Introduction to Hydraulics: elementary lesson for students

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<u>What is hydraulics?</u>

Regarding lakes not only as reservoirs of water but also as storage of deposits makes it possible to read the past environmental change. To execute this trial, we must pay attention to the conveyer of the deposits, that is, rivers because we need to think about not only when but also how sediments were transported.

Hydrology [水文学] is the branch of science concerned with the properties of the earth's water, and especially its movement in relation to land (Oxford Dictionary of English).

Water is very important material in almost all earth science fields. Hydrology deals with physical processes of water as fluid. Fluid is a substance that can flow such as a liquid or a gas. Fluid changes its form continuously without a break or a crack. Sometimes a hard rock can be regarded as fluid, for example folds.

courtesy by Prof. Ishiwatari and Dr. Sumita

Hydrology often needs Hydraulics.

Hydraulics [水理学] is the branch of science and technology concerned with the conveyance of liquids through pipes and channels, especially as a source of mechanical force or control (Oxford Dictionary of English).

This course is the introduction of hydraulics and especially we consider running water, i.e., the water of rivers.

<u>Distribution of Earth's water</u>

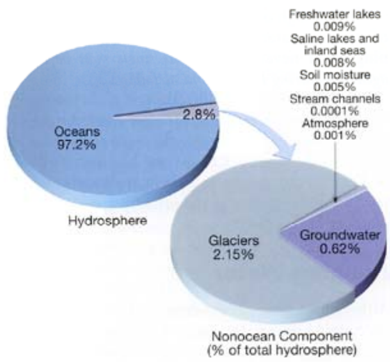
The river water (the water in stream channels) is only 1/10000 % of the total water within Earth.

92.7% -oceans

2.15% -glaciers

0.62% -groundwater

0.03% -others



From "Earth" (Tarbuck and Lutgens.)

Distribution of Earth's water.

By the way, the fresh lake water is about 1/100 % (100 times of river water) of the total water of Earth. Lake Baikal is the 6th largest fresh water lake (31400 km^2) in the world and the largest in Asian area. Lake Baikal is the deepest (1743 m at maximum) in the world and has the pondage (water volume) of 23600 km^3, about 25% of the fresh lake water of the world (91000 km^3). Lake Baikal is the oldest lake in the world, about 25 million years old.

Now, we go back to rivers…

<u>Factors of the stream velocity</u>

There are three main factors contributing to the river flow velocity.

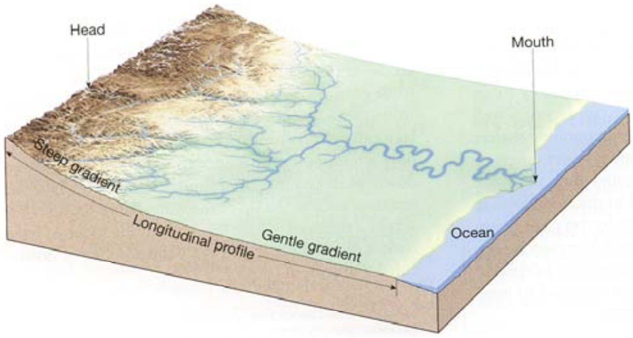
(1) Gradient

(2) Discharge

(3) Shape, size, and roughness of channel

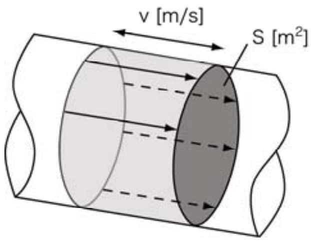
(1) It can be naturally understood that the water on a steep river bed runs faster than that on a gentle bed.

Generally gradient decreases from upstream to downstream in nature.



From “Earth” (Tarbuck and Lutgens:.)

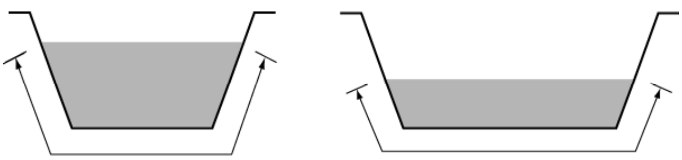
(2) Discharge is defined as
Discharge = cross section × (mean) flow velocity .



Generally speaking, if river flow discharge increases, both the cross section and the flow velocity increase.

The discharge of rivers increases from upstream to downstream because of confluences of tributaries (join of smaller rivers).

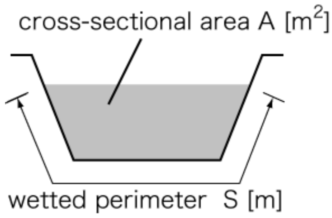
(3) River flows are subject to friction form beds and walls. If the friction increases, the flow velocity decreases. The shape and size of river channel have relation to total friction. The roughness of river beds and walls also influence the friction. Factors of roughness are the surface configuration of the bedrock and/or the grain size of sand or pebbles, and besides bedforms such as ripples and dunes.



The beds and walls exert friction on the flow, the range of which is demonstrated by arrowed segments. The shape of channel pertains to this range.

Parameters in relation to the flow and their definition

Hydraulic radius (hydraulic mean depth) R [径深] and discharge Q [流量] are defined by cross-sectional area of flow A [流積] and wetted perimeter S [潤辺] (see the right figure).



$$R = \frac{A}{S}$$

$$Q = Av \qquad (v\text{: (mean) flow velocity})$$

$$\therefore v = Q/RS$$

R and S are determined by the channel shape for a given discharge, and thus the flow velocity is also determined.

Tractive force [掃流力] of rivers

Tractive force or bottom shear force [底面剪断力] is a drag force due to the friction worked between the fluid and the wall.

Before discussing this subject, we classify the flow state.

Steady flow [定常流] and Non-steady flow [非定常流]:

Steady flow does not change with time.

Non-steady flow is any flow other than steady flow.

Uniform flow [等流]:

Uniform flow is a specific flow of steady flows that has the same physical quantity everywhere.

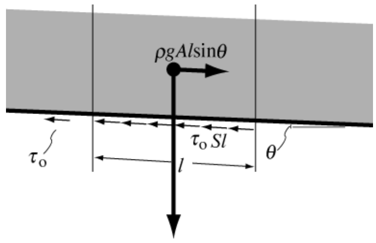
Here, we consider a uniform flow.

The drag force τ_0 has the same dimension as the pressure, force/area. Considering the balance of forces in the flow direction between the weight of fluid and the friction.

$$\tau_o Sl = \rho g A l \sin\theta \qquad (\rho: \text{density of fluid})$$

$$\tau_o = w\frac{A}{S}\sin\theta \qquad (w = \rho g)$$

$$= wR\sin\theta$$



<u>Bernoulli's law (Energy conservation law of fluids)</u>

Three main state of energy:

Kinetic energy [運動エネルギー（能量）]

Potential energy [位置エネルギー（能量）]

Pressure energy [圧力エネルギー（能量）]

Here we consider a "water mass" with the mass m, the volume V, moving at the velocity v, and suffering the pressure p.

Kinetic energy: $\frac{1}{2}mv^2$

Potential energy: mgz where z is vertical coordinate.

Pressure energy: $pV = m\frac{p}{\rho}$ where, ρ is the fluid density.

The energy conservation can be denoted as

$$\frac{1}{2}mv^2 + mgz + m\frac{p}{\rho} = \text{ constant}.$$

Thus,

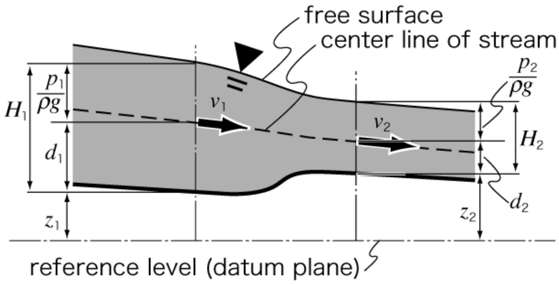
$$E = \frac{v^2}{2g} + z + \frac{p}{\rho g} = \text{ constant}$$

or $$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g}$$ where suffixes indicate positions,

E : total water head; $\frac{v^2}{2g}$: velocity head; z : elevation head (potential head); $\frac{p}{\rho g}$: pressure head.

For flow running in an open channel as the figure, Bernoulli's law can be written as

$$\frac{v_1^{\ 2}}{2g} + z_1 + d_1 + \frac{p_1}{\rho g} = \frac{v_2^{\ 2}}{2g} + z_2 + d_2 + \frac{p_2}{\rho g}$$



In this case,

$$d_1 + \frac{p_1}{\rho g} = H_1$$

$$d_2 + \frac{p_1}{\rho g} = H_2$$

H is flow depth. The mean flow velocity is substitutable for the flow velocity at the center line.

Exercise:

The flow with 200 m³/s discharge runs in a rectangle open channel of 20 m width. At Point 1 with 1 m altitude in the above Figure, the flow depth is 5 m. How much are the velocity and depth at Point 2 with 1.5 m altitude?

Discharge $Q = H_1 W v_1 = H_2 W v_2$ where W is channel width.

Bernoulli's equation is written as

$$\frac{(Q/WH_1)^2}{2g} + z_1 + H_1 = \frac{v_2^{\ 2}}{2g} + z_2 + \frac{Q}{v_2 W}$$

or

$$\frac{(Q/WH_1)^2}{2g} + z_1 + H_1 = \frac{(Q/WH_2)^2}{2g} + z_2 + H_2$$

Putting values into the above eqation,

$$\frac{(200/20/5)^2}{2 \times 9.8} + 1 + 5 = \frac{v_2^{\ 2}}{2 \times 9.8} + 1.5 + \frac{200}{20 v_2}$$

$$\therefore v_2^{\ 3} - 92.2 v_2 + 196 = 0.$$

$$\therefore v_2 \approx 2.2 \text{ [m/s]}$$

$$\therefore H_2 \approx 4.5 \text{ [m]}$$

<u>Head loss</u>

In many cases, energy loss cannot be ignored. In the case of energy loss, Bernoulli's equation is written as

$$\frac{v_1^{\ 2}}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^{\ 2}}{2g} + z_2 + \frac{p_2}{\rho g} + h_l$$

or for open channels

$$\frac{v_1^{\ 2}}{2g} + z_1 + H_1 = \frac{v_2^{\ 2}}{2g} + z_2 + H_2 + h_l$$

where h_l is called head loss.

Head loss is caused by friction and changes of cross-sectional area and shape of channels.

Friction head loss is in proportional to transported distance, l, and to velocity head, $\frac{v^2}{2g}$, and inversely proportional to hydraulic radius, R. Thus, friction head loss, h_f, can be expressed as

$$h_f = f' \frac{l}{R} \frac{v^2}{2g}$$

where f' is the coefficient of friction head loss.

Head loss due to the change of cross-sectional area is called inlet head loss, h_i, and can be written as

$$h_i = f_i \frac{v^2}{2g} \quad .$$

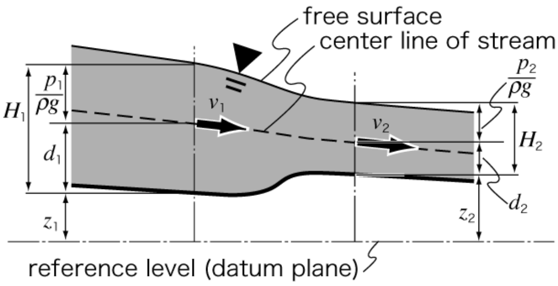
Bend of a channel gives rise to head loss, which is an example of head loss caused by channel shape, and denoted as

$$h_b = f_b \frac{v^2}{2g} \quad .$$

Example:

The inlet head loss of the flow shown in the figure is

$$h_i = f_i \frac{v_2^{\ 2}}{2g} \quad .$$



The value of f_i varies from zero to unity as to the degree of bed change (abrupt or gentle).

Exercise:

The velocity of the flow in a rectangle open channel is 2.5 m/s at Point 1 in the above figure. If the flow velocity at Point 2 is 3 m/s and the coefficient of the inlet head loss is 0.05, how much is the difference of the height of water surface?

$$\frac{v_1^{\ 2}}{2g} + z_1 + H_1 = \frac{v_2^{\ 2}}{2g} + z_2 + H_2 + f_i \frac{v_2^{\ 2}}{2g}$$

$$z_1 + H_1 - (z_2 + H_2) = \frac{v_2^{\ 2}}{2g} + f_i \frac{v_2^{\ 2}}{2g} - \frac{v_1^{\ 2}}{2g}$$

$$= \frac{3^2}{2 \times 9.8}(1 + 1.05) - \frac{2.5^2}{2 \times 9.8}$$
$$= 0.16 \ [\mathrm{m}]$$

<u>Froude number</u>

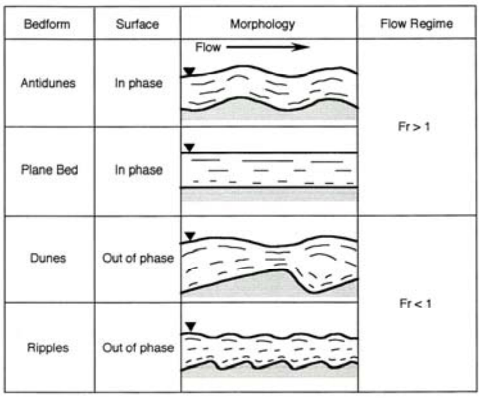
Not only fundamental quantity of flows such as depth or velocity, but also the state of flows is important to consider in hydraulics.

Froude number is dimensionless parameter that is an index for the state of the fluid with a free surface.

$$Fr = \frac{v}{\sqrt{gH}}$$, where H is the water depth.

The flow Fr of which is under unity is called subcritical flow (常流), and the flow of Fr more than 1 is called supercritical flow (射流). $\sqrt{gH}$ is the travel speed of shallow water waves (long waves). Whether the Froude number is less or more than unity determines whether a disturbance can advance upstream or not.

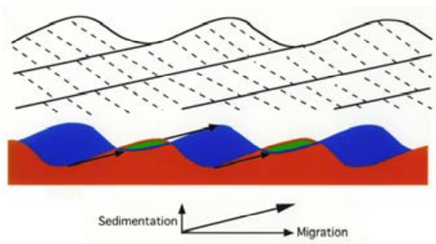
Relation between the Froude number and bedforms is summarized in the below table.



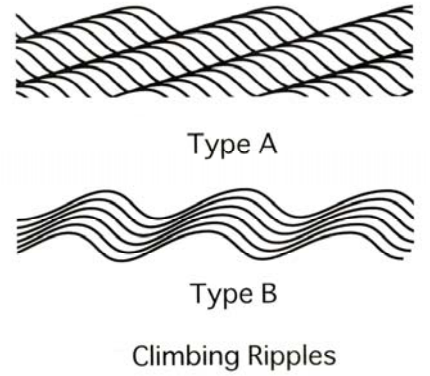
Bedform	Surface	Morphology	Flow Regime
Antidunes	In phase	Flow →	Fr > 1
Plane Bed	In phase		
Dunes	Out of phase		Fr < 1
Ripples	Out of phase		

フルード数　Froude number　$Fr = U/\sqrt{gh}$

Active ripples in shallow water.

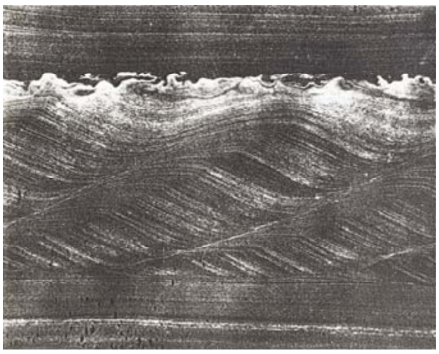


Schematic illustration of cross bed formation due to climbing ripples.



Two types of climbing ripples. The type is determined by the balance of migration speed of ripples and. sedimentation rate

Modern river deposits including climbing ripples structures. From Miall and Smith (1989).



A geologic record of climbing ripples. From “Current Ripples” (Allen, J.R.).

A geologic record of large-scale cross-beds due to dunes. From “Sedimentographica” (Lucchi, FR.).