Dynamic Hysteresis Loop and its Application to a Series Ferroresonance Circuit

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ABSTRACT : A lot of articles concerning ferroresonance phenomena in electric power distribution systems consider a single-value resistor connected in parallel with a nonlinear inductor (described by magnetization characteristic) to represent core losses of the transformer under study. Several studies and measurements as well, proved that such a resistor cannot properly describe the behavior of transformer in saturated area. Furthermore, such a simplification has quite essential effect on the exact birth of ferroresonance. Therefore, better and more complex model of the transformer core losses is necessary. This paper follows suggestions recommended for further improvement of the ferroresonance studies in electric power systems, by considering a dynamic behavior of transformer core losses. Comparison of the results received by using static (single-value resistor) and dynamic (nonlinear resistor) representation of core losses are presented.

Key words: bifurcation diagram, chaotic oscillations, ferroresonance, hysteresis curve, magnetization characteristic, sensitivity analysis

1. INTRODUCTION

A lot of circuits in which resonance can appear, contain windings with ferromagnetic cores. Inductance of these cores is not stable, but due to the non-linearity of magnetic curves it depends on the immediate magnetic state (saturation) of the winding with ferromagnetic core. The phenomenon called ferroresonance can originate from this dependability. The non-linear inductance causes the existence of more than one resonant frequency for given parameter values. The voltage across the capacitor or inductor does not change continuously with the increasing frequency of the supply voltage but jumps suddenly, which causes essential changes in a circuit.

It was shown [1] that the only magnetization curve used for description of magnetic properties of the transformer could exhibit some kind of subharmonic ferroresonance. This kind of ferroresonance does not occur under using an extra resistor, in parallel with the inductor (Fig. 1b), representing transformer core losses. This strongly suggests that for a qualitative understanding of ferroresonant circuits, a 'hysteretic inductance' must be used in the simulations. However, our measurements and field tests mentioned e.g. in [2] show that actual transformers damp transient ferroresonance faster than calculations using a parallel resistor predict. Therefore a nonlinear representation of transformer core losses is necessary to vary the value of core losses according to the actual value of saturation.

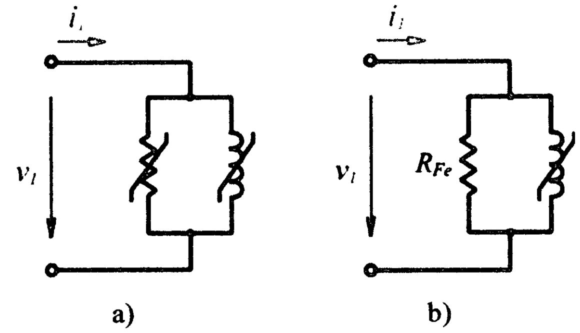


Figure 1 – Dynamic (a) and static (b) representation of core losses

2. METHOD USED FOR DESCRIPTION OF HYSTERESIS CURVE

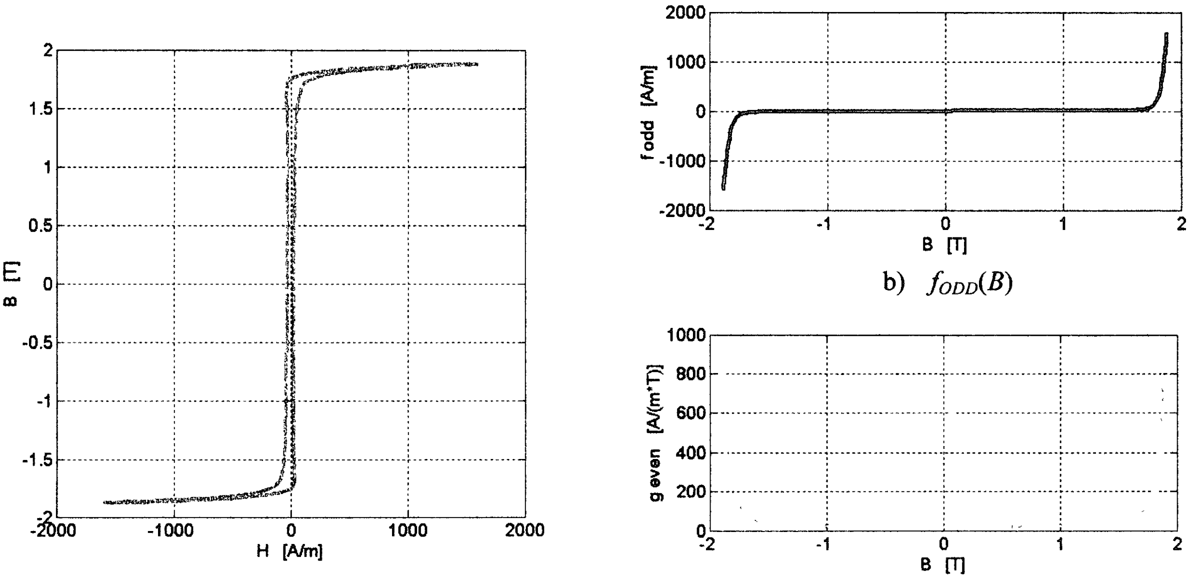
For our purposes, the small transformer with toroidal ferromagnetic core was used. It was designed to withstand high current values under resonance conditions. The base for our mathematical analyses was the fact that the measured voltage and current waveforms (in saturated state) contain all the necessary information concerning the magnetic characteristic of the transformer. The electric and magnetic circuits are coupled with each other in the relations between the current and the magnetomotive force, the voltage and the flux. This fact makes our method applicable to bigger transformer units appearing in electric power systems.

Description of the hysteresis curve [3], [4] was done by means of harmonic spectrum of the measured no-load current as the input for subsequent Chebyshev transformation. As a result of this process it is possible to receive coefficients of the polynomial describing hysteresis curve ($B-H$ characteristic). Equation for magnetic field strength H can be divided into two parts. General description of the field strength H as a function of the magnetic flux density B is given in Eqn. 1.

$$H = a_1 B + a_3 B^3 + \cdots + \left(b_0 + b_2 B^2 + \cdots\right)\frac{dB}{d\theta} = f_{ODD}(B) + g_{EVEN}(B)\frac{dB}{d\theta} \qquad (1)$$

Where: H is the magnetic field strength in the core; B is the flux density in the core; a_1, a_3, ... are the coefficients related with the effective reluctance of the core; b_0, b_2, ... are the coefficients related with the hysteresis loss; $\theta = \omega t$, and $\omega = 2\pi f$.

Graphical representations of those two parts are shown in Fig. 2. The first part is a curve corresponding to the conventional magnetization curve (Fig. 2b) and it is described only by odd terms of the polynomial. What is additional is the second term in the Eqn. 1. This is the part, which adds the hysteresis effect to the magnetization curve. Its characteristic, shown in Fig. 2c, consists of even terms of magnetic flux density and depends on its time behavior as well. Fig. 2a includes both those parts and it is the characteristic used in our analyses.



a) Hysteresis loop

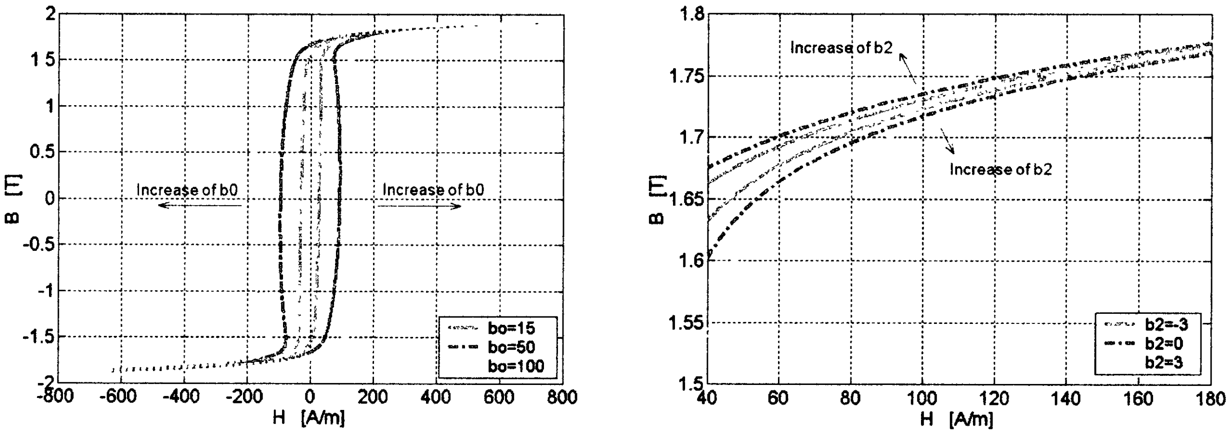
b) $f_{ODD}(B)$

c) $g_{EVEN}(B)$

Figure 2 – Two parts of intensity H and hysteresis loop

How many terms are in Eqn. 1? The answer is simple: The more accurate description is required the more terms should be employed. In reality, the choice is limited by the calculation means used for analysis. In our case we went up to the polynomial of the 35th order that seemed to be sufficient for our purposes. Furthermore, calculated model of the hysteresis curve has been improved to respect the normal magnetization curve [5] and simplified to meet the calculation needs.

Because many terms are used in Eqn. 1, there can be a lot of variations in the shape of hysteresis curve. For example a_0 – the higher is the value of a_0 the lower is the slope of hysteresis curve. Nevertheless, more important in our study are coefficients of the g_{EVEN} function. Besides the effect on the shape of hysteresis curve, they affect also its width and dynamically describe the behavior of core losses. Characteristics of the first two coefficients of g_{EVEN} (b_0 and b_2) are shown in Fig. 3. Coefficient b_0 corresponds to a static coercive force and coefficient b_2 corresponds to a dynamic coercive force [6].



a) Influence of b_0 b) Influence of b_2

Figure 3 – Some characteristic properties of the g_{EVEN} function

For higher values of b_0 the area of hysteresis curve increases. This is the only change it is possible to make when using the static description of transformer core losses. In the case of assuming dynamic description of core losses, it is possible to change also the width of hysteresis curve in saturated part and describe more accurately the knee of hysteresis curve.

Comparing the calculated current waveform with the measured current waveform, one can see the error caused by our calculation process. Accuracy of the method described above can be seen in Fig. 4. Both measured and calculated current waveforms are nearly identical, nevertheless for lower values of the current, some small differences can be found. The lower is the order used in polynomial of Eqn. 1 the bigger difference in current waveforms can be seen. Polynomial of the 35th order seems to be

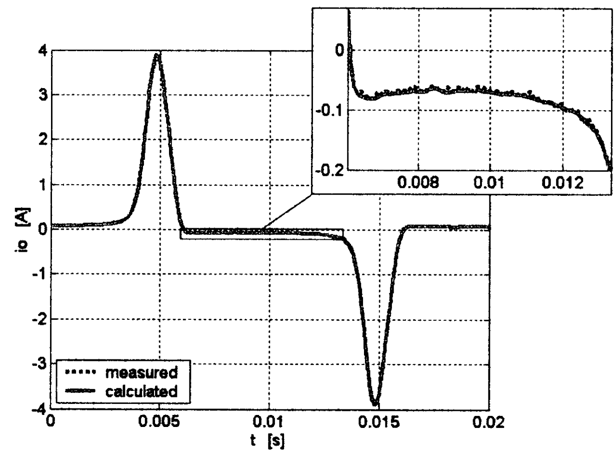


Figure 4 – Comparison of measured and calculated current waveforms

sufficient for our purposes. By using polynomial of this order, the error caused by the Chebyshev transformation [7] is sufficiently small. The error will become negligible if the main area of interest is in a saturated area.

3. SERIES FERRORESONANT CIRCUIT

In power distribution networks, ferroresonance can occur when a de-energized phase of a no-load transformer is suddenly energized through capacitive coupling with the other phases. The corresponding circuit can be simplified to a series RLC circuit. This type of circuit has been used also in our analyses, see Fig. 5. As a transformer under study, the small transformer with toroidal ferromagnetic core it was used.

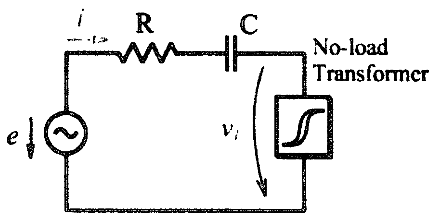


Figure 5 – Series ferroresonant circuit

Many articles concerning the theory of series ferroresonant circuit have been already written, e.g. [8], [9] etc. Equilibrium states of some process create an area of the same or different dimension in the space of these parameters. Observation of an evolution of volt-ampere characteristic for a simple series RLC circuit, depending on the value of series resistance (damping), leads to the surface shown in Fig. 6. This surface is well known in the Catastrophe theory as a cusp catastrophe [10]. Such a surface has been found to be very suitable for ferroresonance studies, because it helps to understand the jump phenomena in an easy way [8].

Simple scheme of the circuit under study consists of the voltage source, resistor *R*, capacitor *C*, and no-load transformer. All of them connected in series; see Fig. 6. In subsequent calculations, values of the system parameters will be fixed, except one or two of them that will be varied in order to observe their influence.

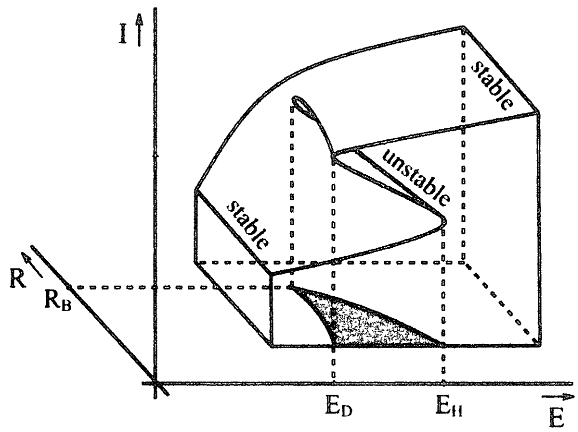


Figure 6 – Graphical solution of the basic series RLC circuit

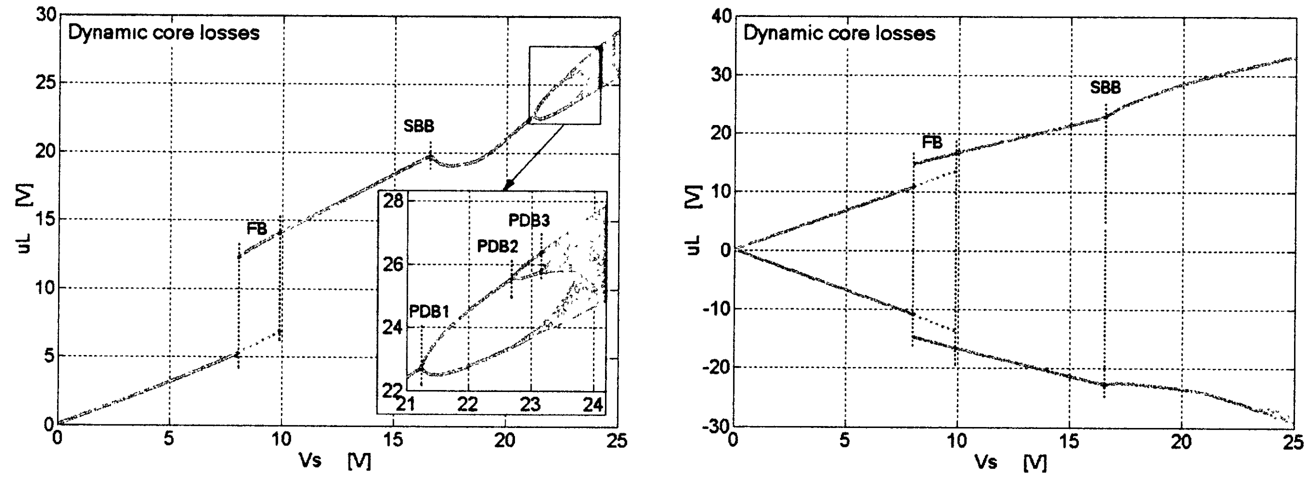
4. CALCULATED RESULTS

Basic outputs from our calculations are Bifurcation diagrams and Sensitivity analyses. Those techniques provide information about the circuit behavior under different source voltage values. The Bifurcation diagram is a plot of Poincare sections [11] and the Sensitivity analysis shows the maximum and the minimum values of the primary voltage (all of them in a steady-state). The points of special interest are points (bifurcation points) where a qualitative change of the preceding solution type can be observed.

For calculations, the circuit shown in Fig. 5 has been used. This circuit was solved by using MATLAB environment (ver. 6.0). The circuit under study has been modeled in Simulink ver. 4 by employing Power System Toolbox ver. 2. In this model, there is a box named 'No-load transformer' that contains information about non-linear characteristic of the no-load transformer under study. Current source driven by a primary voltage together with the calculated polynomial of Eqn. 1 describe its behavior.

The outputs received from calculations under variable source voltage value are shown below. In calculations, the parameter values were: f = 50 Hz, C = 30 μF, R = 0.2 Ω and initial value of the magnetic flux for each calculation was set to correspond to the maximum value of the respective source voltage.

Fig. 7a shows a bifurcation diagram of the primary voltage of transformer. Corresponding sensitivity analysis for the same conditions is shown in Fig. 7b. From these figures it is possible to observe the qualitative changes in the system response. The response evolves slowly when increasing the source voltage V_S from zero value, until its value reach the first bifurcation point. Corresponding to the abbreviations used in Fig. 7, the first bifurcation point is a point where the fold bifurcation FB (sometimes called saddle-node bifurcation) occurs. As it was described in Fig. 6, there is an unstable area in which the system response is very sensitive to initial conditions. Therefore, FB can appear in a certain range of the source voltage value, depending on the initial conditions. This range (from 8.1 V till 9.9 V) is represented by a dotted line in the solution value. The birth of FB is the first case of ferroresonant behavior.



a) b)

Figure 7 – Bifurcation diagram (a) and sensitivity analysis (b) for V_S (dynamic core losses)

From this point, the operating point is working in a saturated area and primary voltage and current waveforms are distorted and have higher magnitudes. Further increasing of the source voltage leads to a symmetry breaking bifurcation (SBB) point. From this point, voltage and current waveforms are not symmetric according to time-axis. This situation can be clearly seen in Fig. 7b. There one can see that maximum values of the primary voltage comparing with minimum values have not the same magnitudes. For values of the source voltage higher then the one corresponding to SBB, the system evolves towards the first period doubling bifurcation PDB1 and subsequently towards the other period doubling bifurcations PDB2, PDB3 etc. till it reach the state with a chaotic behavior.

After the voltage value corresponding to SBB, the system shows higher sensitivity to initial conditions. Each initial condition produces slightly different outputs which can be remarkable especially in Bifurcation diagram. In this diagram the differences are mainly in the value of uL meanwhile the bifurcation points are the same. Therefore, except FB range, Fig. 7 corresponds to the outputs received for the maximum initial condition of the flux for each particular case of the source voltage. Such a state can represents the cases when ferroresonance occurs under some kind of switching events.

Previous figures were related to a simulation assuming dynamic behavior of hysteresis losses of the transformer (according to Fig. 1a). Outputs received from calculations by employing the widely used (static) description of hysteresis losses are a little bit different, see Fig. 8.

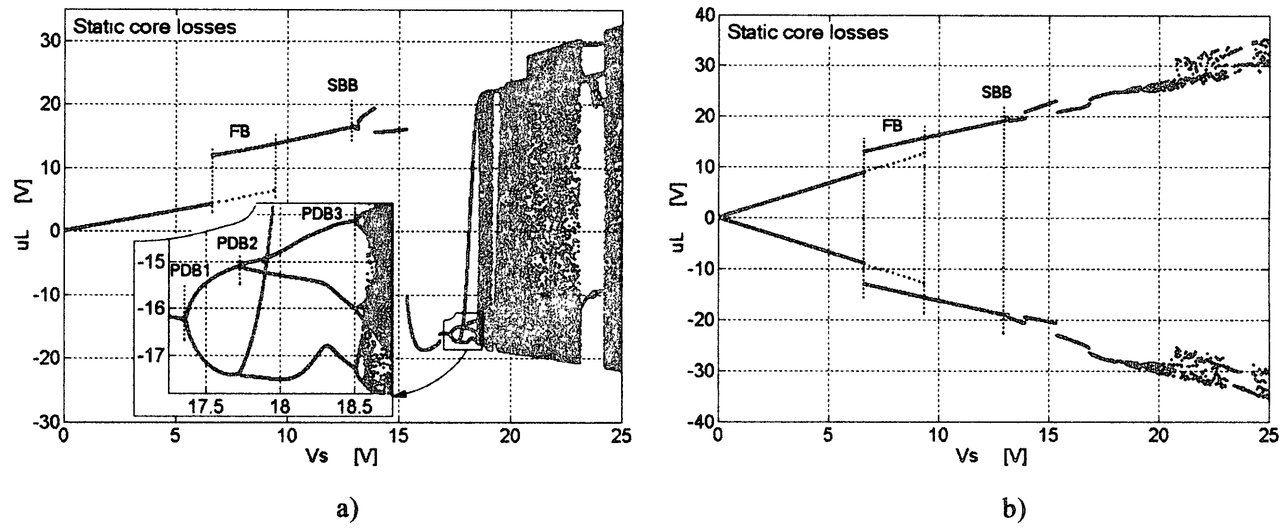


Figure 8 – Bifurcation diagram (a) and sensitivity analysis (b) for V_S (static core losses)

All kinds of bifurcations happen for lower source voltage values than in the case of dynamic core losses, see Tab. 1. Furthermore, because of improper description of damping in saturated state, primary current reach its higher values than in the case of assuming dynamic description of core losses. Especially for higher values of the source voltage, the system is very unstable; the birth of chaotic oscillations evolves in a little bit strange way (from the beginning of period doubling process) but finally behaves according to the Feigenbaum scenario [11]. Such instability is also produced by the high order polynomial used in our calculations.

	Bifurcation						
	FB	SBB	PDB1	PDB2	PDB3	PDB4	PDB5
Dynamic	8.093 V to 9.949 V *)	16.54 V	21.218 V	22.721 V	23.191 V	23.293 V	23.315 V
Static	6.654 V to 9.321 V *)	13.00 V	17.330 V	17.726 V	18.510 V	18.556 V	18.566 V
Dynamic = dynamic description of core losses; Static = static description of core losses; *) Because of possible different initial conditions there is a range in the voltage values							

Table 1 – Values of bifurcation parameter V_S for different bifurcation points

Our calculations also show the transition towards chaotic regime. The period doubling route resulting in chaotic behavior can be described by the sequence of bifurcation parameters λ_k, which obeys a geometric law with a universal constant [11]. This constant, called Feigenbaum constant, has been defined as the limit

$$\delta = \lim_{k \to \infty} \left(\frac{\lambda_k - \lambda_{k-1}}{\lambda_{k+1} - \lambda_k} \right) \approx 4.6692016... \quad (2)$$

However, in practical applications the limit $k \to \infty$ cannot be taken. Nevertheless, an estimate of the Feigenbaum constant can be obtained from a finite sequence of δ.

Using the bifurcation points shown in Tab. 1 as an input for the Eqn. 2, the values of δ can be received. Calculated results are summarized in Tab. 2. From these data it is possible to see the initial strange behavior of period doubling process in the case of static description of core losses. On the other hand, fluent transition towards chaotic behavior can be observed in the case of dynamic representation.

	Value of δ		
	δ_1	δ_2	δ_3
Dynamic	3.1979	4.6078	4.6364
Static	0.5051	17.0435	4.6000

Table 2 – Values of δ considering static and dynamic representation of core losses

In the above analysis only one case of many possible were studied. In reality we can know the value of resistance or capacitance but it can be quite difficult to know exactly and for all possible cases the initial value of the magnetic flux ϕ. For those kinds of purposes, more complex sensitivity analysis is necessary. Such an analysis can be very difficult to interpret because of many variable parameters. Therefore 2D Sensitivity analysis (two parameters are varying) seems to be useful. In the Fig. 9 the source voltage value V_S and the initial value of the magnetic flux are used as variable parameters. The color palette is used to distinguish the maximum value of primary voltage V_1. Nevertheless, the most important for our study is the border between areas of different behavior: area with no possibility of ferroresonance (NF) and area where ferroresonance (F) will occur. The border can be clearly seen due to the ferroresonance phenomena (sudden jump in magnitude of the primary voltage).

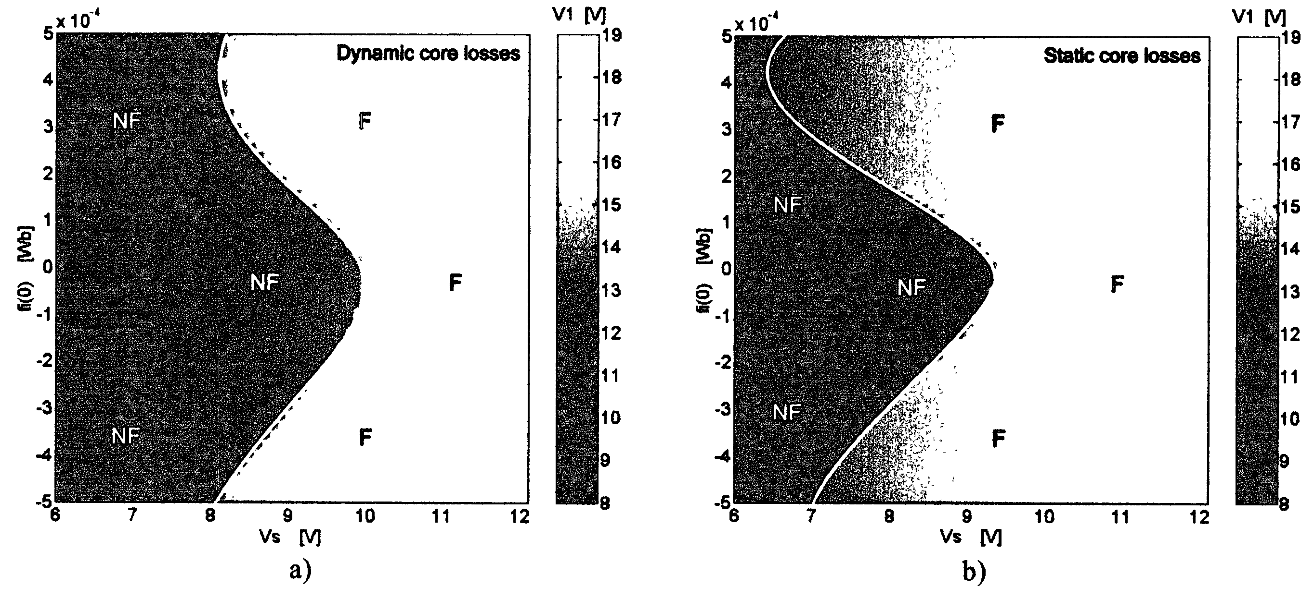


Figure 9 – 2D Sensitivity analysis for varying V_S and $\phi(0)$ using dynamic (a) and static (b) representation of core losses

It is possible to see that the worst cases, represented by an earlier transition into ferroresonance, occur under initial value of the magnetic flux corresponding to the maximum or minimum value of the source voltage. On the other hand, when the initial value of the magnetic flux is zero a higher value of the source voltage is necessary to bring the transformer into ferroresonance. Comparing Fig. 9a (calculated by using dynamic representation of core losses) with Fig. 9b (calculated by using static representation of core losses) already mentioned differences can be clearly seen. Furthermore, the unstable area in Fig. 9 (three possible solutions are available for V_S value depending on the initial condition of the magnetic flux) is smaller in the case of dynamic representation.

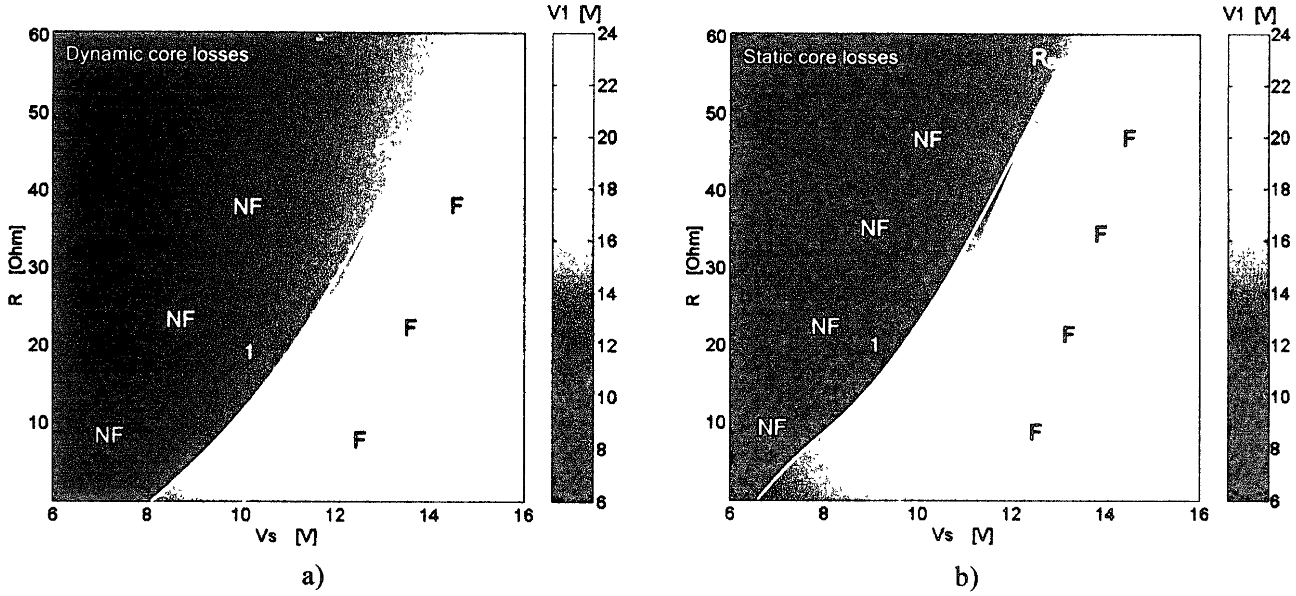


Figure 10 - 2D Sensitivity analysis showing hysteresis effect with variable V_S and R by using dynamic (a) and static (b) representation of core losses

Similar idea can be used for 2D Sensitivity analysis with varying the value of series resistance R together with the value of the source voltage V_S. The capacitance value is fixed – C = 30 μF. In Fig. 10, it is possible to see the unstable areas given by different initial conditions. Lines 1 and 2 create borders between results received by using maximum initial value of the magnetic flux and its zero value. The area between those two lines corresponds to the parameter values for which the system may or may not come into ferroresonance, see R-E plane in Fig. 6. Everything depends on the initial conditions. It is the area of possible ferroresonance (PF). There are also other two areas. One is the area where ferroresonance wil not occur – no ferroresonance (NF), and the other one is the area where ferroresonance will occur - F under all initial conditions of magnetic flux. The critical value of the series resistance R_B is a value, from which the system looses its sensitivity to initial conditions and system does not exhibit any sudden jumps. Yes, system can operate in saturated area, nevertheless there are no ferroresonant oscillations. For series resistance values higher then R_B all of them are successfully damped. The value of R_B are R_B = 43 Ω (dynamic representation) and R_B = 55 Ω (static representation).

As it is possible to see from Fig. 10, the unstable area where there is a possibility of ferroresonance (PF) is bigger in the case of static representation of core losses. This means that transformer, using this kind of representation for calculations, will exhibit higher level of unstability than what actually occurrs in reality. This results in the necessity of considering dynamic behavior of transformer core losses for ferroresonance studies.

4. CONCLUSION

The two kinds of analysis have been presented: one, using the widely used static description of transformer core losses, and the other one using dynamic representation. It was shown that by employing only a static description of core losses there is a higher level of unstability for possible solutions. This result does not correspond to our measured outputs. Real measurement shows higher damping effect and lower level of unstability than what was calculated by using static representation.

Better solution presented in this paper is to employ the dynamic representation of core losses. This method provides results with lower level of unstability and those results are closer to the practical measurements than those received by using static description. Our calculations result in the necessity of using dynamic description of transformer core losses for ferroresonance studies.

It should be emphasized that we do not introduce the most accurate model of transformer. There exist more complex and accurate models [12]. Proposed method provides only one possible way for a qualitative description of the effects of magnetic hysteresis in magnetic circuits and, furthermore, the consequences of assuming the static and the dynamic representation of core losses. In respect to the more complex models of the transformer (which require a lot of hardly-to-obtain information concerning the transformer under study), presented model is based on the measured current and voltage values which are not so difficult to obtain. It has been proved that such a model provides the outputs of ferroresonance studies with sufficient accuracy and makes the method to be applicable for bigger transformer units.

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