Precise Numerical Solution of Soil Consolidation Effect

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Abstract: The aim of this paper is to give an effective precise integration numerical method for soil consolidation effect in Hamilton system. Based on the generalized Biot theory and the finite element discretion method, for soil consolidation effect, the essential equation with u-p form is obtained. Then by introducing into the dual variable of the original variable in the essential equation, the differential equation of soil consolidation effect in Lagrange system is established under Hamilton system. In the process of the concrete computation, the consolidation problem is computed with the help of established time precise integration method in linear dynamic system. Finally an idealized engineering numerical example is given. The numerical example shows that the presented method is effective. Some important valuable conclusions obtained will lay some foundations for the deep researches of theory, computation and engineering application of soil mechanics.

Keywords: consolidation effect, Biot theory, precise numerical solution

1 Basic Equations in Lagrange system

The dynamic differential equation in Lagrange system is[1,2]

$$\sigma_{ij,j} + \rho b_i - \rho \ddot{u}_i - \rho_f \left(\frac{\partial \dot{w}_i}{\partial t} + \dot{w}_k \dot{w}_{i,k}\right) = 0 \tag{1}$$

According to the Darcy theory, the motion equation of the fluid in the porous is

$$-p_{,i} + \rho_f b_i = \rho_f \left(\ddot{u}_i + \frac{1}{n}\left(\frac{\partial \dot{w}_i}{\partial t} + \dot{w}_k \dot{w}_{i,k}\right)\right) + R_i/n = 0 \tag{2}$$

where R_i/n is viscous drag of solid frame to water in the porous.

In Consolidation problem, all the acceleration items in the basic equations can be ignored. The seepage velocity $\dot{w}$ can be represented by u and p . Then Eqs.(1) and (2) can be expressed as

$$\sigma_{ij,j} - \alpha p_{,i} + \rho b_i - \rho \ddot{u}_i = 0 \tag{3}$$

$$-p_{,i} + \rho_f b_f - k^{-1} \dot{w}_i = 0 \tag{4}$$

According to Eq. (4) and the conversation law of the volume variation of the fluid motion, we can obtain

$$[k(-p_{,i} + \rho_f b_i)]_{,i} + \alpha \dot{\varepsilon}_{ii} + \frac{1}{Q}\dot{p} = 0 \tag{5}$$

where only variables u_i and p are included in Eqs. (3) and (5). In order to discrete the above equations, suppose

$$\boldsymbol{u}_i = \boldsymbol{N}_k^u \bar{\boldsymbol{u}}_{ki} \qquad \boldsymbol{p} = \boldsymbol{N}_k^p \bar{\boldsymbol{p}}_k \tag{6}$$

where N_k^u, N_k^p are shape functions of u and p respectively; $\bar{u}_{ki}$ and $\bar{p}_k$ are corresponding value of the nodes.

Substituting Eq.(6) into Eqs.(3) and (5) will lead to the equation in matrix form as follows

$$\bar{\boldsymbol{M}}\ddot{\bar{\boldsymbol{v}}} + \bar{\boldsymbol{C}}\dot{\bar{\boldsymbol{v}}} + \bar{\boldsymbol{K}}\bar{\boldsymbol{v}} = \bar{\boldsymbol{f}} \tag{7}$$

2 Basic Equations in Hamilton system

Introducing vector $\boldsymbol{q} = \bar{\boldsymbol{M}}\dot{\bar{\boldsymbol{v}}} + \bar{\boldsymbol{C}}\bar{\boldsymbol{v}}/2$, the basic equations in Eq.(7) can be transformed as

$$\dot{\boldsymbol{x}} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{f} \tag{8}$$

where $\mathbf{x} = \{ \bar{v}, q \}$, $f = \{0, \bar{f}\}^T$, $H = \begin{bmatrix} -\bar{M}^{-1}\bar{C}/2 & \bar{M}^{-1} \\ \bar{K} - \bar{C}\bar{M}^{-1}\bar{C}/4 & -\bar{C}\bar{M}^{-1}/2 \end{bmatrix}$ is Hamilton matrix.

3 Solution Based on Precise Integration Method

It is assumed that the non-homogenous item is linear in a small time step (t_k, t_{k+1}), which leads

$$\dot{x} = Hx + q_0 + q_1(t - t_k) \tag{9}$$

where q_0, q_1 are the given vector. Suppose that $\Gamma(t - t_k)$ is the solution of the homogenous equation, or

$$\dot{\Gamma} = H\Gamma \ , \ \text{and} \ \Gamma(0) = I \tag{10}$$

The solution of Eq.(8) is

$$x = \Gamma(t - t_k)[x_k + H^{-1}(q_0 + H^{-1}q_1)] - H^{-1}[q_0 + H^{-1}q_1 + q_1(t - t_k)] \tag{11}$$

The corresponding time integration formation is

$$x_{k+1} = T_k[x_k + H^{-1}(q_0 + H^{-1}q_1)] - H^{-1}[q_0 + H^{-1}q_1 + q_1\tau] \tag{12}$$

where the value of T_k is the key in the use of Precise Integration Method, which can be seen in Ref.[3].

4 Numerical Example

A Multi-floor soil with 63 nodes is divided into 6 floors. The soil characteristics of the 2 floors in the bottom are $E_1 = 12.5\times10^5\ N/m^2, \gamma = 0.4, k_{1x} = k_{1y} = 6.1\times10^{-6}\ m/d$. The soil characteristics of the 4 floors upwards are $E_2 = 8.5\times10^5\ N/m^2, \gamma = 0.4, k_{2x} = k_{2y} = 1.22\times10^{-5}\ m/d$. The cases with structure and without structure upward are calculated respectively. When there is structure upward, the bending stiffness of the beam is $EJ_y = 10^8\ Nm^2$. The length of the beam is 3m, and the three parts of the segments are 0.5m, 1.0m and 1.5m respectively. The length of the soil is 17.0m and its height is 12.5m. The results of displacements are shown in Table 1.

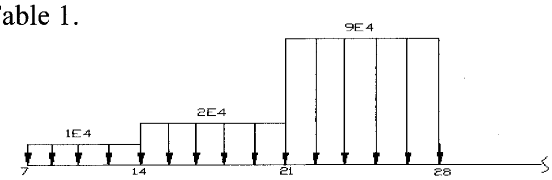

Fig. 1 The loads of the structure

Table 1 The displacements of the nodes(/m)

Node / Disp.	7	14	21	28	35	42	49	56	63
With Structure	-0.3587	-0.3591	-0.3692	-0.3603	-0.0920	-0.0189	0.0299	0.0398	0.0419
Without Structure	-0.3298	-0.3297	-0.4001	-0.3310	-0.0917	-0.0088	0.0277	0.0399	0.0410

5 Conclusions

(1) At the beginning of the process, the settlement is fast. However, the consolidation effect will be stable when a certain time is reached. The numerical examples show that the presented method is effective. The formation is easy and the results have good precision and numerical salability.

(2) The coupling effect of structure and soil can't be ignored, which becomes greater with the structure stiffness increase.

References

[1] Schrefler B.A., Zhang W.X., Pastor M. and Zienkiewicz O.C.. Strain localization modelling and pore pressure in saturated sand samples. Computational Mechanics, 1998, 22: 266-280.

[2] Zhang Hongwu, Zhong Wanxie, Qian Linxi. Variatioal principle for Dynamic consolidation effect analysis of saturated soil. Chinese Journal of Geotechnical Engineering, 1992, 14(3): 20-29 (in Chinese).

[3] Zhong Wanxie, Ouyang Huajiang, Deng Zichen. Computational Structural Mechanics and Optimal Control. Dalian: Dalian University Press, 1993 (in Chinese).