# Suppressing the $\mu$ and neutrino masses by a superconformal force

# Suppressing the $\mu$ and neutrino masses by a superconformal force

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The idea of Nelson and Strassler to obtain a power law suppression of parameters by a superconformal force is applied to understand the smallness of the $\mu$ parameter and neutrino masses in $R$-parity violating supersymmetric standard models. We find that the low-energy sector should contain at least another pair of Higgs doublets, and that a suppression of $\lesssim O(10^{-13})$ for the $\mu$ parameter and neutrino masses can be achieved generically. The superpotential of the low-energy sector happens to possess an anomaly-free discrete $R$ symmetry, either $R_3$ or $R_6$, which naturally suppresses certain lepton-flavor violating processes, the neutrinoless double beta decays and also the electron electric dipole moment. We expect that the escape energy of the superconformal sector is $\lesssim O(10)$ TeV so that this sector will be observable at the CERN Large Hadron Collider (LHC). Our models can accommodate a large mixing among neutrinos and give the same upper bound of the lightest Higgs boson mass as the minimal supersymmetric standard model.

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## I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) contains two Higgs chiral supermultiplets, $H_u$ and $H_d$, and with respect to the standard model (SM) gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ the down-type Higgs doublet $H_d$ has the same quantum numbers as the left-handed lepton doublets $L_i (i=1,2,3)$. Therefore, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions cannot distinguish $H_d$ from $L_i$. What distinguishes them from each other are lepton number and $R$ parity [1], which, however, forbid Majorana neutrino masses. An elegant way to generate small neutrino masses is the seesaw mechanism [2], and if we apply this mechanism without breaking $R$ parity we have to introduce right-handed neutrinos into the MSSM. It has been known for a long time that, once we give up the lepton number as well as $R$-parity conservation, there exist possibilities of generating neutrino masses through mixing with neutralinos without introducing right-handed neutrinos [1,3–9].

In this paper we are concerned with these $R$-parity violating (RPV) models.[1] In the RPV models, there exists no difference among $H_d$ and $L_i$. That is, the $\mu$ term, $H_u H_d$, and the bilinear RPV terms, $H_u L_i$, should be treated on the same footing, which implies that the $\mu$ problem [11] is closely related[2] to the smallness of the neutrino masses [17]. So, unless the $\mu$ problem is solved, the natural neutrino mass in the RPV models will be of the order of a fundamental scale, which is a disaster for the models. Our basic idea, to obtain a small $\mu$ and hence small neutrino masses, is to use a superconformal strong force to drive $\mu$ down to the electroweak scale from a superhigh energy scale. A similar idea has been applied in the Yukawa sector and in the supersymmetry breaking sector by Nelson and Strassler [18] to generate a hierarchical order of the Yukawa couplings at low energies from their anarchical order at a fundamental scale,[3] and at the same time to obtain almost degenerate soft-supersymmetry-breaking (SSB) scalar masses at low energies [21,22].

For our idea to work, we have to couple the Higgs fields to a superconformal sector. However, if the MSSM Higgs multiplets couple to the strong sector, not only $\mu$ but also all the Yukawa couplings are suppressed, which we would like to avoid in this paper. So, we will enlarge the Higgs sector. We introduce another pair of Higgs doublets, $\tilde{H}_u$ and $\tilde{H}_d$, which are supposed to couple to the superconformal sector and are responsible to drive $\mu$ down to the electroweak scale. We will find that a suppression of $\lesssim O(10^{-13})$ can be achieved in this way, and we expect that the escape energy of the superconformal sector is rather a lower scale $\sim O(\text{TeV})$, because otherwise the superconformal suppression would be insufficient to understand the smallness of the $\mu$ and neutrino masses. Since the charged matter in the superconformal sector has nontrivial quantum numbers under $SU(2)_L \times U(1)_Y$, they could be experimentally tested at the CERN Large Hadron Collider (LHC), for instance.

We will explicitly construct realistic low-energy models by imposing anomaly-free discrete $R$ symmetries [23] in the superpotential, while allowing most general, renormalizable supersymmetry-breaking terms. It will turn out that our models can accommodate a large mixing among neutrinos, and that the upper bound of the lightest Higgs boson mass of the MSSM remains unchanged.

## II. SUPERCONFORMAL SECTOR

We assume that the superconformal gauge force that suppresses $\mu$ is based on the gauge group $SU(N_C)$ with a global symmetry $U(N_{TS})_L \times U(N_{TS})_R \times U(N_U)_L \times U(N_U)_R$. The matter content is given in Table I. Note that the representations of the matter chiral supermultiplets in this sector should

[1] See Ref. [10] for recent developments.

[2] See also Refs. [12–16] for various possible solutions for the $\mu$ problem.

[3] The basic mechanism will be explained in the text, and for more details see Ref. [18]. See also Refs. [19,20].

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TABLE I. Field content in the superconformal sector.

| | $SU(3)_C \times SU(2)_L \times U(1)_Y$ | $SU(N_C)$ | $U(N_{TS})_L \times U(N_{TS})_R$ | $U(N_U)_L \times U(N_U)_R$ |
|---|---|---|---|---|
| $T$ | (1,2,−1/2) | $N_C$ | $(N_{TS},1)$ | 1 |
| $\overline{T}$ | (1,2,1/2) | $\overline{N}_C$ | $(1,\overline{N}_{TS})$ | 1 |
| $S$ | (1,1,0) | $N_C$ | $(1,N_{TS})$ | 1 |
| $\overline{S}$ | (1,1,0) | $\overline{N}_C$ | $(\overline{N}_{TS},1)$ | 1 |
| $U$ | (1,1,0) | $N_C$ | 1 | $(N_U,1)$ |
| $\overline{U}$ | (1,1,0) | $\overline{N}_C$ | 1 | $(1,\overline{N}_U)$ |

be real with respect to the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. Otherwise the strong force could break dynamically this symmetry, at least at the escaping energy scale $\Lambda_C$, at which the strong sector is supposed to decouple from the low-energy sector. (We will estimate $\Lambda_C$ later.) This implies that the representation of the new Higgs supermultiplets $\tilde{H}_u$ and $\tilde{H}_d$ that couple to the superconformal sector should also be real with respect to these symmetries. With this remark we now consider the coupling of $\tilde{H}_u$ and $\tilde{H}_d$ to this sector through the renormalizable superpotential

$$W_{SC}=y_U\tilde{H}_u T\overline{S}+y_D\tilde{H}_d\overline{T}S, \tag{1}$$

where we have suppressed all the indices, and the new Higgs doublets $\tilde{H}_u$ and $\tilde{H}_d$ are singlets under $SU(N_C)$, where the $U(1)_Y$ charge of $\tilde{H}_{u(d)}$ is $+(-)1/2$.

Let us briefly explain the mechanism proposed by Nelson and Strassler [18] using our model. According to Seiberg's conjecture [24], a nontrivial infrared fixed point exits in our model[4] if $(3/2)N_C<3N_{TS}+N_U<3N_C$ is satisfied [24]. The anomalous dimension $\gamma_I$ of a chiral supermultiplet $\phi_I$ at the fixed point is related to its charge $R_I$ of an anomaly-free $R$ symmetry through $\gamma_I=(3/2)R_I-1$ [24,25]. (We assume below that $\overline{T}$, $\overline{S}$, and $\overline{U}$ have, respectively, the same anomalous dimensions as $T$, $S$, and $U$.) The point is that the anomalous dimensions can become large negative numbers, because the contribution of gauginos with a positive $R$ charge to the anomaly has to be canceled by that of chiral charged matter supermultiplets with negative $R$ charge. This can also be seen from the Novikov-Shifman-Vainstein-Zakharov $\beta$ function [26]

$$\beta(g)=-\frac{g^3}{16\pi^2}\frac{3N_C-3N_{TS}-N_U+2\Gamma}{1-N_C g^2/8\pi^2},$$

$$\Gamma=N_{TS}(2\gamma_T+\gamma_S)+N_U\gamma_U. \tag{2}$$

So, at the fixed point we obtain

$$\Gamma=-\frac{1}{2}[3N_C-(3N_{TS}+N_U)],$$

$$(\tfrac{3}{2}N_C<3N_{TS}+N_U<3N_C). \tag{3}$$

If we may assume that all the chiral supermultiplets have the same anomalous dimension $\gamma$ for simplicity, we find that

$$\gamma=-\frac{3N_C-(3N_{TS}+N_U)}{2(3N_{TS}+N_U)} \tag{4}$$

at the fixed point, implying that the anomalous dimensions can become negative numbers of $O(1)$. Furthermore, at the superconformal fixed point, the dimension of the superpotential $W_{SC}$ has to be 3, which means that its anomalous dimension should vanish. Therefore, we arrive at

$$\gamma^*=\gamma_{\tilde{H}_u}=\gamma_{\tilde{H}_d}=-2\gamma=\frac{3N_C-(3N_{TS}+N_U)}{3N_{TS}+N_U}<1, \tag{5}$$

which is a positive number of $O(1)$, and for instance, $1/14 \leqslant \gamma^* \leqslant 7/8$ for $SU(5)$.

The crucial point is now that the large positive anomalous dimension $\gamma^*$ carried by the SSM supermultiplets has a large influence on the SSM parameters if their evolution has the form

$$\Lambda\frac{d\mu}{d\Lambda}=\mu\,\gamma_{\tilde{H}_{u,d}}+\cdots, \tag{6}$$

where $\cdots$ stands for other contributions from the SSM, which are assumed to be small at high energies. If the energy decreases from a unification scale $\Lambda_0$ [which may be the Planck scale, string scale or grand unified theory (GUT) scale] to the escaping scale $\Lambda_C$ at which the strong sector decouples due to some dynamics, the parameter $\mu$ receives a strong suppression of the form

$$\mu(\Lambda_C)\simeq\mu(\Lambda_0)[\Lambda_C/\Lambda_0]^{\gamma^*}. \tag{7}$$

This is the mechanism of suppression [18], and we assume that all the massive supersymmetric parameters in the superpotential of the SSM sector enjoy this suppression.

Note, however, that the anomalous dimension at the superconformal fixed point cannot exceed 1, if only one chiral multiplet couples to the parameter. That is,

$$\frac{\mu(\Lambda_C)}{\mu(\Lambda_0)}>\frac{\Lambda_C}{\Lambda_0}, \tag{8}$$

[4]We assume that the supersymmetric SM sector (SSM) couples only weakly to the strong sector and so the conjecture is approximately satisfied.

so that if we would identify $\mu(\Lambda_0)$ with $\Lambda_0$, we would obtain the useless result $\mu(\Lambda_C)>\Lambda_C$. A consequence of this observation is that above the unification scale $\Lambda_0$ the parameter $\mu$ should have already received some suppression mechanism, which yields a suppression of

$$\frac{\mu(\Lambda_0)}{\Lambda_0}\simeq\frac{\mu(\Lambda_C)}{\Lambda_C}\left[\frac{\mu(\Lambda_C)}{\mu(\Lambda_0)}\right]^{1/\gamma^*-1}. \tag{9}$$

The value of $\gamma^*$ is typically $\lesssim 0.8$. Assuming that $1/\gamma^*-1\simeq 0.2$, $\Lambda_C/\mu(\Lambda_C)\simeq 50$, and $\mu(\Lambda_C)/\mu(\Lambda_0)\simeq 10^{-10}$, we obtain a necessary suppression of $\mu(\Lambda_0)/\Lambda_0\simeq 10^{-4}$.

Before we come to construct the SSM sector, let us compute the anomalous dimensions $\gamma^*$ in our model in a semi-nonperturbative way. That is, we use the nonperturbative result for the $\beta$ function of the gauge coupling (2), but for the anomalous dimensions we use the one-loop expression

$$\gamma_{\tilde{H}_u}=\frac{1}{16\pi^2}N_{TS}y_U^2,\quad \gamma_T=\frac{1}{16\pi^2}\left(y_U^2-\frac{N_C^2-1}{N_C}g^2\right), \tag{10}$$

$$\gamma_{\bar{S}}=\frac{1}{16\pi^2}\left(2y_U^2-\frac{N_C^2-1}{N_C}g^2\right),$$

$$\gamma_U=-\frac{1}{16\pi^2}\frac{N_C^2-1}{N_C}g^2, \tag{11}$$

and similarly for $\gamma_{\tilde{H}_d}$ etc.. From $\beta(g)=0$ and $\gamma_{\tilde{H}_u}+\gamma_T+\gamma_{\bar{S}}=\gamma_{\tilde{H}_d}+\gamma_{\bar{T}}+\gamma_S=0$, we obtain

$$\gamma^*=\gamma_{H_u}=\gamma_{H_d}=N_{TS}\frac{3(N_C-N_{TS})-N_U}{N_{TS}+3N_{TS}^2+3N_U+N_{TS}N_U}. \tag{12}$$

The maximal value $\gamma^*_{\rm max}$ for a given gauge group can be computed from Eq. (12). We find for instance

$$\gamma^*_{\rm max}(\mathrm{SU}(3))=\frac{1}{3},\quad \gamma^*_{\rm max}(\mathrm{SU}(5))=\frac{7}{12},$$

$$\gamma^*_{\rm max}(\mathrm{SU}(7))=\frac{5}{7},\quad \gamma^*_{\rm max}(\mathrm{SU}(9))=\frac{26}{33}. \tag{13}$$

Note that the numbers above are not exact results, because we have used only one-loop anomalous dimensions in Eqs. (10) and (11). (In some cases, one-loop anomalous dimensions yield exact results.) So these numbers may receive nonperturbative corrections.

As we have seen in this section, the superconformal force can suppress $\mu$ according to the power law (7). However, the suppression $\mu(\Lambda_C)/\mu(\Lambda_0)$ is not strong, so that only a suppression of $\gtrsim O(10^{-13})$ can be gained from the superconformal force if we assume that $\Lambda_C/\Lambda_0\gtrsim 10^{-16}$ and $\gamma^*\lesssim 0.8$, where we have used relation (9). We therefore cannot identify $\mu(\Lambda_0)$ with the fundamental scale $\Lambda_0$, so we have to assume that a suppression of at least $\lesssim 10^{-3}$ should already exist in the fundamental theory. A representative example is

$$\Lambda_C\simeq 1.8\ \text{TeV},\quad \mu(\Lambda_C)\simeq 10^2\ \text{GeV},$$

$$\mu(\Lambda_0)\simeq 10^{13}\ \text{GeV},\quad \Lambda_0\simeq 10^{17}\ \text{GeV}, \tag{14}$$

where we have assumed that $\gamma^*=0.8$. This should be contrasted to the models of Nelson and Strassler [18], where $\Lambda_C$ is supposed to be between $10^{10}$ and $10^{16}$ GeV. Our models predict a rather lower scale $\sim O(\text{TeV})$, because otherwise the superconformal suppression would be insufficient to understand the smallness of $\mu$ and neutrino masses. Since the charged matter multiplets in the superconformal sector have nontrivial quantum numbers under $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$, they could be produced and seen as new type of hadrons at the LHC.

## III. THE LOW-ENERGY SECTOR

We assume that the low-energy physics can be described by a supersymmetric extension of the SM and that all the supersymmetric mass parameters receive the superconformal suppression. As explained in the Introduction, we must enlarge the matter content of the MSSM for this idea to work, and we have already introduced, in addition to the MSSM Higgs doublets $H_u$ and $H_d$, a new set of Higgs doublets $\tilde{H}_u$ and $\tilde{H}_d$ that couple to the superconformal sector. The SM gauge interactions cannot distinguish $\tilde{H}_u$ from $H_u$ and $\tilde{H}_d$ from $H_d$, and so we would like to find a symmetry that makes it possible to distinguish them from each other and allows in the superpotential the quadratic terms such as $H_d\tilde{H}_u$ and $H_u\tilde{H}_d$, but forbids $H_dH_u$ (which has to be absent, because it cannot enjoy the superconformal suppression). First we consider an ordinary global U(1) or discrete $Z_N$ symmetry,[5] and we assume that the superconformal strong force does not break nonperturbatively the symmetry. This implies that the representations of the charged matter multiplets in the strong sector should be real with respect to the symmetry, that is, the U(1) (or $Z_N$) charge of $\tilde{H}_u$ has to be the opposite sign of that of $\tilde{H}_d$. Consequently, $\tilde{H}_u\tilde{H}_d$ and hence $H_uH_d$ cannot be forbidden by an ordinary global U(1) or discrete $Z_N$ symmetry if $H_d\tilde{H}_u$ and $H_u\tilde{H}_d$ are allowed.

Another possibility is $R$ symmetry, discrete or continuous. We understand under the reality of a $R$ symmetry in the strong sector that the charged matter multiplets $T$, $S$, and $U$ can form a mass term with $\bar{T}$, $\bar{S}$, and $\bar{U}$, respectively. So, their $R$ charge has to be 1, implying that the charge of $\tilde{H}_u$ and $\tilde{H}_d$ has to be zero such that the Yukawa coupling (1) is allowed by the symmetry. We look for an anomaly-free $R$ symmetry along the line of Refs. [23,28], because such a symmetry may descend from a gauge symmetry in a compactified string theory. We denote the $R$ charge of a chiral

[5]By an ''ordinary'' symmetry we mean a symmetry that is not of $R$ symmetry type.

supermultiplet $\phi$ by $R(\phi)$, and impose the following conditions: (1) the reality of $(\tilde{H}_u,\tilde{H}_d)$, which means $R(\tilde{H}_u)=R(\tilde{H}_d)=0$; (2) the presence of $H_d\tilde{H}_u$ and $H_u\tilde{H}_d$; (3) the absence of $H_uH_d$; and (4) the presence of the Yukawa terms $E_i^cL_jH_d$, $D_i^cQ_jH_d$, and $U_i^cQ_jH_u$.

Here $E_i$, $U_i$, and $D_i$ are the right-handed lepton, up-type quark, and down-type quark singlets, and $L_i$ and $Q_i$ are the left-handed lepton and quark doublets ($i=1,2,3$), respectively. An immediate consequence of the reality condition 1 is that $R_2$ ($R$ parity) is ruled out, because this condition implies that $R(H_d)=R(H_u)=2=0(\mathrm{mod}\ 2)$ due to the condition (2), which, however, contradicts condition (3). So we will not consider $R_2$ in the following discussion. Conditions (1) and (2) yield

$$R(H_d)=-R(\tilde{H}_u)+2=2 \quad (\mathrm{mod}\ \ N),$$

$$R(H_u)=-R(\tilde{H}_d)+2=2 \quad (\mathrm{mod}\ \ N), \tag{15}$$

which give

$$R(H_u)+R(H_d)=4 \quad (\mathrm{mod}\ \ N), \tag{16}$$

where we have taken into account the possibility that the $R$ symmetry may be a discrete symmetry $R_N$. The last condition (4) requires

$$\begin{aligned} R(H_u)+R(Q_i)+R(U_j^c)&=R(H_d)+R(Q_i)+R(D_j^c)\\ &=R(H_d)+R(L_i)+R(E_j^c)\\ &=2 \quad (\mathrm{mod}\ \ N). \end{aligned} \tag{17}$$

One can easily see that Eq. (17) requires that the trilinear terms

$$D_i^cQ_j\tilde{H}_d \quad \text{and} \quad U_i^cQ_j\tilde{H}_u \tag{18}$$

should be absent.

There exist mixed non-Abelian gauge anomalies, $R[\mathrm{U}(1)_Y]^2$, $R[\mathrm{SU}(2)_L]^2$, $R[\mathrm{SU}(3)_C]^2$, and $R[\mathrm{SU}(N_C)]^2$, the cubic $R^3$, and mixed gravitational anomalies. The cubic and mixed gravitational anomalies depend on the structure of the massive states in the high-energy theory (so they do not decouple in a certain sense at low energies [23]), while the mixed gauge anomalies should be cancelled by the massless fermions [23,27]. Since we are not interested in the high-energy sector in the present paper, we would like to take into account only the mixed gauge anomalies. Moreover, the $R[\mathrm{U}(1)_Y]^2$ anomaly does not give useful information, because the $\mathrm{U}(1)_Y$ charge is not quantized. With these remarks in mind, we consider $R[\mathrm{SU}(2)_L]^2$ and $R[\mathrm{SU}(3)_C]^2$ only. The anomaly coefficients are given by [23,27,28]

$$\begin{aligned} \mathcal{A}_2=&\frac{3}{2}3[R(Q)-1]+\frac{1}{2}\sum_{i=1}^{3}[R(L_i)-1]+\frac{1}{2}\{[R(H_u)-1]\\ &+[R(H_d)-1]+[R(\tilde{H}_u)-1]+[R(\tilde{H}_d)-1]\}+2, \end{aligned} \tag{19}$$

$$\mathcal{A}_3=\frac{3}{2}\{2[R(Q)-1]+[R(U)-1]+[R(D)-1]\}+3, \tag{20}$$

where we have considered the possibility that the $R$ charge of the leptons may depend on the generation, while we have assumed that for quarks it is independent of the generation. Using now Eqs. (15)–(17), the anomaly coefficients (19) and (20) can be rewritten as

$$2\mathcal{A}_2=\left(-8+\sum_{i=1}^{3}[R(L_i)+9R(Q)]\right) \quad (\mathrm{mod}\ \ N), \tag{21}$$

$$2\mathcal{A}_3=6\left\{1-\frac{1}{2}[R(H_u)+R(H_d)]\right\}=-6 \quad (\mathrm{mod}\ \ N). \tag{22}$$

Equation (22) implies that a continuous $R$ symmetry cannot be anomaly free. So we look for anomaly-free discrete $R$ symmetries $R_N$. For $R_N$, the right-hand side of Eqs. (21) and (22) may be $Nk$ to ensure an anomaly-free symmetry, where $k$ is an arbitrary integer. Therefore, Eq. (22) implies that we can have only $R_3$ or $R_6$ ($R_2$ has already been ruled out). Another immediate consequence is that if $R(L_i)$ is independent of the generation, $2\mathcal{A}_2=Nk$ cannot be satisfied for $N=3$ and 6, because 8 cannot be cancelled by a multiple of 3. In the following discussion we will assume that $L_1$ has a $R$ charge that is different from those of $L_2$ and $L_3$ (although there are other possibilities, e.g., that the $R$ charge of the quarks is generation dependent). The $R[\mathrm{SU}(N_C)]^2$ anomaly results only from the $\mathrm{SU}(N_C)$ gauginos [condition (1) is a consequence of $R(T)=R(\bar{T})=\cdots=R(\bar{U})=1$]:

$$2\mathcal{A}_{N_C}=2T(\mathrm{SU}(N_C))=2N_C, \tag{23}$$

which implies that, because of $R_3$ or $R_6$, only a multiple of 3 for $N_C$ is possible.

We have checked that there exist various solutions, and we would like to give here only two representative solutions in Table II. The models also possess the baryon triality symmetry $B_3$ [23], which is free not only from the mixed non-Abelian gauge anomalies, but also from the cubic as well as the mixed gravitational anomalies.[6]

The superpotential corresponding to the $R_3$ and $R_6$ models takes the form

$$W=W_\mu+W_Y+W_Y', \tag{24}$$

where

[6]The baryon triality is defined as $B_3=2Y-B(\mathrm{mod}\ 3)$ [23], where $Y$ and $B$ are the weak hypercharge and baryon number, respectively. The baryon triality assignment in the superconformal sector is not unique. A possibility is that $B_3(T)=2,B_3(\bar{T})=1$ and all the other superfields have zero charge.

TABLE II. The $R$ charge assignment of two representative models. The last row is the baryon triality [23].

| $R$ | $H_u$ | $H_d$ | $\tilde{H}_u$ | $\tilde{H}_d$ | $L_1$ | $L_{2,3}$ | $E_1^c$ | $E_{2,3}^c$ | $Q$ | $U^c$ | $D^c$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_3$ | 2 | 2 | 0 | 0 | 1 | 2 | 2 | 1 | 0 | 0 | 0 |
| $R_6$ | 2 | 2 | 0 | 0 | 4 | 2 | 2 | 4 | 0 | 0 | 0 |
| $B_3$ | 1 | 2 | 1 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 1 |

$$W_\mu = \tilde{\mu} H_u \tilde{H}_d + \mu_0 H_d \tilde{H}_u + \sum_{i=2,3} \mu_i L_i \tilde{H}_u , \tag{25}$$

$$\begin{aligned} W_Y = &\sum_{i,j=2,3} y_{ij}^e L_i H_d E_j^c + y_{11}^e L_1 H_d E_1^c \\ &+ \sum_{i,j=1}^{3} [y_{ij}^d Q_i H_d D_j^c + y_{ij}^u Q_i H_u U_j^c], \end{aligned}$$

$$\begin{aligned} W_Y' = &\sum_{i=2,3} \Bigg( \lambda_{i11} L_1 L_i E_1^c + \lambda_{23i} L_2 L_3 E_i^c \\ &+ \sum_{j,k=1}^{3} (\lambda'_{ijk} L_i Q_j D_k^c + \tilde{y}_{1i}^e L_1 \tilde{H}_d E_i^c) \Bigg). \end{aligned} \tag{26}$$

The coupling constant $\lambda_{ijk}$ is antisymmetric with respect to the first two indices ($\lambda_{ijk} = -\lambda_{jik}$). The last term $\tilde{y}_{1i}^e L_1 \tilde{H}_d E_i^c$ in $W_Y'$ could cause a flavor changing neutral current (FCNC) problem, but it is not, because $\tilde{y}_{1i}^e$ will be extremely suppressed by the superconformal force. Note that the baryon number violating term $D^c D^c U^c$ is absent in the superpotential. This term is protected by $B_3$ and also by the discrete $R$ symmetry.

To make our model viable we have to take into account supersymmetry breaking. We assume that it appears as the soft-supersymmetry-breaking (SSB) Lagrangian $\mathcal{L}_{soft}$. What about symmetry of $\mathcal{L}_{soft}$? If we impose the same global symmetry $R_3$ or $R_6$ on $\mathcal{L}_{soft}$, the gaugino mass terms for instance are not allowed. This would phenomenologically be a disaster. In the case of the MSSM, the SSB terms satisfy $R_2$ symmetry ($R$ parity), and moreover this symmetry is free of all anomalies. But the superpotential of the MSSM with or without RPV terms has a larger $R$ symmetry than $R_2$, which is free from mixed non-Abelian gauge anomalies. One can convince oneself, for instance, that an anomaly-free $R_4$ or $R_5$ is realized in the superpotential. These discrete symmetries $R_4$ and $R_5$ are assumed to be completely broken by the SSB terms in the case of the MSSM, while the completely anomaly-free $R_2$ is unbroken by the SSB terms. In the present case we therefore assume that the completely anomaly-free $B_3$ is unbroken, while the superpotential symmetry, $R_3$ or $R_6$, is broken by the SSB terms. We thus include all renormalizable SSB terms in $\mathcal{L}_{soft}$ that are consistent with $B_3$. Then the SSB Lagragian is given by

$$\begin{aligned} -\mathcal{L}_{soft} = &\sum_{i,j=1}^{2} (\tilde{m}_u^2)_{ij} H_{ui}^* H_{uj} + \sum_{\alpha,\beta=1}^{5} (\tilde{m}_d^2)_{\alpha\beta} H_{d\alpha}^* H_{d\beta} \\ &+ \sum_{i,j=1}^{3} [(\tilde{m}_Q^2)_{ij} Q_i^* Q_j + (\tilde{m}_U^2)_{ij} U_i^{*c} U_j^c \\ &+ (\tilde{m}_D^2)_{ij} D_i^{*c} D_j^c] + \Bigg[ -\sum_{i=1}^{2} \sum_{\alpha=1}^{5} B_{i\alpha} H_{ui} H_{d\alpha} \\ &+ \sum_{i=1}^{3} \sum_{\alpha,\beta=1}^{5} h_{\alpha\beta i}^e H_{d\alpha} H_{d\beta} E_i^c \\ &+ \sum_{i,j=1}^{3} \left( \sum_{\alpha=1}^{5} h_{ij\alpha}^d Q_i H_{d\alpha} D_j^c + \sum_{k=1}^{2} h_{ijk}^u Q_i H_{uk} U_j^c \right) \\ &+ \text{H.c.} \Bigg], \end{aligned} \tag{27}$$

where the gaugino masses are abbreviated and the same notation has been used for the scalar component of a supermultiplet as the corresponding superfield. We have denoted the Higgs doublets $H_u$ and $\tilde{H}_u$ by $H_{ui}$ with $i=1,2$, and the down-type ones $H_d, \tilde{H}_d$ and $L_i$ ($i=1,2,3$) by $H_{d\alpha}$ with $\alpha = 1, \ldots, 5$, respectively.

The superpotential (26) has various phenomenological consequences. First of all there is no baryon decay as emphasized. ($\lambda''_{ijk}$ in the notation of Ref. [10] vanish identically.) Further various Yukawa couplings vanish:

$$\begin{aligned} &y_{1i} = \lambda_{231} = \lambda_{1ij} = 0 \quad \text{for} \ \ i,j = 2,3, \\ &\lambda'_{1ij} = 0 \quad \text{for} \ \ i,j = 1,2,3. \end{aligned} \tag{28}$$

Therefore, the bounds coming from a certain set of the lepton-flavor violating processes such as $\mu \to e\gamma$, $\mu \to eee$, $\mu$-$e$ conversion in nuclei [29–31], the electron electric dipole moment (EDM) [32], and the neutrinoless double $\beta$ decay [33–35] are automatically satisfied. But the lepton-flavor violating $\tau$ decays as well as various RPV rare leptonic decays of light mesons [29] such as $K_L \to \mu \bar{\mu}$, $K_L \to e\bar{e}$ are allowed, while a certain mode such as $K_L \to e\bar{\mu} + \bar{e}\mu$ is forbidden, giving constraints[7] on the RPV Yukawa couplings [29,10]

$$\begin{aligned} &\lambda_{232} \lambda'_{312,321} \lesssim 3.8 \times 10^{-7}, \\ &\lambda_{121} \lambda'_{212,221}, \lambda_{131} \lambda'_{312,321} \lesssim 2.5 \times 10^{-8}. \end{aligned} \tag{29}$$

These might be considered as prediction of the present model and make it possible to discriminate the model from other RPV models. There are other phenomenological consequences, which we would like to leave for future work.

[7]It is assumed here and above that the mass of all the scalar quarks and leptons is 100 GeV.

## IV. NEUTRINO MASS AND THE LIGHTEST HIGGS BOSON MASS

### A. Neutrino mass and mixing

First we would like to derive the neutralino-neutrino mass matrix $M$ for the superpotential (24) along with the SSB Lagrangian (27). To this end, we define the neutralino vector as

$$\Psi^T=(-i\lambda_1,-i\lambda_2,\psi_u,\psi_{\tilde{u}},\psi_d,\psi_{\tilde{d}},\psi_i),\quad i=1,2,3 \tag{30}$$

where $\lambda_{1,2}$ are the gauginos for $U(1)_Y$ and $SU(2)_L$, and $\psi$'s are the neutral fermionic components of the Higgs and left-handed lepton supermultiplets in an obvious notation. The vacuum expectation values (VEVs) of the neutral bosonic components of the Higgs and left-handed lepton supermultiplets are denoted by $v_I$ with $I=u,\tilde{u},\ldots$, and our normalization of $v$'s can be read off from

$$v=\frac{2M_W}{g}=246\ \text{GeV},\quad v^2=\sum_{I=u,\tilde{u},\ldots} v_I^2, \tag{31}$$

where $g$ is the $SU(2)_L$ gauge coupling constant, and $M_W$ is the $W$ gauge boson mass. We also use the notation $v_0=v_d$, $\rho_I=v_I/v$, $M_{sw}=M_Z\sin\theta_W=M_W\tan\theta_W$, and $M_{cw}=M_Z\cos\theta_W=M_W$, where $\theta$ is the Weinberg angle. Then neutralino-neutrino mass term can be written as $-(1/2)\Psi^T M\,\Psi$, where

$$M=\begin{pmatrix}\mathcal{M}_0 & \mathcal{M}^T\\ \mathcal{M} & 0\end{pmatrix}, \tag{32}$$

$$\mathcal{M}_0=\begin{pmatrix}
M_1 & 0 & M_{sw}\rho_u & M_{sw}\rho_{\tilde{u}} & -M_{sw}\rho_0 & -M_{sw}\rho_{\tilde{d}}\\
0 & M_2 & -M_{cw}\rho_u & -M_{cw}\rho_{\tilde{u}} & M_{cw}\rho_0 & M_{cw}\rho_{\tilde{d}}\\
M_{sw}\rho_u & -M_{cw}\rho_u & 0 & 0 & 0 & \tilde{\mu}\\
M_{sw}\rho_{\tilde{u}} & -M_{cw}\rho_{\tilde{d}} & 0 & 0 & \mu_0 & 0\\
-M_{sw}\rho_0 & M_{cw}\rho_0 & 0 & \mu_0 & 0 & 0\\
-M_{sw}\rho_{\tilde{d}} & M_{cw}\rho_{\tilde{d}} & \tilde{\mu} & 0 & 0 & 0
\end{pmatrix},$$

$$\mathcal{M}=(-M_{sw}\rho_i\ \ M_{cw}\rho_i\ \ 0\ \ \mu_i\ \ 0\ \ 0). \tag{33}$$

Here $\mathcal{M}_0$ is a neutralino mass matrix and a neutralino-neutrino mixing matrix is represented by $\mathcal{M}$. Through this neutralino-neutrino mixing neutrinos can become massive as discussed in the usual RPV models [3–9].

The smallness of the neutrino masses can be achieved in two ways. One possibility is given by a precise alignment of $\vec{\rho}$ and $\vec{\mu}$, in which case the energy scale of $R$-parity violation does not have to be very small, and therefore $\rho_{1,2,3}$ can take $O(1)$ values. As a result, the neutralinos and neutrinos can have a large mixing. The other possibility does not require the precise alignment between $\vec{\rho}$ and $\vec{\mu}$, but the scale $R$-parity violation has to be small compared to the weak scale. In this case all of the elements of $\mathcal{M}\mathcal{M}_0^{-1}$ is smaller than 1, and consequently the neutrino mass matrix can be obtained from the seesaw formula $m_\nu=\mathcal{M}\mathcal{M}_0^{-1}\mathcal{M}^T$.

Let us examine each case in more detail. In our models discussed in the previous sections [see the superpotential (25)], we have $\mu_1=0$. The smallest nonzero eigenvalue $m_{\nu_3}$ of the mass matrix $M$ in the first case can be approximately written as [8]

$$m_{\nu_3}\simeq\frac{M_Z^2(c_w^2M_1+s_w^2M_2)}{\vec{\mu}^2M_1M_2}[\vec{\mu}^2\vec{\rho}^2-(\vec{\mu}\cdot\vec{\rho})^2], \tag{34}$$

where

$$\vec{\mu}=(\mu_0,\mu_1,\mu_2,\mu_3),\quad \vec{\rho}=(\rho_0,\rho_1,\rho_2,\rho_3). \tag{35}$$

Note that $\vec{\mu}$ and $\vec{\rho}$ do not contain $\tilde{\mu}$ and $\rho_{\tilde{u}},\rho_{\tilde{d}}$, respectively. Using the angle $\xi$ made by $\vec{\mu}$ and $\vec{\rho}$ and the GUT motivated relation $M_1/M_2=(5/3)\tan^2\theta_W$, the neutrino mass (34) can be written as

$$m_{\nu_3}\simeq\frac{8}{5}\frac{M_W^2}{M_2}\frac{\sin^2\xi}{1+\tan^2\beta}, \tag{36}$$

where we have defined $|\vec{\rho}|=\cos\beta$, which would coincide with $v_d/(v_u^2+v_d^2)^{1/2}$ of the $R$-parity conserving case if only $H_u$ and $H_d$ would acquire a nonvanishing VEV. To obtain a neutrino mass such as $\lesssim 2.8$ eV satisfying the combined mass bound coming from the tritium $\beta$ decay [36] and various observations of the neutrino oscillation [37], we need $\sin\xi\lesssim 3\times10^{-4}$ for $M_2=1$ TeV and $\tan\beta=10$. It may be interesting to see how the eigenstate $\psi_{\nu_3}$ of the smallest non-vanishing mass $m_{\nu_3}$ is composed. Here we consider only the case in which $\psi_1$ and $\psi_2$ are decoupled (that is, $\mu_1=\mu_2$

$=0$). Since $\vec{\rho}$ has to be almost parallel to $\vec{\mu}$, we make an approximation that $\vec{\rho}\propto\vec{\mu}$, and find that the mass eigenstate is given by

$$\psi_{\nu_3}\simeq\frac{1}{\sqrt{\mu_0^2+\mu_3^2}}(\mu_0\psi_3-\mu_3\psi_d). \tag{37}$$

So the mixing between $\psi_3$ and $\psi_d$ will be large in general, but no mixing occurs with the other neutralinos. There are two zero mass eigenvalues at tree level, but in higher orders in perturbation theory [7,8] this degeneracy is removed and the mixing among all the neutrinos occurs. Although the couplings in the superpotential (26) are restricted by a discrete $R$ symmetry [see Eq. (28)], three neutrinos mix at one-loop order, allowing a variety of mixing among neutrinos depending on the size of the $R$-parity violating parameters. However, we cannot say more about its nature at present.

In the second case the neutrino mass matrix can be obtained from the seesaw formula

$$\begin{aligned} m_\nu&=\mathcal{M}\mathcal{M}_0^{-1}\mathcal{M}^T\\ &=\frac{M_Z^2(c_w^2M_1+s_w^2M_2)}{M_1M_2\mu_0^2}\begin{pmatrix}\Gamma_e^2 & \Gamma_e\Gamma_\mu & \Gamma_e\Gamma_\tau\\ \Gamma_e\Gamma_\mu & \Gamma_\mu^2 & \Gamma_\mu\Gamma_\tau\\ \Gamma_e\Gamma_\tau & \Gamma_\mu\Gamma_\tau & \Gamma_\tau^2\end{pmatrix}, \end{aligned} \tag{38}$$

where $\Gamma_\alpha=-\rho_\alpha\mu_0+\rho_0\mu_\alpha$. The nonzero eigenvalue of this matrix is given by

$$\frac{M_Z^2(c_w^2M_1+s_w^2M_2)|\vec{\Gamma}|^2}{M_1M_2\mu_0^2},$$

which is equivalent to Eq. (34) up to the higher-order terms of $\mu_\alpha$ and $\rho_\alpha$. A possible diagonalization matrix of (38) is[8]

$$\begin{aligned} V_\nu=&\begin{pmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\cos\delta & 0 & \sin\delta\\ 0 & 1 & 0\\ -\sin\delta & 0 & \cos\delta\end{pmatrix}\\ &\times\begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}, \end{aligned} \tag{39}$$

where $\tan\gamma=-\Gamma_\mu/\Gamma_e$, $\tan\delta=\sqrt{\Gamma_e^2+\Gamma_\mu^2}/\Gamma_\tau$, and $\alpha$ is an arbitrary angle. This arbitrariness results from the fact that the mass matrix (38) has two degenerate eigenvalues. Now to find the mixing matrix in the lepton sector $V^{\mathrm{MNS}}$, we remind ourselves that our $R$-charge assignment (see Table II) constrains the mass matrix of the charged leptons to have the form[9]

$$m_l=\begin{pmatrix}m_{ee} & 0 & 0\\ 0 & m_{\mu\mu} & m_{\mu\tau}\\ 0 & m_{\tau\mu} & m_{\tau\tau}\end{pmatrix}. \tag{40}$$

This matrix can allow a maximum mixing in the $e$ and $\mu$ sector, which is favored for the realization of a bimaximal mixing in the lepton sector [38]. (The bimaximal mixing is considered to be a favored form to explain the solar and atmospheric neutrino observation.) Since the mixing matrix $V^{\mathrm{MNS}}$ is given by $V^{\mathrm{MNS}}=V_l^\dagger V_\nu$ ($V_l$ is the diagonalization matrix of the matrix $m_l$), the bimaximal mixing form

$$V^{\mathrm{MNS}}\simeq\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\end{pmatrix} \tag{41}$$

may be obtained if, for instance, $\sin\delta\sim0$ and $\cos(\alpha+\gamma)\sim\sin(\alpha+\gamma)\sim1/\sqrt{2}$. Note that the higher-order corrections resolve the mass degeneracy and hence fix the size of the angle $\alpha$. We expect to obtain results that are similar to those in Ref. [8], in which, as far as the neutrino-neutralino sector is concerned, similar models have been studied. Here we would like to quote their result: Following the notation of Grossman and Haber in Ref. [8], the one-loop contribution $\delta m_\nu$ to the neutrino mass matrix may be written as

$$\begin{aligned} (\delta m_\nu)_{ij}\simeq\frac{1}{32\pi^2}\Bigg[&\sum_{k,p}\lambda_{ikp}\lambda_{jpk}m_k^{(l)}\sin2\phi_k^{(l)}\ln\left(\frac{M_{p_1}^{(l)2}}{M_{p_2}^{(l)2}}\right)\\ &+3\sum_{s,t}\lambda'_{ist}\lambda'_{jts}m_s^{(q)}\sin2\phi_s^{(q)}\ln\left(\frac{M_{t_1}^{(q)2}}{M_{t_2}^{(q)2}}\right)\Bigg], \end{aligned} \tag{42}$$

where $m_k^{(l)}$, $m_s^{(q)}$, $M_{p_1,p_2}^{(l)}$, and $M_{t_1,t_2}^{(q)}$ stand for the lepton, quark, slepton, and squark masses, respectively. Further, it is assumed that the sleptons and squarks are much heavier than the leptons and quarks, and $\phi_k^{(l)}$ and $\phi_k^{(q)}$ are the mixing angles for the mixing between the $L$-type and $R$-type charged sleptons and squarks in each generation, respectively (the flavor mixing has been neglected). It is clear from the one-loop contribution (42) that as long as the couplings $\lambda$ and $\lambda'$ are free parameters, one can obtain in principle any kind of the neutrino mixing matrix $V^{\mathrm{MNS}}$. As for our models presented in Sec. III, there exist indeed certain constraints on

[8]We assume that all the elements of $m_\nu$ are real, and $V_\nu^T m_\nu V_\nu=\text{diagonal}$.

[9]If we take other $R$-charge assignment for the lepton sector, this feature cannot be realized.

them like Eq. (28), but they are not strong enough to predict a model specific structure of the neutrino mass matrix.

### B. The lightest Higgs boson

Since there exist two pairs of Higgs doublets in our models, there exist four $CP$-even neutral, three $CP$-odd neutral and three pairs of charged Higgs bosons that are mixed with the neutral and charged scalar leptons, respectively. Here we are interested in the neutral sector, because we would like to find out the upper bound of the mass of the lightest Higgs boson. We denote the neutral scalar components of $H_u$ and $\tilde{H}_u$ by $h_{ui}$ with $i=1,2$, and those of the down-type ones $H_d$, $\tilde{H}_d$, and $L_i (i=1,2,3)$ by $h_{d\alpha}$ with $\alpha=1,\ldots,5$, respectively. Then the most general renormalizable scalar potential including the SSB terms can be written as

$$V_N=(m_u^2)_{ij}h_{ui}^* h_{uj}+(m_d^2)_{\alpha\beta}h_{d\alpha}^* h_{d\beta}-(B_{i\alpha}h_{ui}h_{d\alpha}+\text{H.c.})+\frac{1}{8}(g^2+g'^2)(h_{ui}^* h_{ui}-h_{d\alpha}^* h_{d\alpha})^2. \tag{43}$$

Since physics is independent of the choice of a basis of the fields, we go to a basis, in which only $h_{u1}$ and $h_{d1}$ have a nonvanishing VEV. Accordingly we define

$$h_{u1}=\frac{1}{\sqrt{2}}(v_u+\varphi_1+i\eta_1), \quad h_{d1}=\frac{1}{\sqrt{2}}(v_d+\varphi_2+i\eta_2),$$
$$h_{u2}=\frac{1}{\sqrt{2}}(\varphi_3+i\eta_3), \quad h_{di}=\frac{1}{\sqrt{2}}(\varphi_{i+2}+i\eta_{i+2}), \quad i=2,\ldots,5 \tag{44}$$

where $\varphi$'s and $\eta$'s are real scalar and pseudoscalar components of the Higgs fields, respectively. In this basis, the mass matrices $M_E^2$ and $M_O^2$ for the $CP$-even and $CP$-odd scalars, respectively, take the form

$$\mathbf{M}_{E,O}^2=\begin{pmatrix}\mathbf{M}_{E,O}^{SM} & \mathbf{B}_{E,O}\\ \mathbf{B}_{E,O}^T & \mathbf{m}_{E,O}\end{pmatrix}, \tag{45}$$

where

$$\mathbf{M}_E^{SM}=\begin{pmatrix}(v_d/v_u)B_{11}+\frac{1}{4}(g^2+g'^2)v_u^2 & -B_{11}-\frac{1}{4}(g^2+g'^2)v_u v_d\\ -B_{11}-\frac{1}{4}(g^2+g'^2)v_u v_d & (v_u/v_d)B_{11}+\frac{1}{4}(g^2+g'^2)v_d^2\end{pmatrix}, \tag{46}$$

$$\mathbf{M}_O^{SM}=\begin{pmatrix}(v_d/v_u)B_{11} & B_{11}\\ B_{11} & (v_u/v_d)B_{11}\end{pmatrix}, \tag{47}$$

$$\mathbf{B}_{E(O)}=\begin{pmatrix}(v_d/v_u)B_{12} & -(+)B_{1j}\\ -(+)B_{12} & (v_u/v_d)B_{1j}\end{pmatrix}, \quad j=2,\ldots,5 \tag{48}$$

$$(\mathbf{m}_E)_{33}=(\mathbf{m}_O)_{33}=(m_u^2)_{22}+\frac{1}{8}(g^2+g'^2)(v_u^2-v_d^2), \tag{49}$$

$$(\mathbf{m}_{E(O)})_{3j+2}=(\mathbf{m}_{E(O)})_{j+23}=-(+)B_{2j}, \quad j=2,\ldots,5$$
$$(\mathbf{m}_E)_{i+2j+2}=(\mathbf{m}_O)_{i+2j+2}=(m_d^2)_{ij}+\frac{1}{8}(g^2+g'^2)(v_d^2-v_u^2)\delta_{ij}, \quad i,j=2,\ldots,5. \tag{50}$$

To derive the above formulas we have used minimum conditions of the scalar potential and also assumed that all the parameters appearing in the scalar potential (43) are real.[10] Note that the upper $2\times2$ matrices $\mathbf{M}_E^2$ and $\mathbf{M}_O^2$ have exactly the same form as those of the MSSM. We see from Eqs. (45)–(50), as in the case of the MSSM, that $\mathrm{Tr}\,\mathbf{M}_E^2=M_Z^2+\mathrm{Tr}\,\mathbf{M}_O^2$ is satisfied, which yields the sum rule at the tree level

$$m_h^2+\sum_{i=1}^{3} m_{Hi}^2+\sum_{i=1}^{3} m_{\tilde{\nu}_+ i}^2=M_Z^2+\sum_{i=1}^{3} m_{Ai}^2+\sum_{i=1}^{3} m_{\tilde{\nu}_- i}, \tag{51}$$

where $m_h$, $m_H$, and $m_{\tilde{\nu}_+}$ stand for the masses of the $CP$-even scalars, and $m_A$ and $m_{\tilde{\nu}_-}$ for the $CP$-odd scalars.

Now we come to discuss the lightest Higgs boson mass $m_h$. To this end, we concentrate on the size of the diagonal

[10]The mass parameters above are not those defined in the original scalar potential (43). They correspond to those in the new basis in which only $h_{u1}$ and $h_{d1}$ acquire a nonvanishing VEV.

elements of $\mathbf{M}_E^2$ and $\mathbf{M}_O^2$, because their smallest eigenvalues cannot be larger than the smallest diagonal elements. The scalar mass squared $m_u^2$ and $m_d^2$ in the scalar potential (43) consist of both the SSB scalar mass squared and the contribution from the superpotential (25). Here we remind ourselves that all the parameters belonging to the mass as well as interaction terms that involve at least one of $\tilde{H}_u$ or $\tilde{H}_d$ are very much suppressed at the escape energy. In particular, the SSB scalar mass squared for $\tilde{H}_u$ or $\tilde{H}_d$ [which we denote by $(\tilde{m}_u^2)_{22}$ and $(\tilde{m}_d^2)_{22}$] vanishes at the superconformal fixed point [21,22,18], if the weakly coupled low-energy sector is switched off. It has been, however, found in Ref. [19] that the low-energy sector has a nontrivial influence on their evolution such that they rather approach, translated into the present case, as

$$(\tilde{m}_u^2)_{22} \simeq (\tilde{m}_d^2)_{22} \to (\gamma^*)^{-1} \frac{3g^2}{8\pi^2} |M_2|^2, \tag{52}$$

where $\gamma^*$ is the anomalous dimension of $\tilde{H}_u$ (or $\tilde{H}_d$) at the fixed point [see Eq. (5)], $M_2$ is the $\mathrm{SU}(2)_L$ gaugino mass, and we have neglected the $\mathrm{U}(1)_Y$ contribution. Below the escape energy $\Lambda_C$, their evolution is dictated by the low-energy sector, and all the couplings that contribute to the evolution, except for the gauge couplings of this sector, are suppressed because of the superconformal force. From this consideration we obtain approximately $(\tilde{m}_u^2)_{22}$ and $(\tilde{m}_d^2)_{22}$ at $M_Z$:

$$(\tilde{m}_u^2)_{22} \simeq (\tilde{m}_d^2)_{22} \simeq \frac{3g^2}{8\pi^2} |M_2|^2 \left[ (\gamma^*)^{-1} + \ln \frac{\Lambda_C}{M_Z} \right], \tag{53}$$

where the quantity in the brackets is a positive number of $\gtrsim O(1)$. Consequently, the total contributions to the diagonal elements in question can be written as

$$(m_u^2)_{22} = (\vec{\mu})^2 + (\tilde{m}_u^2)_{22} + \frac{1}{2} M_Z^2 \cos 2\beta,$$

$$(m_d^2)_{22} = \tilde{\mu}^2 + (\tilde{m}_d^2)_{22} + \frac{1}{2} M_Z^2 \cos 2\beta,$$

where $\tan\beta = v_u / v_d$ is defined in the basis in which all the VEVs except for $H_{u1}$ and $H_{d1}$ vanish [see Eq. (44), and $\vec{\mu}$ is given in Eq. (35)]. It is then obvious that we can make $(m_u^2)_{22}$ and $(m_d^2)_{22}$ arbitrarily large by making the gaugino mass $M_2$ large. Therefore, the smallest eigenvalue of $\mathbf{M}_E^2$ sits in $\mathbf{M}_E^{SM}$, implying that we have the same upper bound of the lightest Higgs boson as in the case of the MSSM,

$$m_h^2 \leq M_Z^2 \cos^2 2\beta, \tag{54}$$

because the matrix $\mathbf{M}_E^{SM}$ [given in Eq. (46)] has exactly the same form as in the MSSM. The tree-level bound (54) should be of course corrected in higher orders in perturbation theory [39,40]. We expect that the correction will be very similar to the case of the MSSM, especially if the other masses are large.

## V. CONCLUSION

In supersymmetric standard models with $R$-parity and lepton number violations, the left-handed lepton and down-type Higgs supermultiplets should be treated on the same footing, unless there exist further quantum numbers that distinguish them from each other. Therefore, the $\mu$ problem in these models is closely related to the question of why the neutrinos are so light. In this paper we have proposed to solve the $\mu$ problem in this class of models by coupling the models to a superconformal gauge force. We found that for this idea to work we have to extend the MSSM so as to contain at least another pair of Higgs doublets, which mediate the superconformal suppression to the MSSM sector. We have shown that a suppression of $\lesssim O(10^{-13})$ for the $\mu$ parameter and neutrino masses can be achieved generically.

We have constrained the form of the superpotential of the low-energy sector by imposing an anomaly-free discrete $R$ symmetry, while we have allowed most general, renormalizable supersymmetry-breaking terms. We have found that the discrete $R$ symmetry automatically suppresses the lepton-flavor violating processes such as $\mu \to e\gamma$, $\mu \to eee$, $\mu$-$e$ conversion in nuclei, the electron electric dipole moment (EDM), and also the neutrinoless double $\beta$ decay. The resulting models can accommodate a large mixing among neutrinos, and it has turned out that the upper bound of the lightest Higgs boson mass of the MSSM remains unchanged in these extended models. Finally we expect that the escape energy of the superconformal sector is $\lesssim O(10)$ TeV so that this sector could be experimentally observed in near feature.

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