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Recently many resonances with spin 3/2 have been observed. If we regard them as elementary particles and want to calculate an S-matrix element in which these particles participate in intermediate states, we must have a knowledge of the propagator of spin 3/2 particle in advance. According to Takahashi and Umezawa[1], the propagator can not be determined in turn without the knowledge of the Lagrangian of the particle.

The Lagrangian of spin 3/2 particle was obtained by Rarita and Schwinger[2] in so-called Rarita-Schwinger formalism and the propagator by Takahashi and Umezawa[1) 2)]. Here we should like to point out that the Lagrangian obtained by Rarita-Schwinger[3] and the propagator derived by Takahashi-Umezawa[1] are only special cases of the most general Lagrangian and propagator which contain an indeterminate constant.

The Lagrangian density we obtained is of the form,

$$\mathcal{L} = \Psi_\mu(x)\, \Lambda_{\mu\nu}(\partial)\, \psi_\nu(x),\quad \Lambda_{\mu\nu}(\partial) = -[(\eth + m)\,\delta_{\mu\nu} + a\,(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) + \gamma_\mu \left\{\frac{1}{2}(3a^2 + 2a + 1)\,\eth - (3a^2 + 3a + 1)\,m\right\} \gamma_\nu], \tag{1}$$

where m denotes the mass of the particle and a is a real constant $\neq -\frac{1}{2}$ and $\eth = \gamma_\mu \partial_\mu$. From the Lagrangian (1) we get the eq. of motion,

$$(\eth + m)\,\psi_\mu + a\,(\gamma_\mu y + \partial_\mu x) + \gamma_\mu \left\{\frac{1}{2}(3a^2 + 2a + 1)\,\eth - (3a^2 + 3a + 1)\,m\right\} x = 0, \tag{2}$$

where $x = \gamma_\alpha \psi_\alpha$ and $y = \partial_\alpha \psi_\alpha$. Applying ∂_μ and γ_μ to eq. (2), we have

$$\left\{(1 + a)\,\eth + m\right\} y + \left\{\frac{1}{2}(3a + 1)\,(a + 1)\,\Box - (3a^2 + 3a + 1)\,m\eth\right\} x = 0, \tag{3}$$

and

$$\{(3a + 1)\,\eth - 3\,(2a + 1)\,m\}\, x + 2y = 0 \tag{4}$$

respectively, where the use has been made of $a \neq -\frac{1}{2}$, in deriving eq. (4).

Eliminating y from eqs. (3) and (4), we can readily show that

$$x = 0 \tag{5}$$

and

$$y = 0 \tag{6}$$

from eq. (4). Substituting these results, we obtain from eq. (2),

$$(\eth + m)\,\psi_\mu = 0, \tag{7}$$

Thus the Lagrangian density (1) is compatible with eq. of motion (6) and (7),

Takahashi and Umezawa[1] have shown that the propagator of the particle can be written as

$$S_{\mu\nu}(x) = -\frac{1}{2} i d_{\mu\nu}(\partial)\,\Delta_F(x), \tag{8}$$

where $d_{\mu\nu}(\partial)$ should satisfy the relation

$$\Lambda_{\lambda\mu}(\partial)\, d_{\mu\nu}(\partial) = (\Box - m^2)\,\delta_{\lambda\nu}. \tag{9}$$

With the relation (9) and the Lagrangian density (1), we can determine $d_{\mu\nu}(\partial)$ as follows,

$$\begin{aligned} d_{\mu\nu}(\partial) = &-(\eth - m)\Big[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3m}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) \\ &- \frac{2}{3m^2}\partial_\mu\partial_\nu\Big] - \frac{a+1}{6(2a+1)^2 m^2}(\Box - m^2)\Big[(3a+1)(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu \\ &+ m\gamma_\mu\gamma_\nu) + (a+1)\Big\{\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu + (\eth - m)\gamma_\mu\gamma_\nu\Big\}\Big], \end{aligned} \tag{10}$$

The Rarita-Schwinger's case corresponds to the choice of $a = -1/3$. The physical meaning of the indeterminate constant is not clear yet. Since the term which contains the indeterminate constant is proportional to the factor $(\Box - m^2)$, the commutation relation $\{\psi_\mu(x), \Psi_\nu(x')\}$ does not depend on the constant a.

Also the Lagrangian of spin 2 particle can contain an indeterminate constant. If we postulate, however, that the propagator should not contain any pole other than that corresponding to the original mass, the indeterminacy disappears in this case and the resulting Lagrangian becomes identical with that obtained by Wentzel[4].

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References

1) Y. Takahashi and H. Umezawa, Prog. Theor. Phys. 9, (1953) 14.

2) H. Umezawa, Quantum Field Theory (1956) § VIII. The expression for $d_{\mu\nu}(\partial)$ is written with a wrong coefficient. The second term should be multiplied by 2.

3) W. Rarita and J. Schwinger, Phys. Rev. **60** (1941) 61.

4) G. Wentzel, Eintührung in die Quantentheorie der Wellenfelder (1943) § VI