New Sliding Puzzle with Neighbors Swap Motion

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Abstract. *The sliding puzzles (15-puzzle, 8-puzzle, 5-puzzle) are known to have 2 kind of puzzle: solvable puzzle and unsolvable puzzle. In this thesis, we make a new puzzle with only 1 kind of it, solvable puzzle. This new puzzle is made by adopting sliding puzzle with several additional rules from M_{13} puzzle; the puzzle that is formed form The Mathieu group M_{13}. This puzzle has a movement that called a neighbors swap motion, a rule of movement that enables every neighboring points to swap. This extra rule make of new puzzle become possible to be solved, whatever the initial state is.*

Keywords: sliding puzzle, 15-puzzle, 8-puzzle, 5-puzzle, M13 puzzle, neighbors swap motion.

1 Introduction

Sliding puzzle is a puzzle that challenges a player to slide on at pieces along certain routes (usually on a board) to establish a certain end-configuration. Unlike other tour puzzles, a sliding puzzle prohibits lifting any pieces on the board. This property distinguish sliding puzzles from rearrangement puzzles. Hence, finding the movements and the paths open up by each movement within the two-dimensional connes of the board are important parts of solving sliding block puzzles.

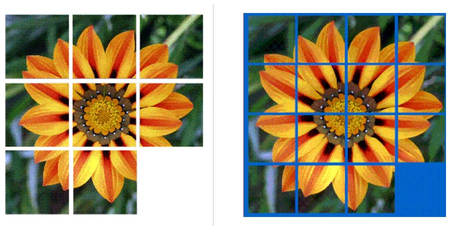

Figure 1: Example of Sliding puzzle (3x3 Sliding Puzzle)

2 15-puzzle

Based on the book written by Edward Hordern in 1986 [4], there is a kind of sliding puzzle that is well known as 15-puzzle. It is a 4x4 square that consists of 15 square tiles in random order and one missing tile. These tiles are numbered from 1 until 15 definitely.

The objective of this game is to set the random tiles to be arranged order by sliding the tiles using the empty space. It is prohibited to lift the tiles, of course. At the end, we will obtain the arrangement of the puzzle with tiles numbered by 1-4 in the first row, 5-8 in the second row, 9-12 in the third row, and 13-15 in the last row added by the empty space.

Generally, there are two type of this puzzle: solvable 15-puzzle and unsolvable 15-puzzle. The unsolvable puzzle is puzzle that have a state that cannot be solved whatever we slide it. Johnson (1879), Story (1879), and Archer (1999) said that the 15-puzzle have state or condition that is impossible to solve. Therefore not all of 15-puzzle can arranged.

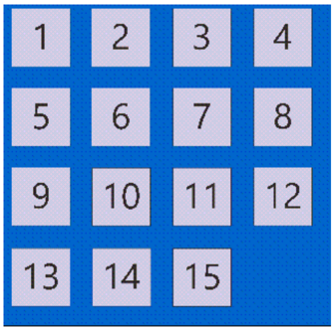

Figure 2: Example of 15-puzzle

3 8-puzzle and 5-puzzle

This sliding puzzle also exists in other size, particularly smaller 8-puzzle and 5-puzzle. Noyes Palmer Chapman is the one who invented and popularized this size of puzzle in 1870s. Similarly with the 15-puzzle, it is also played on a square that consists of some tiles and an empty space, but the size is only 3x3 for 8-puzzle and 2x3 for 5-puzzle. With the purpose of how to solve and the rules are, we have to arrange the tiles by sliding it horizontally or vertically without lifting the tiles, so that they are well ordered (ascending ordered).

Generally, there are two type of this puzzle: solvable puzzle and unsolvable puzzle like 15-puzzle

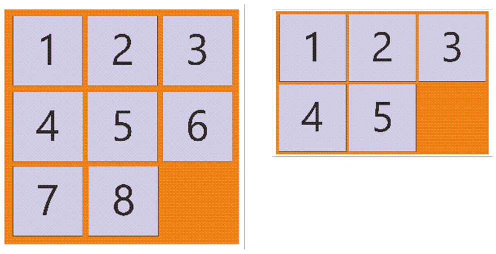

Figure 3: Example of 8-puzzle and 5-puzzle

4 M_{13} Puzzle

M_{13} puzzle is a puzzle that formed form The Mathieu group M_{13}.

M_{13} consists of the permutations of counters that can be obtained by composing finite sequence of moves. This is not a group, since we can only compose two moves if the point left empty after carrying out the first move is the empty point at the start of the second move. More precisely:

the objects of M13 are the 13 positions of the point that's left empty. The morphisms are the permutations of counters that arise from finite sequences of compostable moves.

In How to Play M_{13} by Sebastian Egner and Thomas Beth [3], The Puzzle can be shown as this Figure. The image represent a projective geometry that basic of the M_{13} puzzle. The projective geometry can be thought as the set of one dimensional subspaces of the three dimensional space $\mathbb{F}^3$. A points in this geometry are thirteen one-dimensional subspaces, it indicated by red dot. In addition, the lines at this geometry are thirteen two-dimensional subspaces, it is represented by black triangle. The connection between red dot and triangle is incidence. The red dot is adjacent with triangle if the point is incident to the line.

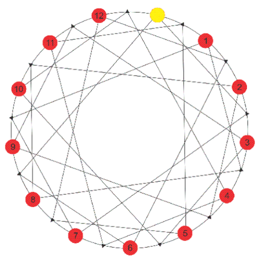

Figure 4: Example of 8-puzzle and 5-puzzle

The puzzle is constructed by putting number 1 until 12 to red dots. The last red dot that doesn't have number called a hole. The basic movement in this puzzle are: 1. Pick a number between 1 and 12, so there is a shortest path to move a number to hole. The path consist one triangle and 2 lines that connect them. so the last state is the picked number became hole and hole became a number that picked before. 2. Exchange the number except a picked number and hole that adjacent to the triangle in the path(from step 1). This movement is called a neighbors swap motion.

The rule in swap motion from M_{13} puzzle swap the adjacency point after we move the target point to the hole. In the M_{13} puzzle we use connectivity between point an triangle, In 8-puzzle and 5-puzzle we use the adjacency between cell.

5 The New 8-puzzle and 5-puzzle

From neighbors swap motion in M_{13} puzzle we can make a new rule with same idea. The idea is a movement that consider with the adjacent with neighbourhood. In this puzzle the rule is adding some extra movement when we slide one of cell.

Assume we move the target cell to empty cell. The movement of new rule is if the number of the cell that is adjacent to the target is 2, the extra movement is swapping between 2 of adjacent cell.

Because 8-puzzle has one more possibilities about number of adjacent cell. Assume we move the target cell to empty cell. The new rule for 8-puzzle become:

1. If the number of the cell that adjacent to the target is 2, the extra movement is swapping between 2 of adjacent cell.
2. If the number of the box that adjacent to the target is 3, the extra movement is swapping only cell that not same row or same column with target cell.

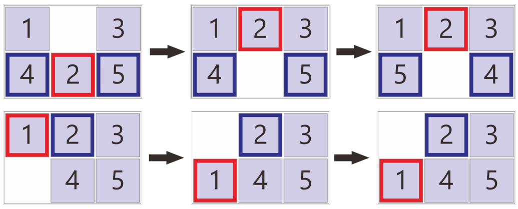

Figure 5: Example of 5-puzzle

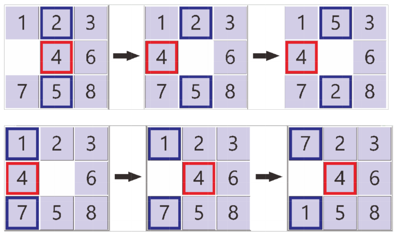

Figure 6: Example of 8-puzzle

6 Solvability of New Puzzle

These new puzzles (new 5-puzzle and new 8-puzzle) can be represented in graph theory as set of vertices and edges. The vertices is a every state of the puzzle (every permutation of the {1,2,3,4,5,0} for 5-puzzle and {1,2,3,4,5,6,7,0} for 8-puzzle) and the edges is the possibilities of the change of the state if we move the hole (0).

Definition 6.1 *$G_5 = (V_5, E_5)$ is a graph with:*

1. *$V_5 :=$ Vertices that represent a state of 5-puzzle,*
2. *$E_5 :=$ Edges that represent possibility between two state.*

Definition 6.2 *$G_5 = (V_8, E_8)$ is a graph with:*

1. *$V_8 :=$ Vertices that represent a state of 8-puzzle,*
2. *$E_8 :=$ Edges that represent possibility between two state.*

With this graph we can find there a possible path between every state. This path indicate there exist a movement/s that makes a state became other state. In other word we can find a possibility path from every state to solved state (state when we finished the puzzle). If there exist that path, we can say that state can be solved.

6.1 Solvability of new 5-puzzle

With definition above we get $|V_5| = 720$ and $|E| = 1680$. And with Breadth-first Search (BFS), we can find path of every state that connect it to solved state. The result is G_5 has 720 vertices that have path to solved state. That means every state can be a solve state. In other word we can solve all of new 5-puzzle. Therefore new 5-puzzle didn't have a unsolvable state.

6.2 Solvability of new 8-puzzle

Theorem 6.1 *Every two adjacent cell in new 8-puzzle can be exchanged.*

Let G_8 is a graph representation of state of 8-puzzle. The existence of path in G_8 from vertex v_1 to v_2 means the possibilities movement that can change the state of puzzle that is represented by v_1 to state that is represented by v_2. If the path exist then the movement that can change that state also exists.

Pick 2 cell in 8-puzzle. We will find whether the cell can exchange without change the position of the other cell. 2 cell can exchange if there exists a movement that change the state from original to exchanged state (only 2 cell are changed) or in other words there exists a path from original state vertex to exchanged state vertex. With BFS we can get path that connects that vertices. Because there exist a path for all case so every two adjacent cell in new 8-puzzle can be exchanged.

Theorem 6.2 *Every state of puzzle can be solved.*

Because every two adjacent cell in puzzle can be exchanged, we can exchange any two cells in this puzzle. Therefore we can make a solved state from any state of puzzle.

7 Summary

The regular 5-puzzle and 8-puzzle have 2 types of puzzle: solvable puzzle and unsolvable puzzle. With the rule/movement from M_{13} puzzle (Neighbors swap motion) we can make new 5-puzzle and new 8-puzzle. These puzzles only have one type of puzzle, solvable puzzle.

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