Superconductivity as Symmetry Breaking. Classical Treatment

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The superconductivity is discussed from the viewpoint of the symmetry breaking in the framework of the classical field theory. The classical Schrödinger field of an electron is assumed to be a Grassmann algebra. The interaction with a radiation field is introduced through the gauge field theory. In the Nambu representation, the Lagrangian of the system is invariant under the rotation about the z axis in the spin space, so that the Goldstone theorem suggests that the condensation occurs on the x–y plane. In the absence of any external field, the system is invariant under the phase transformation. This symmetry breaks due to the condensation, and then the massless Goldstone boson which is the supercurrent will follow. The presence of the photon field, however the photon field itself becomes massive due to the gauge symmetry breaking (Higgs phenomenon), which is the Messner effect.

Since the BCS microscopic theory was established in 1957, this field has progressed well. But it seems that we are still at a distance from the deep understanding of superconductivity, when we think of the recent observations of high temperature superconductors of which the mechanism is not yet established. Before investigating such a specialized subject, it will be summarized the theory of superconductivity from the viewpoints of the recent advance of theoretical physics, especially from the gauge field theory.

The BCS theory begins with Cooper's idea that, if the interaction between electrons is attractive, however small it is, the many electron system will move to a new ground state of which the energy is lower than that of the former. Here we have two aspects. The first problem is concerned with the attractive interaction between electrons. In the ordinary BCS superconductor, this attractive interaction is motivated by the electron–phonon interaction, so that the critical temperature becomes proportional to this phonon frequency. The critical temperature is hard to be above 30 or 40 degrees. The high temperature superconductors of copper oxide are apparently not this case, so that theorists are seeking for a new mechanism of the attractive electron–electron interaction. This viewpoint is important when we design new superconducting materials.

The other, more important aspect lies in the condensation mechanism. The attractive force between electrons yields pairs; the so-called Cooper pair is no longer a fermion, but can rather be considered as a boson. We may say the new ground state previously mentioned is due to the bose condensation. The neat way to take into account this pair formation was invented by Nambu.[2,3] In his representation, the Lagrangian describing electron systems is invariant under the rotation around the σ^3 axis, in the fictitious spin space. Then the Neother theorem ensures the current conservation, especially the charge conservation. Thus the Goldstone theorem suggests the condensation of electrons in the σ^1—σ^2 space. This advanced view really originated from Nambu's work of superconducting. In the present work these will be discussed in the framework of the classical field theory. If the theory is presented quantum mechanically from the beginning, we fear that the story will be hindered by extra and unwanted complexities. Since the superconductivity is a macroscopic phenomenon, even if it be quantum mechanical in its origin, the essential part of it can still be presented in classical terms.

The present treatment proceeds as follows: The preparative discussions up to introducing the photon field by means of the gauge field theory are passed to the appendix. The classic electron field is manipulated in terms of the Grassmann algebra. Since the superconductivity is essentially the symmetry breaking in the gauge invariance, it is crucial to build up the gauge-invariant Lagrangian including the photon field. The text begins by writing this gauge-invariant Lagrangian. The Nambu representation, of which the original form is two-dimensional is extended to four-dimensional. This does not mean a relativistic treatment, but it makes manipulations considerably easier. The superconducting mechanism in which the condensation of the Cooper pairs arises and then is followed by the supercurrent (the Goldstone boson) is clearly shown in this representation. In addition to supercurrent, the Meissner effect is interpreted as a Higgs phenomenon, or the symmetry breaking of the electron field which makes photon massive; in other words, the magnetic field obtains the exponentally decaying term.

General

We start from the gauge-invariant total Lagrangian for the many electron system:

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$$L_{\text{tot}} = \int \mathrm{d}^3\boldsymbol{r}\cdot\frac{i}{2}(u^*\partial_0 u - \partial_0 u^*\cdot u) - \frac{1}{2m}(\partial_i u^* + \mathrm{i}eA_i u^*)(\partial_i u - \mathrm{i}eA_i u)$$
$$-\frac{1}{2}\int \mathrm{d}^3\boldsymbol{r}\int \mathrm{d}^3\boldsymbol{r}'\frac{1}{4\pi\,|\boldsymbol{r}-\boldsymbol{r}'|}u^*(\boldsymbol{r},x_0)u(\boldsymbol{r},x_0)u^*(\boldsymbol{r}',x_0)u(\boldsymbol{r}',x_0)$$
$$+\int \mathrm{d}^3\boldsymbol{r}\frac{1}{2}[(E^{\text{tr}})^2 - \mathrm{B}^2]. \tag{1}$$

This is derived in the appendix. The notations are standard. The terms, u and u^* are the classical electron field which are the Grassmann variables (anti-commuting c-numbers) and obey the Schrödinger equation. The photon field is introduced by the use of the gauge field theory. The Coulomb gauge is employed, so that the electro-magnetic fields are transverse, and the longitudinal part of the electric field gives the instantaneous electron–electron interaction.

In order to discuss the dynamical equation of the electron field u, we need the Hamiltonian of this system.

Hamiltonian. The canonical momentum, π to u, is defined by Eq. 1a in the appendix:

$$\pi = \frac{\mathscr{L}\overleftarrow{\partial}}{\partial(\partial_0 u)} = iu^*. \tag{2}$$

Then the Hamiltonian density is

$$\mathscr{H} = \pi\partial_0 u - \mathscr{L}$$
$$= \frac{1}{2m}(\partial_i u_\sigma^* + \mathrm{i}eA_i u_\sigma^*)(\partial_i u_\sigma - \mathrm{i}eA_i u_\sigma)\}$$
$$+\frac{1}{2}g(u_\sigma^* u_\sigma)(u_{\sigma'}^* u_{\sigma'}) - \frac{1}{2}[(\boldsymbol{E}^{\text{tr}})^2 - \boldsymbol{B}^2], \tag{3}$$

where the coupling constant of the electron–electron interaction is assumed constant g (a delta-function type), since in the system under consideration a strong shielding effect is expected. However the sign of g is not yet fixed. Then the nilpotent character of the Grassmann functions requires a new degree of freedom; then the spin coordinates are reasonably introduced as a new degree of freedom, which are designated by the lower indices, σ or σ' in the above equation. The spin indices will be later given by 1 and 2, which refer to up and down respectively.

In the Absence of External Field. The Hamiltonian in the absence of external field is thus written as

$$\boldsymbol{H} = \int \mathrm{d}^3\boldsymbol{r}[-\frac{1}{2m}u_\sigma^*\partial_i^2 u_\sigma + \frac{1}{2}g(u_\sigma^* u_\sigma)(u_{\sigma'}^* u_{\sigma'})], \tag{4}$$

where, to get the kinetic terms, the integration by parts was carried out.

The equation of motion of the field is given by the use of the Poisson bracket which will be called the classic commutator, indicated by the suffix C. In our case, for the dynamical variables, A and B described by the Grassmann variables, the Poisson bracket is defined as

$$[A,B]_C = \frac{\partial A}{\partial u}\frac{\partial B}{\partial \pi} - \frac{\partial A}{\partial \pi}\frac{\partial B}{\partial u}. \tag{5}$$

Note: supposing the Lagrangian or Hamiltonian be written in the normal order, the differentiations with respect to u^* and u are the left-handed and the right-handed differentiations, respectively. Then the equation of motion of u_σ is obtained as

$$i\partial_0 u = [u_\sigma, H]_C$$
$$= \frac{\partial u_\sigma}{\partial u_\sigma}\frac{\overrightarrow{\partial} H}{\partial u_\sigma^*} - \frac{\partial u_\sigma}{\partial u_\sigma^*}\frac{H\overleftarrow{\partial}}{\partial u_\sigma}$$
$$= \frac{\overrightarrow{\partial} H}{\partial u_\sigma^*}$$
$$= -\frac{1}{2m}\partial_i^2 u + g u_\sigma u_{\sigma'}^* u_{\sigma'}. \tag{6}$$

Rewriting, we have

$$i\partial_0 u_\sigma + \frac{1}{2m}\partial_i^2 u_\sigma = g u_\sigma u_{\sigma'}^* u_{\sigma'}. \tag{7}$$

Similarly

$$-i\partial_0 u_{\sigma'}^* + \frac{\hbar^2}{2m}\partial_i^2 u_{\sigma'}^* = g u_{\sigma'}^* u_\sigma^* u_\sigma. \tag{8}$$

The same results can be obtained from the Euler–Lagrange equation by the use of the Lagrangian density, the integrand in Eq. 1 in the absence of the radiation field.

The above Lagrangian and Hamiltonian are obviously invariant under the phase transformation (the global gauge transformation) with a constant α,

$$u \to u\mathrm{e}^{-i\alpha} \qquad u^* \to u^*\mathrm{e}^{i\alpha}, \tag{9}$$

Then the Neother theorem ensures the current conservation as in Eq. A36 and the charge defined by

$$G = \int \mathrm{d}^3\boldsymbol{r}u_\sigma^* u_\sigma, \tag{10}$$

which is apparent from that

$$i\hbar c\partial_0 G = [G,H]_C = 0. \tag{11}$$

Now we try to estimate the following commutators:

$$[u_\sigma, G]_C = \frac{\partial u_\sigma}{\partial u_\sigma}\frac{\partial G}{\partial u_\sigma^*} - \frac{\partial u_\sigma}{\partial u_\sigma^*}\frac{\partial G}{\partial u_\sigma}$$
$$= \frac{\partial}{\partial u_\sigma^*}\int \mathrm{d}^3\boldsymbol{r}u_\sigma^* u_\sigma$$
$$= u_\sigma, \tag{12}$$

and

$$[u_\sigma^*, G]_C = \frac{\partial u_\sigma^*}{\partial u_\sigma}\frac{\partial G}{\partial u_\sigma^*} - \frac{\partial u_\sigma^*}{\partial u_\sigma^*}\frac{\partial G}{\partial u_\sigma}$$
$$= -\int \mathrm{d}^3\boldsymbol{r}u_\sigma^* u_\sigma\frac{\overleftarrow{\partial}}{\partial u_\sigma}$$
$$= -u_\sigma^*. \tag{13}$$

These non-vanishing results imply the condensation of u_σ or u_σ^* and that the elementary excitation of which the energy is zero will arise with $\boldsymbol{k}=0$, called the Goldstone boson.

Nambu Formalism

Nambu presented a nice method by which the mechanism of superconductivity was made clear.[8,9] The original form of the Nambu representation begins with adopting a spinor representation:

$$\phi^* = (u_1^* u_2), \qquad \phi = \begin{pmatrix} u_1 \\ u_2^* \end{pmatrix}. \tag{14}$$

For the later investigations we note Pauli matrices,

$$\sigma^1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \sigma^2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad(15)$$

and related ones,

$$\sigma^+=\frac{1}{2}(\sigma^1+i\sigma^2),\qquad \sigma^-=\frac{1}{2}(\sigma^1-i\sigma^2).\quad(16)$$

However, in the present investigation, we use an extended representation which exhausts all the possible elements; its usefulness will be clear in the later investigation:

$$\Phi^*=(u_1^*u_2u_1u_2^*),\qquad \Phi=\begin{pmatrix}u_1\\u_2^*\\u_1^*\\u_2\end{pmatrix}.\quad(17)$$

The electron–electron interaction is described in the original Nambu representation as

$$\frac{1}{2}gu_\sigma^*u_\sigma u_{\sigma'}^*u_{\sigma'}\approx\frac{1}{2}g(\phi^*\sigma^3\phi)^2\ \ or$$
$$\approx\frac{1}{2}g(\phi^*\sigma^+\phi)(\phi^*\sigma^-\phi).\quad(18)$$

The former corresponds to the interaction between particle-hole pairs useful for the usual Hartree–Fock calculation, but the latter to the one between the Cooper pairs. In the present work the latter is preferable. This is also rewritten in Φ representation as

$$\frac{1}{2}gu_\sigma^*u_\sigma u_{\sigma'}^*u_{\sigma'}\approx\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi),\quad(19)$$

where

$$\Sigma^+=\begin{pmatrix}\sigma^+&0\\0&-\sigma^-\end{pmatrix},$$
$$\Sigma^-=\begin{pmatrix}\sigma^-&0\\0&-\sigma^+\end{pmatrix},$$
$$\Sigma^3=\begin{pmatrix}\sigma^3&0\\0&-\sigma^3\end{pmatrix}.\quad(20)$$

Using the above, we can write the Lagrangian density in Eq. 1 in a compact form:

$$\mathcal{L}_{\rm tot}=\frac{1}{2}\{i(\Phi^*\partial_0\Phi)-\frac{1}{2m}(D_i\Phi^*)\Sigma^3(D_i\Phi)-\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi)\}$$
$$+\frac{1}{2}[(\boldsymbol{E}^{\rm tr})^2-\boldsymbol{B}^2].\quad(21)$$

It is seen that the Lagrangian is perfectly symmetrized in Φ representation. This is one of the useful points of the four-dimensional representation.

Thus the total Lagrangian in the absence of the external field is

$$\mathcal{L}=\frac{1}{2}\{i(\Phi^*\partial_0\Phi)$$
$$-\frac{1}{2m}(\partial_i\Phi^*)\Sigma^3(\partial_i\Phi)-\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi)\}.\quad(22)$$

The Euler–Lagrange equation given as

$$\frac{\partial_\mu\mathcal{L}}{\partial(\partial_\mu\oplus\Phi^*)}-\frac{\partial\mathcal{L}}{\partial\Phi^*}=0$$

yields the equation of motion,

$$i\partial_0\Phi+\frac{1}{2m}\Sigma^3\partial_i^2\Phi-\frac{1}{2}g[\Sigma^+\Phi\cdot(\Phi^*\Sigma^-\Phi)+\Sigma^-\Phi\cdot(\Phi^*\Sigma^+\Phi)]=0.\quad(23)$$

This is the matrix equation, which is explicitly given in the following:

$$i\partial_0\begin{pmatrix}u_1\\u_2^*\\u_1^*\\u_2\end{pmatrix}+\frac{1}{2m}\partial_i^2\begin{pmatrix}u_1\\-u_2^*\\-u_1^*\\u_2\end{pmatrix}-gu_2u_1\begin{pmatrix}u_2^*\\0\\0\\u_1^*\end{pmatrix}-gu_1^*u_2^*\begin{pmatrix}0\\u_1\\u_2\\0\end{pmatrix}=0.\quad(24)$$

These are of course the same as those obtained by the primitive method. For example, the first line of the above can be read: The particle u_1 couples the hole u_2^* through the potential of the particle pair, u_2u_1. This is the fundamental scheme of the single particle dynamics in the superconductor, and suggests the necessity of the spinor representation of Nambu.

The Hamiltonian formalism works as well. Let us define the canonical momentum to Φ as

$$\Pi=\frac{\mathcal{L}\overleftarrow{\partial}}{\partial(\partial_0\Phi)}=\frac{i}{2}\Phi^*.\quad(25)$$

Then we have the Hamiltonian density:

$$\mathcal{H}=\Pi(\partial_0\Phi)-\mathcal{L}$$
$$=\frac{1}{2m}(\partial_i\Phi^*)\Sigma^3(\partial_i\Phi)+\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi).\quad(26)$$

The spatial integral of the above is the Hamiltonian, $\mathcal{H}$. The partial integration of the first term in the second line is made for the next manipulation. The equation of motion of Φ thus obtained as

$$i\partial_0\Phi=[\Phi,H]_C$$
$$=\frac{\partial\Phi}{\partial\Phi}\frac{\overrightarrow{\partial}H}{\partial\Phi^*}-\frac{\partial\Phi}{\partial\Phi^*}\frac{H\overleftarrow{\partial}}{\partial\Phi}=\frac{\overrightarrow{\partial}H}{\partial\Phi^*}$$
$$=-\frac{\hbar}{2m}\Sigma^3\partial_i^2\Phi+\frac{1}{2}g[\Sigma^+\Phi(\Phi^*\Sigma^-\Phi)+\Sigma^-\Phi(\Phi^*\Sigma^+\Phi)].\quad(27)$$

This is identical to that in Eq. 24.

Symmetry Breaking

The transformation in Eq. 9 is now written as

$$\phi\rightarrow{\rm e}^{-i\sigma^3\alpha}\phi\quad\text{or}\quad\Phi\rightarrow{\rm e}^{-i\Sigma^3\alpha}\Phi.\quad(28)$$

These relations are easily verified by expanding the exponential parts and, after matrix multiplication, summing up terms again.

Thus we understand that the Nambu representation is a spinor representation in the fictitious Σ space. The Lagrangian Eq. 22 or Hamiltonian Eq. 26 is seen to be invariant under the transformation Eq. 28, i.e., under the rotation about the Σ^3 axis.

Let us remember the Neother thorem and the Goldstone theorem. The invariant charge in Eq. 10 is now written by the use of Eq. 20 as

$$G=\frac{1}{2}\int d^3\boldsymbol{r}\,\Phi^*\Sigma^3\Phi. \tag{29}$$

It is instructive to estimate commutators[10)]

$$[\Phi^*\Sigma^+\Phi,\Phi^*\Sigma^3\Phi]_C=-2(\Phi^*\Sigma^+\Phi),$$
$$[\Phi^*\Sigma^-\Phi,\Phi^*\Sigma^3\Phi]_C=2(\Phi^*\Sigma^-\Phi), \tag{30}$$

of which the space integrals are the Goldstone commutators. The non-vanishing results imply that the symmetry beaking arises on the Σ^+–Σ^- plane.

In order to take the above effects into account non-perturbatively, the terms

$$\rho(\Phi^*\Sigma^+\Phi)+\eta(\Phi^*\Sigma^-\Phi)$$

are subtracted from and added to the Lagrangian to modify the free Lagrangian. Then the Lagrangian density in Eq. 22 is rewritten as

$$\mathscr{L}=\mathscr{L}^0+\mathscr{L}',$$
$$\mathscr{L}^0=i(\Phi^*\partial_0\Phi)-\frac{1}{2m}(\partial_i\Phi^*)\Sigma^3(\partial_i\Phi)-\{\rho(\Phi^*\Sigma^+\Phi)+\eta(\Phi^*\Sigma^-\Phi)\},$$
$$\mathscr{L}'=-\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi)+\rho(\Phi^*\Sigma^+\Phi)+\eta(\Phi^*\Sigma^+\Phi), \tag{31}$$

where ρ and η called gap energies characteristic in the theory of superconductivity. They are expressed as

$$\rho=v+\frac{1}{2}f\langle\Phi^*\Sigma^-\Phi\rangle,\qquad \eta=v+\frac{1}{2}f\langle\Phi^*\Sigma^+\Phi\rangle. \tag{32}$$

where f (>0) is the electron–electron repulsion, $\langle\cdots\rangle$ represents an average and v is the single particle interaction not written explicitly in the original Lagrangin (the solid-state physicists sometimes include this in the kinetic term). Precisely speaking, it is an attractive interaction between nuclei and electrons. Two terms in the right hand side of Eq. 32 have almost canceling effect with each other, but we may expect that a slightly attractive contribution will remain. If we use Eqs. 17 and 20, it is found that

$$\langle\Phi^*\Sigma^-\Phi\rangle=2\langle u_2u_1\rangle,\qquad \langle\Phi^*\Sigma^+\Phi\rangle=2\langle u_1^*u_2^*\rangle, \tag{33}$$

so that ρ and η are interpreted as the mass terms for particle and hole pairs, respectively. When u_2u_1 and $u_1^*u_2^*$ will condense to amount constant values, the dynamical equations in Eq. 24 are no longer invariant for the phase transformation. Here $\mathscr{L}^0$ is thus regarded as the free part of Lagrangian in which the Cooper pairs behave freely in the mean fields. One may say this is a simple linearization process, and $\mathscr{L}'$ expresses the electron correlation. However such an argument is rather shallow.

In the quantum field theory, these parameters are determined under the condition that the Green's function for the effective Hamiltonian removes the first order self-energy part.

In the present investigation, this linearization is performed by determining the proper values of ρ and η such that the remaining correlation part is optimized in the following way. The problem is: What happen in this case?

The correlation potential is rewritten as

$$V=-\rho\Phi^*\Sigma^+\Phi-\eta\Phi^*\Sigma^-\Phi+\frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi), \tag{34}$$

then optimized with respect to $\Sigma^+\Phi$ and $\Sigma^-\Phi$ respectively,

$$\frac{V\overleftarrow{\partial}}{\partial(\Sigma^+\Phi)}=-\rho\Phi^*+\frac{1}{2}g\Phi^*(\Phi^*\Sigma^-\Phi)=0, \tag{35}$$

$$\frac{V\overleftarrow{\partial}}{\partial(\Sigma^-\Phi)}=-\eta\Phi^*+\frac{1}{2}g\Phi^*(\Phi^*\Sigma^+\Phi)=0, \tag{36}$$

where the Grassmann character of Φ is used. If we want to know precisely what is going on in our system, it is instructive to investigate the procedures separately, i.e., first, the case of ρ only active (say, the hole pairs condense) is considered. It follows from Eq. 35 that

$$(\Phi^*\Sigma^-\Phi)_0=\frac{2\rho}{g} \tag{37}$$

$$=a^2. \tag{38}$$

One may be apt to confuse the relation Eq. 37 as a tautology of Eq. 32. It is really not, but one should understand the gap equation in the ordinary treatment. Let us understand Eq. 37 to be the relation determining the condensation of the hole pairs. The left hand side, $(\Phi^*\Sigma^-\Phi)_0$ is determined from ρ, which is the function of a tentative average, $\langle\Phi^*\Sigma\Phi\rangle$. Namely Eq. 37 is an integral equation. The sign of $(\Phi^*\Sigma^-\Phi)_0$ should be positive, as indicated by Eq. 38, and ρ is evaluated from the dynamical equation to be negative since it is a kind of stabilization energy, so that the relation Eq. 38 determines the sign of g to be negative, as BCS claimed.

Let us write:

$$(\Phi^*\Sigma^-\Phi)=a^2+(\Phi^*\Sigma^-\Phi)', \tag{39}$$

which means that the density of hole pair shifts to finite values of a^2, and $(\Phi^*\Sigma^-\Phi)'$ is now physical, or is an operator (in the quantum theory). Substituting the above expression into V in Eq. 34 with $\eta = 0$ yields

$$\begin{aligned} V &= -\rho(\Phi^*\Sigma^+\Phi) + \frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi\Sigma^-\Phi) \\ &= -\frac{1}{2}ga^2(\Phi^*\Sigma^+\Phi) + \frac{1}{2}g(\Phi^*\Sigma^+\Phi)[a^2+(\Phi^*\Sigma^-\Phi)'] \\ &= \frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi^*\Sigma^-\Phi)', \end{aligned} \quad (40)$$

where Eq. 38 was used. Thus the bilinear term, $(\Phi^*\Sigma^+\Phi)$ has been completely lost, i.e., the particle-pair $(\Phi^*\Sigma^+\Phi)$ becomes massless (supercurrent). This is an interesting phenomenon: if one field condenses, other fields perpendicular to it become massless.[4)]

In the case of η only active, the procedure proceeds similarly. From Eq. 36 we obtain

$$(\Phi^*\Sigma^+\Phi)_0 = \frac{2\eta}{g} \quad (41)$$

$$= b^2. \quad (42)$$

Now pairs in the particle state condense. If we put

$$(\Phi^*\Sigma^+\Phi) = b^2 + (\Phi^*\Sigma^+\Phi)', \quad (43)$$

the pair $(\Phi^*\Sigma^+\Phi)'$ is physical. Substituting this into V yields

$$\begin{aligned} V &= -\eta(\Phi^*\Sigma^-\Phi) + \frac{1}{2}g(\Phi^*\Sigma^+\Phi)(\Phi\Sigma^-\Phi) \\ &= -\frac{1}{2}ga^2(\Phi^*\Sigma^-\Phi) + \frac{1}{2}g[b^2+(\Phi^*\Sigma^+\Phi)'](\Phi^*\Sigma^-\Phi) \\ &= \frac{1}{2}g(\Phi^*\Sigma^+\Phi)'(\Phi^*\Sigma^-\Phi). \end{aligned} \quad (44)$$

Thus the hole-pair $(\Phi^*\Sigma^-\Phi)$ has been lost, which becomes massless.

If ρ and η are cooperatively active, substituting Eqs. 38 and 42 in V of Eq. 34, we get immediately

$$V = -\frac{1}{2}ga^2b^2 + \frac{1}{2}g(\Phi^*\Sigma^+\Phi)'(\Phi^*\Sigma^+\Phi)'. \quad (45)$$

Namely the both pairs become massless, namely all of the electrons bear supercurrent.

In the Presence of an External Field. The situation considerably changes and we turn to the Higgs phenomena.[4,6)] We must deal with the gauge invariant Lagrangian in Eqs. 21 or 1, and the preceding phase transformation is now interpreted as a gauge transformation. We turn our attention to the term involving A_k^2 in the above equations, and prefer the original expression in Eq. 1 to that in Eq. 21:

$$\begin{aligned} &\frac{1}{2m}(\partial_i u^* + \mathrm{i}eA_i u^*)(\partial_i u + \mathrm{i}eA_i u) \\ &\quad = \frac{1}{2m}\{-u^*\partial_i^2 u + \mathrm{i}eA_i u^*\partial_i u - \mathrm{i}e\partial_i u^* \cdot A_i u\} + \frac{e^2}{2m}u^* u A_i^2. \end{aligned} \quad (46)$$

Here the spin indices are omitted for simplicity and the total derivative is neglected by imaging the surface integral. First we are interested in u^* and u in the last term. These might condense to amount, by combining Eqs. 33, 38, and 42, approximately

$$(u^*)_0 = \frac{b}{\sqrt{2}}, \qquad (u)_0 = \frac{a}{\sqrt{2}}. \quad (47)$$

Now

$$u'^* = u^* - (u^*)_0 \quad \text{and} \quad u' = u - (u)_0 \quad (48)$$

are physical. We thus observe that A_i^2 gains a constant coefficient, $\frac{e^2}{2m}(u^*)_0(u)_0 = \frac{e^2}{4m}ab$, in other words, the photon field becomes massive.

This treatment, which seems reasonable to explain the Messner effect, in turn requires that u^* and u themselves would be massive. Up to now we have limited our consideration to the pair, i.e., the pair condensation or the supercurrent carried by the pair. In the present case dealing with the Meissner effect we must investigate the individual particles. The situation is now a little more complicated. From the mixed terms, for example, $A_i u^* \partial_i u$, we have $\frac{b}{\sqrt{2}}A_i\partial_i u$ which would indicate the propagating field A_i could turn to u, or the massless gauge field A_i eats the massive u to make itself massive. This is qualitatively discussed as follows: Utilizing the gauge-invariant character of the Lagrangian in Eq. 1, at any space-time, we rotate continuously the whole system in the fictitious spin space to a position such that only a special combination, ca. $u+u^*$ is non-vanishing, this being called $u_\parallel$, and another combination, $u_\perp$ vanishes. Then, among the terms in Eq. 46, the terms relating to the former remain and the cross terms vanish. Imaging the term with mass not written in Eq. 46, we can conclude that the photon field gets mass, while on the other hand one of the Goldstone bosons has disappeared.

Finally we want a rough estimation of the Meissner effect. This phenomenon can be understood from the viewpoint of a screening effect. The spatial part of Eq. 24 in the appendix is written as

$$\partial_j F_{ij} = \mathscr{J}_i \quad \text{or} \quad \nabla \times \boldsymbol{B} = \boldsymbol{J}. \quad (49)$$

Assuming that almost all the electrons are condensed, or that u and u^* are rigid, we can neglect the spatial derivatives of them:

$$\nabla \times \boldsymbol{B} = -\frac{e^2}{m^2}\boldsymbol{A}n_s, \quad (50)$$

where $n_s = ab/2$ in the present treatment. Operating $\nabla\times$ on both sides, gives (note $\nabla \cdot \boldsymbol{A} = 0$)

$$\nabla^2 \boldsymbol{B} = \frac{e^2}{mc^2}n_s \boldsymbol{B}. \quad (51)$$

For simplicity, the one-dimensional variation in the half-plane being considered $x \geq 0$, the solution of the form

$$\boldsymbol{B} = \boldsymbol{B}_0 e^{-x/\lambda}, \qquad \lambda = \left[\frac{e^2}{n_s}\right]^{-1/2} \quad (52)$$

is obtained. Assuming $n_s \approx 10^{24}/\mathrm{cm}^3$ we have a penetration depth, $\lambda \approx 10^{-6}$ cm. This is the Meissner effect.

Conclusion

In the standard textbooks of many-body problems, we find the statement that the superconductivity is a typical example of the gauge-symmetry breaking, but it is hard to find a satisfactory explanation for this. The discussions of superconductivity from this viewpoint have been done quantum mechanically, for example, by Leplae at al.[11)] and Cremer.[12)] However we believe the essential part is very simple, as stated in the present work. The concept of symmetry breaking is closely related to the recently advanced gauge field theory, so that the superconductivity can be re-viewed as an interesting example in this region. We have pointed out that the appearance of a supercurrent is not the Bose condensation itself, but a cooperative phenomenon which follows from the Bose condensation. The Meissner effect is directly related to the Bose condensation. As has been mentioned in the introduction, the superconductivity is a macroscopic quantum phenomenon. Even if we treat this classically, the so-called quantum mechanical effect is almost taken into account through the Grassmann character of the classical electron field. However the pure quantum mechanical effect arised from the commutation relation is not yet involved.

According to the author's opinion, the fundamental structure of the theory of supercinductivity has been already well established, and need not to be altered even after the recent discovery of the high-temperature superconductor.[13] The critical temperature seems to mainly depend on the chemical structure of the material in question.

The author remembers the delightful seminar on the many-body problems with excellent professors: S. Hori, E. Yamada, and M. Kimura. M. Kimura once suggested the four-dimensional Nambu representation from which the present idea started, then E. Yamada gave a effective comment about the phase of one of the elements.

Appendix

Here we prepare the preliminary treatments for the text.

Lagrangian. The Lagrangian density (sometimes we call it Lagrangian without confusion) of the Schrödiger field for electron is given as

$$\mathscr{L}= i(u^*\partial_0 u) - \frac{1}{2m}(\partial_i u^*)(\partial_i u), \tag{A1}$$

$$\mathscr{L}= -i(\partial_0 u^*\cdot u) - \frac{1}{2m}(\partial_i u^*)(\partial_i u), \tag{A2}$$

$$\mathscr{L}= \frac{1}{2}i\{u^*\partial_0 u - \partial_0 u^*\cdot u) - \frac{1}{2m}(\partial_i u^*)(\partial_i u), \tag{A3}$$

where the units $\hbar=c=1$ are used, and $\partial_0=\partial/\partial t$. These are different from one another by the total derivative, $i\partial_0(u^*u)$, which does not affect the resulting equation of motion. We can use any one for making mathematics easier. Throughout the present work, the summation convention for repeated indices is used. In Eqs. A1, A2, and A3 we assume u and u^* are Grassmann odds,[1,2] since the classical version of the anticommuting fermion operator is a Grassmann variable,[3] i.e., the anti-commuting c-number. Hereafter the greek suffix $\mu=0,1,2,3$ and the latin suffix $i=1,2,3$.

From the Euler–Lagrange equation,

$$\partial_\mu\left(\frac{\partial\mathscr{L}}{\partial(\partial_\mu u^*)}\right) - \frac{\partial\mathscr{L}}{\partial u^*}=0, \tag{A4}$$

we can obtain the Schrödinger equation:

$$i\partial_0 u + \frac{1}{2m}\partial_i^2 u = 0. \tag{A5}$$

In order to avoid confusion arisen from the anti-commuting property of Grassmann algebra, we set the manipulation rules as follows:

First, let us put fields in the normal order, i.e., the starred fields are left followed by the unstarred ones. Next, unless otherwise stated, the functional differentiation with respect to the starred field is the left differentiation, while the differentiation by the unstarred field is the right differentiation. Notice that differentiation through a field requires an additional minus sign for each step. The differentiation with respect to u leads

$$-i\partial_0 u^* + \frac{1}{2m}\partial_i^2 u^* = 0. \tag{A6}$$

Gauge Transformation.[4,6] Let us try the gauge transformation,

$$u \to u\mathrm{e}^{-i\Lambda}, \qquad u^* \to u^*\mathrm{e}^{i\Lambda}, \tag{A7}$$

where Λ is a function of the space-time x_μ. For infinitesimal transformations, the variations are

$$\delta u = -i\Lambda\, u, \qquad \delta u^* = i\Lambda\, u^*, \tag{A8}$$

and

$$\delta(\partial_\mu u) = -i\Lambda\,(\partial_\mu u) - i\Lambda\,(\partial_\mu\Lambda)u, \tag{A9}$$

$$\delta(\partial_\mu u^*) = i\Lambda\,(\partial_\mu u^*) + i\Lambda\,(\partial_\mu\Lambda)u^*. \tag{A10}$$

Then varying the Lagrangian,

$$\begin{aligned}\delta\mathscr{L}&=\frac{\partial\mathscr{L}}{\partial u}\delta u+\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}\delta(\partial u)\\ &+\delta u^*\frac{\partial\mathscr{L}}{\partial u^*}+\delta(\partial u^*)\frac{\partial\mathscr{L}}{\partial(\partial_\mu u^*)}\end{aligned} \tag{A11}$$

and using the Euler–Lagrange Eq. A4 for the first and third terms, we have

$$\begin{aligned}\delta\mathscr{L}&=\partial_\mu\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}(-i\Lambda\, u)+\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}\\ &[-i\Lambda\,(\partial_\mu u)-i(\partial_\mu\Lambda)u]+(u\to u^*)\\ &=-i\Lambda\,\partial_\mu\left(\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}u\right)+i\Lambda\,\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}\partial_\mu u\\ &-i\Lambda\,\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}\partial_\mu u-i(\partial_\mu\Lambda)\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}u+(u\to u^*),\end{aligned} \tag{A12}$$

where the first term that is the total differential will vanish through the surface integration, and the second and third terms cancel with each other, with the last term remaining. Thus we obtain

$$\delta\mathscr{L}=-i(\partial_\mu\Lambda)\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)}u+i(\partial_\mu\Lambda)u^*\frac{\partial\mathscr{L}}{\partial(\partial_\mu u^*)}. \tag{A13}$$

Then using the explicit form of the Lagrangian for the Schrödinger field in Eqs. A1, A2, and A3 yields

$$\begin{aligned}\delta\mathscr{L}&=(\partial_0\Lambda)u^*u+(\partial_i\Lambda)\frac{1}{2m}[(i\partial_i u^*)u-u^*(i\partial_i u)]\\ &=(\partial_\mu\Lambda)J_\mu,\end{aligned} \tag{A14}$$

where

$$J_0 = u^*u, \tag{A15}$$

$$J_i = \frac{i}{2m}[(\partial_i u^*)u - u^*(\partial_i u)]. \tag{A16}$$

And then J_μ will satisfy the continuity equation,

$$\partial_\mu J_\mu = \partial_0 J_0 - \partial_i J_i = 0 \tag{A17}$$

It is stressed that the result Eq. A14 suggests that the Lagrangian Eqs. A1, A2, and A3 is not invariant under the gauge transformation.

In order to recover the invariance character of Lagrangian, we introduce a field A_μ which is a commuting c-number function of the space-time, and write the interaction Lagrangian as

$$\begin{aligned}\mathscr{L}_1 &= -\mathrm{e}J_\mu A_\mu \\ &= -\mathrm{e}u^* uA_0 + i\frac{\mathrm{e}}{2m}[(\partial_i u^*)u - u^*\partial_i u]A_i, \qquad \text{(A18)}\end{aligned}$$

where the coupling constant e is introduced only for dimensional adjustment and therefore at this step it need not be the electron charge.

As usual we carry out the gauge transformation for A_μ as,

$$A_\mu \to A_\mu + \frac{1}{\mathrm{e}}\partial_\mu \Lambda, \qquad \delta A_\mu = \frac{1}{\mathrm{e}}\partial_\mu\Lambda, \qquad \text{(A19)}$$

followed by

$$\delta\mathscr{L}_1 = -\mathrm{e}(\delta J_\mu)A_\mu - J_\mu\partial_\mu\Lambda. \qquad \text{(A20)}$$

The second term cancels the previous $\delta\mathscr{L}$ in Eq. A14, but we have the first term still remain, which should be removed. To this end δJ_μ is examined:

$$\begin{aligned}\delta J_0 &= \delta(u^* u) = (\delta u^*)u + u^*\delta u \\ &= i\Lambda u^* u - i\Lambda u^* u = 0, \qquad \text{(A21)}\end{aligned}$$

and

$$\begin{aligned}\delta J_i &= \frac{i}{2m}\delta[(\partial_i u^*)u - u^*\partial_i u] \\ &= \frac{1}{m}(\partial_i\Lambda)u^* u, \qquad \text{(A22)}\end{aligned}$$

where Eq. A8 was used. Thus

$$\delta\mathscr{L} + \delta\mathscr{L}_1 = -\frac{1}{m}\mathrm{e}A_i(\partial_i\Lambda)u^* u, \qquad \text{(A23)}$$

which will be canceled by introducing the additional Lagrangian, $\mathscr{L}_2$,

$$\mathscr{L}_2 = -\frac{1}{2m}\mathrm{e}^2 A_i^2 u^* u, \qquad \text{(A24)}$$

The manipulation

$$\delta\mathscr{L}_2 = -\frac{\mathrm{e}^2}{m}A_i\delta A_i u^* u = \frac{\mathrm{e}^2}{m}A_i(\partial_i\Lambda)u^* u, \qquad \text{(A25)}$$

recovers the invariant character of the Lagrangian:

$$\delta\mathscr{L} + \delta\mathscr{L}_1 + \delta\mathscr{L}_2 = 0. \qquad \text{(A26)}$$

Referring to the field A_μ we prepare the gauge-invariant Lagrangian,

$$\mathscr{L}_3 = -\frac{1}{4}F_{\mu\nu}^2, \qquad \text{(A27)}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \qquad \text{(A28)}$$

Thus we obtain the total Lagrangian,

$$\begin{aligned}\mathscr{L}_{\mathrm{tot}} &= \mathscr{L} + \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 \\ &= \frac{i}{2}(u^*\partial_0 u - \partial_0 u^*\cdot u) + \mathrm{e}u^* uA_0 - \frac{1}{2m}(\partial_i u^*)(\partial_i u) + A_i J_k - \frac{1}{2m}\mathrm{e}^2 A_i^2 u^* u \\ &\quad -\frac{1}{4}F_{\mu\nu}^2 \\ &= \frac{i}{2}(u^*\partial_0 u - \partial_0 u^*\cdot u) + \mathrm{e}u^* uA_0 - \frac{1}{2m}(\partial_i u^* + i\mathrm{e}A_i u^*)(\partial_i u - i\mathrm{e}A_i u) \\ &\quad -\frac{1}{4}F_{\mu\nu}^2. \qquad \text{(A29)}\end{aligned}$$

The term $\mathrm{e}u^* uA_0$ in the above is a source of the instantaneous electron–electron interaction (ca. $1/r$), under the Coulomb gauge.

The term,

$$D_i = \partial_i - i\mathrm{e}A_i \qquad \text{(A30)}$$

is called the covariant differentiation, since $\delta(D_i)_u$ behaves as u does. It is seen that, by the use of Eqs. A9, A10, and A19,

$$\begin{aligned}\delta(D_i)u &= \delta(\partial_i - i\mathrm{e}A_i)u \\ &= \delta(\partial_i u) - i\mathrm{e}(\delta A_i)u - i\mathrm{e}A_i(\delta u) \\ &= -i\Lambda(\partial_i - i\mathrm{e}A_i)u \\ &= -i\Lambda(D_i u). \qquad \text{(A31)}\end{aligned}$$

In the course of the present procedure, the inhomogeneous Maxwell equations are obtained by varying $\mathscr{L}_{\mathrm{tot}}$ with A_μ. Namely, the Euler–Lagrange equation

$$\frac{\partial\mathscr{L}}{\partial A_\mu} - \partial_\nu\left[\frac{\partial\mathscr{L}}{\partial(\partial_\nu A_\mu)}\right] = 0, \qquad \text{(A32)}$$

gives

$$\begin{aligned}\partial_\nu F_{\mu\nu} &= u^* u + i\frac{1}{2m}[(D_i u^*)u - u^*(D_i u)] \\ &= e\mathscr{T}_\mu \qquad \text{(A33)}\end{aligned}$$

where $\mathscr{T}_\mu$ are, corresponding to J_μ in Eqs. A15 and A16,

$$\mathscr{T}_0 = u^* u, \qquad \text{(A34)}$$

$$\mathscr{T}_i = -i\frac{1}{2m}[(D_i u^*)u - u^*(D_i u)]. \qquad \text{(A35)}$$

Then it is obvious that

$$\partial_\mu\mathscr{T}_\mu = 0, \qquad \text{(A36)}$$

which means the covariant current conservation in the presence of electromagnetic field.

Coulomb Gauge.[7] We have not yet mentioned about the gauge of A_μ. Hereafter the Coulomb gauge is employed, which will be done by a suitable choice of Λ in Eq. A19. First of all, since the Lagrangian does not contain $\partial_0 A_0$, we regard A_0 as a dependent variable which will be determined later. Then we impose the transversality condition on $\boldsymbol{A}$:

$$\nabla\cdot\boldsymbol{A} = 0, \quad \text{namely } \boldsymbol{A} = \boldsymbol{A}^{\mathrm{tr}}. \qquad \text{(A37)}$$

Noting that

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}, \qquad \text{(A38)}$$

we can come back to the usual Maxell equations. When $\mu = 0$, Eq. A33 is simply

$$\nabla\cdot\boldsymbol{E} = \mathrm{e}\mathscr{T}_0. \qquad \text{(A39)}$$

Since

$$\boldsymbol{E} = -\nabla A_0 - \partial_0\boldsymbol{A}, \qquad \text{(A40)}$$

the Eq. A40 becomes, due to the transversality condition of $\boldsymbol{A}$,

$$\nabla^2 A_0 = -\mathrm{e}\mathscr{T}_0. \qquad \text{(A41)}$$

In the Coulomb gauge, we regard A_0 as a function of ρ, given by a solution of the Laplace equation,

$$A_0(x_\mu) = \int\frac{\mathrm{e}\mathscr{T}_0(x_\mu)}{4\pi\,|\,\boldsymbol{r} - \boldsymbol{r}'\,|}\mathrm{d}^3\boldsymbol{r}', \qquad \text{(A42)}$$

since

$$\nabla^2 \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = -4\pi\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{A43}$$

Let us decompose

$$\boldsymbol{E} = \boldsymbol{E}^{\mathrm{tr}} + \boldsymbol{E}^{\mathrm{l}} \tag{A44}$$

in which the transverse component is

$$\boldsymbol{E}^{\mathrm{tr}} = -\partial_0 \boldsymbol{A} \tag{A45}$$

and the longitudinal component is

$$\boldsymbol{E}^{\mathrm{l}} = -\nabla A_0 \tag{A46}$$

Clearly $\boldsymbol{E}^{\mathrm{l}}$ is irrotational ($\nabla\times\boldsymbol{E}^{\mathrm{l}}=0$) and divergence-free. The partial integration yields

$$\int \boldsymbol{E}^{\mathrm{tr}}\cdot\boldsymbol{E}^{\mathrm{l}}\mathrm{d}^3\boldsymbol{r} = \int \partial_0\boldsymbol{A}\cdot\nabla A_0 \mathrm{d}^3\boldsymbol{r} = 0, \tag{A47}$$

namely the cross term vanishes, so that, without changing the resulting Lagrangian

$$L = \int \mathscr{L}\mathrm{d}^3\boldsymbol{r}, \tag{A48}$$

we obtain

$$\mathscr{L}_3 = -\frac{1}{4}F_{\mu\nu}^2 = \frac{1}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2) = \frac{1}{2}[(\boldsymbol{E}^{\mathrm{tr}})^2 + (\boldsymbol{E}^{\mathrm{l}})^2 - \boldsymbol{B}^2]. \tag{A49}$$

Furthermore

$$\int (\boldsymbol{E}^{\mathrm{l}})^2\mathrm{d}^3\boldsymbol{r} = \int (\nabla A_0)\cdot(\nabla A_0)\mathrm{d}^3\boldsymbol{r} = \int \nabla\cdot(A_0\nabla A_0)\mathrm{d}^3\boldsymbol{r} - \int A_0\nabla^2 A_0\mathrm{d}^3\boldsymbol{r} = \int \mathscr{T}_0 A_0\mathrm{d}^3\boldsymbol{r} \tag{A50}$$

where in the final step Eq. A42 was used. Then using Eq. A45, we obtain

$$L_3 = -\frac{1}{4}\int \mathrm{d}^3\boldsymbol{r}F_{\mu\nu}^2 = \int \mathrm{d}^3\boldsymbol{r}\frac{1}{2}[(\boldsymbol{E}^{\mathrm{tr}})^2 - \boldsymbol{B}^2] + \frac{1}{2}\int \mathrm{d}^3\boldsymbol{r}\int \mathrm{d}^3\boldsymbol{r}'\frac{\mathscr{T}_0(\boldsymbol{r},x_0)\mathscr{T}_0(\boldsymbol{r}',x_0)}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}. \tag{A51}$$

Here it is recognized that the electron–electron interaction buried in $\boldsymbol{E}$ has appeared with the factor $\frac{1}{2}$. We thus obtain the total Lagrangian as

$$L_{\mathrm{tot}} = \int \mathrm{d}^3\boldsymbol{r}\{\frac{i}{2}(u^*\partial_0 u - \partial_0 u^*\cdot u) - \frac{1}{2m}(\partial_i u^* + \mathrm{i}eA_i u^*)(\partial_i u - \mathrm{i}eA_i u)\} - \frac{1}{2}\int \mathrm{d}^3\boldsymbol{r}\int \mathrm{d}^3\boldsymbol{r}'\frac{1}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\times u^*(\boldsymbol{r},x_0)u(\boldsymbol{r},x_0)u^*(\boldsymbol{r}',x_0)u(\boldsymbol{r}',x_0) + \int \mathrm{d}^3\boldsymbol{r}\frac{1}{2}[(\boldsymbol{E}^{\mathrm{tr}})^2 - \boldsymbol{B}^2]. \tag{A52}$$

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