On the Solution of Ordinary Differential Equations by Nomographic Charts

by

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1. Introduction

In the present paper, the author tries to solve a system of simultaneous differential equations of the first order and a single ordinary differential equation of order higher than the first by using the net charts. The author applies the Milne's method and the Runge-Kutta's method to increase the accuracy of the required integrals and to save the troubles of computations. To illustrate the usefulness of this method for finding the first approximate value of the solution, some simple examples are shown.

2. Milne's method[1)]

For the system of simultaneous differential equations of the first order

$$y_j'=F_j\,(x, y_1, y_2, \cdots, y_m)\ \ (j=1,\,2,\cdots,\,m),\ \ (y_j'=\frac{dy_j}{dx}), \tag{1}$$

the values of y_j $(j=1,\,2,\cdots,\,m)$ at the $(n+1)$th step, namely, $y_{j,\,n+1}$ $(j=1,\,2,\,\cdots,\,m)$ are obtained from the following quadrature formulas not containing $y'_{j,n+1}$, say, the equations;

$$y_{j,\,n+1}=y_{j,\,n-3}+\frac{4h}{3}\,(2y'_{j,n-2}-y'_{j,n-1}+y'_{j,n})\ \ (j=1,\,2,\cdots,\,m), \tag{2}$$

where h is the length of the step.

With this approximate values of $y_{j,\,n+1}(j=1,\,2,\cdots,\,m)$, we can then compute an approximate values for $y'_{j,n+1}$ $(j=1,2,\cdots,m)$ from the given differential equations (1).

Using the following quadrature equations

$$y_{j,\,n+1}=y_{j,n-1}+\frac{h}{3}(y'_{j,n-1}+4y'_{j,n}+y'_{j,n+1})-\frac{1}{90}h^5y^{(5)}\quad (j=1,\,2,\cdots,\,m), \tag{3}$$

we have the improved values of $y_{j,n+1}$.

This method should be repeated until the values converge, then the limiting values $y_{j,n+1}$ are the $(n+1)$th step values of y_j $(j=1,\,2,\cdots,\,m)$.

3. Solution of a system of simultaneous differential equations by nomographic charts

We consider the following matrices M_j $(j=1,\,2,\cdots,m)$:

$$M_j=\begin{array}{ccccccc} (\bar{1}) & (\bar{2}) & & (\overline{n-3}) & (1) & & (n) \\ \left\|\begin{array}{c} f_1^{(j)}(t_1^{(j)}) \\ g_1^{(j)}(t_1^{(j)}) \\ h_1^{(j)}(t_1^{(j)}) \end{array}\right. & \begin{array}{c} f_2^{(j)}(t_2^{(j)}) \\ g_2^{(j)}(t_2^{(j)}) \\ h_2^{(j)}(t_2^{(j)}) \end{array} & \begin{array}{c} \cdots \\ \cdots \\ \cdots \end{array} & \begin{array}{c} f_{n-3}^{(j)}(t_{n-3}^{(j)}) \\ g_{n-3}^{(j)}(t_{n-3}^{(j)}) \\ h_{n-3}^{(j)}(t_{n-3}^{(j)}) \end{array} & \begin{array}{c} p_{1,2}^{(j)}(u_1,u_2) \\ q_{1,2}^{(j)}(u_1,u_2) \\ r_{1,2}^{(j)}(u_1,u_2) \end{array} & \begin{array}{c} \cdots \\ \cdots \\ \cdots \end{array} & \left.\begin{array}{c} p_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \\ q_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \\ r_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \end{array}\right\| \end{array}, \tag{4}$$

where $n=(m+2)-[\frac{m+2}{2}]$, $[\quad]$ is the Gauss's symbol, when m is odd, then u_{2n} is equal to zero and $(u_1, u_2, \cdots, u_{2n})$ is one of $(m+2)$! substitutions

$$\begin{pmatrix} x, & y_1, & y_2, & \cdots, & y_m, & y_j' \\ u_1, & u_2, & u_3, & \cdots, & u_{2n-1}, & u_{2n} \end{pmatrix}.$$

We assume the following two conditions in M_j :

1°. $f_l^{(j)}$, $g_l^{(j)}$, $h_l^{(j)}$, $p_{\lambda\mu}^{(j)}$, $q_{\lambda\mu}^{(j)}$ and $r_{\lambda\mu}^{(j)}$ are continuous functions of respective independent variable. $(j=1, 2, \cdots, m)$, $(l=1, 2, \cdots, n-3)$, $(\lambda=1, 3, \cdots, 2n-1)$, $(\mu=2, 4, \cdots, 2n)$.

2°. Each triplet of column vectors $\{(\bar{1}), (1), (2)\}$, $\{(\bar{1}), (\bar{2}), (3)\}$, $\{(\bar{2}), (\bar{3}), (4)\}, \cdots$, $\{(\overline{n-4}), (\overline{n-3}), (n-2)\}$ and $\{(\overline{n-3}), (n-1), (n)\}$ is linearly dependent, respectively.

Then the matrices M_j $(j=1, 2, \cdots, m)$ are called Massau's $(m+2)$ variable chart matrices of separable type. From the second condition, we have the following relations

$$\det(M_{j,k})=0 \quad (j=1, 2, \cdots, m;\ k=1, 2, \cdots, n-2), \tag{5}$$

where

$$M_{j,1}=\begin{Vmatrix} f_1^{(j)}(t_1^{(j)}) & p_{1,2}^{(j)}(u_1,u_2) & p_{3,4}^{(j)}(u_3,u_4) \\ g_1^{(j)}(t_1^{(j)}) & q_{1,2}^{(j)}(u_1,u_2) & q_{3,4}^{(j)}(u_3,u_4) \\ h_1^{(j)}(t_1^{(j)}) & r_{1,2}^{(j)}(u_1,u_2) & r_{3,4}^{(j)}(u_3,u_4) \end{Vmatrix},\quad M_{j,2}=\begin{Vmatrix} f_1^{(j)}(t_1^{(j)}) & f_2^{(j)}(t_2^{(j)}) & p_{5,6}^{(j)}(u_5,u_6) \\ g_1^{(j)}(t_1^{(j)}) & g_2^{(j)}(t_2^{(j)}) & q_{5,6}^{(j)}(u_5,u_6) \\ h_1^{(j)}(t_1^{(j)}) & h_2^{(j)}(t_2^{(j)}) & r_{5,6}^{(j)}(u_5,u_6) \end{Vmatrix},$$

$$\cdots\cdots,\quad M_{j,k}=\begin{Vmatrix} f_{k-1}^{(j)}(t_{k-1}^{(j)}) & f_k^{(j)}(t_k^{(j)}) & p_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2}) \\ g_{k-1}^{(j)}(t_{k-1}^{(j)}) & g_k^{(j)}(t_k^{(j)}) & q_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2}) \\ h_{k-1}^{(j)}(t_{k-1}^{(j)}) & h_k^{(j)}(t_k^{(j)}) & r_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2}) \end{Vmatrix},\quad \cdots\cdots,$$

$$\cdots\cdots,\quad M_{j,n-3}=\begin{Vmatrix} f_{n-4}^{(j)}(t_{n-4}^{(j)}) & f_{n-3}^{(j)}(t_{n-3}^{(j)}) & p_{2n-5,2n-4}^{(j)}(u_{2n-5},u_{2n-4}) \\ g_{n-4}^{(j)}(t_{n-4}^{(j)}) & g_{n-3}^{(j)}(t_{n-3}^{(j)}) & q_{2n-5,2n-4}^{(j)}(u_{2n-5},u_{2n-4}) \\ h_{n-4}^{(j)}(t_{n-4}^{(j)}) & h_{n-3}^{(j)}(t_{n-3}^{(j)}) & r_{2n-5,2n-4}^{(j)}(u_{2n-5},u_{2n-4}) \end{Vmatrix},\quad \cdots\cdots,$$

$$M_{j,n-2}=\begin{Vmatrix} f_{n-3}^{(j)}(t_{n-3}^{(j)}) & p_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2}) & p_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \\ g_{n-3}^{(j)}(t_{n-3}^{(j)}) & q_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2}) & q_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \\ h_{n-3}^{(j)}(t_{n-3}^{(j)}) & r_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2}) & r_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n}) \end{Vmatrix},\quad (j=1,2,\cdots,m) \tag{6}$$

and $M_{j,k}$ is called a Massau's curvilinear net chart matrix of the third order.

If the system of simultaneous differential equation of the first order

$$F_j(x, y_1, y_2, \cdots, y_m, y_j')=0 \quad (j=1, 2, \cdots, m) \tag{7}$$

are represented by the above Massau's $(m+2)$ variable chart matrices satisfying the relation (6), we have a group of alignment charts being composed of curvilinear functional scales and curvilinear nets. The net charts $M_{j,1}$, $M_{j,k}$ $(k=2, 3, \cdots, n-3)$ and $M_{j,n-2}$ $(j=1, 2, \cdots, m)$ are represented by the following relations in Cartesian coordinates (ξ, η).

For $M_{j,1}$,

$$(t_1^{(j)}) : \quad \xi=\frac{f_1^{(j)}(t_1^{(j)})}{h_1^{(j)}(t_1^{(j)})}, \quad \eta=\frac{g_1^{(j)}(t_1^{(j)})}{h_1^{(j)}(t_1^{(j)})};$$

$$(u_1,u_2) : \quad \xi=\frac{p_{1,2}^{(j)}(u_1,u_2)}{r_{1,2}^{(j)}(u_1,u_2)}, \quad \eta=\frac{q_{1,2}^{(j)}(u_1,u_2)}{r_{1,2}^{(j)}(u_1,u_2)};$$

$$(u_3,u_4) : \quad \xi=\frac{p_{3,4}^{(j)}(u_3,u_4)}{r_{3,4}^{(j)}(u_3,u_4)}, \quad \eta=\frac{q_{3,4}^{(j)}(u_3,u_4)}{r_{3,4}^{(j)}(u_3,u_4)}.$$

For $M_{j,k}$,

$$(t_{k-1}^{(j)}) : \quad \xi=\frac{f_{k-1}^{(j)}(t_{k-1}^{(j)})}{h_{k-1}^{(j)}(t_{k-1}^{(j)})}, \quad \eta=\frac{g_{k-1}^{(j)}(t_{k-1}^{(j)})}{h_{k-1}^{(j)}(t_{k-1}^{(j)})};$$

$$(t_k^{(j)}) : \quad \xi=\frac{f_k^{(j)}(t_k^{(j)})}{h_k^{(j)}(t_k^{(j)})}, \quad \eta=\frac{g_k^{(j)}(t_k^{(j)})}{h_k^{(j)}(t_k^{(j)})};$$

$$(u_{2k+1},u_{2k+2}) : \quad \xi=\frac{p_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2})}{r_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2})}, \quad \eta=\frac{q_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2})}{r_{2k+1,2k+2}^{(j)}(u_{2k+1},u_{2k+2})}$$

$$(k=2,3,\cdots,n-3).$$

For $M_{j,n-2}$,

$$(t_{n-3}^{(j)}) : \quad \xi=\frac{f_{n-3}^{(j)}(t_{n-3}^{(j)})}{h_{n-3}^{(j)}(t_{n-3}^{(j)})}, \quad \eta=\frac{g_{n-3}^{(j)}(t_{n-3}^{(j)})}{h_{n-3}^{(j)}(t_{n-3}^{(j)})};$$

$$(u_{2n-3},u_{2n-2}) : \quad \xi=\frac{p_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2})}{r_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2})}, \quad \eta=\frac{q_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2})}{r_{2n-3,2n-2}^{(j)}(u_{2n-3},u_{2n-2})};$$

$$(u_{2n-1},u_{2n}) : \quad \xi=\frac{p_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n})}{r_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n})}, \quad \eta=\frac{q_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n})}{r_{2n-1,2n}^{(j)}(u_{2n-1},u_{2n})}. \tag{8}$$

By the principle of the net charts, three points $\{(t_1^{(j)}),(u_1,u_2),(u_3,u_4)\}$, $\{(t_{k-1}^{(j)}),(t_k^{(j)}),(u_{2k+1},u_{2k+2})\}$ and $\{(t_{n-3}^{(j)}),(u_{2n-3},u_{2n-2}),(u_{2n-1},u_{2n})\}$ $(k=2,3,\cdots,n-3)$ are collinear, respectively. Therefore, when the values $x, y_1, y_2, \cdots, y_m$ are given, we can find out the values $y_j'(j=1,2,\cdots,m)$, namely, by $m(n-2)$ charts, we can know values of $y'_{j,n+1}(j=1,2,\cdots,m)$ and the corrected values $y_{j,n+1}$ $(j=1,2,\cdots,m)$.

4. Solution of the ordinary differential equation of higher order than the first by nomographic charts.

By the theory of differential equations, the ordinary differential equations of order higher than the first can always are replaced by an equivalent system of the first order simultaneous differential equation. Therefore, the above method can be applied. But we may try to solve it by using the Runge-Kutta's method[2]. Runge-Kutta's scheme for ordinary differential equation is shown by the following tables:

1. $y''=f(x, y, y')$.

x	y	$hy'=v_1$	$k_\nu=\frac{h^2}{2}f\left(x,y,\frac{v_1}{h}\right)$	correction
x_0	y_0	v_{10}	k_1	
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}k_1$	$v_{10}+k_1$	k_2	$k=\frac{1}{3}(k_1+k_2+k_3)$
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}k_1$	$v_{10}+k_2$	k_3	$k'=\frac{1}{3}(k_1+2k_2+2k_3+k_4)$
x_0+h	$y_0+v_{10}+k_3$	$v_{10}+2k_3$	k_4	
$x_1=x_0+h$	$y_1=y_0+v_{10}+k$	$v_{11}=v_{10}+k'$		

2. $y'''=f(x, y, y', y'')$.

x	y	$hy'=v_1$	$\frac{h^2}{2}y''=v_2$	$k_\nu=\frac{h^3}{6}f\left(x,y,\frac{v_1}{h},\frac{2v_2}{h^2}\right)$
x_0	y_0	v_{10}	v_{20}	k_1
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}v_{20}+\frac{1}{8}k_1$	$v_{10}+v_{20}+\frac{3}{4}k_1$	$v_{20}+\frac{3}{2}k_1$	k_2
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}v_{20}+\frac{1}{8}k_1$	$v_{10}+v_{20}+\frac{3}{4}k_1$	$v_{20}+\frac{3}{2}k_2$	k_3
x_0+h	$y_0+v_{10}+v_{20}+k_3$	$v_{10}+2v_{20}+3k_3$	$v_{20}+3k_3$	k_4
$x_1=x_0+h$	$y_1=y_0+v_{10}+v_{20}+k$	$v_{11}=v_{10}+2v_{20}+k'$	$v_{21}=v_{20}+k''$	

where $k=\frac{1}{20}(9k_1+6k_2+6k_3-k_4)$, $k'=k_1+k_2+k_3$ and $k''=\frac{1}{2}(k_1+2k_2+2k_3+k_4)$.

3. $y^{(\mathrm{IV})}=f(x, y, y', y'', y''')$.

x	y	$hy'=v_1$	$\frac{h^2}{2}y''=v_2$	$\frac{h^3}{6}y'''=v_3$	$k_\nu=\frac{h^4}{24}f\left(x,y,\frac{v_1}{h},\frac{2v_2}{h^2},\frac{6v_3}{h^3}\right)$
x_0	y_0	v_{10}	v_{20}	v_{30}	k_1
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}v_{20}+\frac{1}{8}v_{30}+\frac{1}{16}k_1$	$v_{10}+v_{20}+\frac{3}{4}v_{30}+\frac{1}{2}k_1$	$v_{20}+\frac{3}{2}v_{30}+\frac{3}{2}k_1$	$v_{30}+2k_1$	k_2
$x_0+\frac{1}{2}h$	$y_0+\frac{1}{2}v_{10}+\frac{1}{4}v_{20}+\frac{1}{8}v_{30}+\frac{1}{16}k_1$	$v_{10}+v_{20}+\frac{3}{4}v_{30}+\frac{1}{2}k_1$	$v_{20}+\frac{3}{2}v_{30}+\frac{3}{2}k_1$	$v_{30}+2k_2$	k_3
x_0+h	$y_0+v_{10}+v_{20}+v_{30}+k_3$	$v_{10}+2v_{20}+3v_{30}+4k_3$	$v_{20}+3v_{30}+6k_3$	$v_{30}+4k_3$	k_4
$x_1=x_0+h$	$y_1=y_0+v_{10}+v_{20}+v_{30}+k$	$v_{11}=v_{10}+2v_{20}+3v_{30}+k'$	$v_{21}=v_{20}+3v_{30}+k''$	$v_{31}=v_{30}+k'''$	

where $k=\frac{1}{15}(8k_1+4k_2+4k_3-k_4)$, $k'=\frac{1}{5}(9k_1+6k_2+6k_3-k_4)$, $k''=2(k_1+k_2+k_3)$ and $k'''=\frac{2}{3}(k_1+2k_2+2k_2+k_4)$.

If the differential equations

$$G_1(x, y, y', y'')=0,\ G_2(x, y, y', y'', y''')=0 \text{ or } G_3(x, y, y', y'', y''', y^{(IV)})=0$$

are represented by the above Massau's curvilinear net chart determinant, respectively, by using the net charts, we can find out the values k_1, k_2, k_3 and k_4 in the above tables in the same way as in §3. Therefore, we can know the value $y=y_1$ at $x=x_1 =x_0+h$.

5. Examples

1°. We try to solve a system of simultaneous differential equations

$$y_1'=y_1-x(y_1-y_2),\quad y_2'=y_2+x(y_1-y_2),$$

where the initial conditions are $y_1=1$ and $y_2=0$ at $x=0$.

The chart matrices for these equations are represented by

$$M_1=\begin{Vmatrix} x & 0 & 1 \\ y_1' & y_1 & y_2 \\ 1 & 1 & 1 \end{Vmatrix} \quad \text{and} \quad M_2=\begin{Vmatrix} 1-x & 1 & 0 \\ y_2' & y_2 & y_1 \\ 1 & 1 & 1 \end{Vmatrix}.$$

These charts are shown in Fig.1. The solutions $y_{1,1}, y_{2,1}$ of our method are compared numerically with the solutions $y_{1,2}, y_{2,2}$ of exact solutions $y_1=\frac{e^x}{2}(1+e^{-x^2}), y_2=\frac{e^x}{2}(1-e^{-x^2})$ in the table 1.

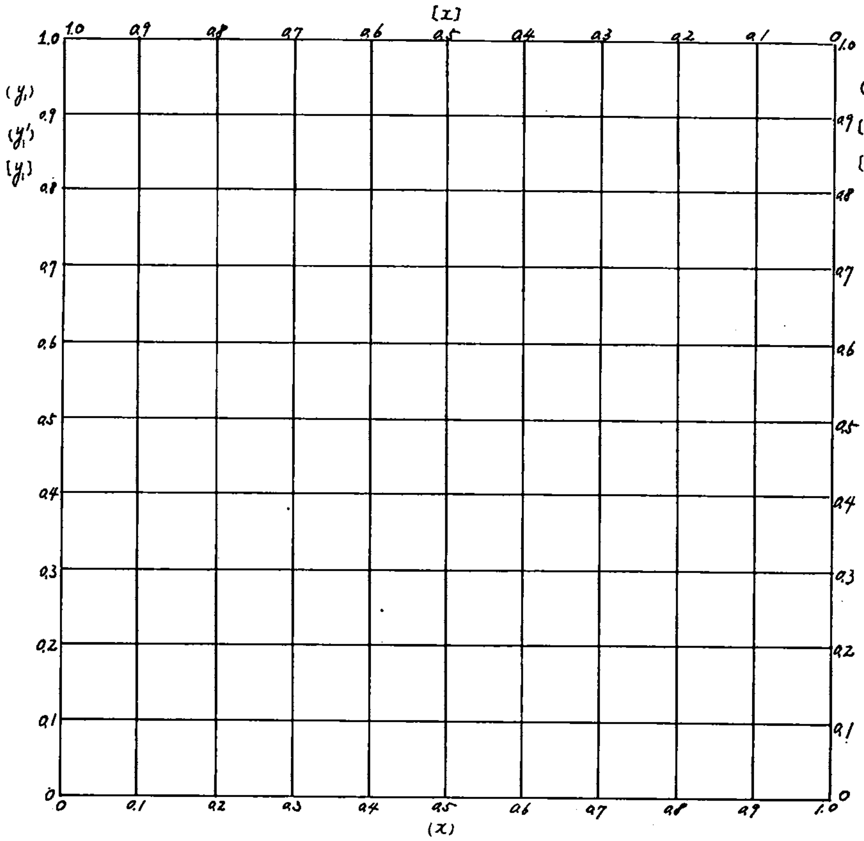

Fig. 1. Relation between $\{(x), (y_1), (y_2), (y_1')\}$ and $\{[x], [y_1], [y_2], [y_2']\}$.

Table. 1

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y_{1,1}$	1	1.100	1.198	1.291	1.382	1.466	1.547	1.624	1.700	1.777	1.859
$y_{2,1}$	0	0.006	0.024	0.058	0.110	0.182	0.275	0.390	0.526	0.683	0.859
$y_{1,2}$	1	1.0997	1.1975	1.2918	1.3815	1.4664	1.5467	1.6237	1.6995	1.7769	1.8591
$y_{2,2}$	0	0.0055	0.0239	0.0581	0.1103	0.1823	0.2754	0.3900	0.5260	0.6827	0.8591

To use the Milne's method we have to know the starting values. In this example we used the next starting values.

x	y_1	y_2	y_1'	y_2'
−0.3	0.709	0.032	0.912	−0.171
−0.2	0.803	0.016	0.960	−0.141
−0.1	0.900	0.005	0.990	−0.085
0	1	0	1	0

2°. Solve a system of simultaneous differential equations

$$y_2 y_1'^2+(y_2 x+x-y_2^2-y_1 y_2)y_1'-y_1 y_2=0,\quad y_2'(y_1+y_2)-x(y_1+y_2-1)-y_1+y_2=0,$$

where the initial conditions are $y_1=0, y_2=1$ at $x=0$.

The chart matrices for these equations are

$$M_{1,1}=\begin{Vmatrix} 0 & y_1'+1 & 1 \\ t & y_1'^2 & y_1 \\ 1 & 1 & 1 \end{Vmatrix},\quad M_{1,2}=\begin{Vmatrix} 0 & y_2+1 & 1 \\ t & y_2^2 & x \\ 1 & 1 & 1 \end{Vmatrix} \text{ and } M_2=\begin{Vmatrix} 0 & y_1+y_2 & 1 \\ x & y_1-y_2 & y_2' \\ 1 & 1 & 1 \end{Vmatrix}.$$

These charts are shown in Fig.2. We used the starting values in the following table.

x	y_1	y_2	y_1'	y_2'
−0.2	−0.220	1.243	1.200	−1.435
−0.1	−0.105	1.110	1.100	−1.210
0	0	1	1	−1
0.1	0.095	0.910	0.900	−0.810
0.2	0.180	0.837	0.803	−0.642

By our method, we obtain the next table 2.

Table 2

x	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_1	0.256	0.323	0.382	0.435	0.484	0.529	0.571	0.611
y_2	0.781	0.737	0.706	0.686	0.647	0.672	0.677	0.690

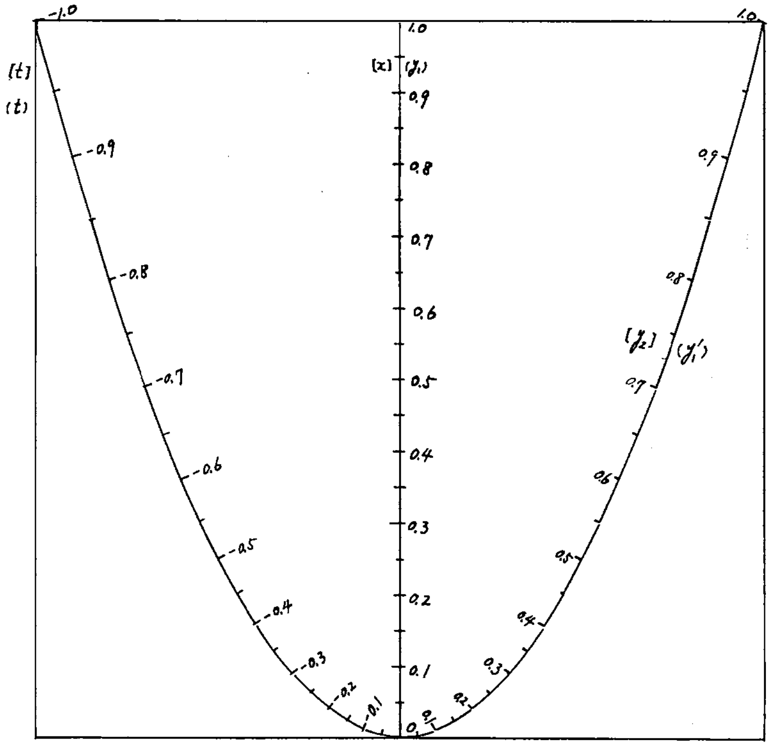

Fig. 2. 1. Relation between $\{[x], [y_2], [t]\}$ and $\{(y_1), (y_1'), (t)\}$.

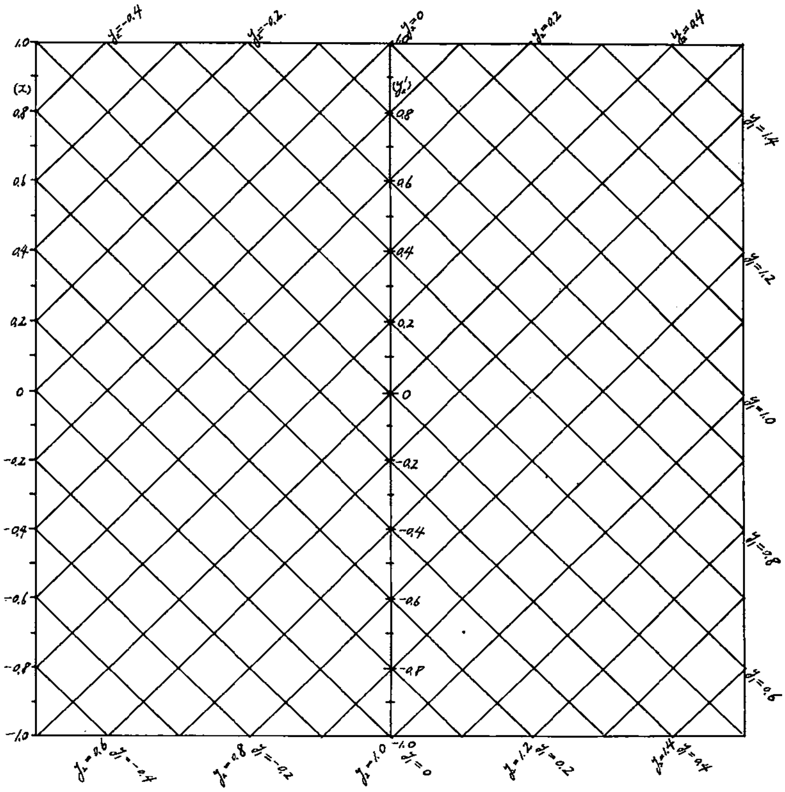

Fig. 2. 2. Relation between $\{(x), (y_1), (y_2), (y_2')\}$.

3°. We consider the ordinary differential equation of the second order

$$xy''+(b-x)y'-ay=0.$$

The chart matrix for this equation is

$$M=\begin{Vmatrix} l_2 y'' & l_1 y' & l_1 l_2 \frac{ay}{x} \\ 1 & 0 & l_1 \\ 1 & 1 & l_1+l_2\left(\frac{b}{x}-1\right) \end{Vmatrix}.$$

When we put $b=2$ and $a=-2$, we have the equation

$$xy''+(2-x)y'+2y=0.$$

This chart is shown in Fig. 3. By our method, we have the following values under the initial conditions $y=-0.5$, $y'=0.5$ at $x=1$.

Table 3

x	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
y	−0.636	−0.637	−0.619	−0.588	−0.548	−0.5	−0.448	−0.392	−0.333	−0.272	−0.210

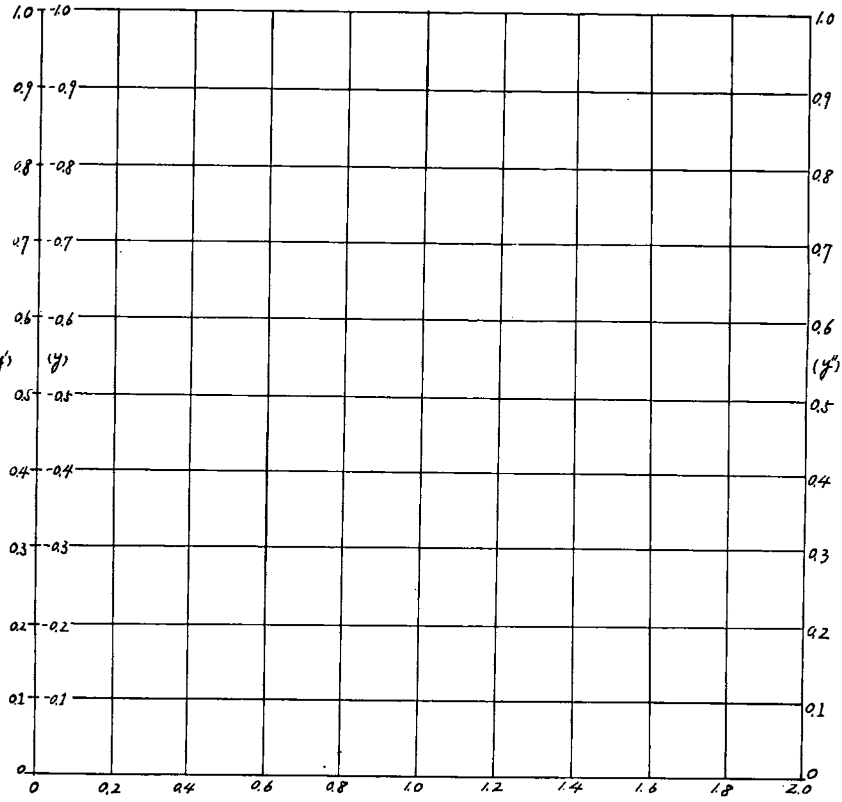

Fig. 3. Relation between $\{(x), (y), (y'), (y'')\}$.

4°. Trying to solve the ordinary differential equation of the third order

$$(x+y+1)(2y'+1)y'''+y'''-(x+y)y''+(x+1)(y'+1)=0,$$

where the initial conditions are $y=1$, $y'=-\frac{1}{2}$ and $y''=\frac{1}{2}$ at $x=0$.

The chart matrix for this equation is

$$M=\left\| \begin{matrix} 1 & -y' & \frac{x+y+2}{2(x+y+1)} \\ -y''' & y'' & \frac{x+1}{2(x+y+1)} \\ 1 & 1 & 1 \end{matrix} \right\| .$$

This chart is shown in Fig.4. By our method, we have the following values.

Table 4

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	1	0.953	0.910	0.872	0.839	0.811	0.786	0.766	0.749	0.736	0.726

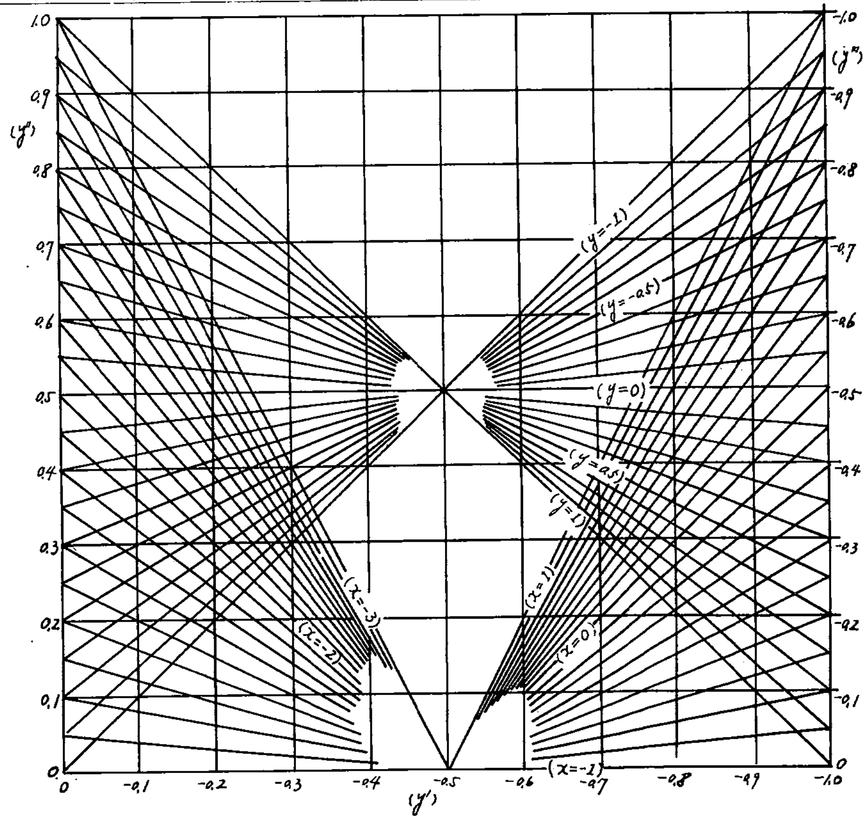

Fig. 4. Relation between $\{(x), (y), (y'), (y''), (y''')\}$.

5°. We try to solve the ordinary differential equation of the fourth order

$$y^{(IV)}y'''y'-(1+y''')\{(x-1)(1-y')+yy'\}+y'y''y'''=0,$$

where the initial conditions are $y=0, y'=1, y''=0$ and $y'''=-1$ at $x=0$.

The chart matrices are

$$M_1=\left\| \begin{matrix} 0 & 1 & y' \\ -x+1 & t & yy' \\ 1 & 1 & 1 \end{matrix} \right\| \quad \text{and} \quad M_2=\left\| \begin{matrix} 1 & 0 & y'''+1 \\ t & y^{(IV)} & y''\,y''' \\ 1 & 1 & 1 \end{matrix} \right\|$$

and the associated charts are shown in Fig. 5.

By our method, we have the following table.

Table 5

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	0	0.100	0.199	0.295	0.389	0.479	0.565	0.644	0.717	0.783	0.841

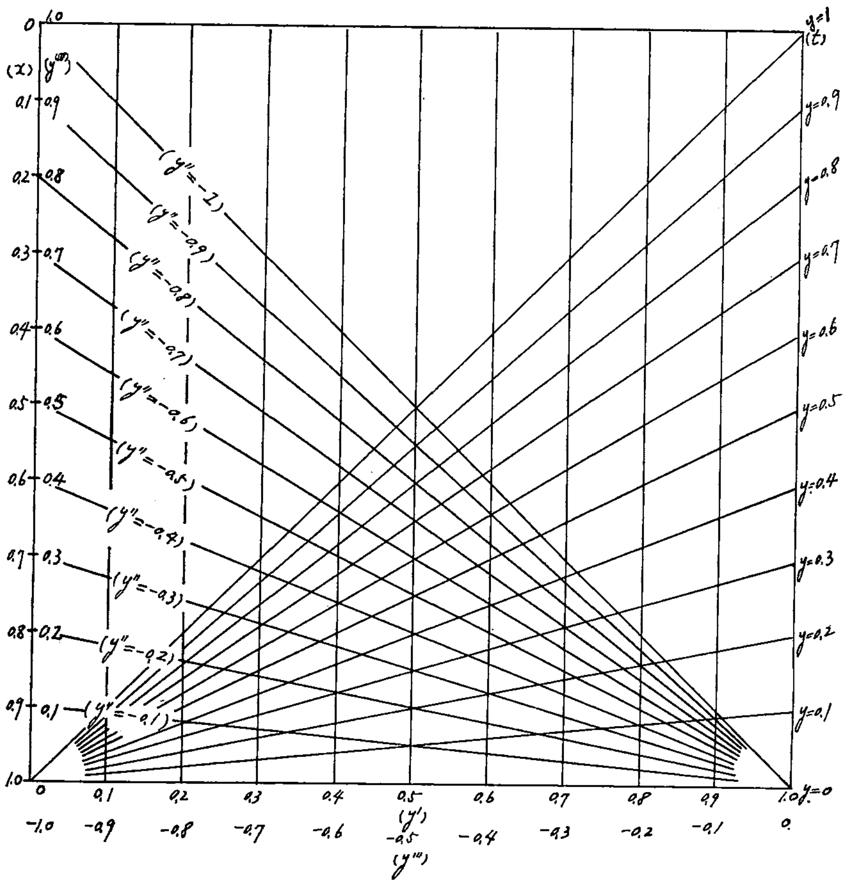

Fig. 5. Relation between $\{(x), (y), (y'), (y''), (y'''), (y^{(IV)})\}$.

The original charts of our examples have been drawn on the millimeter section papers 700×1000mm.

6. Conclusion

In this paper the author has found the method to obtain the approximate solution of the ordinary differential equations by using the net charts. This method will be useful when derivatives are involved implicitly and numerical computation is hard to be carried out. But our method being graphical, we can not expect the higher accurate solution.

References

1) Milne, W. E. : Numerical solution of Differential equations, New York (1953), p. 65.

2) Zurmühl, R. : Runge-Kutta-Verfahren zur numerischen Integration von Differentialgleichungen n-ter Ordnung. Z. A. M. M. Bd. 28 (1948), 173-182.