A Theoretical Consideration on Latitudinal and Seasonal Variations of the Ionosphere F2.

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A Theoretical Consideration on Latitudinal and Seasonal Variations of the Ionosphere F_2.*

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1. Introduction.

The variations of the ionosphere E and F_1 are considered to be regular, while that of F_2 is regarded to be irregular or implausible. This means that the layers E and F_1 obey the Chapman's law (1) in their diurnal, seasonal and latitudinal variations with their electron densities varying generally after $\cos\chi$, where χ is the sun's zenith distance, and that the latitudinal variation of electron density of the layer F_2, however, has, as shown in Fig. 1, a sunken portion in the equatorial region and convex portions in the regions of intermediate latitudes and its seasonal one has, as generally known, two minima a year in summer and winter, varying discordant from Chapman's law of $\cos\chi$.

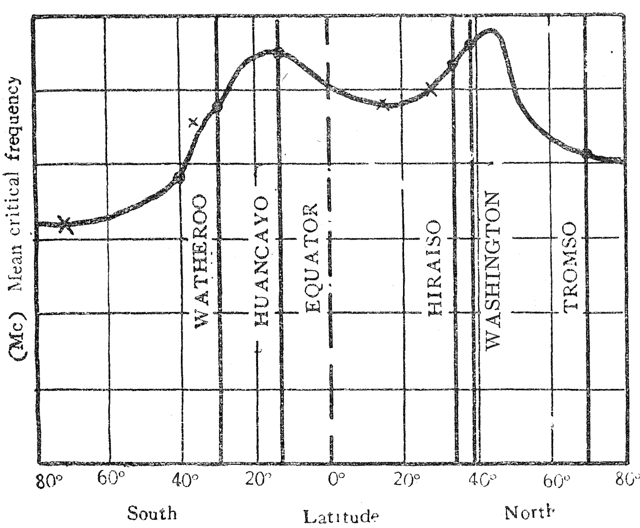

Fig. 1. Critical frequency versus latitude
(December—January)

The variation of F_2, however, just doesn't follow the law of $\cos\chi$ but varies regularly every year with its two annual maxima and minima; it can never be called irregular.

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What can be done for the interpretation of the F_2's variation? Aren't some necessary factors lacking in Chapman's theory? The Chapman's law takes only the ionization and recombination caused by the solar radiation into consideration and assumes an isothermal distribution of the earth atmosphere.

Beside above, we have some factors to be taken into consideration, such as

(a) Expansion and contraction of the earth atmosphere,

What we observe is the electron density or the number of electrons in a unit volume, which decreases by expansion when the temperature gets higher by absorbing solar radiation and increases by contraction

(b) Circulation of upper atmosphere,

(c) Superposition and splitting of layers F_1 and F_2,

The relative positions of F_1 and F_2 cause an increase of electron density by superposition and a decrease by splitting.

(d) Influence of terrestial magnetism.

Though all the factors should be taken into consideration and it is desirable to treat the atmosphere dynamically, the author, for a qualitative purpose, investigated the factor (a) which was considered to be of the most influence after the model of an ellipsoidal distribution of the earth's upper atmosphere with its major axis along the equatorial direction. The boundary of troposphere and stratosphere is $9\,km$ high at the poles and $17\,km$ at the Equator, and the thickness of the layer F_2 is 50-$60\,km$ in the regions of intermediate latitudes when it is about $150\,km$ at the equatorial regions. Judging from these facts, the above model may well not necessarily be absurd. In short, after the model of an ellipsoidal distribution of the atmosphere expanding in the upper air near the equatorial plane (or the direction where the solar zenith distance is zero), how the latitudinal distribution and the seasonal variation would be is investigated in our present paper.

2. Ellipsoidal Distribution of the Earth's Atomosphere.

The equation of an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

becomes in polar coordinates

$$\frac{r^2}{a^2}\cos^2\theta+\frac{r^2}{b^2}\sin^2\theta=1$$

and its eccentricity is

$$e=\sqrt{\frac{a^2-b^2}{a^2}}\,.$$

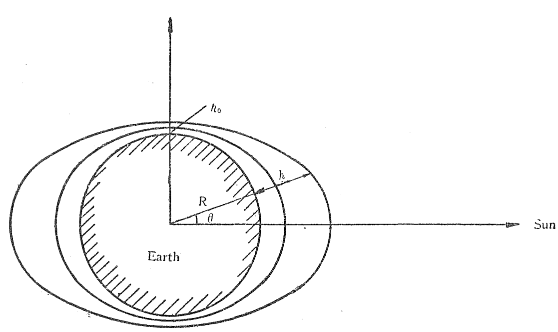

Fig. 2. Ellipsoidal distribution of the atmosphere

Let the earth's radius be represented by R, height of the ellipsoid at the poles by h_0 and its height above the earth by h, neglect the terms of orders over e^4 and assume that $R \gg h_0$ (in such problems of a height 300–400km as of the ionosphere, various integrals from infinity are considered not to be contributed much from the domains very high where the air density is very small), and we have

$$h = h_0 \left(1 + \frac{R}{2h_0} e^2 (h_0) \cos\theta\right) \qquad \cdots\cdots (1)$$

We assume now that e is zero on the earth's surface coinciding with a sphere and gets greater as height increases. We have started with an idea of an ellipsoid and, with some neglections, will assume hereafter such a distribution as represented by (1). By assuming a constant pressure upon the above distribution, we have

$$\exp\left(-\frac{h}{H}\right) = \exp\left(-\frac{h_0}{H_0}\right)$$

$$\therefore \quad \frac{h}{H} = \frac{h_0}{H_0}$$

$$H = H_0 \left\{1 + \frac{R}{2h_0} e^2 (h_0) \cos^2\theta\right\}$$

or, since $H = kT/mg$, we have

$$T = T_0 \left\{1 + \frac{R}{2h_0} e^2 (h_0) \cos^2\theta\right\}$$

which is similar to the empirical formula for the temperature distribution on the earth.

Therefore, the pressure and the density are given by

$$\left.\begin{aligned} p &= p_0 \exp\left[-\frac{h}{H_0 \{1 + \frac{R}{2h_0} e^2 (h_0) \cos^2\theta\}}\right] \\ \rho &= \rho_0 \frac{1}{\{1 + \frac{R}{2h_0} e^2 (h_0) \cos^2\theta\}} \exp\left[-\frac{h}{H_0 \{1 + \frac{R}{2h_0} e^2 (h_0) \cos^2\theta\}}\right] \end{aligned}\right\} \cdots\cdots (2)$$

3. A Case in which e (h_0) is identical to $\varepsilon\sqrt{h_0}$.

When e (h_0) is identical to $\varepsilon \sqrt{h_0}$, the above formulae take their simplest form with the coefficient of $\cos^2\theta$ independent of h_0, ε now being nothing but a constant.

We will treat this simplest case as an example of ellipsoidal distribution.

In this case, we have

$$p = p_0 \exp\left[-\frac{h}{H_0\{1 + \frac{R}{2}\varepsilon^2 \cos^2\theta\}}\right]$$

$$\rho = \rho_0 \frac{1}{\{1 + \frac{R}{2}\varepsilon^2 \cos^2\theta\}} \exp\left[-\frac{h}{H_0 \{1 + \frac{R}{2}\varepsilon^2 \cos^2\theta\}}\right]$$

By putting

$$\frac{R}{2}\varepsilon^2 = k$$

which is a constant independent of height, we have

$$\left.\begin{array}{l} p=p_0 \exp\left\{-\dfrac{h}{H_0(1+k\cos^2\theta)}\right\} \\ \rho=\rho_0 \dfrac{1}{(1+k\cos^2\theta)} \exp\left\{-\dfrac{h}{H_0(1+k\cos^2\theta)}\right\} \end{array}\right\} . \quad \cdots\cdots(3)$$

This distribution may, in a certain sense, be called a generalized form of Chapman's distribution $\rho=\rho_0 e^{-\frac{h}{H}}$, the temperature being constant vertically at a certain point but not constant for different latitudes. In this case, $H=H_0(1+k\cos^2\theta)$ not being a function of h, the above may be integrated as easily as in the case of Chapman's distribution.

Let S be the intensity of solar radiation and A the mass absorption coefficient, χ solar zenith distance. Then $dS/dh=\sec\chi\cdot SA\rho$,

$$dS/S=\sec\chi\cdot A\rho_0\frac{1}{1+k\cos^2\theta}\exp\left\{-\frac{h}{H_0(1+k\cos^2\theta)}\right\}dh,$$

$$S=S_\infty \exp\left[-\sec\chi\cdot A\rho_0 H_0 \exp\left\{-\frac{h}{H_0(1+k\cos^2\theta)}\right\}\right],$$ while ionization is

$$I=\beta AS\rho=\beta\left(\frac{dS}{dh}\right)\cos\chi,$$ where β is the number of electrons produced by a unit energy of absorbed radiation.

Then,

$$I=S_\infty \beta A\rho_0 \frac{1}{1+k\cos^2\theta}\exp\left[-\frac{h}{H_0(1+k\cos^2\theta)} -\sec\chi A\rho_0 H_0 \exp\left\{-\frac{h}{H_0(1+k\cos^2\theta)}\right\}\right]. \quad \cdots\cdots(4)$$

We then obtain the height of maximum ion production.

$$\frac{dI}{dh}=S_\infty A\beta\rho_0\frac{1}{1+k\cos^2\theta}\exp\left[-\frac{h}{H_0(1+k\cos^2\theta)}-\sec\chi\cdot A\rho_0 H_0 \exp\left\{-\frac{h}{H_0(1+k\cos^2\theta)}\right\}\right]$$
$$\times\left[-\frac{1}{H_0(1+k\cos^2\theta)}+\sec\chi\cdot A\rho_0 H_0\times\frac{1}{H_0(1+k\cos^2\theta)}\exp\left\{\frac{h}{H_0(1+k\cos^2\theta)}\right\}\right].$$

Therefore, the condition $dI/dh=0$ becomes

$$\sec\chi\cdot A\rho_0 H_0 \exp\left\{-\frac{h}{H_0(1+k\cos^2\theta)}\right\}=1 .$$

Hence, denoting by h_{max} the height where the maximal ion production I_{max} takes place, we have

$$h_{max}=H_0(1+k\cos^2\theta)\log(\sec\chi\cdot A\rho_0 H_0) \quad \cdots\cdots(5)$$

and

$$I_{max}=\frac{\beta S_\infty}{eH_0}\;\frac{\cos\chi}{1+k\cos^2\theta} \quad \cdots\cdots(6)$$

The electron density changes in such a manner that

$$\frac{dN}{dt} = I - \alpha N^2 \qquad \text{for recombination}$$

$$\frac{dN}{dt} = I - \beta' N \qquad \text{for attachment}$$

and, in total

$$\frac{dN}{dt} = I - \alpha N^2 - \beta' N.$$

In a stationary state,

$$\left.\begin{aligned} N_{max} &= \sqrt{\frac{I_{max}}{\alpha}} = \sqrt{\frac{\beta S_\infty}{\alpha e H_0}} \sqrt{\frac{\cos\chi}{1+k\cos^2\theta}} && \text{for recombination} \\ N_{max} &= \frac{I_{max}}{\beta'} = \frac{\beta}{\beta'} \frac{S_\infty}{eH_0} \frac{\cos\chi}{1+k\cos\theta} && \text{for attachment} \end{aligned}\right\} \cdots\cdots(7)$$

The diurnal variations may not be treated as stationary states. But such problems of long periods as latitudinal or seasonal variations may well, in general, be treated as stationary states without great errors regarding N_{max} to be related uniquely by (7) to the ion production. Let us then calculate the total number of electrons produced in a $1cm^2$-column from the earth's surface to the upper air.

$$\int I\, dh = S_\infty\, \beta A \rho_0 \frac{1}{1+k\cos^2\theta} \int \exp\Big[-\frac{h}{H_0(1+k\cos^2\theta)} - \sec\chi \cdot A\rho_0 H_0 \exp\Big\{-\frac{h}{H_0(1+k\cos^2\theta)}\Big\}\Big] dh.$$

This integration may be carried out if a simple one

$$\int \exp\Big\{-\frac{h}{a} - b\exp\Big(-\frac{h}{a}\Big)\Big\} dh.$$

is obtained.

Put

$$e^{-\frac{h}{a}} = x \qquad \text{or} \qquad -\frac{1}{a} e^{-\frac{h}{a}} dh = dx$$

and we have

$$\int \exp\Big\{-\frac{h}{a} - b\exp\Big(-\frac{h}{a}\Big)\Big\} dh = -a\int x e^{-bx} \frac{dx}{x} = -a\int e^{-bx}\, dx$$

$$= \frac{a}{b} e^{-bx} + C = \frac{a}{b} \exp\Big\{-b\exp\Big(-\frac{h}{a}\Big)\Big\} + C$$

In our case,

$$a = H_0\,(1+k\cos^2\theta), \qquad b = \sec\chi \cdot A\rho_0 H_0.$$

Therefore,

$$\int I\mathrm{d}h = S_\infty\, A\rho_0 \frac{1}{1+k\cos^2\theta}$$

$$\times \frac{\mathrm{H}_0(1+k\cos^2\theta)}{\sec\chi \cdot A\rho_0\mathrm{H}_0}\exp\left[-\sec\chi \cdot A\rho_0\mathrm{H}_0\exp\left\{-\frac{h}{\mathrm{H}_0(1+k\cos^2\theta)}\right\}\right]+\mathrm{C}$$

$$= S_\infty\,\beta\cos\chi\,\mathrm{exp}\left[-\sec\chi \cdot A\rho_0\mathrm{H}_0\exp\left\{-\frac{h}{\mathrm{H}_0(1+k\cos^2\theta)}\right\}\right]+\mathrm{C}\cdots(8)$$

$$\int_0^\infty I dh = \beta\, S_\infty \cos\chi\,\left(1-\mathrm{e}^{-\sec\chi\cdot A\rho_0\mathrm{H}_0}\right).\cdots\cdots(9)$$

By integrating the electron production up to the height of the maximum electron density, we have

$$\int_0^{h_{max}} I dh = \beta S_\infty \cos\chi\,\left(\mathrm{e}^{-1}-\mathrm{e}^{-\sec\chi\cdot A\rho_0\mathrm{H}_0}\right).\cdots\cdots(10)$$

Therefore, the ratio of total numbers of electrons from the earth's surface up to h_{max} and to the infinity is given by

$$\frac{\int_0^{h_{max}} I dh}{\int_0^\infty I dh} = \frac{\mathrm{e}^{-1}-\mathrm{e}^{-\sec\chi\cdot A\rho_0\mathrm{H}_0}}{1-\mathrm{e}^{-\sec\chi\cdot A\rho_0\mathrm{H}_0}}.\cdots\cdots(11)$$

Generally speaking, it is supposed that $A\rho_0\mathrm{H}_0 \gg 1$. Therefore, neglecting $\mathrm{e}^{-\sec\chi\cdot A\rho_0\mathrm{H}_0}$ in comparison with 1,

$$\frac{\int_0^{h_{max}} I dh}{\int_0^\infty I dh} \doteqdot \frac{1}{\mathrm{e}} = \frac{1}{2.718}\cdots\cdots(12)$$

$$\int_0^\infty I dh \doteqdot \beta S_\infty \cos\chi, \qquad \int_0^{h_{max}} I dh \doteqdot \frac{\beta S_\infty}{\mathrm{e}}\cos\chi.$$

Therefore, it is concluded that about 1/3 of the electrons exist below the height of maximum electron production and about 2/3 above it. The equations (9), (10), (11) and (12) are identical to what are derived from Chapman's distribution.

4. Latitudinal Variation in Ellipsoidal Distribution.

The maximum electron production and its height are given by (5) and (6), in which only H in Chapman's equation is substituted by $\mathrm{H}_0\,(1+k\cos^2\theta)$, being identical to a temperature distribution $\mathrm{T}=\mathrm{T}_0(1+k\cos^2\theta)$.

We have, thus far, considered the expansion to have taken place only along the equatorial plane. But if we consider the major axis move along with seasons to the directions where the sun's zenith distance is zero, θ in the foregoing equation is substituted by $\theta-\delta$, where δ represents the sun's declination. Thus, (5) and (6) become

$$h_{max} = \mathrm{H}_0\,(1+k\cos^2(\theta-\delta))\,\log\,(\sec\chi \cdot A\rho_0\mathrm{H}_0)\cdots\cdots(13)$$

$$I_{max}=\frac{\beta S_\infty}{eH_0}\frac{\cos\chi}{\{1+k\cos^2(\theta-\delta)\}}\cdots\cdots(14)$$

(a) Latitudinal Variation of Electron Density.

According to Chapman's theory,

$$I_{max}=\frac{\beta S_\infty}{eH}\cos\chi$$

which just differs from ours by the term of latitude in the denominater $1+k\cos^2(\theta-\delta)$.

Since $\cos\chi=\sin\theta\sin\delta+\cos\theta\cos\delta\cos\phi$, we have, for $\phi=0$ or at noon, $\cos\chi=\cos(\theta-\delta)$ As $\delta=0$ at Equinox, Chapman's theory gives the maximum of I_{max} at the Equator which decreases towards north and south after $\cos\theta$.

The experimental results tell us, however, that the maximum electron density is, on the contrary, greater for intermediate latitudes than the neighbourhood of the Equator as shown in Fig. 1. This phenomenon can by no means be interpreted by Chapman's theory. How will it be in our present case?

If we confine ourselves to consider the values at noon alone, (14) becomes

$$I_{max}=\frac{\beta S_\infty\cos(\theta-\delta)}{eH_0\{1+k\cos^2(\theta-\delta)\}}$$

$$\frac{dI_{max}}{d(\theta-\delta)}=\frac{\beta S_\infty}{eH_0}\frac{\cos(\theta-\delta)\times 2k\cos(\theta-\delta)\sin(\theta-\delta)-\{1+k\cos^2(\theta-\delta)\}\sin(\theta-\delta)}{\{1+k\cos^2(\theta-\delta)\}^2}$$

and the condition $dI_{max}/d(\theta-\delta)=0$ becomes

$$\sin(\theta-\delta)\{1-k\cos^2(\theta-\delta)\}=0$$

Thus, I_{max} has its maximum at $\cos(\theta-\delta)=\sqrt{1/k}$

and minimum at $\sin(\theta-\delta)=0$,

which becomes at equinox

$$\left.\begin{aligned}\text{maximum}:\ \cos\theta&=\sqrt{1/k}\\ \text{and minimum}:\ \sin\theta&=0\end{aligned}\right\}\cdots\cdots(15)$$

Therefore, if $k>1$, it follows that maxima take place at $\theta=\arccos\sqrt{1/k}$ in the north and the south and a minimum at $\theta=0$ or at the Equator, which is qualitatively in accordance with the observation. The greater k becomes, the more prominent the maxima and the higher their latitudes. If $0<k<1$, the maximum takes place nowhere else but at $\theta=0$ and it is flatter than the curve of $\cos\chi$ for $k=0$.

For $\phi=0$, $\delta=0$ or at noon in equinox

$$I_{max}=\frac{\beta S_\infty}{eH_0}\frac{\cos\theta}{1+k\cos^2\theta}$$

may be calculated and its value at the Equator may be adjusted to give such a curve as Fig. 3.

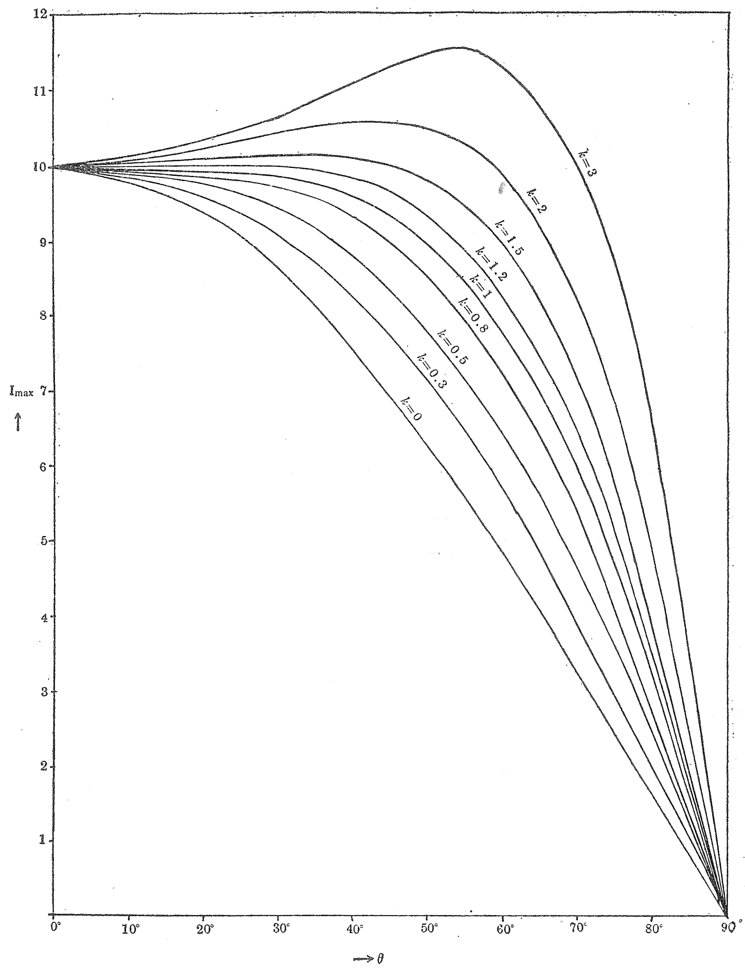

Fig. 3. I_{max} versus latitude $I_{max} \propto \frac{\cos\theta}{1+k\cos^2\theta}$

(b) Latitudinal Variation of h_{max} where I_{max} takes place.

The height h_{max} where I_{max} takes place is, according to Chapman's theory, given by

$$h_{max} = \mathrm{H}\log(\sec\chi \cdot A\rho_0\mathrm{H})$$

which becomes at noon in equinox

$$h_{max} = \mathrm{H}\log\ (\sec\theta \cdot A\rho_0\mathrm{H}).$$

As seen in this expression, h_{max} is expected to be the lowest at the Equator and to get higher as the latitude increases. The fact is, however, entirely opposite for the layer F_2, the minimum apparent layer height being the highest at the Equator and getting lower as the latitude increases as roughly tabulated below:

θ	0°	20°	40°	60°	80°
$h\ (F_2)$	400km	380km	330km	280km	260km

Our present theory gives

$$h_{max} = H_0\ (1 + k\cos^2\theta)\ \log\ (\sec\theta \cdot A\rho_0 H_0)$$

in which, if k is greater than a certain value, $\log\ (\sec\theta \cdot A\rho_0 H_0)$ increases, when θ increases from zero no more rapidly than $1 + k\cos^2\theta$ decreases, so that h_{max} has its maximum at the Equator to decrease when θ increases and finally, near the poles, increases rapidly affected by the remarkable increases of $\log(\sec\theta \cdot A\rho_0 H_0)$. Thus, except for the regions near the poles, we have obtained a result close to the observed facts. In order to carry out the calculation, we have to assume certain values of H_0 and $A\rho_0$. In Fig. 4, some examples of the calculation are given. Therefore, we hope that our theory may be considered to accord with the facts better than that of Chapman for the latitudinal variations

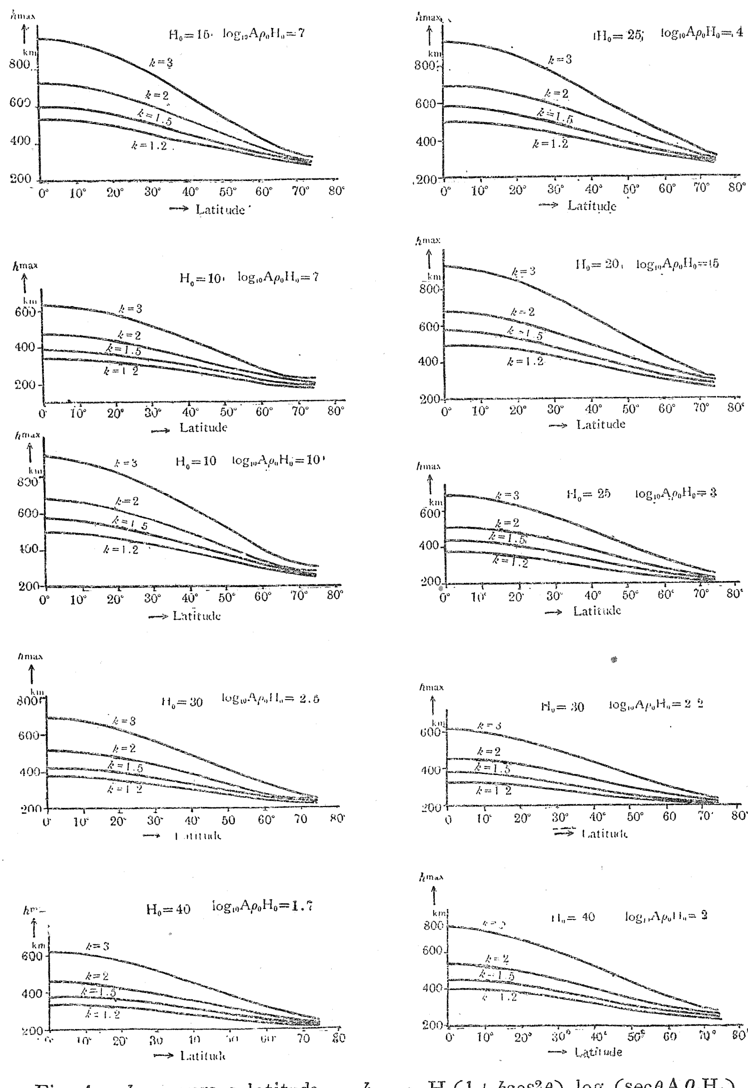

Fig. 4. h_{max} versus latitude $\qquad h_{max} = H_0(1 + k\cos^2\theta)\ \log\ (\sec\theta A\rho_0 H_0)$

of electron density and height of the layer F_2.

We have investigated the simplest case k const. But, generally speaking, k (h_0) is considered to be a function of h_0. By taking proper functions as $k(h_0)$, we may not only treat an isothermal case but also take vertical temperture distribution into consideration.

At any rate, the latitudinal variations of the layer F_2 seem to be interpreted qualitatively by assuming greater expansion near the Equator than near the poles.

5. Seasonal Variation in Ellipsoidal Distribution.

According to Chapman's theory, the seasonal variation is also given by $I_{max} \propto \cos\chi$; maximum electron density in summer and minimum in winter. In reality, however, the result of observation at intermediate latitudes gives a minimum in summer, maxima in spring and autumn and the second minimum in winter, which is another reason why the variation of the layer F_2 is said to be complicated and implausible.

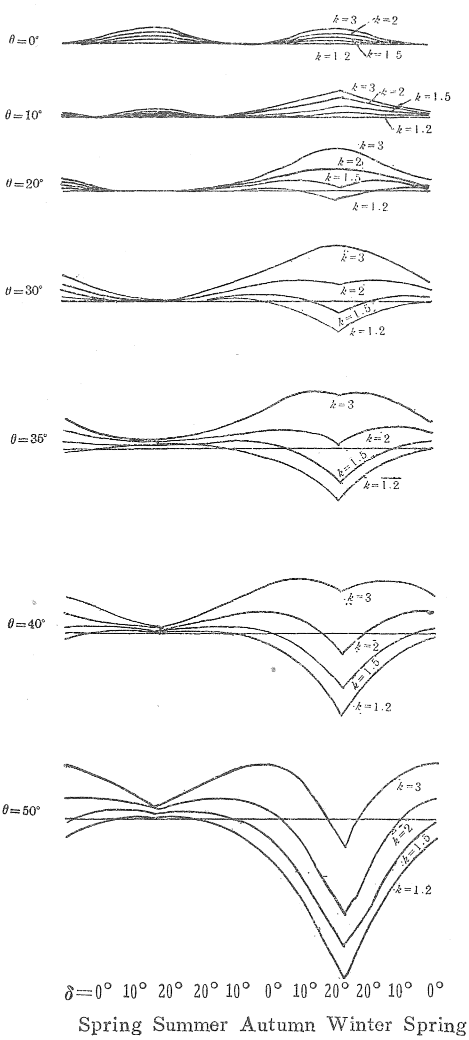

Fig. 5. Seasonal variation of I_{max} at various latitudes

We have in our theory, besides the term $\cos\chi$, a term $1/(1+k\cos^2(\theta-\delta))$ derived from the assumption that the direction of the major axis move along with seasons to points where the sun's zenith distance is zero. Put $\phi=0$ for noon, and we have

$$I_{max}=\frac{\beta S_\infty}{eH}\cdot\frac{\cos(\theta-\delta)}{1+k\cos^2(\theta-\delta)}$$

where δ represents the solar declination wandering between the angles $-23.5°$ and $+23.5°$.

This is plotted together with various values of θ in Fig. 5, from which it is learned that, on the northern hemisphere,

1. At the Equator, though the amplitude is very small, maxima are found in summer and winter and minima in spring and autumn.
2. As θ gets greater gradually, the maximum in summer gets less and less and, at least, vanishes to give a minimum. The maximum in winter gradually grows up.
3. As θ gets still greater, a concavity appears gradually to the maximum in winter.
4. The minimum of winter gets deeper as θ increases and that of summer vanishes to leave a maximum in summer and a mini-

mum in winter. Only one maximum and one minimum remain.

What have been qualitatively explained above vary, naturally, according to the value of k. In the southern hemisphere, they are just opposite in its seasons.

Though extremely qualitative, two maxima and minima in a year at intermediate latitudes and their phases seem to be in good accordance with the fact.

6. Total Ionization $\int_0^\infty I dh$ and $\int_0^{h_{max}} I dh$.

Our theory above differs from that of Chapman in such points that, in its latitudinal variation,

1. I_{max} has a shallow minimum at the Equator, maxima at certain latitudes between the Equator and the poles and decreases rapidly near the poles,
2. The layer F_2 is highest at the Equator, gets lower gradually and higher again rapidly near the poles,

and in its seasonal variations,

3. At intermediate latitudes, I_{max} has its minimum in summer, second minimum in winter and maxima in spring and autumn giving results comparatively colser to the facts.

Let us further have a test if our distribution is a plausible one by calculating total electron production from the earth's surface up to the infinity and up to h_{max}.

There are already calculated and given by (9) and (10). We see from these equations that $\int_0^\infty I dh$ and $\int_0^{h_{max}} I dh$ are related merely to $\cos\chi$ not containing the term $1+k\cos^2(\theta-\delta)$. Identical expressions are obtained by integrating after Chapman's theory. As the term $\exp(-\sec\chi\cdot A\rho_0 H_0)$ is supposed to be much smaller than both 1 and $1/e$, the total number of electrons in a column which is 1 cm^2 in cross section, as I_{max} in Chapman's theory does, has its maximum at the Equator and decreases toward the poles in its latitudinal behavior and shows its maximum in summer and minimum in winter varying proportionally to $\cos\chi$ in its seasonal one.

$$\int_0^\infty I dh \doteqdot \beta\, S_\infty \cos\chi, \qquad \int_0^{h_{max}} I dh \doteqdot \frac{\beta\, S_\infty}{e} \cos\chi.$$

These results are rather sensible and may be considered to provide us with a basis for the plausibleness of our theory. It is already described that $\int_0^{h_{max}} I dh$ is nearly equal to $\frac{1}{e}\int_0^\infty I dh$.

7. Conclusions.

We assumed that the distribution of the earth's atmosphere was not uniform but ellipsoidal most expanding at the Equator (or where the sun's zenith distance was zero) and that the pressure on the earth's surface was constant everywhere giving its density

$$\rho=\rho_0\frac{1}{1+k\cos^2(\theta-\delta)}\exp\left[-\frac{h}{H_0\{1+k\cos^2(\theta-\delta)\}}\right]$$

to take into consideration the atmospheric expansion to some extent, and, after this model, we obtained the latitudinal and seasonal variations. The results have been found to be qualitatively in good accordance with the observed facts listed below:

1. The electron density has a concavity in the neighbourhood of the Equator and maxima at intermediate latitudes.
2. The apparent height is highest at the Equator and gets lower toward higher latitudes.
3. At the intermediate latitudes, there are two minima in summer and winter and maxima in spring and fall.

Furthermore, $\int_0^{\infty} I dh$ and $\int_0^{h_{max}} I dh$ were found by calculation to be generally proportional to cos χ, which suggests that the distribution is plausible, and about 1/e of the total number of electrons was found to be below h_{max}.

We have treated here the simplest case among the possible ellipsoidal distributions.

It may be more interersting to study the problem taking into consideration the vertical temperature distribution putting $k(h_0)=f(h_0)$.

It is desirable to treat the atmosphere dynamically taking into consideration its motion, expansion, contraction, circulation etc, and to construct a perfect theory with superposition and splitting of the layers F_1 and F_2 and dynamical influences of terrestial magnetism upon the electrons.

1. Chapman : Proc. Roy. Soc. A43. 26, 483 (1931)