New Approach to the "Lasting-tilt" of Sperry Type Gyro-Compass

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New Approach to the "Lasting-Tilt" of Sperry Type Gyro-Compass

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§ 1. Introduction.

Sperry Type Gyro-Compass, owing to its mechanism, even if on a fixed place, generally does not keep the axle in the meridian and horizontal : that is, *the resting position* of the north pointing end of the Gyro, which is adjusted for the equator, deviates easterly from the meridian and upwards from the horizon on north latitude, westerly and downwards on south latitude. This azimuthal resting deviation, as is known, is usually called *latitude correction* and given by the formula $\alpha_0 = -\gamma \tan \lambda$; where λ is the angle of latitude at any place in question, γ so called *eccentric angle* (a constant owing to the damping device), α_0 and λ are taken positive for westerly deviation and north latitude respectively. And, hereafter, the author will call this resting rise or dip "*lasting-tilt*", and denote it β_0.

Of course, the fact that the compass axle does not point to due north, is greatly unfavorable when we use it on board. Actually by using the latitude corrector we get true north on the compass card, but it must be remembered that the Gyro-axle itself rests on a deviated and tilted position.

So far as the north pointing character of compass card is concerned the use of latitude corrector is sufficient ; but the fact that the Gyro-axle settles down in the tilted resting position, causes the chance of indefinite deflection based on ship's movement, and is not desirable.

Now, since the earth is always turning underneath the Gyro-Compass which tends to maintain its axle direction in space, it follows that the axle will be left behind on the east side of the meridian, unless a precessional velocity can be imparted to it by some couple. In the cace of well-adjusted pendulous Gyro, lasting-tilt β_0 is just caused for giving this couple by the gravitational force.

So that, by adding a small weight, *a compensation weight*, to one side of the case containing the Gyro-wheel, we can introduce a turning couple, apart from the gravity effect on the Gyro-system itself, and thus keep the Gyro-axle horizontal, and still in the meridian(1). This seems to be very convenient, but we can never forget that the effect of earth's rotation is dependent on latitude and so the compensation weight is a function

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(1) cf. H. Crabtree ; Spinning Tops and Gyroscopic Motion (Loqgmans, Green and Co.). p. 160 ; p. 171.

of latitude. So, for any desired latitude α_0 and β_0 can be made to disappear by adding a suitable compensation weight to one side of the rotor case ; unless, however, the amount or position of this weight is altered, a change of latitude introduces α_0 and β_0, and necessitates a correction. In this case, as α_0 is not only a function of γ and λ but also a function of the compensation weight, we can't use a definite latitude corrector. Moreover, to change the compensation weight during navigation is practically impossible. So that, the above mentioned compensation is not used for practical purpose.

We researched on *the compensation weight mounted on the Mercury Ballistic Reservoir*, instead of the rotor case, and found that its effect appeared only in lasting-tilt β_0 and not in latitude correction α_0 : i.e. by adding a suitable compensation weight to the south-side mercury ballistic reservoir, for any desired latitude we can keep the Gyro-Compass pointing a direction determined by $-\gamma\tan\lambda$, and still with its axle horizontal. Thus, using a definite latitude corrector we can both get true north on compass-card and reduce lasting-tilt.

§ 2. Coordinates-system, Symbols, and some assumptions for simplification.

1) Following calculation is based on the rectangular coordinates-system x, y, z, origin of which we take in the center of rotation of Gyro-wheel, and the positive direction of x, y and z represents East, North and Zenith respectively. Axial vector of rotation and angular momentum is obeyed to "right-handed screw rule."

2) Symbols employed in this investigation are as follows ;

$\alpha=$ Azimuthal angle between north end of Gyro-axle and north end of meridian (positive with the west),

$\beta=$ Angle of inclination of Gyro-axle to the horizontal (positive with elevation of the north end of the axle),

R= Radius of the earth (=6370.3km.),

T*= 1 sidereal day (=86164.1 sec.),

$\omega=$ Angular velocity of earth's rotation,

$$\omega=\frac{2\pi}{T^*}=0.729212\times10^{-4}\text{rad/sec.}=15.04''/\text{sec.},$$

$\lambda=$ Angle of latitude on which Gyro-compass is placed (positive with north latitude),

$g=$ Acceleration of gravity,

H= Angular momentum of Gyro-wheel about its axle (southward pointing vector),

$\gamma=$ Eccentric angle (refer Fig. 1),

D= The righting co-efficient of the mercury ballistic,

$$D=\frac{1}{2}A\rho g l^2$$

where,

A= effective cross sectional area of the mercury reservoir,

$\rho=$ density of mercury,

$l=$ distance between the center of north and south mercury reservoir,

O= The center of rotation of Gyro-wheel, origin of the coordinates system,

G= Position of the eccentric pivot,

$a=$ Length of $\overline{\text{OG}}$,

$T_0=$ Period of undamped Gyro $(=2\pi\sqrt{H/D\omega\cos\lambda})$,

$\alpha_0=$ Value of α in resting position, i.e. latitude correction,

$\beta_0=$ Value of β in resting position, i.e. lasting-tilt,

$\lambda_s=$ Latitude for which ballistic deflection of compass is "dead-beat",

$$T_{0(\lambda=\lambda_s)}=2\pi\sqrt{H/D\omega\cos\lambda_s}=2\pi\sqrt{R/g}\fallingdotseq 84.5\text{min.},$$

$m=$ Compensation mass,

$mgr=$ Compensation torque,

$\lambda_0=$ Latitude for which compensation weight is added,

$\alpha_{0m}=$ Latitude correction with compensation weight,

$\beta_{0m}=$ Lasting-tilt with compensation weight,

$v_e=$ Component of ship's velocity in East direction,

$v_n=$ Component of ship's velocity in North direction,

$\delta=$ Change in azimuthal angle due to speed of ship ; Speed correction

$[=v_n/(R\omega\cos\lambda+v_e)]$.

3) Simplification of the problem.

Here we do not discuss oscillation about the y axis (North-South line), because it has not any important meaning for the usual azimuthal Compass. And, in accordance with the general method for technical researches for the Gyro-Compass, we have following assumptions ;

a) We take it for granted that the angular momentum of every moving part is a negligible quantity compared with that of the Gyro-wheel itself about its axle, i.e. that the direction of the total angular momentum is always in accord with the Gyro-axle(1).

b) Compared with the "dynamical inertia" H^2/D, the moment of inertia of Compass system —— statical —— is neglected ; we do not take the nutational motion into consideration.

c) The following research would be limited on finding the equilibrium position —— resting position —— and the small oscillation about it, i.e. $\sin\alpha\fallingdotseq\alpha$ and $\sin\beta\fallingdotseq\beta$.

d) Compared with the righting co-efficient of the mercury ballistic D, we neglect the meridian-righting co-efficient $H\omega\cos\lambda$.

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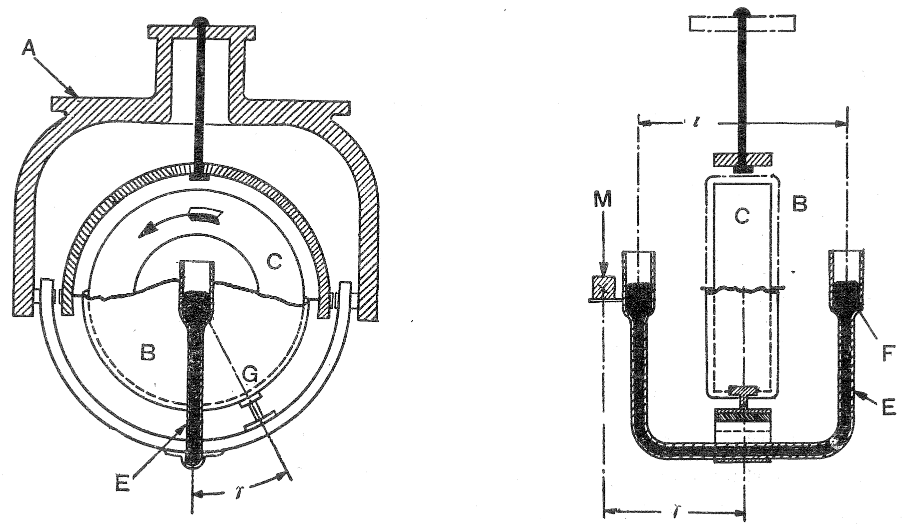
(1) The correct speed for the rotation of the wheel in the Sperry Gyro-Compass is 8,600 or 6,000 revolutions per minute giving an angular momentum about the axle of wheel, for which the angular momentum due to earth's rotation or ship's movement relative to the earth is neglected.

§ 3. The fundamental equations for the damped Gyro-Compass placed on a fixed position.

Taking the construction of the damping arrangement of the Sperry-Type Gyro-Compass(1) into account, referring to Fig. 1 and 2, and considering above-mentioned assumptions, we have :

$$H\left(\frac{d\alpha}{dt}+\omega\sin\lambda-\omega\cos\lambda.\beta\right)=D\,\beta$$

$$H\left(\frac{d\beta}{dt}+\omega\cos\lambda.\alpha\right)=-D\,\beta\tan\gamma$$



(from the South side) (from the East side)

Fig. 1

A ; Phantom, **B** ; Rotor case, **C** ; Gyro-wheel (rotor), **E** ; Mercury Ballistic,
F ; Mercury, **G** ; Eccentric Pivot, **M** ; Compensation Weight.

(The phantom follows every movement of the Gyro about the vertical axis. This may be automatically accomplished by means of electrical contacts controlling a motor which will rotate the phantom about the vertical axis in such manner as to follow all movements of the Gyro about that axis).

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(1) Gyro case containing Gyro-wheel itself is non-pendulous, while the mercury ballistic system suspended from an outer frame —— phantom —— is pendulous. If we connect the mercury ballistic to the Gyro case by means of an eccentric pivot, as shown at G in Fig. **1**, the ballistic will act on the Gyro about both the horizontal and the vertical axes, thus causing the Gyro system to precess toward the meridian and back toward the horizontal.

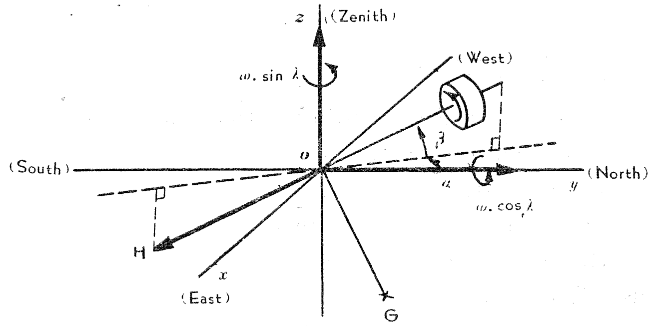

Fig. 2

Further, cosidering the assumption that $H\,\omega\cos\lambda <<< D$, and taking the fact that $\gamma = 1^\circ.36'$ for our Gyro-compass into account, the fundamental equations for the damped Gyro-compass placed on a fixed position are as follows ;

$$H\left(\frac{d\alpha}{dt} + \omega\sin\lambda\right) = D\beta \qquad (1)$$

$$H\left(\frac{d\beta}{dt} + \omega\cos\lambda.\alpha\right) = -D\gamma\beta \qquad (2)$$

So that the Gyro-axel finally settles down in the resting position (α_0, β_0), where

$$\alpha_0 = -\gamma\tan\lambda \qquad \text{(latitude correction)} \qquad (3)$$

$$\beta_0 = H\omega\sin\lambda/D \qquad \text{(lasting-tilt)}. \qquad (4)$$

§ 4. Further Discussion on β_0.

1) From the relation (4), it is clear that the lasting-tilt β_0 is proportional to $\sin\lambda$ provided that $D = \frac{1}{2}A\rho g l^2$ is constant, but if we change D and naturally A, β_0 also varies with the change of D.

2) While, the equations of motion for the undamped Gyro-Compass $(\gamma = 0)$[1] are ;

$$H\left(\frac{d\alpha}{dt} + \omega\sin\lambda\right) = D\beta \qquad (5)$$

$$H\left(\frac{d\beta}{dt} + \omega\cos\lambda.\alpha\right) = 0. \qquad (6)$$

and by eliminating β, the equation of azimuthal swing becomes

$$\frac{H^2}{D}\cdot\frac{d^2\alpha}{dt^2} + H\,\omega\cos\lambda.\alpha = 0.$$

Consequently, if T_0 be the time of a complete oscillation of the undamped Gyro axle, given by the above equation, then

$$T_0 = 2\pi(H/D\,\omega\cos\lambda)^{\frac{1}{2}}. \qquad (7)$$

So that,

$$1/D = \left(\frac{T_0}{2\pi}\right)^2\cdot\frac{\omega\cos\lambda}{H}. \qquad (8)$$

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(1) $\gamma = 0$ is accomplished by using the electro-magnetic damping cut-out device attached to the mercury ballistic system.

Hence,

$$\beta_0 = \frac{1}{2}\left(\frac{\omega}{2\pi}\right)^2 T_0^2 \sin 2\lambda. \tag{9}$$

3) As it is well known, to make the ballistic deflection of the Gyro-Compass "dead-beat", there must be the following relation ;

$$T_0 = 2\pi(R/g)^{\frac{1}{2}} \doteqdot 84.5\text{min.}.$$

If this condition is satisfied for all latitudes, then

$$\beta_0 = 1.72 \times 10^{-3} \sin 2\lambda$$

namely β_0 takes its maximum value at $\lambda = 45°$.

4) From the equation (7), it is clear that to get $T_0 = 84.5$ min. for all latitudes, there must be a relation

$$D\cos\lambda = (2\pi/84.5\text{min.})^2 \cdot H/\omega = \text{const.},$$

because H is a constant for a Gyro-Compass ; i.e. the effective cross sectional area of the mercury reservoir must be inversely proportional to cos λ. On the other hand, it is actually impossible to change A during the operation of the Gyro-Compass without any occurrence of undesirable deflection. Accordingly, as to the Gyro-Compass which is so constructed that we can change A at will, we deduce A for a desired latitude by considering the course of the ship and must make the handling of changing A, during stay in harbour, and not during navigation. And a certain type of the Sperry Compass has such construction that it has undamped period of 84.5 min. at some definite standard latitude λ_s. If we denote the period of the undamped Gyro at the latitude λ_s for which ballistic deflection of compass is dead-beat T_{os}, then

$$T_{os} = 2\pi(H/D\,\omega\cos\lambda_s)^{\frac{1}{2}} = 84.5\text{ min.}.$$

Hence the period of the undamped Gyro at any latitude λ is,

$$\begin{aligned} T_0 &= 2\pi(H/D\,\omega\cos\lambda)^{\frac{1}{2}} = T_{os}(\cos\lambda_s/\cos\lambda)^{\frac{1}{2}} \\ &= 84.5 \times 60(\cos\lambda_s/\cos\lambda)^{\frac{1}{2}}\text{ sec.}. \end{aligned} \tag{10}$$

So that,

$$\begin{aligned} \beta_0 &= \frac{1}{2}\cdot(84.5\times 60)^2\cdot\left(\frac{\omega}{2\pi}\right)^2\cdot\left(\frac{\cos\lambda_s}{\cos\lambda}\right)\cdot\sin 2\lambda \\ &= (84.5\times 60)^2\cdot\frac{1}{T^{*2}}\cdot\cos\lambda_s\sin\lambda. \end{aligned} \tag{11}$$

§ 5. "$\beta_0 \to 0$" Adjustment.

As above mentioned, the axle of Gyro has the "lasting-tilt" $\beta_0 = H\omega\sin\lambda/D$. Here we will modify this relation as follws,

$$D\beta_0 = H\omega\sin\lambda. \tag{12}$$

Considering the equation (12), it is clear that the lasting-tilt β_0 has following physical meaning. Since the earth is always turning about the vertical axis, underneath the compass, with angular velocity $\omega\sin\lambda$, to keep the Gyro-axle in the resting position (relative to the earth) there must be a torque about the horizontal axis of Gyro[1] just enough to cause the precession about the vertical axis with the angular velocity $\omega\sin\lambda$.

(1) We call the line, which is perpendicular to Gyro-rotor-axle and lies in horizontal plane when Gyro is not in motion, the horizontal axis of Gyro.

As the angular momentum of the Gyro is H, this torque about the horizontal axis of the Gyro must be $H\omega\sin\lambda$. Meanwhile, the Gyro-axle at its resting position has the lasting-tilt β_0. Accordingly, at the resting position, mercury ballistic always acts a couple $D\beta_0$ about the horizontal axis of Gyro. This couple $D\beta_0$, due to the lasting-tilt β_0, is nothing else but the above-mentioned required torque.

So that, if we introduce this required torque $H\omega\sin\lambda$ about the horizontal axis of Gyro by means of any other special mechanism, then the lasting-tilt β_0 can be made to disappear.

But, as already mentioned, to introduce this torque by adding a compensation weight to one side of the rotor case is impracticable.

Here, to the middle point of the south-side mercury ballistic reservoir, we add the compensatation weight, mass of which is m gm and which causes the torque about the horizontal axis of Gyro with an amount of

$$mgr = H\omega\sin\lambda, \tag{13}$$

when the Gyro axle is horizontal. When arranged in this manner, this compensation weight causes such torque on the Gyro as follows ;

1) about the horizontal axis of Gyro,

$$mgr\cos\beta \doteqdot mgr,$$

2) about vertical axis of Gyro,

$$(mgr\cos\beta/a\cos\gamma)\ a\sin\gamma \doteqdot mgr\gamma.$$

So that, in this case, the equations of the damped Gyro are ;

$$H\left(\frac{d\alpha}{dt}+\omega\sin\lambda\right)=D\beta+mgr \tag{14}$$

$$H\left(\frac{d\beta}{dt}+\omega\cos\lambda.\alpha\right)=-D\gamma\beta-mgr\gamma \tag{15}$$

By resolving these equations, we get the new resting position ;

$$\alpha_{0m}=-\gamma\tan\lambda=\alpha_0 \tag{16}$$

$$\beta_{0m}=H\omega\sin\lambda/D-mgr/D \tag{17}$$

By considering the relation (13), that is $H\omega\sin\lambda=mgr$, we get

$$\beta_{0m}=0. \tag{18}$$

That is, by placing a suitable compensation weight, which is determined by the relation (13), on the south-side mercury ballistic reservoir, artificial couple can be introduced, causing the precession necessary to keep the Gyro-axle horizontal without giving any change of the latitude correction $\alpha_0=-\gamma\tan\lambda$.

Here we must pay attention to the fact that, as it is clear from the relation $mr=H\omega\sin\lambda/g$, the compensation couple is a function of $\sin\lambda$, and it is actually impossible to change the amount or position of this compensation weight during the operation of the Gyro without any cause of the undesirable deflection of the axle. If we do this "$\beta_0\to0$"

adjustment for a definite latitude λ_0 by adding the compensation torque m_0gr, then the resting positition for any latitude λ is,

$$\alpha_{0m} = -\gamma\tan\lambda = \alpha_0 \tag{19}$$

$$\begin{aligned}\beta_{0m} &= \mathrm{H}\omega\sin\lambda/\mathrm{D} - m_0gr/\mathrm{D} \\ &= \mathrm{H}\omega\sin\lambda/\mathrm{D} - \mathrm{H}\omega\sin\lambda_0/\mathrm{D} = \beta_0 - \beta_{0\,(\lambda=\lambda_0)}\,.\end{aligned} \tag{20}$$

§ 6. Effect of the ship's movement.

Hitherto we have considered the compass at rest relatively to the earth. When, however, it is mounted on board ship, certain corrections are necessary owing to the motion of the vessel.

1) *With a constant velocity.*

If there is a component of ship's velocity in East direction, v_e, then the angular velocity of earth's rotation becomes apparently

$$\omega' = \omega + v_e/\mathrm{R}\cos\lambda.$$

And if there is a component of ship's velocity in North direction, v_n, then the apparent angular velocity of earth's rotation about the East-West line, $-v_n/\mathrm{R}$, is introduced.

Taking the above consideration into account, the equations of motion for the damped Gyro-Compass on a ship, with no compensation weight, now became,

$$\mathrm{H}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \omega'\sin\lambda\right) = \mathrm{D}\beta \tag{21}$$

$$\mathrm{H}\left(\frac{\mathrm{d}\beta}{\mathrm{d}t} + \omega'\cos\lambda.\alpha - \frac{v_n}{\mathrm{R}}\right) = -\mathrm{D}\gamma\beta. \tag{22}$$

So the resting position of the Gyro-axle is given by

$$\alpha_{0\mathrm{v}} = -\gamma\tan\lambda + v_n/\mathrm{R}\omega'\cos\lambda$$

$$\beta_{0\mathrm{v}} = \mathrm{H}\omega'\sin\lambda/\mathrm{D},$$

where $v_n/\mathrm{R}\omega'\cos\lambda$ is generally called a speed correction and is denoted by δ.

$$\delta = \frac{v_n}{\mathrm{R}\omega\cos\lambda + v_e} \tag{23}$$

δ is no way dependent on any particular Gyro-Compass, but is only a geometrical relation between the ship's velocity and the earth's rotation, and is eliminated on the Compass-Card making use of the speed-corrector. By using the symbol δ, α_0 and β_0 the resting position can be shown as follows ;

$$\alpha_{0\mathrm{v}} = \alpha_0 + \delta \tag{24}$$

$$\beta_{0\mathrm{v}} = \beta_0 + (\mathrm{H}/\mathrm{DR})\tan\lambda.v_e = \beta_0\{1 + (v_e/\mathrm{R}\cos\lambda)/\omega\} \tag{25}$$

If the compensation weight is added as in the above mentioned manner, then the equations of motion are

$$\mathrm{H}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \omega'\sin\lambda\right) = \mathrm{D}\beta + mgr. \tag{26}$$

$$\mathrm{H}\left(\frac{\mathrm{d}\beta}{\mathrm{d}t} + \omega'\cos\lambda.\alpha - \frac{v_n}{\mathrm{R}}\right) = -\mathrm{D}\gamma\beta - mgr\gamma\,. \tag{27}$$

So that the Gyro axle finally settles down in the resting position $(\alpha_{0\mathrm{v}m}, \beta_{0\mathrm{v}m})$, where

$$\alpha_{0vm}=\alpha_{0v}=\alpha_0+\delta \tag{28}$$

$$\beta_{0vm}=\beta_{0v}-mgr/\mathrm{D}=\beta_0\{1+(v_e/\mathrm{R}\cos\lambda)/\omega\}-mgr/\mathrm{D}$$
$$=\beta_{0m}+(\mathrm{H/DR})\tan\lambda.v_e. \tag{29}$$

By cosidering the above mentioned relations (3), (4), (19), (20), (24), (25), (28) and (29), we can conclude that the effect of the compensation weight, which is added to the mercury ballistic reservoir, is independent of the ship's movement with a constant velocity : i.e. compared with the case when the Gyro is placed on a fixed position, if it is mounted on a ship traveling with a constant velocity, then the value of α in resting position increases by an amount of δ, and the value of β in resting position by an amount of (H/DR) $\tan\lambda.v_e$, both independent of the compensation weight. Moreover, by taking actual sppeed of a ship into account, we can understand that the value of speed correction term for the lasting-tilt, $(\mathrm{H/DR})\tan\lambda.v_e=\beta_0(v_e/\mathrm{R}\cos\lambda)/\omega$, is about 10% or so of β_0 at most, and generally smaller than 3% of β_0. And the numerical value of β_0 is usually smaller than $10'$ as we shall see in the next paragraph. So that we may neglect the speed correction term for the lasting-tilt. Accordingly, even if the Gyro is mounted on a ship traveling with a constant velocity, by adding the compensation weight with our method, the relation, $\beta_0 \fallingdotseq 0$, can be maintained without any change of latitude correction and speed correction.

2) *With acceleration.*

As is well-known, when the ship moves with acceleration, we get "$\gamma=0$" through the Damping-cut-out Device, that is, we make the Gyro undamp, and make its undamped period as follows, $\mathrm{T}_0=2\pi(\mathrm{R}/g)^{\frac{1}{2}}\fallingdotseq 84.5$ minutes ; i.e. the Gyro-compass is so constructed that the ballistic deflection shall be amount exactly to the difference between the angle δ of the old course and the angle δ of the new course, so that the compass comes immediately to its new resting position, quite "dead-beat".

Here we consider the effect of acceleration upon the compensation weight. Even if there is acceleration in East-West direction, the compensation weight gives no action to the Gyro-wheel itself, because it is mounted on the mercury ballistic reservoir which is suspended from the outer-frame (Phantom). If there is acceleration in North direction, $\mathbf{b}_\mathrm{N}$, the horizontal line in the meridian, apparently dips by the amount of $\tan^{-1}(\mathbf{b}_\mathrm{N}/g)\fallingdotseq \mathbf{b}_\mathrm{N}/g$ at its north end ; that means, the Gyro axle rises apparently by the amount of $\mathbf{b}_\mathrm{N}/g$. But $\mathbf{b}_\mathrm{N}/g$ is such a small value as $\cos(\mathbf{b}_\mathrm{N}/g)\fallingdotseq 1$ and therefore we may consider that the torque about the horizontal axis of Gyro caused by the compensation weight is constant independently of the acceleration $\mathbf{b}_\mathrm{N}$. Moreover, if $\gamma=0$, the Gyro becomes undamping whether with compensation weight or without.

Accordingly the effect of acceleration upon the Gyro, caused by the compensation weight can be considered to be nothing.

§ 7. Applications.

1) *In the case of Sperry Type Gyro Compass whose angular momentum is,*

$H = 2.86 \times 10^9 gm.cm^2/sec.$

(Moment of inertia of the Gyro-wheel about its axle is

$I = 3.18 \times 10^6$ gm. cm^2

Speed for the rotation of the Gyro-wheel is 8,600/min.)

(The term, used by Japanese Navy formerly, was 5th type Sperry Gyro Compass).

The Gyro Compass of this type is so constructed that it has undamped period of 84.5 minutes for the following standard latitude λ_s. (Accordingly λ_s means the latitude for which Ballistic Deflection of Compass is "dead-beat").

λ_s	D	Range of Latitude
16°27′	$D_1 = 0.847 D_2$	0° ~ 27.5°
35°40′	D_2	27.5° ~ 45°
52°15′	$D_3 = 1.33 D_2$	45° ~ 54°
56°42′	$D_4 = 1.48 D_2$	54° ~ 66°

If λ means any latitude in question, then, as already mentioned, we get

$$\beta_0 = \frac{1}{2}(84.5 \times 60)^2 \cdot \frac{1}{T^{*2}} \cdot (\cos\lambda_s/\cos\lambda)\sin 2\lambda$$

$$= (84.5 \times 60/T^*)^2 \cos\lambda_s \sin\lambda.$$

These results are given in the next Table and Ffg. 3 (full-line).

λ	β_0	λ	β_0	λ	β_0	λ	β_0
0°	0′	27.5°	4′28″	45°	5′09″	54°	5′17″
5°	1′	30°	4′50″	50°	5′35″	55°	5′21″
10°	1′59″	35°18′	5′35″	52°15′	5′46″	56°42′	5′28″
15°	2′57″	35°40′	5′38″	54°	5′54″	60°	5′40″
16°27′	3′14″	40°	6′13″			66°	5′58″
20°	3′54″	45°	6′50″			70°	6′08″
25°	4′49″						
27.5°	5′16″						

The compensation mass "m_0" required for "$\beta_0 \to 0$" adjustment at Yokosuka Harbour ($\lambda = 35°18'$N, $g = 979.8$cm/sec²) is

$$m_0 = (H\omega\sin\lambda/gr)_{Yokosuka} \fallingdotseq (1.23 \times 10^2 gm.cm)/r.$$

Considering the construction of the Gyro-Compass of this type, we take,

$$r = 13.4\text{cm}$$

then, compensation mass "m_0" = 9.18 gm.

If we adjust the Gyro-Compass in this way, then the values of β_{0m} at various latitudes are

$$\beta_{0m} = \beta_0 - m_0 gr/D$$

while

$$m_0 gr/D_2 = (\beta_0)_{Yokosuka} = 5'35''$$

Hence

$$m_0gr/\mathrm{D}_1 = 5'35''/0.847 \fallingdotseq 6'36''$$
$$m_0gr/\mathrm{D}_3 = 5'35''/1.33 \fallingdotseq 4'12''$$
$$m_0gr/\mathrm{D}_4 = 5'35''/1.48 \fallingdotseq 3'46''$$

Therefore, $\beta_{0m} = \beta_0 - k$

where

from	0°	to 27.5°	'k'	is	6'36''
	27.5°	45°			5'35''
	45°	54°			4'12''
	54°	60°			3'46''

The results obtained from these formulae will be shown in Fig.3 (dotted-line).

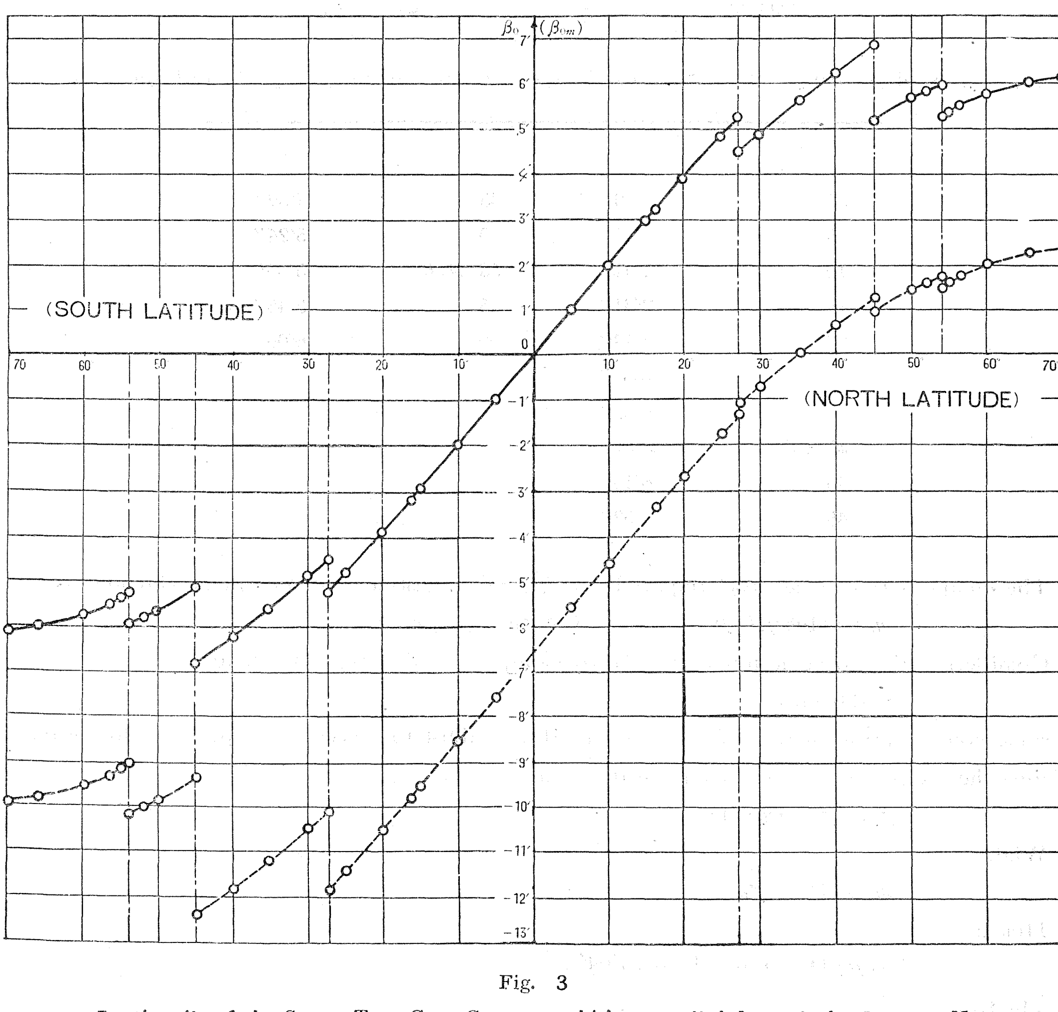

Fig. 3

Lasting-tilt of the Sperry Type Gyro Compass, which was called formerly by Japanese Navy "5th type".

(—— shows β_0 ; ······ shows β_{0m} when "$\beta_0 \to 0$" adjustment is made at Yokosuka Harbour.)

2) *In the case of Sperry Type Gyro Compass whose angular momentum is* $H=1.38\times10^9$ *gm.cm²/sec.*

$$\left(\begin{array}{l}\text{Moment of inertia of the Gyro-wheel about its axle is}\\ \qquad I=2.20\times10^6\ \text{gm.cm}^2\\ \text{Speed for the rotation of the Gyro-wheel is } 6000/\text{min.}\end{array}\right)$$

(The term used by Japanese Navy formerly, was Sperry Gyro Compass Type No.2.)

The Gyro Compass of this type is so constructed that it has undamped period of 84.5 minutes for the following standard latitude λ_s.

λ_s	D	Range of Latitude
38°18′	D_1	0° ~ 45°
53°44.6′	$D_2=1.38D_1$	45° ~ 60°

The values of β_0 are given in the next Table and are shown in Fig. 4 (full-line).

λ	β_0	λ	β_0
0°	0″	45°	4′59″
5°	51″	50°	5′24″
10°	1′41″	53°44.6′	5′40″
15°	2′31″	55°	5′46″
20°	3′19″	60°	6′06″
25°	4′05″		
30°	4′51″		
35°18′	5′37″		
40°	6′15″		
45°	6′52″		

The compensation mass "m_0" required for "$\beta_0\to0$" adjustment at Yokosuka Harbour is

$$m_0=(H\omega\sin\lambda/gr)_{Yokosuka}\doteqdot(58.8\text{gm.cm})/r.$$

Considering the construction of the Gyro Compass of this type, we take

$$r=20\ \text{cm},$$

then, compensation mass "m_0" $=2.94$ gm. If we adjust the Gyro Compass in this way, then the values of β_{0m} at various latitudes are

$$\beta_{0m}=\beta_0-mgr/D\,.$$

While

$$m_0gr/D_1=(\beta_0)_{Yokosuka}=5'37''.$$

Hence

$$m_0gr/D_2=5'37''/1.38\doteqdot4'04''.$$

Therfore,

$$\beta_{0m}=\beta_0-k$$

where

from 0° to 45°, 'k' is 5′37″
45° 60°, 4′04″

The results obtained from these formulae will be shown in Fig. 4 (dotted-line).

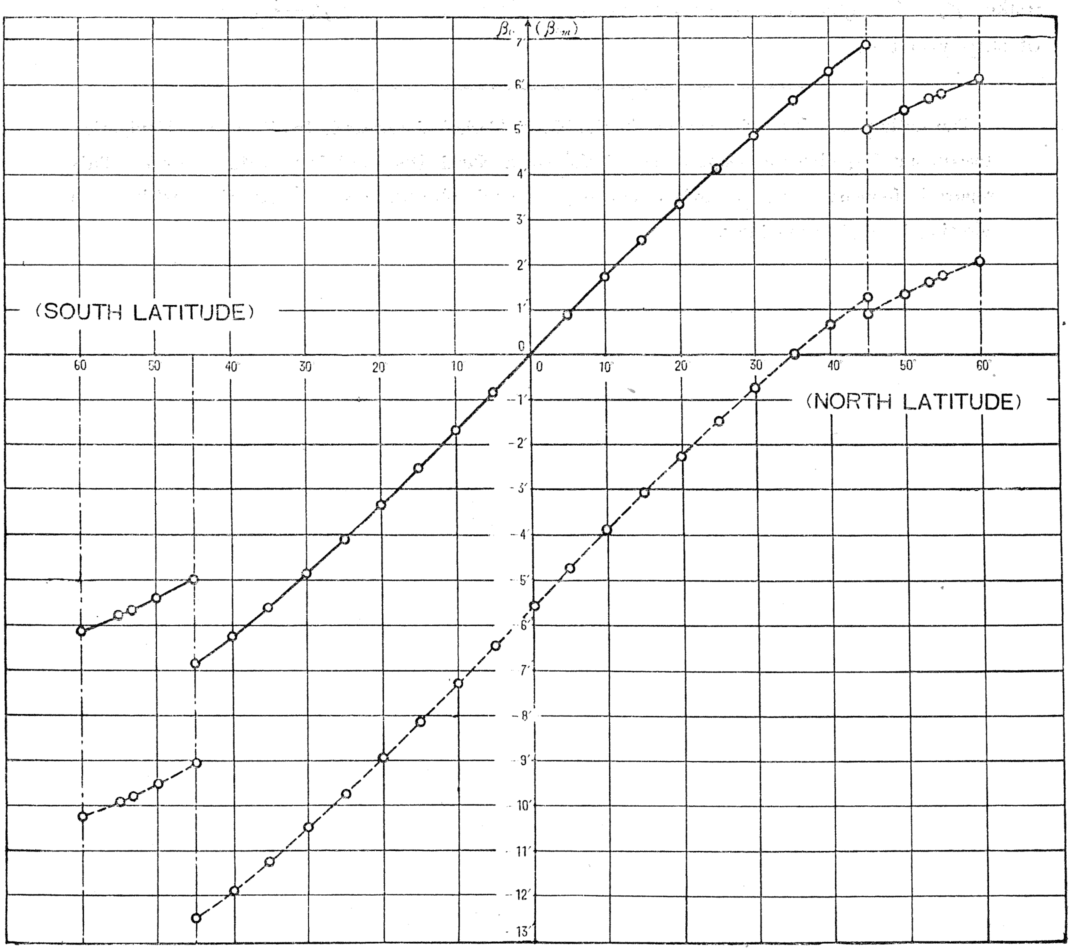

Fig. 4

Lasting-tilt of the Sperry Type Gyro Compass which was called formerly by Japanese Navy "Type No. 2".

(—— shows β_0 ; ······ shows β_{0m} when "$\beta_0 \to 0$" adjustment is made at Yokosuka Harbour.)

§ 8. Conclusion.

The "Lasting-tilt" of Sperry Type Gyro-Compass can be eliminated by mounting a suitable compensation weight on the center of the south-side mercury ballistic reservoir, without any chage of azimuthal resting position. Further, even when the compensation weight has not correct value, no change of azimuthal resting position will be caused. The above-mentioned results hold whether the Gyro-Compass is placed on a fixed position or mounted on a traveling ship with any speed and course. These results have also been proved by experimental observations. Though the compensation weight is a function of a latitude, practically we can not change its amount or position, during the operation of the Gyro-Compass, in other word, during a navigation. For what latitude we must make "$\beta_0 \to 0$" adjustment, must be determined by considering $\beta_0 - \lambda$ curve and the course of ship you take.

The author was formerly engaged in the Naval Technical Research Institute and Navigation Instrument Experimental Department of Yokosuka Naval Dockyard from 1942 to 1945. This report is founded on the researches and experimental observations at that time with some additional supplements later.