Application of New Inverse Analysis Method to Crust Deformation of Japanese Islands

Proceedings: International Symposium of the Kanazawa University 21st-Century COE Program Vol. 1 (2003)
Edited by Naoto KAMATA

Application of New Inverse Analysis Method to Crust Deformation of Japanese Islands

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Abstract - A new inverse analysis method, called stress inversion, has been developed for identifying a stress distribution for a body whose constitutive relations are only partially known. This paper presents brief explanation of the stress inversion, and discusses the applicability to the Japanese Islands by making use of the nationwide GPS array data. Results of numerical computation of finding regional stress increment in the Japanese Islands are presented.

I. Introduction

It is of primary significance to continuously monitor the crust deformation of the Japanese Islands for earthquake prediction researches [1]. While kinematical quantities such as displacement and strain are measured by taking advantage of Global Positioning System (GPS), dynamic quantity, stress, cannot be estimated from strain. This is because constitutive relations or stress-strain relations of the Japanese Islands are not known.

Identification of stress distribution is also important for, say, small materials or biological organism [2]. While strains are somehow measured, stress cannot be estimated unless the constitutive relations are known.

The authors have been proposing a new inverse analysis method for identifying stress distribution for a body whose constitutive relations are only partially known [3,4,5]. When the body is in state of plane stress or strain, the inverse analysis method, called the stress inversion, is able to find three stress components which satisfy equilibrium by measuring strain distribution.

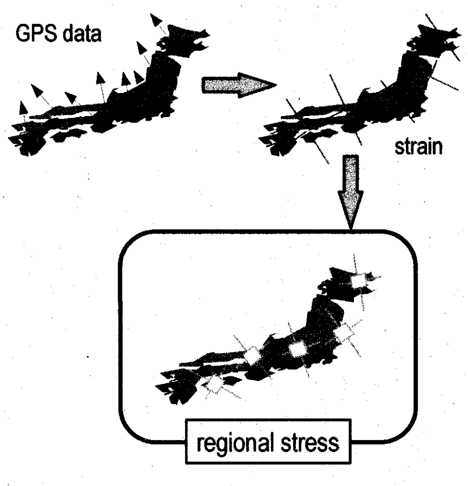

Fig. 1. Inversion of regional stress using GPS data.

In this paper, we briefly explain the stress inversion. The applicability of the stress inversion is then discussed. Distribution of regional stress rate is computed from strain rate distribution which is deduced from the GPS array data of displacement rate.

II. Formulation of Stress Inversion

A. Problem Setting

We consider a body in state of plane stress. There are three non-trivial stress components for this body. Since the three components satisfy two equations of equilibrium, we can determine them if one condition is assigned. When strain distribution is measured, the one condition is derived from one constitutive relation. This is the basic idea of the stress inversion [4].

To apply the stress inversion, therefore, we need to known one constitutive relation for the material which the body consists of; here, we assume that a linear relation between 2D volumetric strain and stress is given, i.e.,

$$\sigma_{11} + \sigma_{22} = \kappa(\varepsilon_{11} + \varepsilon_{22}) \tag{1}$$

Also, data of a strain distribution on the surface of the body and traction distribution along the boundary of the body are needed; see Fig. 1 for a schematic view of the problem setting.

B. Airy's Stress Functions

Three stress components which satisfy two equations of equilibrium are given as the second-order derivative of one single function, Airy's stress function. That is,

$$\begin{aligned} \sigma_{11} &= a_{,22} \\ \sigma_{22} &= a_{,11} \\ \sigma_{12} &= -a_{,12} \end{aligned} \tag{2}$$

where is Airy's stress function and are the stress components.

A governing equation for a is derived by substituting Eq. (2) into Eq. (1). This is a Poisson equation. When traction along the boundary is measured, the following boundary conditions are derived:

$$n_1 a_{,1} + n_2 a_{,2} = -n_1 r_2 + n_2 r_1 \tag{3}$$

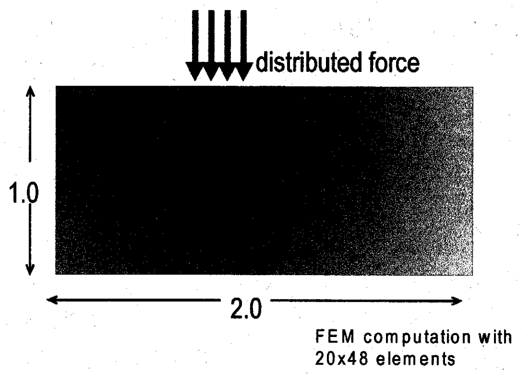

Fig. 2. Simulation of material sample test.

Here, r_i is the resultant force that is given by integrating t_i .along the boundary

A boundary value problem is thus posed for a. It should be emphasized that the problem becomes linear even for the material is non-linear; the effects of the non-linearity appears in the input data of strain.

We present an example of the stress inversion [5]. A material sample test is simulated; elasto-plastic material is used. The distribution of identified shear stress is compared with the exact one in Fig. 3. As is seen, the agreement is satisfactory. Figure 4 presents measured strain which is plotted in the principle stress plane to show the plastic constitutive relations; an ellipse corresponds to yield locus. It is seen that the stress inversion succeeds to reproduce unknown plastic constitutive relations.

III. Analysis of GPS Array Data

The nation-wide GPS array has been operating in Japan, and displacement increments are daily measured at almost 1,000 sites; see Fig.5. Errors in measuring horizontal displacement increment are less than 1[cm].

In analyzing the GPS data, we assume that the Japanese Islands are in state of plane stress. This assumption is acceptable since the data are for displacement increment during a time scale of a few years, even though the total stress varies vertically, We also assume Eq. (1) as one a-priori known constitutive relation; non-elastic deformation is caused by sliding of faults or fault systems which causes shear deformation only.

We apply the stress inversion to the GPS array data which are temporally filtered; the data are during 1999 and linear regression is taken to get rid of noises of daily measurement; see Fig. 6. The distribution of regional strain increment is plotted in Fig. 7; the first and second invariants, i.e., volumetric and maximum shear, are used. In Fig. 8, the distribution of identified stress increment, the first and second invariants, is plotted. From the comparison of Figs. 7 and 8, it is seen that the distribution pattern of the stress increment and the strain increment is different since the regional stress increment satisfies the equilibrium. While the validity of the identified stress increment distribution is not verified, it is at least shown that the boundary value problem of the stress inversion is solvable when the strain increment data of the GPS array are used and the regional stress distribution satisfying the equilibrium equations are found.

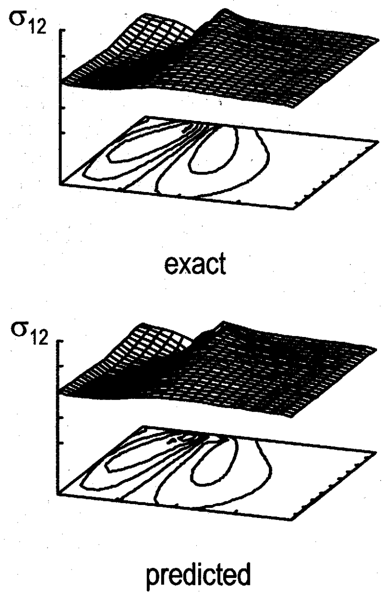

Fig. 3. Distribution of shear stress.

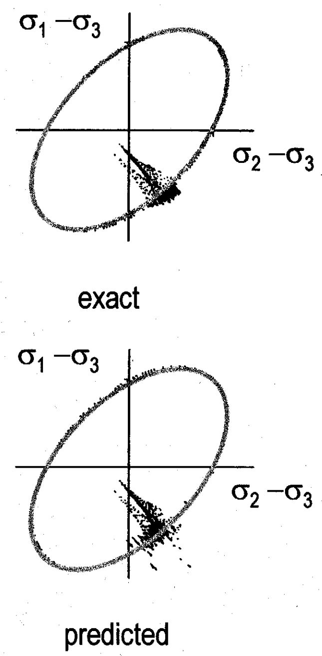

Fig. 4. Plastic strain plotted in principle stress plane: ellipse corresponds to yield locus.

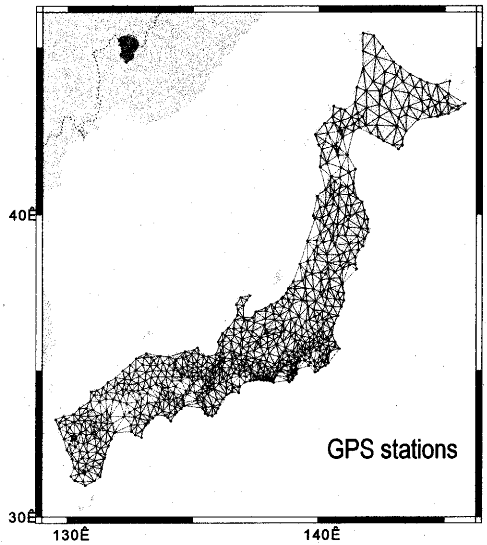

Fig. 5. nationwide GPS array.

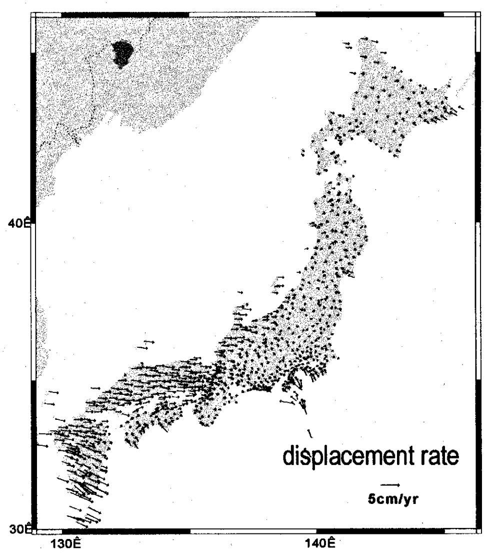

Fig. 6. Temporally filtered data of displacement increment.

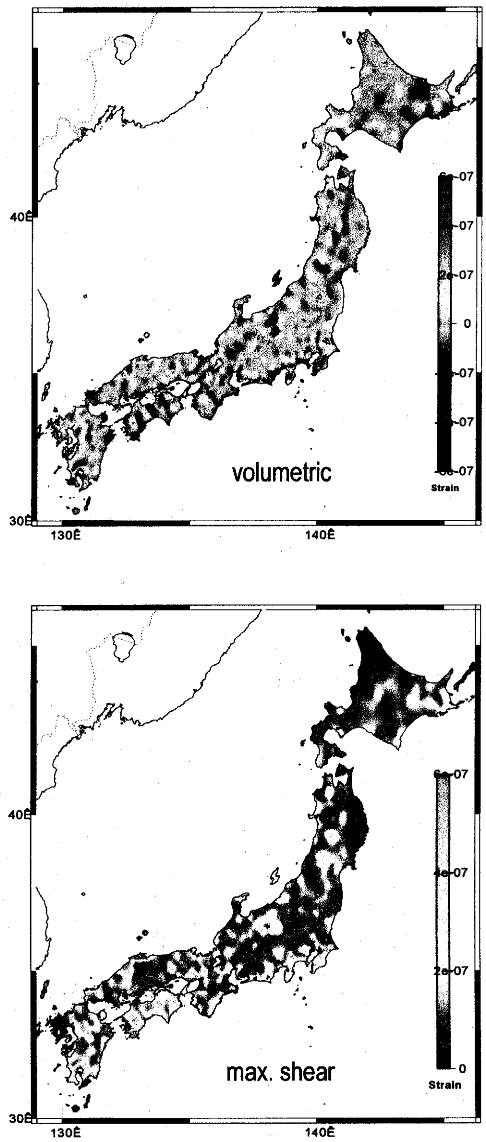

Fig. 7. Distribution of regional strain increment.

As a usage of the stress inversion, we seek to identify the regional rigidity. In the results shown above, the coefficient between the volumetric stress and strain increment is uniform all over the Japanese Islands. Now, we find a distribution of the coefficient such that the ratio between the shear stress and strain increment is proportional to the constant. The distribution of the identified coefficient is shown in Fig. 9.

IV. Concluding Remarks

The applicability of the stress inversion to the GPS array data of displacement increment is presented, although the validity of the identified regional stress increment is not verified. The stress inversion is being used to continuously monitor the crust deformation of the Japanese Islands.

Acknowledgements

This research is supported partially by Grant-in-Aid for Scientific Research, the Japan Society of the Promotion of Science.

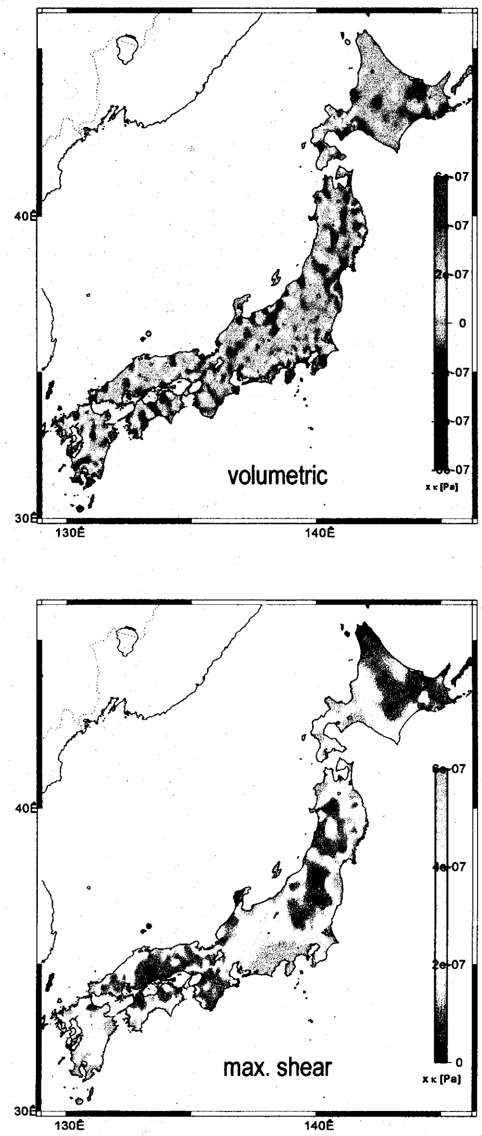

Fig. 8. Distribution of regional stress increment.

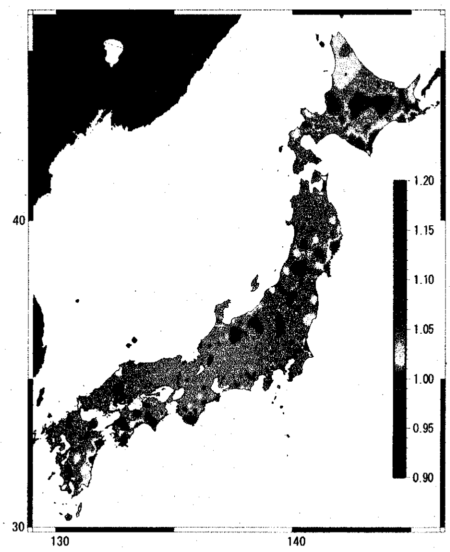

Fig. 9. Distribution of regional rigidity.

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