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We propose a model for the mass matrices of quarks and leptons based on two Abelian flavor symmetries. One is assumed to be broken at a high energy region near the Planck scale. It is used for the Froggatt-Nielsen mechanism in both quark and charged lepton sectors. The other symmetry remains unbroken to the multi-TeV region. The mixing among neutrinos and gauginos including that of the new Abelian symmetry generates nonzero masses and mixing among neutrinos. A bimaximal scheme for the neutrino oscillation can be realized together with suitable masses and CKM mixing in the quark sector. A rather large value of the MNS matrix element $V_{e3}^{\rm MNS}$ is predicted. Although FCNC constraints on this flavor dependent Abelian symmetry seem to be evaded, the typical FCNC processes are expected to be observed in the near future.

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I. INTRODUCTION

The origin of flavor mixing of quarks and leptons is one of the most important problems beyond the standard model (SM). Recently the existence of nontrivial lepton mixing has been strongly suggested through atmospheric and solar neutrino observations whose results can be explained by assuming neutrino oscillations [1–3]. The predicted flavor mixing is much bigger than that of the quark sector. The explanation of this feature is a challenge to the grand unified theory (GUT) and a lot of work has been done by now [4,5]. In many models the flavor mixing in both sectors is considered to be controlled by the Froggatt-Nielsen mechanism [6],[1] and the smallness of the neutrino mass is explained by the celebrated seesaw mechanism [8]. In such scenarios the origin of flavor mixing is eventually related to the physics at high energies such as the Planck scale and an intermediate scale.

In this paper we propose an alternative possibility based on a different origin of the flavor mixing of neutrinos in a supersymmetric model with an extended gauge structure. In our scenario the flavor mixing in the quark and charged lepton sectors is considered to have its origin in the high energy region due to the usual Froggatt-Nielsen mechanism. On the other hand, the flavor mixing in the neutrino sector is assumed to come from mixing with the extended gaugino sector by an extra U(1) gauge symmetry [9]. The mixing is induced by *R*-parity violation [10,11]. Its origin might be considered to be related to physics concerning an effective supersymmetry breaking in the TeV region. Since neutrino masses are related to gauge interactions in the present model, the number of free parameters can be greatly reduced as compared with the ordinary seesaw scenario where the neutrino masses are explained by unconstrained Yukawa couplings. One of the interesting points of the model is that the large mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solution for the solar neutrino problem can be consistently accommodated together with the small flavor mixing and the qualitatively favorable mass eigenvalues in the quark sector. In addition, although the large mixing scheme for the solar and atmospheric neutrino problems is obtained, a rather large value of the Maki-Nakagawa-Sakata (MNS) matrix element $V_{e3}^{\rm MNS}$ is definitely predicted. This feature seems to be favored by the recent super-Kamiokande observations and it may be examined in a future experiment [12]. The scenario could be consistent with the gauge coupling unification since it has an SU(5) GUT structure when we switch off the low energy extra U(1) gauge interaction which plays the part of flavor symmetry together with the Froggatt-Nielsen type U(1) symmetry. For convenience, we will use the SU(5) representations to classify the fields in the following discussion.

In the next section we define the flavor symmetry of the model. In Sec. III the mass matrices in the quark and charged lepton sectors are discussed. Flavor changing neutral current (FCNC) constraints are also examined for the nonuniversal couplings of an extra neutral gauge boson here. The neutrino mass matrix is studied in Sec. IV. We show that an almost bimaximal mixing is derived in this model. We also discuss the *R*-parity violation here. Section V is devoted to the summary.

II. ABELIAN FLAVOR SYMMETRY

We consider a model with two Abelian flavor symmetries $U(1)_F\times U(1)_X$. The $U(1)_F$ is considered to be broken near the Planck scale and used to generate the mass hierarchy and flavor mixing through the Froggatt-Nielsen mechanism. It may be considered to be an anomalous U(1) symmetry. On the other hand, the $U(1)_X$ is assumed to remain unbroken to the TeV region.[2] We assume that this low energy extra $U(1)_X$ gauge field has flavor diagonal but nonuniversal couplings. Different charges of $U(1)_X$ are assigned to the **5*** fields be-

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[1]There are many works in which the Abelian flavor symmetry is discussed. As examples, see [7] and references therein.

[2]Additional U(1) symmetries are known to appear very often in the heterotic superstring models and it can play a useful role in supersymmetric models [13].

longing to the different generations.[3] Since its breaking scale is in the TeV region, we cannot use this symmetry for the Froggatt-Nielsen mechanism to induce the hierarchical structure of quark mass matrices since its breaking scale is too small as compared to the Planck scale. However, since some of the content of the minimal supersymmetric SM (MSSM) is assumed to have its charge in a generation dependent way, it can generate an additional nontrivial texture in the mass matrices.

We adopt the following charge assignments of $U(1)_F \times U(1)_X$ to the chiral superfields of quarks and leptons:

$$\mathbf{10}_f \equiv (q,u^c,e^c)_f: \ (3,2,0), \quad (\alpha,\alpha,\alpha),$$

$$\mathbf{5}^*_f \equiv (d^c,l)_f: \ (1,0,0), \quad (q_1,q_1,q_2), \tag{1}$$

where $f=1-3$ and the numbers in parentheses stand for the charges for each generation. We need no right-handed neutrino since in the present scenario neutrino masses are considered to be generated through the mixing with gauginos. In general, the introduction of $U(1)_X$ to the MSSM requires additional chiral superfields to cancel the gauge anomaly which causes a nontrivial constraint on the charge assignment. Since it is required to be radiatively broken at the TeV region, we need at least a new SM singlet chiral superfield whose scalar component causes the spontaneous breaking of $U(1)_X$ by its vacuum expectation value (VEV). Taking account of these facts, as the Higgs chiral superfield sector we consider the following content:

$$\mathbf{5}^a \equiv (D;H_2)^a: \ ((0,0);(0,0)), \quad ((x,z);(p,r)),$$

$$\mathbf{5}^{*a} \equiv (\bar{D};H_1)^a: \ ((0,0);(0,0)), \quad ((y,w);(q,s)),$$

$$\mathbf{1}_0 \equiv S_0: \ (-1), \quad (0),$$

$$\mathbf{1}_i \equiv S_i: \ (0,\ldots,0), \quad (Q_1,\ldots,Q_6), \tag{2}$$

where $a=1-2$ and $i=1-6$. From the charge assignment of $U(1)_X$ for $\mathbf{5}^a$ and $\mathbf{5}^{*a}$ we find that the SU(5) symmetry is explicitly broken unless $U(1)_X$ is switched off. Thus SU(5) may be considered to have a meaning only as a classification group in the model. The choice of additional chiral superfields and their charges should guarantee SM gauge coupling unification, proton stability, and anomaly cancellation of $U(1)_X$. The coupling unification of the SM gauge group is expected to be satisfied as a result of the SU(5) structure.[4]

[3] This kind of charge assignment of $U(1)_X$ has often been discussed in a different context. For example, it has been assumed to explain the small neutrino mass and proton stability in [14].

[4] The coupling unification requires two additional $SU(2)_L \times U(1)_Y$ vectorlike fields such as H_1 and H_2. However, since they can be trivial under $U(1)_X$ and obtain masses, for example, due to the Giudice and Masiero mechanism [15], they are irrelevant to our discussion.

Moreover, the $U(1)_X$ can prohibit the couplings between quarks and extra colored fields D^a and $\bar{D}^a$ and guarantees proton longevity.

Because of other phenomenological requirements some conditions should be imposed on the $U(1)_X$. It should allow the presence of necessary Yukawa couplings to generate the masses of quarks and leptons. For this requirement we impose the conditions

$$2\alpha+p=0, \quad \alpha+q_1+q=0, \quad \alpha+q_2+s=0, \tag{3}$$

and then we have the following Yukawa couplings:

$$W_{\text{Yukawa}} = \sum_{f,f'=1}^{3} y^u_{ff'} u^c_f H_2^1 q_{f'} + \sum_{f'=1}^{3} \left\{ \sum_{f=1,2} (y^d_{ff'} d^c_f H_1^1 q_{f'} + y^e_{ff'} e^c_f H_1^1 l_{f'}) + y^d_{3f'} d^c_3 H_1^2 q_{f'} + y^e_{3f'} e^c_3 H_1^2 l_{f'} \right\}. \tag{4}$$

We also require the following conditions on the Higgs chiral superfield sector:

$$\begin{aligned} &p+q+Q_1=0, \quad p+s+Q_2=0,\\ &x+y+Q_3=0, \quad Q_1+Q_2+Q_3=0,\\ &r+q+Q_4=0, \quad r+s+Q_5=0,\\ &z+w+Q_6=0, \quad Q_4+Q_5+Q_6=0. \end{aligned} \tag{5}$$

Here we consider the situation that the scalar components of the SM singlet fields S_3 and S_6 get VEVs in the TeV region radiatively through the couplings with extra colored fields D^a and $\bar{D}^a$ and then the $U(1)_X$ gauge field becomes massive [13]. Moreover, the last conditions in each line of Eq. (5) allow the trilinear couplings $S_1S_2S_3$ and $S_4S_5S_6$ in the superpotential. Since these trilinear couplings are accompanied by scalar trilinear couplings which break the supersymmetry softly, it can be expected that they generally induce the VEVs to other singlet fields S_i. As a result all of the extra colored fields D^a and $\bar{D}^a$ and the doublet Higgs fields become massive at that scale through the couplings with S_i. The mixings of the doublet Higgs fields in the superpotential can be written as

$$(H_2^1,H_2^2)\begin{pmatrix} \kappa_1\langle S_1\rangle & \kappa_2\langle S_2\rangle \\ \kappa_4\langle S_4\rangle & \kappa_5\langle S_5\rangle \end{pmatrix}\begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}. \tag{6}$$

If $\kappa_5\langle S_5\rangle/\kappa_4\langle S_4\rangle$ is equal to $-\kappa_1\langle S_1\rangle/\kappa_2\langle S_2\rangle$, the eigenstates of the mixing matrix (6) can be identified as $(H_2^1, H_1^l \equiv \sin\zeta H_1^1 + \cos\zeta H_1^2)$ and $(H_2^2, H_1^h \equiv -\cos\zeta H_1^1 + \sin\zeta H_1^2)$ where $\tan\zeta = \kappa_1\langle S_1\rangle/\kappa_2\langle S_2\rangle$. Since H_2^1 has a coupling with the top quark as shown in Eq. (4) and a mixing with H_1^l through Eq. (6), only the first set of Higgs fields is expected to get the VEVs and it works like the usual Higgs fields. In the following discussion we take this as a basic assumption since the mixing in the H_1^a sector can play an important role

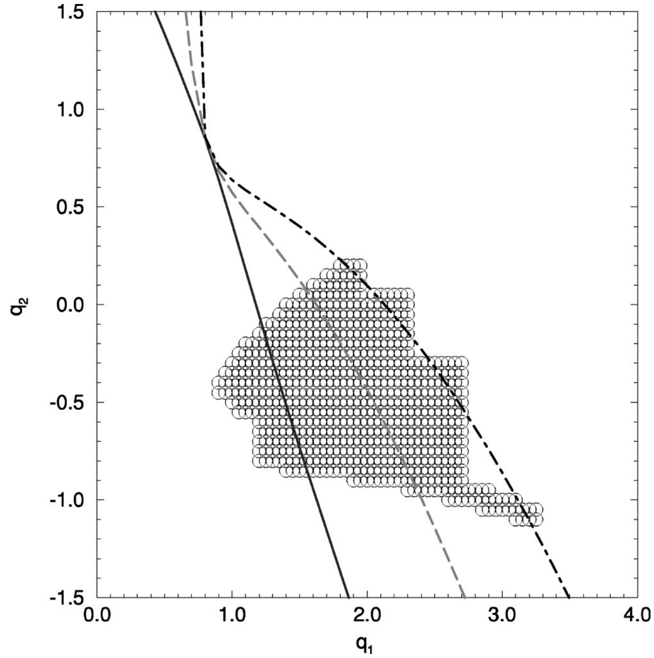

FIG. 1. Scatter plot of the $\mathrm{U}(1)_X$ charge of neutrinos which can give the solution to the atmospheric and solar neutrino problems. Solid, dashed, and dot-dashed lines correspond to $x=1.5$, 3, and 4.5, respectively.

in deriving the MNS matrix due to the effect on the charged lepton mass matrix. It may be useful to note that this assumption is related to the tunings of the scalar potential for S_i.

Anomaly free conditions for $\mathrm{U}(1)_X$ can impose an additional constraint. If we require $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_X$ to be anomaly free under the conditions (3) and (5), the $\mathrm{U}(1)_X$ charges q_1 and q_2 will be constrained into the restricted region. The details are discussed in the Appendix and we will show a few examples of the solutions in Fig. 1 below.

III. MASS AND MIXING OF QUARKS AND CHARGED LEPTONS

The $\mathrm{U}(1)_F$ symmetry controls the flavor mixing structure by regulating the number of field S_0 contained in each nonrenormalizable term through the so-called Froggatt-Nielsen mechanism. As a result the effective Yukawa couplings $y_{ff'}$ in Eq. (4) have a hierarchical structure. In fact, if the singlet field S_0 gets the VEV $\langle S_0\rangle$, the suppression factor for the Yukawa couplings can appear as a power of $\lambda=\langle S_0\rangle/M_{\mathrm{pl}}$. Here M_{pl} is the Planck scale. Using the $\mathrm{U}(1)_F$ charges introduced above, we can obtain the mass matrices of quarks and charged leptons in the usual way. However, there is additional structure coming from the $\mathrm{U}(1)_X$ constraints that are realized by the condition (3) and also a composition of the doublet Higgs field H_1^l as discussed above. Taking them into account we can write the mass matrices for the quarks and the charged leptons as follows:

$$M_u\sim\begin{pmatrix}\lambda^6 & \lambda^5 & \lambda^3\\ \lambda^5 & \lambda^4 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix}\langle H_2^1\rangle,$$

$$M_d\sim\begin{pmatrix}\lambda^4\sin\zeta & \lambda^3\sin\zeta & \lambda\sin\zeta\\ \lambda^3\sin\zeta & \lambda^2\sin\zeta & \sin\zeta\\ \lambda^3\cos\zeta & \lambda^2\cos\zeta & \cos\zeta\end{pmatrix}\langle H_1^l\rangle, \tag{7}$$

where the above mass matrices are written in the basis of $\overline{\psi_R}m_D\psi_L$. We do not consider the CP phases here. In Eq. (7) we abbreviate the order 1 coupling constants by using the similarity symbol. In the quark sector the mass eigenvalues and the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be found after some inspection as

$$m_u:m_c:m_t=\lambda^6:\lambda^4:1,$$

$$m_d:m_s:m_b=\lambda^4\sin\zeta:\lambda^2\cos\zeta:\cos\zeta,$$

$$V_{us}\sim\lambda,\quad V_{ub}\sim\lambda^3,\quad V_{cb}\sim\lambda^2. \tag{8}$$

If we require that the order 1 couplings should be in the range $(\sqrt{\lambda},1/\sqrt{\lambda})$, the lower bound of $\tan\beta\equiv\langle H_2^1\rangle/\langle H_1^l\rangle$ is estimated as $\tan\beta\gtrsim 40\lambda\cos\zeta$. On the charged lepton sector we can find the mass eigenvalues by noting that an SU(5) relation such as $M_e^T=M_d$ is also satisfied in the present model. The ratio of mass eigenvalues is the same as that of the down quark sector and thus

$$m_e:m_\mu:m_\tau=\lambda^4\sin\zeta:\lambda^2\cos\zeta:\cos\zeta. \tag{9}$$

The result has some different features from the ones presented in Ref. [5] in the down quark and charged lepton sectors. These come from the charge assignment for **5*** and the composition of the Higgs field H_1^l. If we take λ as the Cabibbo mixing angle ~ 0.22, these results seem to describe nicely the experimental data for the mass eigenvalues and the CKM mixing angles as long as $\cos\zeta\sim\sin\zeta$ is satisfied, except for m_u and m_e, which are predicted to be somewhat larger. Also, in our framework we cannot overcome this common defect with the usual scheme based on a kind of $\mathrm{U}(1)_F$ symmetry. The value of $\sin\zeta$ is, in principle, determined by the scalar potential of the singlet fields S_i which is briefly discussed below Eq. (5). From its structure the above value of $\sin\zeta$ can be expected to be realized without unnatural tunings of the scalar potential for S_i.

We define the diagonalization matrix $\tilde{U}$ of the charged lepton mass matrix in a basis where $\tilde{U}^\dagger M_e^\dagger M_e\tilde{U}$ is diagonal. Then $\tilde{U}$ can be approximately written as

$$\tilde{U}=\begin{pmatrix} 1 & 0 & \lambda \sin\zeta \\ -\lambda \sin^2\zeta & \cos\zeta & \sin\zeta \\ -\lambda \sin\zeta\cos\zeta & -\sin\zeta & \cos\zeta \end{pmatrix}, \tag{10}$$

where the CP phase in the charged lepton sector is taken to be zero although there can be some sources for it.

The breaking scale of $\mathrm{U}(1)_X$ is assumed to be in the TeV region and thus we have a rather light Z' that can impose constraints on the model. The nonuniversal couplings of Z' with **5*** may induce large flavor changing neutral currents. A detailed analysis for this issue has been done in [16] and we can apply the discussion to the present model. In the model the Z' interaction term relevant to the nonuniversal couplings of **5*** can be written in the mass eigenstates as

$$\mathcal{L}_{Z'}=-g_1\left[\frac{g_X}{g_1}\cos\xi J^{\mu}_{(2)}-\sin\xi J^{\mu}_{(1)}\right]Z^{(2)}_{\mu},$$

$$J^{\mu}_{(2)}=\sum_{ij}\left[\overline{\nu}_{L_i}B^{\nu_L}_{ij}\nu_{L_j}+\overline{l}_{L_i}B^{l_L}_{ij}l_{L_j}+\overline{d}_{R_i}B^{l_R}_{ij}d_{R_j}\right],$$

$$B^{\psi}_{ij}=V^{\psi\dagger}\,\mathrm{diag}(q_1,q_1,q_2)V^{\psi}, \tag{11}$$

where $J^{\mu}_{(1)}$ is the SM weak neutral current and ξ is a Z-Z' mixing angle. V^{ψ} is a unitary matrix used to diagonalize the mass matrix of ψ. In the present model $V^{d_R}=V^{l_L}$ is satisfied because of the SU(5) relation and they are represented by Eq. (10). Thus the relevant B^{ψ}_{ij} can be estimated as

$$|B^{l_L}_{12}|^2=|B^{d_R}_{12}|^2=(q_1-q_2)^2\lambda^2\sin^4\zeta\cos^2\zeta,$$

$$|B^{l_L}_{13}|^2=|B^{d_R}_{13}|^2=(q_1-q_2)^2\lambda^2\cos^4\zeta\sin^2\zeta,$$

$$|B^{l_L}_{23}|^2=|B^{d_R}_{23}|^2=(q_1-q_2)^2\cos^2\zeta\sin^2\zeta. \tag{12}$$

On the other hand, the experimental constraints on these values are also estimated in [16]. The relevant constraints to the present model can be summarized as follows.[5] The coherent μ-e conversion and the decays $\tau\to 3e,3\mu$ require

$$w^2(|B^{l_L}_{12}|^2)<4\times 10^{-14},$$

$$w^2(|B^{l_L}_{13}|^2)<2\times 10^{-5},$$

$$w^2(|B^{l_L}_{23}|^2)<10^{-5}, \tag{13}$$

and from the lepton flavor violating meson decays such as $K_L\to\mu^{\pm}e^{\mp}$ we obtain

$$y^2|B^{l_L}_{12}|^2|B^{d_R}_{12}|^2<10^{-14}. \tag{14}$$

The lepton flavor conserving meson decays such as $K_L\to\mu^+\mu^-$ impose

$$w^2|\mathrm{Re}[B^{d_R}_{12}]|^2<3\times 10^{-11}, \tag{15}$$

and decays of B^0 into $\mu^+\mu^-$ and $\pi^0\mu^+\mu^-$ give

$$w^2|B^{d_R}_{13}|^2<10^{-5},\quad w^2|B^{d_R}_{23}|^2<3\times 10^{-6}. \tag{16}$$

In addition, from the experimental results on meson mass splittings we know that the conditions

$$y|\mathrm{Re}[(B^{d_R}_{12})^2]|<10^{-8},$$

$$y|\mathrm{Re}[(B^{d_R}_{13})^2]|<6\times 10^{-8},$$

$$y|\mathrm{Re}[(B^{d_R}_{23})^2]|<2\times 10^{-6} \tag{17}$$

should be satisfied. In these conditions we use the definitions

$$w=\frac{g_X}{g_1}(\rho_1-\rho_2)\sin\xi\cos\xi,$$

$$y=\left(\frac{g_X}{g_1}\right)^2(\rho_1\sin^2\xi+\rho_2\cos^2\xi), \tag{18}$$

where $\rho_i=M_W^2/M_{Z_i}^2\cos^2\theta_W$. If we apply the above value of λ and $\sin\zeta\sim 1/\sqrt{2}$ to Eq. (12), the most stringent constraints on w and y are obtained as[6]

$$w^2(q_1-q_2)^2<7\times 10^{-12},\quad y(q_1-q_2)^2<2\times 10^{-6}. \tag{19}$$

If $|q_1-q_2|$ takes a value of order 1, w and y should be smaller than 10^{-6}. They require $\sin\xi<10^{-6}$ and $M_{Z_2}>100$ TeV. If both the Z' mass and the mixing with the ordinary Z take these boundary values, we could observe the effect of the nonuniversal couplings of Z' in the coherent μ-e conversion and the meson mass splittings. The values of the charges q_1 and q_2 will be discussed from the viewpoint of the explanation of the solar and atmospheric neutrino problems in the next section.

IV. NEUTRINO MASS AND MIXING

For the neutrino mass and mixing we adopt the scenario proposed in [9]. Using the $\mathrm{U}(1)_X$ charges defined in Eq. (1), the coupling between the neutrinos and the $\mathrm{U}(1)_X$ gaugino is

[5] We do not consider the CP violating effect here.

[6] In this discussion we assume that the $\mathrm{U}(1)_X$ charge is normalized in the same way as $\mathrm{U}(1)_Y$.

given by $i\sqrt{2}g_X\Sigma_\alpha q_\alpha(\tilde{\nu}_\alpha^*\lambda_X\nu_\alpha-\overline{\lambda}_X\overline{\nu}_\alpha\tilde{\nu}_\alpha)$. We do not consider the kinetic term mixing between the U(1) gauginos [17]. If we take this effect into account, off-diagonal elements appear in the gaugino mass matrix. The $\mathrm{U}(1)_X$ can play a crucial role in the generation of nonzero neutrino masses due to the above mentioned interaction since ν_τ has a charge q_2 different from other neutrinos ν_e and ν_μ whose charges are defined by q_1. If sneutrinos get the VEV u due to the R-parity violation, the mixing among neutrinos and gauginos appears as $\mathcal{L}_{\rm mass}=-\frac{1}{2}(\mathcal{N}^T\mathcal{M}\mathcal{N}+\mathrm{H.c.})$ and

$$\mathcal{M}=\begin{pmatrix} 0 & m^T \\ m & M \end{pmatrix}, \quad m=\begin{pmatrix} a_2 & a_1 & b \\ a_2 & a_1 & b \\ a_2 & a_1 & c \end{pmatrix},$$

$$M=\begin{pmatrix} M_2 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_X \end{pmatrix}, \tag{20}$$

where $\mathcal{N}^T=(\nu_\alpha,-i\lambda_{W_3},i\lambda_Y,-i\lambda_X)$. We use the definition such that $a_l=(g_l/\sqrt{2})u$, $b=\sqrt{2}g_Xq_1u$, and $c=\sqrt{2}g_Xq_2u$. If we assume u is much smaller than the gaugino masses M_A, we can obtain the light neutrino mass matrix from it by using the generalized seesaw formula. It can be written as

$$M_\nu=m^TM^{-1}m=\begin{pmatrix} m_0+\epsilon^2 & m_0+\epsilon^2 & m_0+\epsilon\delta \\ m_0+\epsilon^2 & m_0+\epsilon^2 & m_0+\epsilon\delta \\ m_0+\epsilon\delta & m_0+\epsilon\delta & m_0+\delta^2 \end{pmatrix}, \tag{21}$$

where m_0,ϵ, and δ are defined by

$$m_0=\frac{g_2^2u^2}{2M_2}+\frac{g_1^2u^2}{2M_1}, \quad \epsilon=\frac{\sqrt{2}g_Xq_1u}{\sqrt{M_X}}, \quad \delta=\frac{\sqrt{2}g_Xq_2u}{\sqrt{M_X}}. \tag{22}$$

If the neutrinos have the same $\mathrm{U}(1)_X$ charge, we find that there is only one nonzero mass eigenvalue as in the usual R-parity violating scenario [10,11]. The interesting aspect of this mass matrix is that it is defined only by the gaugino mass $M_A(A=l,X)$, the gauge couplings g_A, the $\mathrm{U}(1)_X$ charges q_α, and the VEV u of sneutrinos. Moreover, if we use an analysis based on the renormalization group equations (REGs), the number of free parameters can be reduced. In fact, the number of remaining free parameters related to the neutrino mass becomes only five, including the numerical coefficients, as seen later.

Here we define the mass eigenstates ν_i by $\nu_\alpha=U_{\alpha i}\nu_i$. The mass eigenvalue m_2 is zero and nonzero mass eigenvalues are represented as

$$m_{1,3}=\frac{1}{2}\{(3m_0+2\epsilon^2+\delta^2) \pm\sqrt{(m_0+2\epsilon^2-\delta^2)^2+8(m_0+\epsilon\delta)^2}\}. \tag{23}$$

Here we should note that the gaugino mass can have a CP phase which depends on the supersymmetry breaking mechanism. Because of it we can consider both the possibilities $|m_1|<|m_3|$ and $|m_1|>|m_3|$ depending on the sign of M_A. The diagonalization of the matrix (21) gives

$$U=\begin{pmatrix} \frac{1}{\sqrt{2}}\cos\theta & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\sin\theta \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{24}$$

where one of the mixing angles $\sin\theta$ is defined as

$$\sin^2 2\theta=\frac{8(m_0+\epsilon\delta)^2}{(m_0+2\epsilon^2-\delta^2)^2+8(m_0+\epsilon\delta)^2}. \tag{25}$$

If we recall that the MNS matrix that controls the neutrino oscillation phenomena is defined as $V^{\rm MNS}=U^T\tilde{U}$, we find that it can be written by using Eqs. (10) and (24) as

$$\begin{pmatrix} \frac{\cos\theta}{\sqrt{2}}(1-\lambda\sin^2\zeta)-\lambda\sin\theta\sin\zeta\cos\zeta & -\frac{1}{\sqrt{2}}(1+\lambda\sin^2\zeta) & -\frac{\sin\theta}{\sqrt{2}}(1-\lambda\sin^2\zeta)-\lambda\cos\theta\sin\zeta\cos\zeta \\ \frac{\cos\theta}{\sqrt{2}}\cos\zeta-\sin\theta\sin\zeta & \frac{\cos\zeta}{\sqrt{2}} & -\frac{\sin\theta}{\sqrt{2}}\cos\zeta-\cos\theta\sin\zeta \\ \frac{\cos\theta}{\sqrt{2}}(1+\lambda)\sin\zeta+\sin\theta\cos\zeta & \frac{1}{\sqrt{2}}(1-\lambda)\sin\zeta & -\frac{\sin\theta}{\sqrt{2}}(1+\lambda)\sin\zeta+\cos\theta\cos\zeta \end{pmatrix}. \tag{26}$$

Now we study the oscillation phenomena in the present model. It is well known that the transition probability due to the neutrino oscillation $\nu_\alpha \to \nu_\beta$ after the flight length L is written by using the matrix elements of (26) as

$$\mathcal{P}_{\nu_\alpha\to\nu_\beta}(L)=\delta_{\alpha\beta}-4\sum_{i>j} V^{\rm MNS}_{\alpha i}V^{\rm MNS}_{\beta i}V^{\rm MNS}_{\alpha j} \times V^{\rm MNS}_{\beta j}\sin^2\left(\frac{\Delta m^2_{ij}}{4E}L\right), \qquad (27)$$

where $\Delta m^2_{ij}=|m_i^2-m_j^2|$. If we remember that $m_2=0$, we find that Δm^2_{12} or Δm^2_{23} should be relevant to the atmospheric neutrino problem. Unless both $|\cos\theta|\sim 1$ and $|\sin\zeta|\sim|\cos\zeta|$ are not satisfied, the relevant amplitude $-4\Sigma_{j=1,3}V^{\rm MNS}_{\mu 2}V^{\rm MNS}_{\tau 2}V^{\rm MNS}_{\mu j}V^{\rm MNS}_{\tau j}$ to $\nu_\mu\to\nu_\tau$ is too small to explain the atmospheric neutrino deficit. Thus we need to consider cases such as $\cos\theta\sim 1$ and $\sin\zeta\sim\cos\zeta\sim 1/\sqrt{2}$ here.[7] In this case we obtain

$$V^{\rm MNS}\sim\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{\lambda}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \qquad (28)$$

This MNS matrix is just the one representing the bimaximal mixing [18–20]. In our model the large mixing between ν_e and ν_μ has its origin in the neutrino sector and is related to the value of $\cos\theta$. On the other hand, the large mixing between ν_μ and ν_τ comes from the charged lepton sector and is relevant to the value of $\cos\zeta$. This feature makes it possible to reconcile the MNS matrix with the best fit value of the large mixing angle (LMA) MSW solution with the super-Kamiokande data of the solar neutrino by allowing a nonzero $V^{\rm MNS}_{e3}$ value. This is a different situation from other bimaximal mixing models based on the Zee model [24]. On the squared mass differences, the atmospheric neutrino deficit should be relevant to both Δm^2_{13} and Δm^2_{23}. On the other hand, the solar neutrino deficit should be related to $\nu_e\to\nu_\mu$ with Δm^2_{12} and $\nu_e\to\nu_\tau$ with Δm^2_{12}. This means that the normal hierarchy $|m_2| \lesssim |m_1| \ll |m_3|$ should be satisfied and M_A is required to have a negative sign in Eq. (20).

The CHOOZ experiment [21] constrains a component $V^{\rm MNS}_{e3}$ of the MNS mixing matrix since the amplitude of the contribution to $\nu_e\to\nu_x$ with Δm^2_{23} or Δm^2_{13} always contains it. The model escapes the constraint since $|V^{\rm MNS}_{e3}|$ takes the value $\lambda/2$ which satisfies its upper bound 0.16 [22]. This value of $V^{\rm MNS}_{e3}$ is an important prediction of our model. Future experiments might make it possible to check the validity of the model through its observation. In the model the effective mass parameter that appears in the formula for the rate of neutrinoless double β decay [23] is estimated as

$$|m_{ee}|=|\sum_j |V^{\rm MNS}_{ej}|^2 e^{i\phi_j}m_j|\sim\frac{1}{2}|m_1|+\frac{\lambda^2}{4}|m_3|. \qquad (29)$$

Since $|m_1|=\sqrt{\Delta m^2_{\rm solar}}$ and $|m_3|=\sqrt{\Delta m^2_{\rm atm}}$, it seems unlikely that m_{ee} will be within reach in the near future.

Whether the above values of $\cos\theta$ and mass eigenvalues m_i can be consistently realized is a crucial point for the model. We can study it by using the model parameters M_A, g_A, q_α, and u. In the usual soft supersymmetry breaking scenario the gaugino mass is considered to be universally produced as M_0 at the unification scale. Its low energy value is determined by the RGEs. If we use the one-loop RGEs, the gaugino mass at a scale μ can be expressed as

$$M_2(\mu)=\frac{M_0}{g_U^2}g_2^2(\mu), \qquad M_1(\mu)=\frac{5}{3}\frac{M_0}{g_U^2}g_1^2(\mu), \qquad (30)$$

where we assume the gauge coupling unification at the scale M_U as usual and define the value of the gauge coupling at M_U as g_U. It is not unnatural to consider the gauge coupling of $\mathrm{U}(1)_X$ and its gaugino mass to be the same as those of $\mathrm{U}(1)_Y$.[8] If we adopt this simplified possibility, we find that $m_{1,3}$ and $\sin^2 2\theta$ can be written by using only q_α and $(g_U^2/|M_0|)u^2$ as

$$m_{1,3}=\frac{3}{5}\left(-(2+2q_1^2+q_2^2) \pm\sqrt{(2+2q_1^2+q_2^2)^2-\frac{16}{3}(q_1-q_2)^2}\right)\frac{g_U^2u^2}{|M_0|},$$

$$\sin^2 2\theta=\frac{8(2+3q_1q_2)^2}{(2+6q_1^2-3q_2^2)^2+8(2+3q_1q_2)^2}, \qquad (31)$$

where we take $M_0<0$ as mentioned above. The structure of the mass spectrum and the flavor mixing is controlled by the $\mathrm{U}(1)_X$ charge. The gaugino mass M_0 and the VEV u of sneutrinos are relevant to the mass eigenvalues only in the form of an overall factor $(g_U^2/|M_0|)u^2$. In order to realize the value in the range of 2×10^{-3} eV$^2\lesssim\Delta m^2_{13}\simeq\Delta m^2_{23}\lesssim 6\times10^{-3}$ eV^2 that is required by the atmospheric neutrino deficit, $(g_U^2/|M_0|)u^2$ needs to be in the range of 0.003 eV $\lesssim(g_U^2/|M_0|)u^2 \lesssim 0.014$ eV. If we take $|M_0|\sim 100$ TeV,[9] and $g_U\sim 0.72$, for example, it shows that the sneutrino VEV

[7] The former condition requires restricted values for q_1 and q_2 as is seen below. The latter seems to be naturally satisfied as noted before.

[8] In the superstring context the freedom of an Abelian Kac-Moody level can make this possible.

[9] The gaugino mass is required to be larger than the value usually considered. This comes from the M_{Z_2} value imposed by the FCNC constraint, which was discussed in the previous section.

should be $u \sim 0.76-1.6$ MeV. The remaining freedom that we can use to explain the solar neutrino deficit is only the $\mathrm{U}(1)_X$ charge of $\mathbf{5}_f^*$.

In Fig. 1 we plot the value of the $\mathrm{U}(1)_X$ charge to realize the LMA for explanation of the solar neutrino deficit, which requires $\cos\theta \sim 1$ as discussed above. Here we require $\cos\theta > 0.98$ and also $0.1\times 10^{-4}\ \mathrm{eV}^2 \lesssim \Delta m_{12}^2 \lesssim 6\times 10^{-4}\ \mathrm{eV}^2$ to draw the figure. It shows that a reasonable value of the $\mathrm{U}(1)_X$ charge can realize the LMA. We should also note that the $\mathrm{U}(1)_X$ charges obtained here can satisfy the FCNC constraint given in Eq. (19) coming from the nonuniversal couplings of Z' as long as $\sin\xi < 10^{-6}$ and $M_{Z_2} > 100$ TeV are satisfied.

One of the unsolved important points is the origin of the small VEV u of sneutrinos. As mentioned previously, it should be around $O(1)$ MeV, which is much smaller than the weak scale. In the MSSM there are arguments about lepton number violation due to the VEVs of sneutrinos in the vicinity of the weak scale [25] and also some authors point out that the neutrino mass produced by them can be sufficiently small [26]. However, in our scenario we need much smaller VEVs of sneutrinos than the weak scale. To consider such a possibility it is useful to note that the small VEVs of sneutrinos could be obtained if there were bilinear R-parity violating terms $\epsilon L_\alpha H_2^1$ with a sufficiently small ϵ. For example, as such a candidate we may consider nonrenormalizable terms that are consistent with the $\mathrm{U}(1)_F \times \mathrm{U}(1)_X$ symmetry, such as

$$\frac{\bar{N}N}{M_{\mathrm{pl}}^2}\frac{S_0}{M_{\mathrm{pl}}} S_{\epsilon_1} L_1 H_2^1, \quad \frac{\bar{N}N}{M_{\mathrm{pl}}^2} S_{\epsilon_2} L_2 H_2^1, \quad \frac{\bar{N}N}{M_{\mathrm{pl}}^2} S_{\epsilon_3} L_3 H_2^1, \tag{32}$$

where new SM singlet fields S_{ϵ_α} are introduced for the $\mathrm{U}(1)_X$ invariance. If an intermediate scale is induced through the D- and F-flat direction $\bar{N} = N$ of other singlet fields $N, \bar{N}$ [13] and also S_{ϵ_α} acquires the VEV at the TeV scale, the appropriate ϵ term might be obtained. However, their equality is not guaranteed in the present example. Once we find that ϵ terms could exist, we can check that the small u is realized by minimizing the scalar potential. Since $\langle H_1^l \rangle$ and $\langle H_2^1 \rangle$ can be treated as constants in the present case, the value of u derived from the potential minimization is approximately expressed as

$$u \sim \epsilon \frac{\mu \langle H_1^l \rangle + B_\epsilon \langle H_2^1 \rangle}{m^2}, \tag{33}$$

where B_ϵ is a soft supersymmetry breaking parameter related to the $\epsilon L_\alpha H_2^1$ terms and m^2 is the soft scalar mass of sneutrinos, which is taken to be universal here. From this we find that a sufficiently small u can be obtained as long as ϵ is small enough and the μ parameter, B_ϵ, and m^2 take values of the order of the weak scale. We need to check whether these conditions are satisfied at the true vacuum by taking account of the radiative correction. Even if the VEVs of sneutrinos are not equal and Eq. (20) is somewhat modified, which may be expected in the case of Eq. (32), there are two nonzero mass eigenvalues and a similar result to those obtained above might be derived as long as the deviations from equal u are not very large. Quantitative analysis of this aspect is also necessary to know the viability of the model. The supersymmetry breaking scenario is also important for the model. These issues are now under investigation.

V. SUMMARY

We have proposed a scenario for the origin of the mixing of quarks and leptons in the supersymmetric model with an extra $\mathrm{U}(1)_X$ symmetry. The scenario is based on the usual Froggatt-Nielsen mechanism for the quark and charged lepton sectors. On the other hand, the mixing of neutrinos is considered to come from mixing among neutrinos and gauginos induced by the R-parity violation. Since the free parameters in the neutrino mass matrix are related to the gauge interactions, their number can be reduced effectively. As a result we can study it by including the order 1 coefficients which are completely free parameters in most other models.

In this model we could obtain two nonzero mass eigenvalues for neutrinos at the tree level. The atmospheric and solar neutrino deficits can be simultaneously explained by the usual mass hierarchy scenario. In particular, the large mixing angle MSW solution for the solar neutrino problem can be realized consistently with the small quark mixing only by tuning the $\mathrm{U}(1)_X$ charge of the neutrinos. One of the important features of the model is the definite value of V_{e3}^{MNS} which takes a rather large value ~ 0.11 within the CHOOZ constraint. Thus the present model may be checked through observations of both the value of V_{e3}^{MNS} and certain FCNC processes induced by the nonuniversal couplings of Z' to quarks and leptons.

The scenario gives an alternative possibility to the flavor mixing of quarks and leptons as compared to the usual one. That is, although the quarks and charged leptons have the origin of their mixing in the high energy region, the neutrinos may have it in the low energy region. The most crucial unsolved problem is to clarify the R-parity violation and the realization of the small sneutrino VEVs quantitatively. Further investigation of these problems seems to be necessary to see whether our model works well in a realistic way.

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APPENDIX

Nontrivial anomaly free conditions in the model are summarized as

$$\mathrm{SU}(3)^2\mathrm{U}(1)_X: \quad 9\alpha + 2q_1 + q_2 + x + y + z + w = 0,$$

$$\mathrm{SU}(2)^2\mathrm{U}(1)_X: \quad 9\alpha + 2q_1 + q_2 + p + q + r + s = 0,$$

$$
\begin{aligned}
\mathrm{U}(1)_Y^2\mathrm{U}(1)_X:&\quad 45\alpha+5(2q_1+q_2)+2(x+y+z+w)\\
&\qquad +3(p+q+r+s)=0,\\
\mathrm{U}(1)_Y\mathrm{U}(1)_X^2:&\quad p^2-q^2+r^2-s^2+y^2-x^2+w^2-z^2=0,\\
\mathrm{U}(1)_X^3:&\quad 30\alpha^3+5(2q_1^3+q_2^3)+3(x^3+y^3\\
&\qquad +z^3+w^3)+2(p^3+q^3+r^3+s^3)\\
&\qquad +\sum_{i=1}^{6}Q_i^3=0.
\end{aligned}
\tag{A1}
$$

Combining Eqs. (A1) except for the last one with Eqs. (3) and (5), we can express other parameters using q_1, q_2, and x as follows:

$$
\begin{aligned}
&\alpha=-\frac{1}{9}(2q_1+q_2),\quad p=\frac{2}{9}(2q_1+q_2),\\
&q=\frac{1}{9}(-7q_1+q_2),\quad r=\frac{1}{9}(q_1+5q_2),\\
&s=\frac{1}{9}(2q_1-8q_2),\\
&y=-\frac{1}{3}(q_1-q_2)-x,\quad z=-\frac{1}{3}(q_1-q_2)+x,\\
&w=\frac{2}{3}(q_1-q_2)-x,\\
&Q_1=\frac{1}{3}(q_1-q_2),\quad Q_2=-\frac{2}{3}(q_1-q_2),\\
&Q_3=\frac{1}{3}(q_1-q_2),\\
&Q_4=\frac{2}{3}(q_1-q_2),\quad Q_5=-\frac{1}{3}(q_1-q_2),\\
&Q_6=-\frac{1}{3}(q_1-q_2).
\end{aligned}
\tag{A2}
$$

If we use Eqs. (A2) in the last one of Eqs. (A1) to solve it numerically, we can plot the solutions in the (q_1,q_2) plane for each value of x. In Fig. 1 we show this for three values of x.

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