having a problem with triviality. That is, for a phenomenological viable model we have to break S_3 explicitly if we do not introduce further Higgs fields.

(ii) Among the real nonequivalent soft S_3 breaking masses (28), (29), and (31) that can be characterized according to discrete symmetries, only the S_2' invariant case (29) with the S_2' invariant VEVs (6) can satisfy the phenomenological requirement of (i).

(iii) Even for the most general quartic Higgs potential with the most general real S_3 breaking masses (21), the S_2' invariant VEVs (6) can correspond to a local minimum and satisfy the phenomenological requirement of (i).

(iv) The Pakvasa-Sugawara VEVs (91) can be a local minimum in the case of the most general quartic Higgs potential with the most general complex S_3 breaking masses and can satisfy the phenomenological requirement of (i).

(v) In a minimal supersymmetric extension with the S_2' invariant, real soft S_3 breaking masses in the B sector, the phenomenological requirement of (i) can be satisfied with the S_2' invariant VEVs (105), where the other B parameters are also assumed to be real. These B terms violate supersymmetry as well as S_3 softly. This possibility to introduce S_2 violating soft terms in the B sector only is consistent with renormalizability. The lower bound of the lightest Higgs boson is the same as in the MSSM.

It is a very difficult task to test the Higgs sector experimentally. However, as we see from (57) and (60), in the case of the S_2' invariant soft breaking with the S_2' invariant VEVs (6), there are basically only two masses m_{h_H} and m_{h_-} for four neutral and two charged heavy Higgs bosons. This may be experimentally tested because their couplings to the fermions are fixed [8,9].

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