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生　年　月　日	
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学位授与の要件	課程博士（学位規則第 4 条第 1 項）
学位授与の題目	On the Association Schemes of Type II Matrices（タイプII行列に附随するアソシエーションスキームについて）
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学位論文要旨

Abstract

A type II matrix is a matrix which generates a spin model for a link invariant. An important connection between type II matrices and association schemes was first suggested by Jaeger, and it was explicitly formulated by Jaeger, Matsumoto and Nomura. It is known that a type II matrix W is associated with the Nomura algebra $\mathcal{N}(W)$, the Bose-Mesner algebra of a commutative association scheme. We are interested in what sort of association schemes arise as $\mathcal{N}(W)$. Suzuki and the author show that imprimitive association schemes are associated with so-called generalized tensor products of type II matrices. We have only few examples of type II matrices whose $\mathcal{N}(W)$ is primitive. We also study $\mathcal{N}(W)$ for the type II matrix W constructed on the conference graph, which gives an example of the type II matrix associated with a primitive association scheme. We show that if the size of the type II matrix W is greater than 9, $\mathcal{N}(W)$ is the trivial association scheme.

Summary

Type II Matrices form a class of matrices having certain properties related to both topological and combinatorial objects. The study of type II matrices first arose from that of spin models, which was introduced by Jones to construct invariants for knots and links. A spin model is a matrix (or a set of matrices) satisfying three conditions; type I, type II and type III, each of which corresponds to the Reidemeister moves I, II, and III respectively. Type II matrices are those satisfying the type II condition. If a type II matrix satisfies the type III condition, it becomes a spin model. At first, we were not conscious of the concept of type II matrices, which are merely matrices generating spin models. Jaeger noticed that there is an interesting relation between spin models and association schemes, and Jaeger, Matsumoto and Nomura formulated it explicitly. They showed that given a type II matrix W, we can define what is called Nomura algebra $\mathcal{N}(W)$, which becomes a Bose-Mesner algebra of a commutative association scheme. Since then combinatorists become conscious of type II matrices by way of association schemes.

The motivation for our study comes from the classification problem of spin models and type II matrices. For classification of spin models, that of type II matrices is crucial. We can classify small type II matrices, i.e., at most size 5 up to type II equivalence by direct calculations. For larger type II matrices, direct calculations do not seem appropriate way to classify them. We introduce another method of classification using their Nomura algebras. Association schemes of small orders have been classified already by Hanaki, Hirasaka, Miyamoto, Nomiyama, et al. We classify type II matrices in terms of which association schemes their Nomura algebras coincide with. In the process of classification, Suzuki and the author obtain our first result. We define generalized tensor products of matrices and show that a type II matrix can be decomposed into a generalized tensor product of small type II matrices if and only if its Nomura algebra is imprimitive:

Theorem 0.1. *Let W be a type II matrix of size mn. Let J_n be the all 1 matrix of size $n \geq 2$, and let I_m be the identity matrix of size $m \geq 2$. Then the following are equivalent.*

(*i*) *There exists a permutation matrix S such that $J_n \otimes I_m \in \mathcal{N}(SW)$.*

(ii) $\mathcal{N}(W)$ *is an imprimitive Bose-Mesner algebra with a system of imprimitivity having blocks of size* n.

(iii) W *is type II equivalent to a generalized tensor product* $(U_1, U_2, \ldots, U_m) \otimes (V_1, V_2, \ldots, V_n)$, *where* $U_1, U_2, \ldots, U_m$ *and* $V_1, V_2, \ldots, V_n$ *are type II matrices of size* n *and* m *respectively.*

By applying this result, we can classify type II matrices of size at most 10 in terms of Nomura algebras.

We are also interested in type II matrices whose Nomura algebras are primitive. We have only few examples of such type II matrices. Jaeger introduced a method of constructing of type II matrices on strongly regular graphs, a special class of distance-regular graphs. Using his method, we obtain type II matrices on strongly regular graphs, especially on conference graphs and determine their Nomura algebras, which become examples of type II matrices related to primitive association schemes:

Theorem 0.2. *Let* W *be the type II matrix constructed on the conference graph with parameters* $(4\mu+1, 2\mu, \mu-1, \mu)$. *If* $\mu > 2$, *then* $\mathcal{N}(W)$ *is trivial, i.e., the Bose-Mesner algebra of the class* 1 *association scheme.*

Our goal is to characterize association schemes obtained from type II matrices as completely as possible. We need more information about primitive cases, which is expected to be studied further in the near future.

学位論文審査結果の要旨

提出された学位論文ならびに学位申請書類にもとづき，1月22日審査会を開催し，口頭発表，面接審査を行ったうえ，以下の通り判定した。

本論文の主要結果は以下の通りである。

(1) typeⅡ matrixに附随するassociation schemeがin primitiveであることと，そのtypeⅡ matrixがよりサイズの小さいtypeⅡ matricesのgeneralized tensor productとなることとは，同値であることを示した。

(2) conference graph上のtypeⅡ matrixに附随するassociation schemeはtrivialであること，従って特にprimitiveであることを示した。

(1)は，typeⅡ matrixをよりサイズの小さいtypeⅡ matricesに分解するための基本原理を与えたもので，typeⅡ matrix研究において，極めて重要な位置をしめる仕事である。

(2)は，conference graph上のtypeⅡ matrixがよりサイズの小さなtypeⅡ matricesに分解出来ないことを示すものである。この性質を持つtypeⅡ matrixはあまり知られておらず，(2)の結果は，今後，新領域の開拓につながる可能性を秘めている。

本論文は，代数的組合せ論，低次元トポロジーに新たな知見をもたらすものであり，博士論文に十分に値するものと判定した。