Neutralino decay in the μ-problem solvable extra U(1) models

Neutralino decay in the μ-problem solvable extra U(1) models

Daijiro Suematsu*
Department of Physics, Kanazawa University, Kanazawa 920-11, Japan

(Received 20 August 1997; published 12 January 1998)

We study neutralino decay in the supersymmetric extra U(1) models which can solve the μ problem. In these models the neutralino sector is extended at least into six components by an extra U(1) gaugino and a superpartner of a Higgs singlet. Focusing on its two lower mass eigenstates $\widetilde{\chi}_2^0$ and $\widetilde{\chi}_1^0$, decay processes such as a tree-level three-body decay $\widetilde{\chi}_2^0 \to \widetilde{\chi}_1^0 f\overline{f}$ and a one-loop radiative decay $\widetilde{\chi}_2^0 \to \widetilde{\chi}_1^0 \gamma$ are estimated. We investigate the condition under which the radiative decay becomes the dominant mode and also numerically search for such parameter regions. In this analysis we take account of the Abelian gaugino kinetic term mixing. We suggest that the gaugino mass relation $M_W \sim M_Y$ may not be necessary for the radiative decay dominance in the extra U(1) models. [S0556-2821(98)04703-1]

PACS number(s): 12.60.Jv, 14.80.Ly

I. INTRODUCTION

Recently the standard model (SM) has been confirmed to incredible accuracy through the precise measurements at the CERN e^+e^- collider LEP. Nevertheless, it has still not been considered the fundamental theory of particle physics and physics beyond the SM is eagerly explored. Along this line the supersymmetrization of the SM is now considered as the most promising extension [1]. However, even in this minimal supersymmetric standard model (MSSM) there remain some theoretically unsatisfactory features in addition to the existence of too many parameters. The famous one is known as the μ problem [2]. The MSSM has a supersymmetric Higgs mixing term $\mu H_1 H_2$. To cause an appropriate radiative symmetry breaking at the weak scale [3], we should put $\mu \sim O(G_F^{-1/2})$ by hand, where G_F is a Fermi constant. Although in the supersymmetric models its typical scale is generally characterized by the supersymmetry breaking scale M_S which is usually taken as the 1 TeV region, there is no reason why μ should be such a scale because it is usually considered to be irrelevant to supersymmetry breaking. A reasonable way to answer this issue is to consider the origin of the μ scale as some result of supersymmetry breaking [4]. One such solution is the introduction of a singlet field S, replacing $\mu H_1 H_2$ by a Yukawa type coupling $\lambda S H_1 H_2$. If S gets a vacuum expectation value (VEV) of order 1 TeV as a result of renormalization effects on the soft supersymmetry breaking parameters, $\mu \sim O(G_F^{-1/2})$ will be realized dynamically as $\mu = \lambda \langle S \rangle$. As is well known, such a scenario can be available by introducing a κS^3 term into the superpotential and a lot of work has been done on this type of model [5], where the superpotential of S is composed of the terms $\lambda S H_1 H_2 + \kappa S^3$. At the price of the introduction of a new parameter κ, a κS^3 term can prohibit the appearance of a massless axion and also guarantee the stability of the potential for the scalar component of S. The introduction of an extra $U(1)_X$ symmetry which is broken by a SM singlet field S can effectively play the same role as the introduction of the κS^3 term [6]. A D term for this $U(1)_X$ induces a quartic term of S in the scalar potential. The axion is absorbed by this extra $U(1)_X$ gauge boson and disappears from the physical spectrum. Moreover, this extra $U(1)_X$ automatically forbids the appearance of $\mu H_1 H_2$ in the original Lagrangian, and also if we assume the unification of gauge coupling constants, we need no new parameter such as κ. Thus models extended with an extra $U(1)_X$ symmetry can be considered as one of the most simple and promising extensions of the MSSM. Their phenomenological aspects have also been studied by various authors [6–9].

The extra $U(1)_X$ models have an another interesting aspect if they are supersymmetrized. Their supersymmetrization introduces the extra neutralino candidates in addition to the ones of the MSSM, that is, an extra $U(1)_X$ gaugino λ_X and a superpartner $\widetilde{S}$ of the Higgs singlet S. Confirmation of the extra gauge structure is one of the main parts of the study of extension of the SM. It is well known that the extra $U(1)_X$ gauge structure is often induced from a more fundamental theory such as superstring theory [9]. However, recent precise measurements at the LEP and the direct search at the Tevatron suggest that the lower bound of the extra neutral gauge boson is rather large and it may be difficult to find its existence directly in the near future [10]. If supersymmetry is what exists in nature, there may be a new possibility to find its existence in a completely different way [11]. Even if the mass of extra neutral gauge boson is too large to observe in near future collider experiments, its superpartner sector may open a window to find its existence. The study of the neutralino sector is interesting from the viewpoint not only of the investigation of supersymmetry but also of the search for extra gauge structure. In particular, we should note that the gauge coupling of this extra $U(1)_X$ to ordinary matter fields is rather large compared with ordinary Yukawa couplings[1] (instead of top Yukawa) and then the neutralino sector can be substantially affected by this inclusion in a suitable parameter region.

*Email address: suematsu@hep.s.kanazawa-u.ac.jp

[1]It should also be noted that the Yukawa coupling λ of $\lambda S H_1 H_2$ can be large enough compared with ordinary Yukawa couplings.

TABLE I. The charge assignment of extra U(1)'s which are derived from E_6. These charges are normalized as $\Sigma_{i\in\mathbf{27}}Q_i^2=20$.

Fields	SU(3)×SU(2)	Y	Q_ψ	Q_χ	Q_η
Q	(3,2)	$\frac{1}{3}$	$\sqrt{\frac{5}{18}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{3}$
U^c	(3*,1)	$-\frac{4}{3}$	$\sqrt{\frac{5}{18}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{3}$
D^c	(3*,1)	$\frac{2}{3}$	$\sqrt{\frac{5}{18}}$	$\frac{3}{\sqrt{6}}$	$\frac{1}{3}$
L	(1,2)	-1	$\sqrt{\frac{5}{18}}$	$\frac{3}{\sqrt{6}}$	$\frac{1}{3}$
E^c	(1,1)	2	$\sqrt{\frac{5}{18}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{3}$
H_1	(1,2)	-1	$-2\sqrt{\frac{5}{18}}$	$-\frac{2}{\sqrt{6}}$	$\frac{1}{3}$
H_2	(1,2)	1	$-2\sqrt{\frac{5}{18}}$	$\frac{2}{\sqrt{6}}$	$\frac{4}{3}$
g	(3,1)	$-\frac{2}{3}$	$-2\sqrt{\frac{5}{18}}$	$\frac{2}{\sqrt{6}}$	$\frac{4}{3}$
$\overline{g}$	(3*,1)	$\frac{2}{3}$	$-2\sqrt{\frac{5}{18}}$	$-\frac{2}{\sqrt{6}}$	$\frac{1}{3}$
S	(1,1)	0	$4\sqrt{\frac{5}{18}}$	0	$-\frac{5}{3}$
N	(1,1)	0	$\sqrt{\frac{5}{18}}$	$-\frac{5}{\sqrt{6}}$	$-\frac{5}{3}$

In this paper we treat the neutralino decay in the extra $U(1)_X$ models since it may be one of the important subjects along the above-mentioned direction. The lightest neutralino is a candidate of the lightest supersymmetric particle. Thus if R parity is conserved, the neutralino decay modes such as $\tilde{\chi}_2^0\to\tilde{\chi}_1^0 f\bar{f}$ and $\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma$ are expected to appear as a subprocess of the decay of supersymmetric particles, where $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ are the two lower neutralino mass eigenstates. These decay processes have been calculated in the case of the MSSM under suitable conditions [12].

Recently, some attention has been attracted to this process in relation to the Collider Detector at Fermilab (CDF) $ee\gamma\gamma+\not{E}_T$ event [13]. Especially, related to this type of event, it seems to be a very interesting subject under what condition $\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma$ can become the dominant mode [13,14]. This is because it can give us fruitful information on the parameters of supersymmetric models as stressed in [13]. Since this type of process is a typical one which may be observed in the near future, its detailed study in the μ-problem solvable extra $U(1)_X$ models will be useful. The estimation of the widths $\Gamma(\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma)$ and $\Gamma(\tilde{\chi}_2^0\to\tilde{\chi}_1^0 f\bar{f})$ in the extra $U(1)_X$ models can be modified from that in the MSSM because there are new components λ_X and $\tilde{S}$ contained in the neutralino mass eigenstate $\tilde{\chi}_i^0$. Additionally, in the multi-U(1) models Abelian kinetic term mixing can occur as suggested in Refs. [15–17]. As a result of this Abelian kinetic term mixing, there are some changes in the interactions between neutralinos and ordinary matter fields [11]. This should be taken into account in the analysis of these processes. Because of these effects, the $\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma$ dominant condition is also expected to be altered from the MSSM one. If we take the lesson brought from the study of the CDF-type event seriously, this analysis may give us important information for model building on additional gauge structure and also Planck scale physics.

The organization of this paper is the following. In Sec. II, we present examples of the μ-problem solvable extra $U(1)_X$ models derived from the superstring inspired E_6 models. After that we give a brief review of the Abelian gaugino mixing whose effect is taken into account in the later analysis. We also examine the neutral gauge boson and Higgs sector to constrain the parameters of the models in terms of their present experimental mass bounds. In Sec. III, mass eigenstates and their couplings to the matter fields of the extended neutralino sector are studied. Based on these preparations the decay widths $\Gamma(\tilde{\chi}_2^0\to\tilde{\chi}_1^0 f\bar{f})$ and $\Gamma(\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma)$ are estimated. We also study under what condition the radiative decay mode becomes the dominant one, which is crucially relevant to the CDF-type event. In Sec. IV, these decay widths are numerically estimated and we show what kind of parameter region is crucial for the radiative decay dominance. Section V is devoted to a summary.

II. EXTRA $U(1)_X$ MODELS

A. μ-problem solvable models

There can be many low energy extra $U(1)_X$ models. In these models we are especially interested in μ-problem solvable extra $U(1)_X$ models. From such a point of view, it seems to be natural to examine models which satisfy the condition mentioned in the Introduction. That is, the extra $U(1)_X$ symmetry should be broken by the VEV of the SM singlet S which has a coupling to the ordinary Higgs doublets H_1 and H_2 such as λSH_1H_2. In these models the μ scale is naturally related to the mass of the extra $U(1)_X$ boson and then they seem to be very interesting from the phenomenological viewpoint too.[2] So we confine our attention to this class of models derived from the superstring inspired E_6 models.

There are two classes of extra $U(1)_X$ models derived from superstring inspired E_6 models. The rank six models have two extra U(1)'s besides the SM gauge structure. They can be expressed as the appropriate linear combinations of $U(1)_\psi$ and $U(1)_\chi$ whose charge assignments for **27** of E_6 are given in Table I. There is also a rank five model called the η model. Its charge assignment is also listed in Table I. As

[2]There is also a possibility that the μ term is realized by a nonrenormalizable term $\lambda(S\bar{S}/M_{\rm Pl}^2)^n SH_1H_2$ because of some discrete symmetry [18]. In such a case $\langle S\rangle$ should be large in order to realize the appropriate μ scale. As a result there is not the low energy extra gauge symmetry which can be relevant to the present experimental front. Because of this reason, we do not consider this possibility.

TABLE II. The charge assignment of the extra $U(1)_X$ which remains unbroken after the VEV of N becomes nonzero. They are obtained as $Q_{\xi_\pm}=(\pm\sqrt{15}/4)Q_\psi \pm \frac{1}{4}Q_\chi$.

Fields	SU(3)×SU(2)	Y	$Q_{\xi_\pm}$
Q	(3,2)	$\frac{1}{3}$	$\pm\frac{1}{\sqrt{6}}$
U^c	(3*,1)	$-\frac{4}{3}$	$\pm\frac{1}{\sqrt{6}}$
D^c	(3*,1)	$\frac{2}{3}$	$\pm\frac{2}{\sqrt{6}}$
L	(1,2)	-1	$\pm\frac{2}{\sqrt{6}}$
E^c	(1,1)	2	$\pm\frac{1}{\sqrt{6}}$
H_1	(1,2)	-1	$\mp\frac{3}{\sqrt{6}}$
H_2	(1,2)	1	$\mp\frac{2}{\sqrt{6}}$
g	(3,1)	$-\frac{2}{3}$	$\mp\frac{2}{\sqrt{6}}$
$\bar{g}$	(3*,1)	$\frac{2}{3}$	$\mp\frac{3}{\sqrt{6}}$
S	(1,1)	0	$\pm\frac{5}{\sqrt{6}}$
N	(1,1)	0	0

seen from this table, there is a SM singlet S which has the coupling $\lambda S H_1 H_2$. The η model clearly satisfies the above-mentioned condition. On the other hand, in the rank six models this condition imposes a rather severe constraint on the extra $U(1)_X$ in the low energy region. In this type of model a right-handed sneutrino N also has to get the VEV to break the gauge symmetry into the SM one. If we try to explain the smallness of the neutrino mass in this context, N should get a sufficiently large VEV. In fact, in the case that N has a conjugate chiral partner $\bar{N}$, a sector of $(N,\bar{N})$ has a D-flat direction and then they can get a large VEV without breaking supersymmetry [19]. This VEV can induce the large right-handed Majorana neutrino mass through the nonrenormalizable term $(N\bar{N})^n/M_{\rm Pl}^{2n-3}$ in the superpotential and then the seesaw mechanism is applicable to yield the small neutrino mass [6,20]. However, this usually breaks the direct relation between the μ scale and the mass of the extra neutral gauge boson because the VEV of N also contributes to the latter. In order to escape this situation and obtain the extra $U(1)_X$ satisfying our condition, we need to construct a $U(1)_X$ by taking a linear combination of $U(1)_\psi$ and $U(1)_\chi$ [6,16,20]. As such examples, we can construct two low energy extra $U(1)_X$ models. They are shown in Table II. The difference between them is the overall sign.[3] In these models the right-handed sneutrinos have no charge of this low energy extra $U(1)_X$. This is a different situation from the rank five η model. Thus using the D-flat direction of another extra $U(1)$, the right-handed sneutrino gets a large VEV which breaks this extra $U(1)$ symmetry and also can induce large Majorana masses for the right-handed neutrinos. This mechanism may also be related to the inflation of the universe and the baryogenesis as discussed in [22]. As a result of this symmetry breaking at the intermediate scale, only one extra $U(1)_X$ remains as the low energy symmetry. We will concentrate on these three $U(1)_X$ models ($X=\eta,\xi_\pm$) in the following study.

We focus our attention to the minimally extended part of these models with an extra $U(1)_X$ and a SM Higgs singlet S. Other extra matter fields such as color triplet fields $(g,\bar{g})$ and the right-handed neutrino N, which are introduced associated with the extension, are irrelevant to the present purpose and we can neglect them. Thus the relevant parts of the superpotential and soft supersymmetry breaking terms are

$$W=\lambda S H_1 H_2 + h_U Q U^c H_2 + h_D Q D^c H_1 + h_E L E^c H_1 + \cdots,$$

$$\mathcal{L}_{\rm soft}=-\sum_i m_i^2|\phi_i|^2+(A\lambda S H_1 H_2+\text{H.c.}+\cdots)$$
$$+\frac{1}{2}\left(M_W\sum_{a=1}^{3}\hat{\lambda}_W^a\hat{\lambda}_W^a+M_Y\hat{\lambda}_Y\hat{\lambda}_Y+M_X\hat{\lambda}_X\hat{\lambda}_X\right.$$
$$\left.+M_{YX}\hat{\lambda}_Y\hat{\lambda}_X+\text{H.c.}\right), \tag{1}$$

where ϕ_i represents the scalar component of each chiral superfield contained in the models. M_W, M_Y, and M_X are the gaugino masses.[4] We assume the Yukawa coupling λ and soft supersymmetry breaking parameters to be real, for simplicity.

B. Abelian gaugino mixing

Next we briefly review a particular feature in the neutralino sector caused by the Abelian gauge kinetic term mixing in the supersymmetric multi-U(1) models. In supersymmetric models gauge fields are extended to vector superfields

$$V_{\rm WZ}(x,\theta,\bar{\theta})=-\theta\sigma_\mu\bar{\theta}V^\mu+i\theta\theta\bar{\theta}\bar{\lambda}-i\bar{\theta}\bar{\theta}\theta\lambda+\tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \tag{2}$$

[3]As discussed in Refs. [6,20], Q_{ξ_-} can also be obtained only by changing the field assignments for Q_χ. This insight allows us to construct new models, which can induce an interesting neutrino mass matrix [21] by using the charge assignments Q_χ and Q_{ξ_-} for the different generations [20]. However, in this paper we shall not consider such models for simplicity.

[4]In this expression we introduced the Abelian gaugino mass mixing as M_{YX}, which might exist as the tree-level term at the Planck scale and also be yielded through quantum effects.

where we used the Wess-Zumino gauge. A gauge field strength is included in the chiral superfield constructed from $V_{\rm WZ}$ in the well-known procedure

$$W_\alpha(x,\theta)=(\overline{DD})D_\alpha V_{\rm WZ}$$
$$=4i\lambda_\alpha-4\theta_\alpha D+4i\theta^\beta\sigma_{\nu\alpha\dot\beta}\sigma^{\dot\beta}_{\mu\beta}(\partial^\mu V^\nu-\partial^\nu V^\mu)$$
$$-4\theta\theta\sigma_{\mu\alpha\dot\beta}\partial^\mu\overline{\lambda}^{\dot\beta}. \qquad (3)$$

Here we should note that W_α of the Abelian gauge group is gauge invariant itself. In terms of these superfields the supersymmetric gauge invariant Lagrangian can be written as

$$\mathcal{L}=\tfrac{1}{32}(W^\alpha W_\alpha)_F+[\Phi^\dagger\exp(2g_0QV_{\rm WZ})\Phi]_D, \qquad (4)$$

where $\Phi=(\phi,\psi,F)$ is the chiral superfield and represents matter fields. Its generalization to the multi-U(1) case is straightforward. The supersymmetric gauge kinetic parts are obtained by using chiral superfields W^a_α and W^b_α for $\mathrm{U}(1)_a\times\mathrm{U}(1)_b$ as

$$\frac{1}{32}(\hat W^{a\alpha}\hat W^a_\alpha)_F+\frac{1}{32}(\hat W^{b\alpha}\hat W^b_\alpha)_F+\frac{\sin\chi}{16}(\hat W^{a\alpha}\hat W^b_\alpha)_F. \qquad (5)$$

Here we introduced the mixing term between the different U(1)'s. This can be canonically diagonalized by using the transformation,

$$\begin{pmatrix}\hat W^a\\ W^b\end{pmatrix}=\begin{pmatrix}1 & -\tan\chi\\ 0 & 1/\cos\chi\end{pmatrix}\begin{pmatrix}W^a\\ W^b\end{pmatrix}. \qquad (6)$$

This transformation affects not only the gauge field sector but also the sector of gauginos $\lambda_{a,b}$ and auxiliary fields $D_{a,b}$.[5] As easily seen from the form of the last term in Eq. (4), the change induced in the interactions of gauginos with other fields through this transformation can be summarized as

$$g^0_aQ_a\hat\lambda^a+g^0_bQ_b\hat\lambda^b=g_aQ_a\lambda^a+(g_{ab}Q_a+g_bQ_b)\lambda^b, \qquad (7)$$

where $\lambda_{a,b}$ are canonically normalized gauginos. The charges of $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_b$ are represented by Q_a and Q_b. The couplings g_a, g_{ab}, and g_b are related to the original ones g^0_a and g^0_b as

$$g_a=g^0_a,\quad g_{ab}=-g^0_a\tan\chi,\quad g_b=\frac{g^0_b}{\cos\chi}. \qquad (8)$$

These coupling constants at the weak scale will be determined by using the renormalization group equations from the initial values at the high energy scale [16,23]. However, such a study is beyond our present purpose and we will treat them as parameters in the later analysis.

[5]This shift in the D term changes the scalar potential and can affect the symmetry breaking at the weak scale. However, we will not refer to this problem here.

C. Neutral gauge sector

In the previously introduced extra $\mathrm{U}(1)_X$ models, the gauge symmetry of the electroweak sector at the low energy region is $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_X$. In order to obtain the correct symmetry breaking for these models, we assume that Higgs fields get VEV's as follows:

$$\langle H_1\rangle=\begin{pmatrix}v_1\\0\end{pmatrix},\quad\langle H_2\rangle=\begin{pmatrix}0\\v_2\end{pmatrix},\quad\langle S\rangle=u, \qquad (9)$$

where $v_1^2+v_2^2=(246\ \mathrm{GeV})^2(\equiv v^2)$ is assumed. For simplicity, all VEV's are assumed to be real. Under these settings in order to constrain the parameters of the models, we investigate some features of the gauge boson sector.

For this purpose we need to determine the physical states at and below the weak scale [17]. The mass mixing between two neutral gauge fields appears associated with the spontaneous symmetry breaking due to the VEV's of Eq. (9) around the weak scale. In the present models the charged gauge sector is the same as that of the MSSM. In the neutral gauge sector we introduce the Weinberg angle θ_W in the usual way,[6]

$$Z_\mu=\cos\theta_W W^3_\mu-\sin\theta_W B_\mu,\quad A_\mu=\sin\theta_W W^3_\mu+\cos\theta_W B_\mu. \qquad (10)$$

Here we used the canonically normalized basis (Z_μ,X_μ) so that A_μ has already been decoupled from (Z_μ,X_μ). The mass matrix of the neutral gauge fields (Z_μ,X_μ) can be written as

$$\begin{pmatrix}m_Y^2 & m_{YX}^2\\ m_{YX}^2 & m_X^2\end{pmatrix}, \qquad (11)$$

where each element is expressed as

$$m_Y^2=m_Z^2,$$

$$m_{YX}^2=m_Z^2 s_W\tan\chi+\frac{\Delta m^2}{\cos\chi},$$

$$m_X^2=m_Z^2s_W^2\tan^2\chi+2\Delta m^2 s_W\frac{\sin\chi}{\cos^2\chi}+\frac{M_{Z'}^2}{\cos^2\chi}. \qquad (12)$$

In this expression m_Z^2, Δm^2, and $M_{Z'}^2$ represent the values of corresponding components in case of no kinetic term mixing $(\chi=0)$,

$$m_Z^2=\tfrac{1}{2}(g_W^2+g_Y^2)v^2,$$

$$\Delta m^2=\tfrac{1}{2}(g_W^2+g_Y^2)^{1/2}g_Xv^2(Q_1\cos^2\beta-Q_2\sin^2\beta),$$

[6]In the following we use the abbreviated notation $s_W\equiv\sin\theta_W$ and $c_W\equiv\cos\theta_W$.

$$M_{Z'}^2 = \frac{1}{2} g_X^2 (Q_1^2 v_1^2 + Q_2^2 v_2^2 + Q_S^2 u^2). \tag{13}$$

The mass matrix Eq. (11), can be diagonalized by introducing a mixing angle ξ. The canonically normalized mass eigenstates are written by using χ and ξ. Their concrete expressions are given in Appendix A. We also present there the interaction Lagrangian of the neutral gauge bosons and matter fermions for later use.

The mixing angle introduced for the diagonalization of the mass matrix, Eq. (11), is given by

$$\tan 2\xi = \frac{-2\cos\chi(m_Z^2 s_W \sin\chi + \Delta m^2)}{M_{Z'}^2 + 2\Delta m^2 s_W \sin\chi + m_Z^2 s_W^2 \sin^2\chi - m_Z^2 \cos^2\chi}. \tag{14}$$

In general the mixing angle ξ is severely constrained to be small enough by the precise measurements at the LEP [10]. From the study of radiative symmetry breaking it has been known that $\tan\beta \gtrsim 1$ is generally favored. In fact, it has been shown in Ref. [6] that suitable radiative symmetry breaking could occur for $1.4 \lesssim \tan\beta \lesssim 2.1$ in the ξ_- model. We will adopt

$$\tan\beta \sim 1.5 \tag{15}$$

as its typical value throughout this paper. Therefore, in the case of $\sin\chi = 0$, since $Q_1\cos^2\beta \simeq Q_2\sin^2\beta$ is not satisfied in the present three models, we need to consider the possibility that the small ξ is realized because of $\Delta m^2 \ll M_{Z'}^2$, which is equivalent to $v_1^2, v_2^2 \ll u^2$. If $\sin\chi \neq 0$, however, there may be a new possibility to satisfy the smallness of ξ even if $\Delta m^2 \ll M_{Z'}^2$ is not satisfied. Such a situation can be expected to occur if the condition

$$\sin\chi \sim -\frac{\Delta m^2}{m_Z^2 s_W} = \frac{g_X}{(g_W^2 + g_Y^2)^{1/2} s_W}(Q_1\cos^2\beta - Q_2\sin^2\beta) \tag{16}$$

is valid. In this case $Q_1\cos^2\beta \simeq Q_2\sin^2\beta$ is not required unlike the $\sin\chi = 0$ case but instead of that a tuning of $\sin\chi$ becomes necessary. The constraint on the value of u also becomes very weak. Since this possibility for small m_{YX}^2 compared with m_Z^2 is interesting enough for the explanation of the smallness of $|\xi|$, we will also consider the case with such a mixing angle $\sin\chi$ in the following discussion as one of the typical examples.

The present model-independent bound on the mixing angle ξ is $|\xi| < 0.01$ [24]. If we impose this bound on the models, we can restrict the allowed u range in each model. Here it should be noted that the mixing angle ξ has no λ dependence. In order to show this constraint coming from the neutral gauge sector, we plot the contours of the mixing angle $|\xi| = 0.01$ for each model in the $(\sin\chi, |u|)$ plane in Fig. 1. The lower regions of the contours are forbidden in each model. It is noticeable that rather small value of $|u|$ is generally allowed in $\xi_\pm$ models in comparison with the η model. From this figure we find that the kinetic term mixing $\sin\chi$ can affect the lower bound of $|u|$ substantially. In the η model the larger $\sin\chi$ reduces the required bound of $|u|$ values. In $\xi_\pm$ models[7] there are special values of $\sin\chi$ which make the lower bound of $|u|$ very small, as anticipated in Eq. (16). Thus in these models a rather light extra Z^0 may be possible.[8]

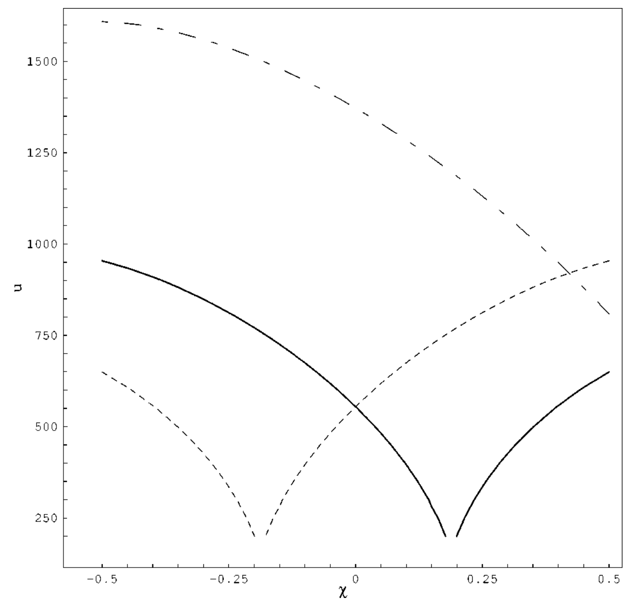

FIG. 1. The allowed region in the $(\sin\chi, |u|)$ plane due to the constraint on the mixing angle ξ between the extra $U(1)_X$ and the ordinary Z^0. The contours of $|\xi| = 0.01$ for three models are drawn. ξ_-, ξ_+, and η models correspond to solid, dashed, and dot-dashed lines, respectively. The lower region of each contour is forbidden.

Related to the fact that a rather large $|u|$ is generally required except for the case with the special $\sin\chi$ value, it will be useful to recall again the origin of the μ scale in the present models. In these models the vacuum expectation value u is relevant to the μ scale. Based on this feature we may need to put an upper bound on λ to keep μ a suitable scale from the viewpoint of radiative symmetry breaking as discussed in [6]. If we use the present Higgs mass bounds, however, λ can be effectively constrained as shown in the following subsection.

D. Higgs sector

The Higgs sector is changed from that of the MSSM due to the existence of the singlet S and its coupling $\lambda S H_1 H_2$ to

[7] $\xi_\pm$ models have a symmetric feature mutually with respect to the sign of $\sin\chi$ so that they are expected to show similar behavior in their phenomenology. This comes from their characteristics of the charge assignments.

[8] In this case the extra $U(1)_X$ gaugino is also expected to affect largely rare phenomena such as $\mu \to e\gamma$ and the electric dipole moment (EDM) of an electron [11].

the Higgs doublets H_1 and H_2. Its brief study can give us some useful information on the allowed region of parameter space [25,26]. If we take account of the Abelian gauge kinetic term mixing, the scalar potential for the Higgs sector can be written as

$$V=\frac{1}{8}(g_W^2+g_Y^2)(v_1^2+v_2^2)^2+\frac{1}{8}\Big\{g_Y\tan\chi(v_1^2-v_2^2)+\frac{g_X}{\cos\chi}[Q_1v_1^2+Q_2v_2^2+Q_Su^2]\Big\}^2+\lambda^2v_1^2v_2^2+\lambda^2u^2v_1^2+\lambda^2u^2v_2^2+m_1^2v_1^2+m_2^2v_2^2+m_S^2u^2-2A\lambda uv_1v_2, \quad (17)$$

where Q_1, Q_2, and Q_S represent the extra $U(1)_X$ charges of the Higgs chiral superfields H_1, H_2, and S. At the minimum of this potential, the mass matrices for the Higgs sector are given as follows.

Charged Higgs scalar sector:

$$\left[\frac{1}{2}m_Z^2c_W^2\sin 2\beta\left(1-\frac{2\lambda^2}{g_W^2}\right)+A\lambda u\right]\begin{pmatrix}\tan\beta & 1\\ 1 & \cot\beta\end{pmatrix}. \quad (18)$$

Neutral Higgs scalar sector:

$$\begin{pmatrix}
m_Z^2\cos^2\beta\left(1+\frac{\zeta_1^2}{\tilde{g}^2}\right)+A\lambda u\tan\beta & \frac{m_Z^2}{2}\sin 2\beta\left(-1+\frac{\zeta_1\zeta_2+4\lambda^2}{\tilde{g}^2}\right)-A\lambda u & \frac{m_Z\cos\beta}{\tilde{g}}\left(u\frac{\zeta_1\zeta_3+4\lambda^2}{\sqrt{2}}-\sqrt{2}A\lambda\tan\beta\right)\\
\frac{m_Z^2}{2}\sin 2\beta\left(-1+\frac{\zeta_1\zeta_2+4\lambda^2}{\tilde{g}^2}\right)-A\lambda u & m_Z^2\sin^2\beta\left(1+\frac{\zeta_2^2}{\tilde{g}^2}\right)+A\lambda u\cot\beta & \frac{m_Z\sin\beta}{\tilde{g}}\left(u\frac{\zeta_2\zeta_3+4\lambda^2}{\sqrt{2}}-\sqrt{2}A\lambda\cot\beta\right)\\
\frac{m_Z\cos\beta}{\tilde{g}}\left(u\frac{\zeta_1\zeta_3+4\lambda^2}{\sqrt{2}}-\sqrt{2}A\lambda\tan\beta\right) & \frac{m_Z\sin\beta}{\tilde{g}}\left(u\frac{\zeta_2\zeta_3+4\lambda^2}{\sqrt{2}}-\sqrt{2}A\lambda\cot\beta\right) & \frac{1}{2}\zeta_3^2u^2+\frac{A\lambda}{u}\frac{m_Z^2}{\tilde{g}^2}\sin 2\beta
\end{pmatrix}, \quad (19)$$

where $\tilde{g}=\sqrt{g_W^2+g_Y^2}$ and we define ζ_1, ζ_2, and ζ_3 as[9]

$$\zeta_1=g_Y\tan\chi+\frac{g_XQ_1}{\cos\chi},$$

$$\zeta_2=-g_Y\tan\chi+\frac{g_XQ_2}{\cos\chi},$$

$$\zeta_3=\frac{g_XQ_S}{\cos\chi}. \quad (20)$$

The overall factor of a mass matrix of the charged Higgs sector is somehow changed from that of the MSSM due to the coupling λSH_1H_2. However, the mass eigenstate of a charged Higgs scalar can be obtained in the same form as the MSSM case,

$$H^{\pm}=\sin\beta H_1^{\pm}+\cos\beta H_2^{\mp *}, \quad (21)$$

and its mass eigenvalues are expressed as

$$M_{H^\pm}^2=m_Z^2c_W^2\left(1-\frac{2\lambda^2}{g_W^2}\right)+\frac{A\lambda u}{\sin\beta\cos\beta}. \quad (22)$$

The λ^2 term is added to the MSSM one and then the charged Higgs boson mass takes a smaller value than that of the MSSM for the same value of $\mu=\lambda u$. On the other hand, the neutral Higgs boson mass matrix is too complex to be diagonalized analytically. However, if we note that the smallest eigenvalue of the matrix is smaller than the smallest diagonal component, we can find the tree-level upper bound of the lightest neutral Higgs boson mass. By diagonalizing the 2×2 submatrix at the upper left corner of Eq. (19) we can obtain [26,6]

$$m_{h^0}^2\leq m_Z^2\left[\cos^2 2\beta+\frac{2\lambda^2}{g_W^2+g_Y^2}\sin^2 2\beta+\frac{1}{g_W^2+g_Y^2}(\zeta_1\cos^2\beta+\zeta_2\sin^2\beta)^2\right]. \quad (23)$$

The first two terms correspond to the bound which is derived from the usually studied model extended with a gauge singlet.

As easily seen from these results, these Higgs boson masses have a crucial dependence on λ and u. One of the important differences between the present models and the MSSM comes from the fact that the μ term is replaced by the Yukawa coupling λSH_1H_2. If we impose the present experimental bounds on the Higgs boson masses, useful constraints can be obtained in the (λ,u) plane. The present mass bounds on both the charged Higgs and the lightest neutral Higgs bosons are $\sim$44 GeV [27]. We use this bound and show the allowed region in the (λ,u) plane in Fig. 2. Since it is found to be insensitive to the models and also the $\sin\chi$ value, we take the ξ_- model with $\sin\chi=0$ as an example. Here for the lightest neutral Higgs boson we used the result obtained by numerical diagonalization of the mass matrix,

[9]It may be useful to note that the sign of ζ_1, ζ_2, and ζ_3 is reversed between a ξ_+ model with $\sin\chi$ and a ξ_- model with $-\sin\chi$.

Eq. (19). It should be noted that only the $u>0$ region is allowed. This is completely dependent on our choice ($A>0$) for the sign of A.[10]

Additional important constraints on μ can be obtained from the condition in the (μ, M_W) plane coming from the search of the neutralinos and charginos at the LEP [28]. If we assume $\tan\beta \sim 1.5$, the allowed region in this plane is roughly estimated as[11]

$$|\mu|, M_W \gtrsim 40\ \text{GeV} \quad (\text{for } \lambda u>0),$$
$$|\mu|, M_W \gtrsim 100\ \text{GeV} \quad (\text{for } \lambda u<0). \tag{24}$$

The chargino sector in the present model is not altered from the MSSM and then these conditions on μ can be used as the constraint for λ and u. Thus the allowed region of the (λ, u) plane is found to be determined by the lower bound of the lightest neutral Higgs boson mass for all models. It corresponds to the surrounded region by the dashed lines in Fig. 2. If we combine this with the result obtained from Fig. 1, we can restrict the allowed region in the (λ, u) plane for each model with a certain $\sin\chi$ value. We will use this fact later.

III. DECAY WIDTH OF $\tilde{\chi}_2^0$ INTO $\tilde{\chi}_1^0$

A. Neutralino sector

In this subsection we examine the structure of the neutralino sector and also define the mass eigenstates of the chargino and squark-slepton sector, which are necessary for the calculation of the neutralino decay. Starting from the superpotential and soft supersymmetry breaking terms given in Eq. (1) and using the canonically normalized basis defined by Eq. (6), we can write down the modified quantities from the MSSM, which are relevant to the neutralino sector, that is, the neutralino mass matrix and the gaugino-fermion-sfermion interaction terms. If we take the canonically normalized gaugino basis $\mathcal{N}^T = (-i\lambda_W^3, -i\lambda_Y, -i\lambda_X, \tilde{H}_1, \tilde{H}_2, \tilde{S})$ and define the neutralino mass term as $\mathcal{L}_{\text{mass}}^n = -\frac{1}{2}\mathcal{N}^T\mathcal{M}\mathcal{N} + \text{H.c.}$, the 6×6 neutralino mass matrix $\mathcal{M}$ can be expressed as

$$\begin{pmatrix} M_W & 0 & 0 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta & 0 \\ 0 & M_Y & C_1 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta & 0 \\ 0 & C_1 & C_2 & C_3 & C_4 & C_5 \\ m_Z c_W \cos\beta & -m_Z s_W \cos\beta & C_3 & 0 & \lambda u & \lambda v \sin\beta \\ -m_Z c_W \sin\beta & m_Z s_W \sin\beta & C_4 & \lambda u & 0 & \lambda v \cos\beta \\ 0 & 0 & C_5 & \lambda v \sin\beta & \lambda v \cos\beta & 0 \end{pmatrix}, \tag{25}$$

where v and u are defined by Eq. (9). Matrix elements C_1–C_5 are components which are affected by the kinetic term mixing. They are represented as

$$C_1 = -M_Y \tan\chi + \frac{M_{YX}}{\cos\chi},$$
$$C_2 = M_Y \tan^2\chi + \frac{M_X}{\cos^2\chi} - \frac{2M_{YX}\sin\chi}{\cos^2\chi},$$
$$C_3 = \frac{1}{\sqrt{2}}\left(g_Y \tan\chi + \frac{g_X Q_1}{\cos\chi}\right) v \cos\beta,$$
$$C_4 = \frac{1}{\sqrt{2}}\left(-g_Y \tan\chi + \frac{g_X Q_2}{\cos\chi}\right) v \sin\beta,$$
$$C_5 = \frac{1}{\sqrt{2}}\frac{g_X Q_S}{\cos\chi} u. \tag{26}$$

Neutralino mass eigenstates $\tilde{\chi}_i^0 (i=1-6)$ are related to $\mathcal{N}_j$ through the mixing matrix U_{ij} as

$$\tilde{\chi}^0 = U^T \mathcal{N}. \tag{27}$$

The change in the gaugino interactions can be confined to the extra $\mathrm{U}(1)_X$ gaugino sector and new interaction terms can be expressed as

$$\frac{i}{\sqrt{2}}\Bigg[\tilde{\psi}^*\left(-g_Y Y \tan\chi + \frac{g_X Q_X}{\cos\chi}\right)\lambda_X \psi$$
$$-\left(-g_Y Y \tan\chi + \frac{g_X Q_X}{\cos\chi}\right)\overline{\lambda_X}\overline{\psi}\tilde{\psi}$$

[10] The A and u dependence of the Higgs mass eigenvalues is included in the terms, which are composed of Au and even powers of each of them. Thus the sign of u is related to that of A. Here it should also be noted that in the present notation $u>0$ corresponds to the ordinary $\mu<0$ case.

[11] It should be noted that this restriction has been derived under some assumptions, for example, the gaugino unification relation $M_Y = \frac{5}{3}\tan^2\theta_W M_W$. However, we will apply them for the general M_Y and M_W here. These constraints correspond to the condition for the chargino mass $m_1^c \gtrsim 65$ GeV.

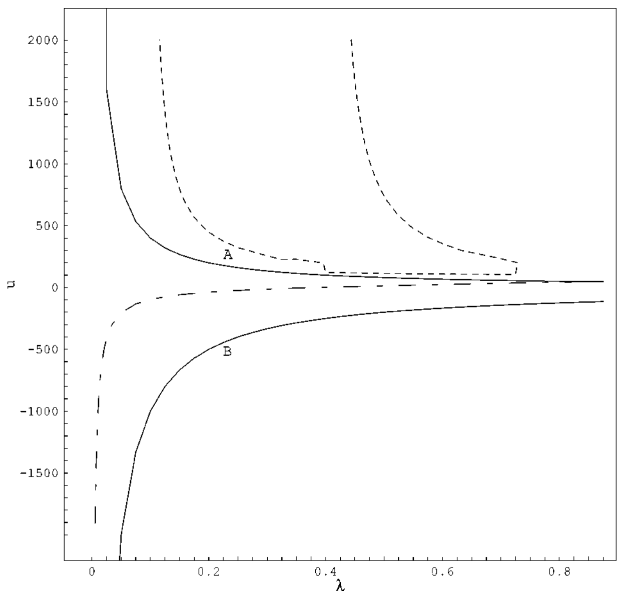

FIG. 2. The allowed region in the (λ, u) plane for the ξ_- model with $\sin\chi=0$. The contours of the present mass bounds of the lightest neutral Higgs scalar and the charged Higgs scalar are shown by the dashed and dot-dashed lines, respectively. The surrounded region by the dashed lines and the upper region of the dot-dashed one are allowed. The solid lines represent the boundary ($A, \lambda u=40$ GeV; and $B, \lambda u=-100$ GeV) coming from the experimental searches of charginos and neutralinos. The region sandwiched between them is forbidden.

$$+H^*\left(-g_Y Y\tan\chi+\frac{g_X Q_X}{\cos\chi}\right)\lambda_X\tilde{H}$$

$$-\left(-g_Y Y\tan\chi+\frac{g_X Q_X}{\cos\chi}\right)\overline{\lambda_X}\overline{\tilde{H}}H\Bigg], \qquad (28)$$

where ψ and $\tilde{\psi}$ represent quarks or leptons and squarks or sleptons. Higgs fields (H_1, H_2, S) are summarized as H and the corresponding Higgsinos $(\tilde{H}_1, \tilde{H}_2, \tilde{S})$ are denoted as $\tilde{H}$. The charges of $U(1)_Y$ and $U(1)_X$ are denoted as Y and Q_X. As a result, the parts corresponding to the gaugino component of the neutralino $\tilde{\chi}_i^0$-fermion-sfermion vertices are represented by the factors

$$Z_i^L(Y,Q_X)=-\frac{1}{\sqrt{2}}\Bigg[g_W U_{1i}\tau_3+g_Y Y U_{2i}$$

$$+\left(-g_Y Y\tan\chi+\frac{g_X Q_X}{\cos\chi}\right)U_{3i}\Bigg],$$

$$\overline{Z_i^R}(Y,Q_X)=-\frac{1}{\sqrt{2}}\left[g_Y Y U_{2i}+\left(-g_Y Y\tan\chi+\frac{g_X Q_X}{\cos\chi}\right)U_{3i}\right], \qquad (29)$$

where the suffixes L and R stand for the chirality of the coupled matter fields ψ and their charges are defined in terms of the left-handed chiral basis as presented in Tables I and II.

Additionally, it is also useful to define the chargino and squark mass eigenstates here for the forthcoming calculation. Taking account of Eq. (1), the chargino mass terms are given as

$$\mathcal{L}_{\rm mass}^c=-(H_1^-,-i\lambda^-)\begin{pmatrix}-\lambda u & \sqrt{2}m_Z c_W\cos\beta\\ \sqrt{2}m_Z c_W\sin\beta & M_W\end{pmatrix}$$

$$\times\begin{pmatrix}H_2^+\\ -i\lambda^+\end{pmatrix}. \qquad (30)$$

The mass eigenstates $\tilde{\chi}_i^\pm$ are defined in terms of the weak interaction eigenstates through the unitary transformations,

$$\begin{pmatrix}\tilde{\chi}_1^+\\ \tilde{\chi}_2^+\end{pmatrix}\equiv W^{(+)\dagger}\begin{pmatrix}H_2^+\\ -i\lambda^+\end{pmatrix}, \quad \begin{pmatrix}\tilde{\chi}_1^-\\ \tilde{\chi}_2^-\end{pmatrix}\equiv W^{(-)\dagger}\begin{pmatrix}H_1^-\\ -i\lambda^-\end{pmatrix}. \qquad (31)$$

Squarks and sleptons are also relevant to the neutralino decay. When we consider this subject, all flavors can be treated in the same way except for the top quark sector. If they appear in the internal lines, the top squark may be especially important because of the largeness of its Yukawa couplings and then we only consider the top squark sector in such cases. However, in the neutralino decay modes which contain the ordinary fermions in the final states, the top quark is too heavy to be included in them and it is irrelevant to such processes.

In the following analysis we do not consider flavor mixing in the squark and slepton sectors, for simplicity. Thus the sfermion mass matrices can be reduced to the 2×2 form for each flavor. This 2×2 sfermion mass matrix can be written in terms of the basis $(\tilde{f}_L, \tilde{f}_R)$ as

$$\begin{pmatrix}|m_f|^2+M_L^2+D_L^2 & m_f(A_f+\lambda u R_f)\\ m_f^*(A_f^*+\lambda u R_f) & |m_f|^2+M_R^2+D_R^2\end{pmatrix}, \qquad (32)$$

where m_f and $M_{L,R}^2$ are the masses of ordinary fermion f and its superpartners $\tilde{f}_{L,R}$, respectively. We assume that $M_{L,R}^2$ is universal for all flavors. R_f is $\cot\beta$ for the up sector and $\tan\beta$ for the down sector. Soft supersymmetry breaking parameters A_f are the dimensionful coefficients of three scalar partners of the corresponding Yukawa couplings. D_L^2 and D_R^2 represent the D-term contributions, which are modified in the present models as follows:

$$D_L^2=\pm\tfrac{1}{2}m_Z^2\cos 2\beta[1-(1\pm Y)s_W^2]$$

$$+\tfrac{1}{4}g_X^2 Q_X'(Q_1'v_1^2+Q_2'v_2^2+Q_S'u^2),$$

$$D_R^2=-\tfrac{1}{2}m_Z^2 s_W^2 Y\cos 2\beta+\tfrac{1}{4}g_X^2 Q_X'(Q_1'v_1^2+Q_2'v_2^2+Q_S'u^2), \qquad (33)$$

where the upper sign in D_L corresponds to the up-sector sfermions and the lower one to down-sector sfermions. The

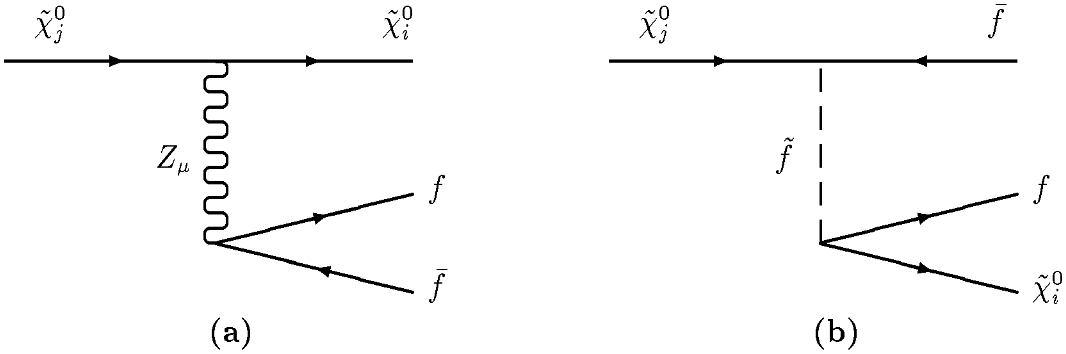

FIG. 3. Diagrams contributing to the tree-level three-body decay $\tilde{\chi}_j^0 \to \tilde{\chi}_i^0 f\overline{f}$.

primed charge Q'_X stands for the modified charge due to the kinetic term mixing and defined as $g_X Q'_X = -g_Y Y \tan\chi + g_X Q_X/\cos\chi$. We should note that these D-term contributions cannot be neglected in the extra $U(1)_X$ where u tends to be large. In such cases it will be useful to note that the positivity condition of the sfermion masses may induce no condition on the soft scalar masses. We define the mass eigenstates $(\tilde{f}_1, \tilde{f}_2)$ as

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \equiv V^{f\dagger} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}. \tag{34}$$

Under our assumption for the reality of soft SUSY parameters the above chargino and sfermion mass matrices are real and then $W^{(\pm)}$ and V^f become the orthogonal matrices.

By now we have finished the preparations for the calculation of neutralino decay in the present models. If R parity is conserved and the lightest neutralino is the lightest superparticle, the decay of the next-to-lightest neutralino $\tilde{\chi}_2^0$ into the lightest neutralino $\tilde{\chi}_1^0$ can be expected to appear in the various superparticle decay processes. As the representative decay modes of $\tilde{\chi}_2^0$ into $\tilde{\chi}_1^0$, the tree-level three-body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$ and the one-loop radiative decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma$ have been calculated in the MSSM framework [12,14,29]. In these studies, which decay mode of these becomes dominant has been shown to be crucially dependent on the composition of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ and then on the SUSY parameters. It is very interesting that the one-loop decay mode can easily dominate the tree-level process in the suitable parameter region. As was recently stressed in Ref. [13], if a CDF-type event relevant to $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma$ happens to be observed dominantly instead of $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$, it can constrain the SUSY parameter space severely. In the following part of this section we shall analyze the decay widths of $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma$ in the present extra $U(1)_X$ models and also qualitatively discuss the condition on the SUSY parameters for $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma$ dominance.

There exist other decay modes like two-body decay into the lightest Higgs $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 h^0$ and the cascade decay mediated through the chargino as $\tilde{\chi}_2^0 \to \tilde{\chi}_1^+ (e\overline{\nu_e}) \to \tilde{\chi}_1^0 \overline{e}\,\nu_e (e\overline{\nu_e})$. If the h^0 is light enough for the threshold to be opened satisfying $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} > m_{h^0}$, the first one can be a relevant mode. The second one may not be suppressed if $\tilde{\chi}_1^+$ is lighter than $\tilde{\chi}_2^0$ even in the case that $\tilde{\chi}_2^0$ is composed of the same ingredients as the case where $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$ is suppressed. Although these points should be taken into account in the analysis, through the numerical calculation of the mass eigenvalues h^0 at least seems to be heavy enough not to open the threshold in the parameter region $(\lambda u, m_Z)$ which we are interested in. For the chargino mediated cascade decay the threshold can be opened but the existence of its suppression mechanism has been pointed out in Ref. [14]. Therefore, in this paper we concentrate our attention on the comparison of $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$. For this purpose we shall first calculate the decay width of both modes. We are particularly interested in the case of rather small neutralino masses since in such a case these neutralino decays may be observed in an experiment in the near future.

B. $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\overline{f}$

There are two types of diagrams which contribute to the tree-level three-body decay. They are shown in Fig. 3. The top quark cannot be a final state so that the contribution from diagram 3(b) is generally suppressed by the small Yukawa coupling. The phase space integral can be analytically done in the limit that the mass of the final state fermion f is zero. This seems to be generally a rather good approximation and we adopt this result of the phase space integral in the present estimation. Thus the decay width for this process can be expressed as[12]

$$\Gamma(\tilde{\chi}_j^0 \to \tilde{\chi}_i^0 f\overline{f}) = \frac{1}{96\pi^3}\left[\frac{m_j^4 - m_i^4}{m_j^3}(m_j^4 + m_i^4 - 8m_j^2 m_i^2) + 24 m_j m_i^4 \ln\frac{m_j}{m_i}\right] \sum_{\alpha=1}^{4} \mathcal{F}_\alpha^2, \tag{35}$$

where the vertex factors $\mathcal{F}_\alpha$ can be expressed by using the mixing matrix element U_{ij} in the neutralino sector as

[12]It should be noted that in the limit of $m_f \to 0$ there is no interference term such as $F_{f_L}^{(1)} F_{f_R}^{(1)}$ between the different fermion chiralities in $\mathcal{F}_\alpha^2 (\alpha=1,2)$.

$$\mathcal{F}_1=\frac{1}{4m_{Z_1}^2}\left[\left(g^{(1)}+\frac{g_X}{2\cos\chi}Q_1\sin\xi\right)U_{4j}U_{4i}+\left(-g^{(1)}+\frac{g_X}{2\cos\chi}Q_2\sin\xi\right)U_{5j}U_{5i}+\frac{g_X}{2\cos\chi}Q_S\sin\xi U_{6j}U_{6i}\right](F_{f_L}^{(1)}+F_{f_R}^{(1)}),$$

$$\mathcal{F}_2=\frac{1}{4m_{Z_2}^2}\left[\left(g^{(2)}+\frac{g_X}{2\cos\chi}Q_1\cos\xi\right)U_{4j}U_{4i}+\left(-g^{(2)}+\frac{g_X}{2\cos\chi}Q_2\cos\xi\right)U_{5j}U_{5i}+\frac{g_X}{2\cos\chi}Q_S\cos\xi U_{6j}U_{6i}\right](F_{f_L}^{(2)}+F_{f_R}^{(2)}),$$

$$\mathcal{F}_3^{(f=U)}=\sum_{\alpha=1,2}\frac{1}{8M_{\tilde{f}_\alpha}^2}\{[Z_{1j}^L(Y,Q_X)U_{5i}+Z_{1i}^L(Y,Q_X)U_{5j}]h_fV_{1\alpha}^{f2}-[\overline{Z_{1j}^R}(Y,Q_X)U_{5i}+\overline{Z_{1i}^R}(Y,Q_X)U_{5j}]h_fV_{2\alpha}^{f2}-2h_f^2U_{5j}U_{5i}V_{2\alpha}^fV_{1\alpha}^f$$
$$+[Z_{1j}^L(Y,Q_X)\overline{Z_{1i}^R}(Y,Q_X)+Z_{1i}^L(Y,Q_X)\overline{Z_{1j}^R}(Y,Q_X)]V_{1\alpha}^fV_{2\alpha}^f\}, \tag{36}$$

where

$$g^{(1)}=\frac{g_W}{2c_W}(\cos\xi+s_W\tan\chi\sin\xi),$$

$$g^{(2)}=\frac{g_W}{2c_W}(-\sin\xi+s_W\tan\chi\cos\xi). \tag{37}$$

$\mathcal{F}_1$ and $\mathcal{F}_2$ comes from diagram 3(a). The mass eigenvalues of the neutral gauge bosons are expressed as m_{Z_1} and m_{Z_2}. $M_{\tilde{f}_\alpha}^2$ is the mass eigenvalue of the sfermion mass matrix, Eq. (32). The effective neutral current couplings, $F_{f_L}^{(1)}$, etc., are deviated from ones of the MSSM due to the existence of the extra $\mathrm{U}(1)_X$ and the Abelian gauge kinetic term mixing. Their concrete expressions are presented in Appendix A. $Z_i^L(Y,Q_X)$ and $\overline{Z_i^R}(Y,Q_X)$ are defined by Eq. (29). Diagram 3(b) gives $\mathcal{F}_{3,4}^f$ and $\mathcal{F}_4^{(f=D,E)}$ is obtained by replacing $Z_{1j}^L(Y,Q_X)$, $\overline{Z_{1j}^R}(Y,Q_X)$ and U_{5j} in $\mathcal{F}_3^{(f=U)}$ with $Z_{2j}^L(Y,Q_X)$, $\overline{Z_{2j}^R}(Y,Q_X)$ and U_{4j}, respectively.

It is useful to examine under what condition this decay width can be suppressed based on Eqs. (35) and (36). As was noticed up to now [14], a dynamical suppression can happen depending on the composition of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ which is determined by the SUSY parameters. For the contribution from $\mathcal{F}_1$ and $\mathcal{F}_2$ they are suppressed unless both $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ are dominated by Higgsinos. In $\mathcal{F}_3$ and $\mathcal{F}_4$ there are contributions from both the Higgsino and gaugino components in $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ and then it seems to be difficult to expect the suppression due to the neutralino composition. However, there is a crucial suppression due to the small Yukawa coupling and also the small left-right mixing V_{12}^f in the sfermion mass matrix. These features can be summarized as follows. The dynamical suppression appears effectively in such a case that one of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ is dominated by gauginos and the other is dominated by Higgsinos. Although this is the same as the MSSM situation, there is a noticeable feature in the present extra $\mathrm{U}(1)_X$ models. In the case of the $\tilde{S}$ dominated neutralino, it has no mixings with λ_W and λ_Y. Moreover, it has no couplings with ordinary fermions. If this is the case, it is not necessary for the gaugino dominated neutralino to be an almost pure photino in order to suppress this three-body decay unlike the MSSM. Later this point will be discussed in more detail again.

C. $\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma$

Next we proceed to the calculation of the one-loop radiative decay $\tilde{\chi}_2^0\to\tilde{\chi}_1^0\gamma$. This has already been studied in the MSSM framework [12]. From gauge invariance, as suggested in [30], it is easily found that the effective interaction describing this process is given as

$$\mathcal{L}_{\mathrm{eff}}=\mathcal{G}\overline{\tilde{\chi}_j^0}\sigma_{\mu\nu}\tilde{\chi}_i^0F^{\mu\nu}. \tag{38}$$

Using this effective coupling $\mathcal{G}$, the decay width is written as

$$\Gamma(\tilde{\chi}_j^0\to\tilde{\chi}_i^0\gamma)=\frac{|\mathcal{G}|^2(m_j^2-m_i^2)^3}{2\pi m_j^3}, \tag{39}$$

where m_i and m_j are the masses of $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$. Our main problem is the estimation of the effective coupling $\mathcal{G}$. One-loop diagrams contributing this coupling are given in Fig. 4. In diagrams 4(1a) and 4(1b), only the top squark contribution cannot be neglected because of its large Yukawa coupling. After some algebraic manipulation, it is obvious that this coupling can be obtained as the coefficient of $\not{q}\cdot\not{\epsilon}$ terms where q_μ and ϵ_μ are the momentum and the polarization vector of photon. In case of small neutralino masses $m_{i,j}\ll m,M$ where m and M, respectively, represent the masses of fermions and bosons in the internal lines, the neutralino mass dependence disappears from these one-loop amplitudes. Its only dependence on the neutralino sector comes through the mixing matrix U_{ij} of the neutralino sector. The effective coupling $\mathcal{G}$ can be summarized as follows:

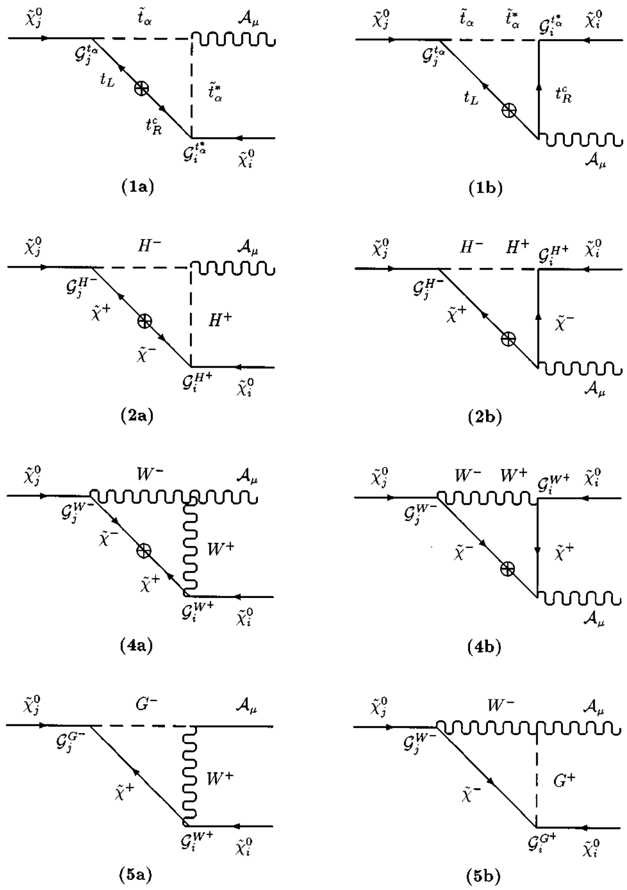

FIG. 4. One-loop diagrams contributing to $\widetilde{\chi}_j^0 \to \widetilde{\chi}_i^0 \gamma$. The chirality flip occurs at the fermion internal lines and/or Yukawa vertices. In (2a) and (2b) we show representative ones.

$$\mathcal{G} = -\frac{e}{32\pi^2}\left(\sum_{\alpha=1,2} \frac{3}{m_t} f\left(\frac{M_{\tilde{t}_\alpha}^2}{m_t^2}\right) G_1^\alpha + \sum_{\alpha=1,2} \frac{1}{m_\alpha}\left\{ f\left(\frac{M_{H^\pm}^2}{m_\alpha^2}\right) G_2^\alpha + f\left(\frac{M_W^2}{m_\alpha^2}\right) G_3^\alpha - \left[4I\left(\frac{M_W^2}{m_\alpha^2}\right) + 3J\left(\frac{M_W^2}{m_\alpha^2}\right)\right] G_4^\alpha + \frac{M_W}{2m_\alpha} J\left(\frac{M_W^2}{m_\alpha^2}\right) G_5^\alpha \right\}\right), \quad (40)$$

where m_α and $M_{H^\pm}$ stand for the masses of the charginos and the charged Higgs boson. The charged Higgs boson mass expression is presented in Eq. (22). The first and second summations should be taken for the top squark mass eigenstates and the chargino mass eigenstates, respectively. Each term with a vertex factor G_i^α comes from Feynman diagram numbered with i in Fig. 4 and their concrete expressions are presented in Appendix B. Kinematical functions $f(r)$, $I(r)$, and $J(r)$ are defined as

$$f(r) = \frac{1}{1-r}\left[1 + \frac{r}{1-r} \ln r \right], \quad (41)$$

$$I(r) = \frac{1}{2(1-r)^2}\left[1 + r + \frac{2r}{1-r} \ln r \right], \quad (42)$$

$$J(r) = \frac{1}{2(1-r)^2}\left[-3 + r - \frac{2}{1-r} \ln r \right]. \quad (43)$$

For checking this formula, we assume that $\lambda u, M_W, M_Y, M_X \ll m_Z$ and the top squark mass matrix is diagonal ($V_{\alpha\beta} = \delta_{\alpha\beta}$). In such a case, for $W_{\alpha\beta}^{(\pm)}$, the situation is the same as the MSSM and they can be taken as[13]

$$W_{12}^{(+)} = -W_{21}^{(+)} = W_{11}^{(-)} = W_{22}^{(-)} = 1,$$

$$W_{11}^{(+)} = W_{22}^{(+)} = W_{12}^{(-)} = W_{21}^{(-)} = 0. \quad (44)$$

Mass eigenvalues of charginos are approximately written as

$$m_1^c = \sqrt{2} m_Z c_W \cos\beta, \quad m_2^c = \sqrt{2} m_Z c_W \sin\beta. \quad (45)$$

For U_{ij}, if we put $g_X = 0$ and $\lambda \to 0$ but keeping $\mu(=\lambda u)$ constant, U_{ij} can be approximated as

$$U_{1i} = s_W, \quad U_{2i} = c_W, \quad U_{4j} = \sin\beta,$$

$$U_{5j} = \cos\beta, \quad \text{other } U_{ij} = 0,$$

$$Z_i^L(Y) = -\sqrt{2} g_2 s_W Q_{\rm em}, \quad \overline{Z_i^R}(Y) = \sqrt{2} g_2 s_W Q_{\rm em},$$

$$Z_j^L(Y) = \overline{Z_j^R}(Y) = 0. \quad (46)$$

Using these expressions, it can be easily checked that $\mathcal{G}$ is reduced to the MSSM result calculated in this parameter setting [12].

This feature of Eq. (40) is rather similar to the one of the MSSM. As easily seen from the structure of G_i^α in Appendix B, there is no special neutralino configuration in which the drastic suppression mechanism works for $\Gamma(\widetilde{\chi}_j^0 \to \widetilde{\chi}_i^0 \gamma)$ unlike $\Gamma(\widetilde{\chi}_j^0 \to \widetilde{\chi}_i^0 \overline{f} f)$. This is an important feature to consider the neutralino decay processes.

D. Radiative decay dominant condition

As was clarified through the study of the CDF event $ee\gamma\gamma + \not{E}_T$ [13], neutralino decay can give the valuable information on the SUSY parameters. Based on a naive perturbative sense, as $\widetilde{\chi}_2^0 \to \widetilde{\chi}_1^0 \gamma$ is the higher order process compared with $\widetilde{\chi}_2^0 \to \widetilde{\chi}_1^0 f \overline{f}$, the former is expected to be largely suppressed by the small couplings compared with the latter.[14] However, in the present case the neutralinos are complicatedly composed of various ingredients and two decay modes imply a different feature depending on their compositions which are determined by the SUSY parameters. If the signature of the radiative decay mode is dominantly ob-

[13]Here the sign conventions are taken so as to make both mass eigenvalues positive.

[14]It has been suggested that there is also a kinematical suppression of the three-body decay when $\widetilde{\chi}_2^0$ and $\widetilde{\chi}_1^0$ are nearly degenerate $m_j^2 - m_i^2 \ll m_j^2$ [14]. However, in our study we will not refer to such a parameter region.

served, the SUSY parameter space can be strictly restricted due to the suppression condition of the tree-level three-body decay. Thus it will be useful to study how this situation can be changed in the extra $U(1)_X$ models.

For this investigation it is convenient to rewrite the neutralino mass matrix, Eq. (25), in terms of the usual photino and Higgsino basis which is often used in the MSSM case. It can be written as

$$\begin{pmatrix} M_W s_W^2 + M_Y c_W^2 & (M_W - M_Y) s_W c_W & c_W C_1 & 0 & 0 & 0 \\ (M_W - M_Y) s_W c_W & M_W c_W^2 + M_Y s_W^2 & -s_W C_1 & m_Z & 0 & 0 \\ c_W C_1 & -s_W C_1 & C_2 & C_3 \cos\beta - C_4 \sin\beta & C_3 \sin\beta + C_4 \cos\beta & C_5 \\ 0 & m_Z & C_3 \cos\beta - C_4 \sin\beta & -\lambda u \sin 2\beta & \lambda u \cos 2\beta & 0 \\ 0 & 0 & C_3 \sin\beta + C_4 \cos\beta & \lambda u \cos 2\beta & \lambda u \sin 2\beta & \lambda v \\ 0 & 0 & C_5 & 0 & \lambda v & 0 \end{pmatrix}, \tag{47}$$

where we define the neutralino basis of this matrix as $(-i\lambda_1, -i\lambda_2, -i\lambda_3, \tilde{H}_a, \tilde{H}_b, \tilde{H}_c)$. Throughout this study we assume that the gaugino masses M_W and M_Y take a value smaller than 200 GeV.

In the MSSM case the radiative decay dominant condition is expressed as [13,14]

$$M_W \simeq M_Y, \quad \tan\beta \simeq 1. \tag{48}$$

The second one is natural from the viewpoint of radiative symmetry breaking and we assume that it is satisfied in our study as mentioned before. The first one is nontrivial but it may not be necessarily required strictly in some parameter region as pointed out in Ref. [14]. As easily seen from the part of Eq. (47) corresponding to the MSSM neutralino sector, we find that in the MSSM with the condition (48) the almost pure photino λ_1 and the one of Higgsinos $\tilde{H}_b$ become the lower two neutralino mass eigenstates as far as M_W, $M_Y, \lambda u \ll m_Z$. This situation realizes the suppression of the three-body decay as discussed in the last part of Sec. III B. On the other hand, this kind of suppression of three-body decay seems not to be realized in the present extra $U(1)_X$ models even if the above condition is satisfied. This is because of the existence of the extra $U(1)_X$ gaugino which has mixings with every neutralino component. Thus in order to suppress the tree-level three-body decay it is necessary to resolve this mixing effectively and produce a purely Higgsino-type neutralino. Although various possibilities may be considered, we are particularly interested in the case with $M_W \neq M_Y$.

The first possibility is to make λ_1 and/or λ_2 decouple from one of the Higgsinos by imposing

$$C_1 \simeq 0, \quad u \gg v, \tag{49}$$

in addition to Eq. (48). The first one requires $M_Y \sin\chi = M_{YX}$ and it is always satisfied in the case of no kinetic term mixing. The second one should be usually satisfied in the extra $U(1)_X$ models to overcome the small mixing condition on ξ as discussed in the previous section. As shown in Tables I and II, $C_3 \sin\beta \pm C_4 \cos\beta \simeq 0$ cannot be satisfied in the present extra $U(1)_X$ models. However, if u is large enough, C_5 becomes large and as a result $C_3 \sin\beta \pm C_4 \cos\beta \simeq 0$ can be effectively satisfied. Under this situation the Higginos $\tilde{H}_b$ can decouple from λ_1 and λ_2. The value of λ is related to which neutralinos become the lower two neutralino mass eigenstates and then it seems not to be severely restricted by requiring radiative decay dominance. As easily seen from the above mass matrix, λ_3 and $\tilde{H}_c$ tend to decouple from other fields under the condition (49) and the situation is reduced to the MSSM one. This feature of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ is expected to be similar to the one of the MSSM. When the composition of these states is interchanged, the same suppression is also expected to occur. In this possibility it should be noted that $M_W \sim M_Y$ will not be necessarily required like the MSSM as far as M_X takes a similar value as M_W and M_Y. In the $\xi_\pm$ model with a suitable $\sin\chi$ value, a large u is not necessarily needed. In such a case, although the Z' becomes rather light, radiative decay dominance cannot be expected. In this case $\sin\chi = 0$ seems to be preferable for radiative decay dominance.

The second possibility is to make the lightest neutralino an almost pure $\tilde{S}$. As mentioned in the Sec. III B, $\tilde{S}$ has no mixings with λ_W and λ_Y and also no couplings with ordinary fermions. Thus if we consider the situation that the next-to-lightest neutralino is the mixture of λ_W and λ_Y and the lightest neutralino is dominated by $\tilde{S}$, three-body decay can be suppressed. This gives a new window which does not require the condition $M_W \sim M_Y$. A very light neutralino dominated by $\tilde{S}$ is considered in a different context in Ref. [31]. To realize this situation it is necessary to impose

$$C_2 \gg C_5, \quad C_1 \simeq 0, \ \lambda u > m_Z. \tag{50}$$

The first one means that M_X needs to be rather large compared with u. We need a particular supersymmetry breaking mechanism which can realize the large hierarchy among soft gaugino masses such as $M_X \gg M_Y$. If $u \gg v$, which is generally the preferable situation for the extra $U(1)_X$ models, the next-to-lightest neutralino is almost a mixture of λ_1 and λ_2 (i.e., λ_W and λ_Y) and also the lightest neutralino $\tilde{H}_c$ which is purely $\widetilde{S}$. Starting from this case, we can get other compositions for the lightest neutralino which realize radiative decay dominance by shifting the values of M_X and u. If we assume $u \gtrsim v$, the lightest neutralino becomes a mixture of $\widetilde{H}_b$ and $\widetilde{H}_c$. This situation can be realized in the $\xi_\pm$ models with a suitable $\sin\chi$ value as found from Fig. 1. If the condition $C_2 \gg C_5$ is changed into $C_5 \gg C_2 \gg v$, which is equivalent to $u \gg M_X \gg v$, the lightest neutralino becomes $\widetilde{H}_b$ and the situation becomes similar to the MSSM case except that $\widetilde{\chi}_2^0$ does not need to be a photinolike state but is enough to be any states composed of λ_W and λ_Y. It should be noted that these new possibilities are related to the large $\mu(>m_Z)$ and/or $\sin\chi\neq 0$ case, where λ_X and $\widetilde{S}$ can play a crucial role. In the $\sin\chi\neq 0$ case, $C_1 \simeq 0$ requires the existence of nonzero M_{YX}. The validity of this condition should be checked by using renormalization group equations (RGEs) in each model.

IV. NUMERICAL ANALYSIS

The arguments in the previous section are qualitative ones on the suppression mechanism for the three-body decay of $\widetilde{\chi}_2^0$ compared with radiative decay. It is necessary to proceed with numerical calculations to treat the subtlety of the parameter dependences and also restrict in more quantitative way the SUSY parameter space where radiative neutralino decay becomes the dominant mode. As suggested above, there may be a new window of the SUSY parameters in the present extra $U(1)_X$ models and it may be possible to escape the constraint, Eq. (48), on the gaugino mass in the MSSM. To clarify this we compare the two decay modes numerically. In the study of this direction the most interesting parameters are the gaugino masses. In addition to them, u and λ will be also important in the present models because they are relevant to the extra Z^0 mass and also the μ scale.

Before going to the numerical analysis of these decay widths, it will be useful to summarize the allowed parameter region. We have already presented constraints on λ and u in Figs. 1 and 2. By combining these results, for typical values of $\sin\chi$ the allowed region of u is roughly estimated as

$$\sin\chi=0\begin{cases}\eta \text{ model:} & u\gtrsim 1375 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.42,\\ \xi_+ \text{ model:} & u\gtrsim 550 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.53,\\ \xi_- \text{ model:} & u\gtrsim 550 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.53,\end{cases}$$

$$\sin\chi=0.2\begin{cases}\eta \text{ model:} & u\gtrsim 1200 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.42,\\ \xi_+ \text{ model:} & u\gtrsim 775 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.52,\\ \xi_- \text{ model:} & u\gtrsim 200 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.66,\end{cases}$$

$$\sin\chi=-0.2\begin{cases}\eta \text{ model:} & u\gtrsim 1525 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.42,\\ \xi_+ \text{ model:} & u\gtrsim 200 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.66,\\ \xi_- \text{ model:} & u\gtrsim 775 \text{ GeV}, \quad 0.1\lesssim\lambda\lesssim 0.52,\end{cases} \tag{51}$$

where $M_W \gtrsim 40$ GeV should be satisfied. Here we should note that $\sin\chi$ affects the neutralino decay widths, Eqs. (35) and (39), not only directly through the vertex factors and the mixing matrix but also indirectly through determining the lower bound of u. For the soft supersymmetry breaking parameters we assume typical values as follows:

$$A=A_f=200 \text{ GeV}, \ M_L=M_R=200 \text{ GeV}. \tag{52}$$

Additionally, $g_Y=g_X$ and $M_{YX}=0$ are also assumed.[15] The gaugino mass M_X is treated as a free parameter and also the gaugino masses M_W and M_Y are assumed to take not so large values such as 40 GeV$\lesssim M_W$, $M_Y \lesssim 200$ GeV. Under this parameter setting, the branching ratio $Br\equiv\Gamma(\widetilde{\chi}_2^0\to\widetilde{\chi}_1^0\gamma)/[\Gamma(\widetilde{\chi}_2^0\to\widetilde{\chi}_1^0 f\bar{f})+\Gamma(\widetilde{\chi}_2^0\to\widetilde{\chi}_1^0\gamma)]$ is studied in the (λ,u) and (M_W,M_Y) planes for typical values of $\sin\chi$ and M_X. Through this study we found that the decay width of the radiative decay is in the rather wide range $O(10^{-6}-10^{-10})$ GeV depending on the parameters. Although from the viewpoint of experimental detectability it may be possible to restrict further the parameter region based on the absolute value of $\Gamma(\widetilde{\chi}_2^0\to\widetilde{\chi}_1^0\gamma)$, we are interested mainly in the radiative decay dominance conditions and then we focus our attention only on the Br value here. It should also be noted that Br gives the same value for the ξ_- model with $\sin\chi$ and the ξ_+ model with $-\sin\chi$.

At first we examine Br under the condition of $M_W=M_Y=M_X$ ($\lesssim 200$ GeV) in the (λ,u) plane. As an example, we take the ξ_- model which has a rather small lower bound of u. In this model it is expected that there is no severe restriction on the value of λ. In fact numerical studies show that $Br>0.98$ is realized almost through the entire region which satisfies the constraints coming from Figs. 1 and 2, although for a certain μ value around ~ 600 GeV there is a shallow valley where Br gives a slightly smaller value compared with other region. That valley moves in the (λ,u) plane by the order of $\lambda u\sim O(10^1-10^2)$ GeV following a change of the value of $\sin\chi$ from -0.2 to 0.2. This shift originating from the change of $\sin\chi$ becomes larger as M_X becomes larger. When M_X becomes larger, $Br<0.90$ occurs at a small λ region such as $\lambda\lesssim 0.2$. These qualitative features are found to be common to all models. The difference between the η model and $\xi_\pm$ model is that the latter can have a smaller bound of u. As a result, for the same value of λ, μ in $\xi_\pm$ models can take smaller values than that in the η model. In such a small μ region Br has the tendency to become smaller as far as the small gaugino masses are assumed. This is because the gaugino-Higgsino mixing cannot be extracted in the lower lying neutrino eigenstates. Anyway we can safely conclude that radiative decay dominance is good enough in the whole region of (λ,u) as far as $M_W=M_Y=M_X$ is satisfied.

Next we proceed to the study of M_W and M_Y dependence of Br. For this purpose we estimate Br in the (M_W,M_Y) plane. In Fig. 5 we show the results for the ξ_- model as an example. The global feature of this kind of plot seems to be characterized by the value of μ $(=\lambda u)$ if M_X is fixed. In the case of $\lambda u \lesssim m_Z$ [Figs. 5(a), 5(b), and 5(c)], $M_W \simeq M_Y$ seems not to be severely required. This point has been already pointed out in the MSSM case [14]. However, in this

[15] Although these should be determined in terms of a RGE analysis, we make these assumptions only for simplicity.

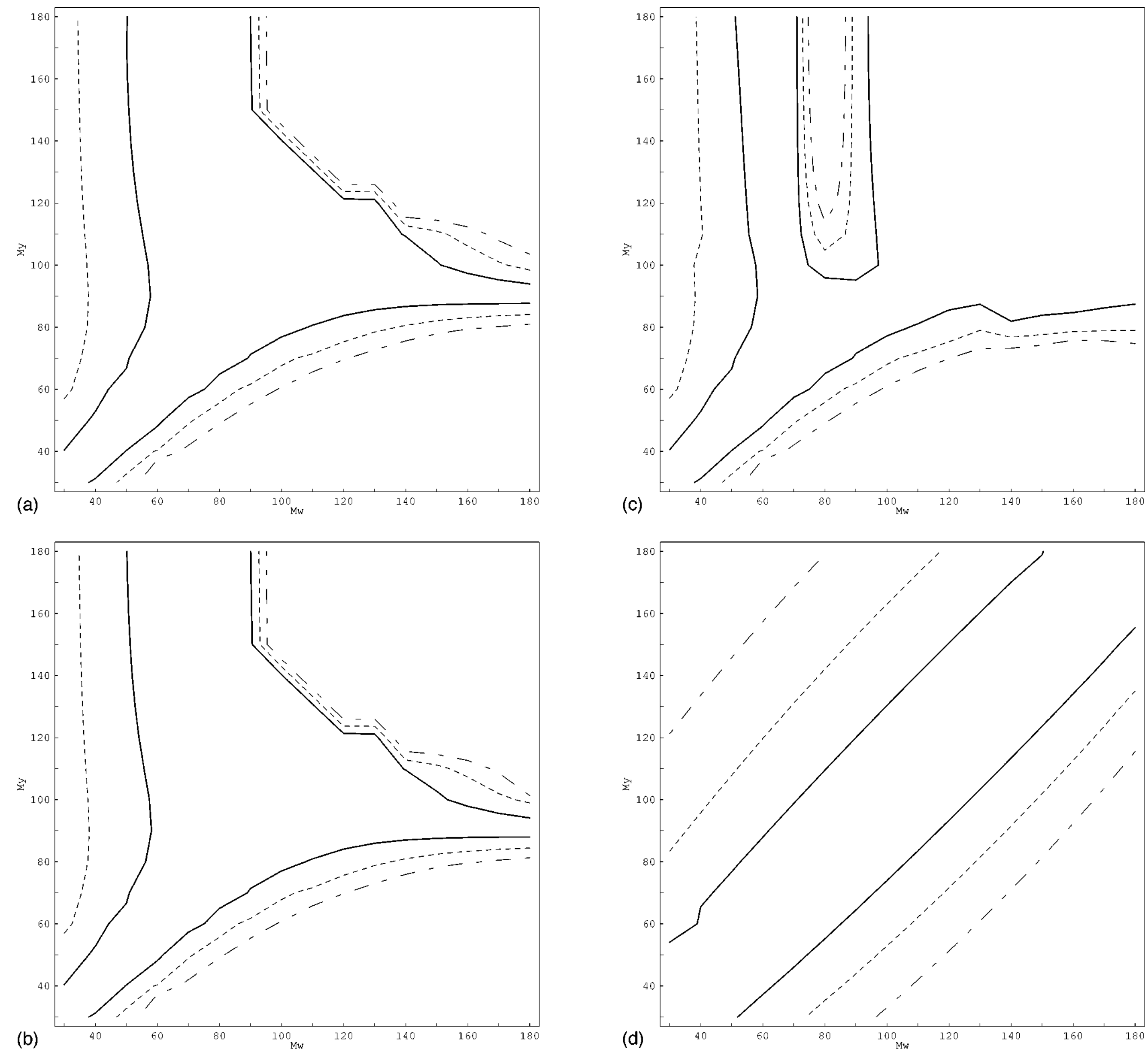

FIG. 5. (a) The contours of the branching ratio $Br=0.9$, 0.7, and 0.5 of the ξ_- model with $\sin\chi=0$ in the (M_W,M_Y) plane, which are represented by solid, dashed, and dot-dashed lines, respectively. Parameters are set as $\lambda=0.15$, $u=600$ GeV and $M_X=50$ GeV. (b) The same contours of Br as (a). Parameters are set as $\lambda=0.15$, $u=600$ GeV, and $M_X=400$ GeV. (c) The same contours of Br as (a). Parameters are set as $\lambda=0.15$, $u=600$ GeV, and $M_X=1000$ GeV. (d) The same contours of Br as (a). Parameters are set as $\lambda=0.5$, $u=600$ GeV, and $M_X=50$ GeV. (e) The same contours of Br as (a). Parameters are set as $\lambda=0.5$, $u=600$ GeV, and $M_X=400$ GeV. (f) The same contours of Br as (a). Parameters are set as $\lambda=0.5$, $u=600$ GeV, and $M_X=1000$ GeV.

model the larger violation of the relation $M_W\simeq M_Y$ seems to be allowed compared with the MSSM case. When M_X becomes larger compared with M_W and M_Y, the $Br>0.9$ region shrinks into the smaller M_W,M_Y region and also there appears a new $Br>0.9$ region in the large M_W,M_Y domain, where $M_W\sim M_Y$ is not required. These behaviors of Br may be understood as follows. Accompanied by a change of M_X, a level crossing occurs between $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ and then their ingredients are interchanged. And in the region of M_X where the separation between $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ is large enough, $Br>0.9$ is realized. In the case of $\lambda u>m_Z$ [Figs. 5(d), 5(e), and 5(f)], when M_X is smaller compared with m_Z, the $Br>0.9$ region appears as a beltlike zone around the $M_W\sim M_Y$ line but the width of this region is not so narrow. This means that the next-to-lightest neutralino should be the almost photino λ_1 to realize radiative decay dominance and then $M_W\simeq M_Y$ is preferable. Under this condition the mixture of λ_2, λ_3, $\tilde{H}_a$, $\tilde{H}_b$, and $\tilde{H}_c$ can decouple from λ_1. As M_X becomes larger, the $Br>0.9$ region has the tendency to occupy a wider space where $M_W\simeq M_Y$ is not required. The reason for this Br behavior can be understood from the qualitative arguments in the previous section. Although we show here the results for only one model, we have checked that other models also showed similar qualitative features. So these results can be considered as qualitatively general ones.

Finally we would like to stress that in the extra $U(1)_X$

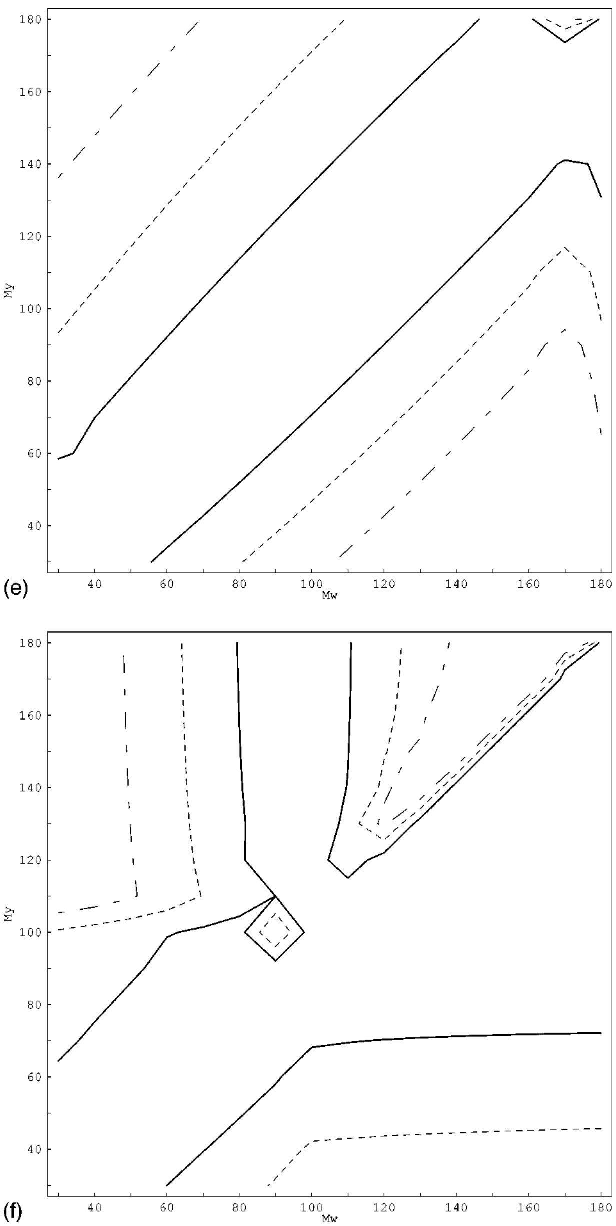

FIG. 5. (*Continued*).

models there is a wide parameter region where radiative decay becomes the dominant mode of the neutralino decay. This region contains a new possibility such that the relation $M_W \sim M_Y$ is completely violated in comparison with the corresponding parameter space to the case of the MSSM [14]. This can be possible because of the existence of λ_X and $\tilde{S}$. The neutralino decay may give us various information on the extra gauge structure.

V. SUMMARY

We studied the decay of the next-to-lightest neutralino into the lightest neutralino in the extended models with an extra $U(1)_X$ and a SM Higgs singlet S, which can solve the μ problem as the result of its radiative symmetry breaking. In this study we took account of the Abelian gaugino kinetic term mixing. At first we investigated the neutral gauge sector and Higgs sector in order to constrain the parameter space of the models. Through this analysis we showed that the VEV $\langle S \rangle$ and the Yukawa coupling λ of the Higgs singlet S were constrained in the suitable region. Next the width of the one-loop radiative decay and the tree-level three-body decay were calculated. Based on those results the suppression condition of the three-body decay was qualitatively discussed and we suggested that there could be a new possibility to escape the constraint on the gaugino masses $M_W \simeq M_Y$ for the realization of such a suppression in the MSSM. This is due to the existence of the extra $U(1)_X$ gaugino and the singlet field S. For a more quantitative analysis the branching ratio of radiative decay was numerically estimated in the (λ, u) and (M_W, M_Y) planes. As a result we found that the μ problem solvable extension with the extra $U(1)_X$ could largely modify the parameter space which realizes radiative decay dominance from that of the MSSM. Especially, it was pointed out that the condition $M_W \simeq M_Y$ for the gaugino masses is not necessarily required for radiative decay dominance as far as M_X is large enough. In the extra $U(1)_X$ models slepton and squark decays which contain the above processes as subprocesses can be largely affected by the existence of the extra gauge bosons and the Higgs singlet. These results seem to be interesting for future accelerator experiments. In the supersymmetric models the extension with extra U(1)'s may have interesting and fruitful phenomena in their superpartner sector and its extra gauge structure may be seen through the study of the superpartner sector. Further study of this aspect will be worthy enough.

ACKNOWLEDGMENTS

The author would like to thank J. Kubo for useful discussions. This work was partially supported by a Grant-in-aid for Scientific Research from the Ministry of Education, Science and Culture (No. 08640362).

APPENDIX A

In this appendix we give the concrete expressions of the interaction Lagrangian of the neutral gauge sector. Original states which are not canonically normalized are represented by the mass eigenstates $(\mathcal{A}^\mu, Z_1^\mu, Z_2^\mu)$ as

$$\hat{A}^\mu = \mathcal{A}^\mu - c_W \tan\chi(\sin\xi Z_1^\mu + \cos\xi Z_2^\mu),$$

$$\hat{Z}^\mu = (\cos\xi + s_W \tan\chi \sin\xi) Z_1^\mu + (-\sin\xi + s_W \tan\chi \cos\xi) Z_2^\mu,$$

$$\hat{X}^\mu = \frac{\sin\xi}{\cos\chi} Z_1^\mu + \frac{\cos\xi}{\cos\chi} Z_2^\mu, \tag{A1}$$

where $\mathcal{A}^\mu$ stands for the real photon field and Z_1^μ is understood as Z^μ observed at the LEP. Using these mass eigenstates, the interaction terms of these gauge fields with ordinary quarks and leptons in this model can be expressed as

$$\mathcal{L}_{\rm int} = J_\mu^{\rm em} \mathcal{A}^\mu + j_\mu^{(1)} Z_1^\mu + j_\mu^{(2)} Z_2^\mu,$$

$$j_\mu^{(1)} = F_{f_L}^{(1)} \overline{f_L} \gamma_\mu f_L + F_{f_R}^{(1)} \overline{f_R} \gamma_\mu f_R,$$

$$j_\mu^{(2)}=F_{f_L}^{(2)}\overline{f_L}\gamma_\mu f_L+F_{f_R}^{(2)}\overline{f_R}\gamma_\mu f_R, \tag{A2}$$

where the coefficients $F_{f_L}^{(1)}$, etc., are defined as

$$F_{f_L}^{(1)}=(\tau^3-2Q_{\rm em}s_W^2)g^{(1)}-eQ_{\rm em}c_W\tan\chi\sin\xi+\frac{g_X}{2\cos\chi}Q_X^{f_L}\sin\xi,$$

$$F_{f_R}^{(1)}=-2Q_{\rm em}s_W^2g^{(1)}-eQ_{\rm em}c_W\tan\chi\sin\xi+\frac{g_X}{2\cos\chi}Q_X^{f_R}\sin\xi,$$

$$F_{f_L}^{(2)}=(\tau^3-2Q_{\rm em}s_W^2)g^{(2)}-eQ_{\rm em}c_W\tan\chi\cos\xi+\frac{g_X}{2\cos\chi}Q_X^{f_L}\cos\xi,$$

$$F_{f_R}^{(2)}=-2Q_{\rm em}s_W^2g^{(2)}-eQ_{\rm em}c_W\tan\chi\cos\xi+\frac{g_X}{2\cos\chi}Q_X^{f_R}\cos\xi. \tag{A3}$$

$Q_X^{f_L}$ and $Q_X^{f_R}$ stand for the U$(1)_X$ charges of f_L and f_R.

APPENDIX B

We give here the concrete expressions of the vertex factors $G_i^\alpha(i=1-5)$ in Eq. (40):

$$G_1^\alpha=-\frac{2}{3}\left\{V_{1\alpha}^\dagger V_{2\alpha}^\dagger\left[\overline{Z_j^R}\left(-\frac{4}{3}\right)Z_{1i}^L\left(\frac{1}{3}\right)-\overline{Z_i^R}\left(-\frac{4}{3}\right)Z_{1j}^L\left(\frac{1}{3}\right)\right]-h_UV_{1\alpha}^\dagger V_{1\alpha}\left[U_{5j}Z_{1i}^L\left(\frac{1}{3}\right)-U_{5i}Z_{1j}^L\left(\frac{1}{3}\right)\right]\right.$$
$$\left.-h_UV_{2\alpha}^\dagger V_{2\alpha}\left[U_{5i}\overline{Z_j^R}\left(-\frac{4}{3}\right)-U_{5j}\overline{Z_i^R}\left(-\frac{4}{3}\right)\right]\right\}, \tag{B1}$$

$$G_2^\alpha=\frac{\sin 2\beta}{2}\{g_W^2W_{2\alpha}^{(+)}W_{2\alpha}^{(-)}(U_{4i}U_{5j}-U_{5i}U_{4j})+W_{1\alpha}^{(+)}W_{1\alpha}^{(-)}[Z_{2i}^L(-1)Z_{1j}^L(1)-Z_{2j}^L(-1)Z_{1i}^L(1)]-g_WW_{2\alpha}^{(+)}W_{1\alpha}^{(-)}$$
$$\times[U_{5j}Z_{2i}^L(-1)-U_{5i}Z_{2j}^L(-1)]-g_WW_{2\alpha}^{(-)}W_{1\alpha}^{(+)}[U_{4i}Z_{1j}^L(1)-U_{4j}Z_{1i}^L(1)]\}+\lambda\sin^2\beta\{g_WW_{2\alpha}^{(-)}W_{1\alpha}^{(+)}$$
$$\times(U_{4j}U_{6i}-U_{6j}U_{4i})+W_{1\alpha}^{(+)}W_{1\alpha}^{(-)}[Z_{2i}^L(-1)U_{6j}-Z_{2j}^L(-1)U_{6i}]\}+\lambda\cos^2\beta\{g_WW_{2\alpha}^{(+)}W_{1\alpha}^{(-)}(U_{5i}U_{6j}$$
$$-U_{6i}U_{5j})+W_{1\alpha}^{(+)}W_{1\alpha}^{(-)}[Z_{1j}^L(1)U_{6i}-Z_{1i}^L(1)U_{6j}]\}, \tag{B2}$$

$$G_4^\alpha=-\frac{g_W^2}{\sqrt{2}}\left[-W_{2\alpha}^{(+)\dagger}W_{1\alpha}^{(-)\dagger}(U_{1i}U_{4j}-U_{4i}U_{1j})+W_{1\alpha}^{(+)\dagger}W_{2\alpha}^{(-)\dagger}(U_{5i}U_{1j}-U_{1i}U_{5j})\right.$$
$$\left.+\frac{1}{\sqrt{2}}W_{1\alpha}^{(+)\dagger}W_{1\alpha}^{(-)\dagger}(U_{5i}U_{4j}-U_{4i}U_{5j})\right], \tag{B3}$$

$$G_5^\alpha=g_W\left[\cos\beta\left(g_WW_{2\alpha}^{(-)\dagger}W_{2\alpha}^{(-)}(U_{1i}U_{4j}-U_{4i}U_{1j})+\frac{1}{\sqrt{2}}W_{1\alpha}^{(-)\dagger}W_{1\alpha}^{(-)}[U_{4j}Z_{2i}^L(-1)-U_{4i}Z_{2j}^L(-1)]\right.\right.$$
$$-\frac{\lambda}{\sqrt{2}}W_{1\alpha}^{(+)\dagger}W_{1\alpha}^{(+)}(U_{5j}U_{6i}-U_{6j}U_{5i})+W_{1\alpha}^{(-)}W_{2\alpha}^{(-)\dagger}[U_{1j}Z_{2i}^L(-1)-U_{1i}Z_{2j}^L(-1)]$$
$$\left.-\lambda W_{1\alpha}^{(+)}W_{2\alpha}^{(+)\dagger}(U_{1j}U_{6i}-U_{6j}U_{1i})\right)+\sin\beta\left(-g_WW_{2\alpha}^{(+)\dagger}W_{2\alpha}^{(+)}(U_{1i}U_{5j}-U_{5i}U_{1j})\right.$$
$$+\frac{1}{\sqrt{2}}W_{1\alpha}^{(+)\dagger}W_{1\alpha}^{(+)}[U_{5j}Z_{1i}^L(1)-U_{5i}Z_{1j}^L(1)]-\frac{\lambda}{\sqrt{2}}W_{1\alpha}^{(-)\dagger}W_{1\alpha}^{(-)}(U_{4j}U_{6i}-U_{6j}U_{4i})$$
$$\left.\left.-W_{1\alpha}^{(+)}W_{2\alpha}^{(+)\dagger}[U_{1j}Z_{1i}^L(1)-U_{1i}Z_{1j}^L(1)]-\lambda W_{1\alpha}^{(-)}W_{2\alpha}^{(-)\dagger}(U_{1j}U_{6i}-U_{6j}U_{1i})\right)\right], \tag{B4}$$

where in these equations we abbreviate the U$(1)_X$ charges in the expression of $Z_i^L(Y,Q_X)$ and $\overline{Z_i^R}(Y,Q_X)$. G_3^α can be obtained by making a replacement such as $\sin\beta\to\cos\beta$ and $\cos\beta\to-\sin\beta$ in G_2^α.

[1] For a review, see, for example, H.-P. Nilles, Phys. Rep. **110**, 1 (1984); H. E. Haber and G. C. Kane, *ibid.* **117**, 75 (1985).
[2] J. E. Kim and H. P. Nilles, Phys. Lett. **138B**, 150 (1984).
[3] K. Inoue, K. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1982); L. Alvarez-Gaume, J. Polchinski, and M. B. Wise, Nucl. Phys. **B221**, 495 (1983).
[4] J. E. Kim and H. P. Nilles, Phys. Lett. B **263**, 79 (1991); E. J. Chun, J. E. Kim, and H. P. Nilles, Nucl. Phys. **B370**, 105 (1992); J. A. Casas and C. Muñoz, Phys. Lett. B **306**, 228 (1993); G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).
[5] H. P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. **129B**, 364 (1983); J.-P. Derendinger and C. A. Savoy, Nucl. Phys. **B237**, 364 (1984); L. E. Ibáñez and J. Mas, Nucl. Phys. **B286**, 107 (1987); J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. D **39**, 844 (1989).
[6] D. Suematsu and Y. Yamagishi, Int. J. Mod. Phys. A **10**, 4521 (1995).
[7] J. F. Gunion, L. Roszkowski, and H. E. Haber, Phys. Rev. D **38**, 105 (1988).
[8] For recent works, see, example, M. Cvetič and P. Langacker, Phys. Rev. D **54**, 3570 (1996); T. Gherghetta, T. A. Kaeding, and G. L. Kane, Report No. UM-TH-96-18, hep-ph/9701343; M. Cvetič, D. A. Demir, J. R. Espinosa, L. Everett, and P. Langacker, Phys. Rev. D **56**, 2861 (1997).
[9] For a review, see, for example, J. Hewett and T. G. Rizzo, Phys. Rep. **183**, 193 (1989); F. Zwirner, Int. J. Mod. Phys. A **3**, 49 (1988).
[10] For example, M. Cvetič and S. Godfrey, in *Proceedings of Electro-weak Symmetry Breaking and Beyond the Standard Model*, edited by T. Barklow, S. Dawson, H. Haber, and J. Siegrist (World Scientific, Singapore, 1995); K. Maeshima, in *ICHEP'96 Proceedings of the 28th International Conference on High Energy Physics*, Warsaw, Poland, 1996, edited by Z. Ajduk and A. Wroblewski (World Scientific, Singapore, 1997).
[11] D. Suematsu, Report No. KANAZAWA-97-06, hep-ph/9705405; Mod. Phys. Lett. A **12**, 1709 (1997).
[12] H. Komatsu and J. Kubo, Phys. Lett. **157B**, 90 (1985); Nucl. Phys. **B263**, 265 (1986); H. E. Haber and D. Wyler, Nucl. Phys. **B323**, 267 (1989).
[13] S. Ambrosanio, G. L. Kane, G. D. Kribs, S. P. Martin, and S. Mrenna, Phys. Rev. Lett. **76**, 3498 (1996); Phys. Rev. D **55**, 1372 (1997).
[14] S. Ambrosanio and B. Mele, Phys. Rev. D **55**, 1399 (1997).
[15] B. Holdom, Phys. Lett. **166B**, 196 (1986); K. Choi and J. E. Kim, Phys. Lett. **165B**, 71 (1985).
[16] T. Matsuoka and D. Suematsu, Prog. Theor. Phys. **76**, 901 (1986).
[17] K. S. Babu, C. Kolda, and J. March-Russell, Phys. Rev. D **54**, 4635 (1996); K. R. Dienes, C. Kolda, and J. March-Russell, Nucl. Phys. **B492**, 104 (1997).
[18] N. Haba, C. Hattori, M. Matsuda, T. Matsuoka, and D. Mochinaga, Prog. Theor. Phys. **92**, 153 (1994); G. Cleaver, M. Cvetič, J. R. Espinosa, L. Everett, and P. Langacker, Report No. UPR-0750-T, hep-ph/9705391.
[19] M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B254**, 549 (1985); T. Matsuoka and D. Suematsu, Nucl. Phys. **B274**, 106 (1986); Prog. Theor. Phys. **76**, 886 (1986).
[20] D. Suematsu, Prog. Theor. Phys. **96**, 611 (1996).
[21] D. Suematsu, Phys. Lett. B **392**, 413 (1997).
[22] D. Suematsu and Y. Yamagishi, Mod. Phys. Lett. A **10**, 2923 (1995).
[23] F. del Aguila, G. D. Coughlan, and M. Quirós, Nucl. Phys. **B307**, 633 (1988).
[24] L3 Collaboration, O. Adriani *et al.*, Phys. Lett. B **306**, 187 (1993); DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **65**, 603 (1995).
[25] H. E. Haber and M. Sher, Phys. Rev. D **35**, 2206 (1987).
[26] M. Drees, Phys. Rev. D **35**, 2910 (1987).
[27] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996), p. 225.
[28] OPAL Collaboration, K. Ackerstaff *et al.*, Phys. Lett. B **389**, 616 (1996); ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **72**, 549 (1996).
[29] A. H. Chamseddine, P. Nath, and R. Arnowitt, Phys. Lett. **129B**, 445 (1983); J. Ellis, J.-M. Frère, J. S. Hagelin, G. L. Kane, and S. T. Petcov, *ibid.* **132B**, 436 (1983); A. Bartl, E. Fraas, and W. Majerotto, Nucl. Phys. **B278**, 1 (1986).
[30] N. Cabibbo, G. R. Farrar, and L. Maiani, Phys. Lett. **105B**, 155 (1981).
[31] B. de Carlos and J. R. Espinosa, Phys. Lett. B **407**, 12 (1997).