Weak Interactions in the Theory of Unitary Symmetry

Akira WAKASA*, Shoichi HORI**, Takao TATI**
and Shin CHIBA**

(Recieved October 20, 1964)

Recently one of the authors[1] proposed a methpd to extend the $\triangle I=\frac{1}{2}$ rule for hadronic decays to the unitary space. Here we try to incorporate leptonic decays in the theory of unitary symmetry.

If we write an infinitisimal rotation operator in a representation space of SU (3) as[2]

$$U=1+iS, \qquad (1)$$

then the generator S is decomposed to

$$S=\sum_{i=1}^{3}\alpha_i t_i+\sum_{i=1}^{3}\beta_i u_i+\sum_{i=1}^{3}\gamma_i v_i. \qquad (2)$$

3×3 matrices t_i, u_i, and v_i are rotation operators in three dimensional real spaces, where t_3, u_3, v_3 are not linearlly independent, but satisfy the relation

$$t_3+u_3+v_3=0. \qquad (3)$$

Elementary particles and higher resonances can be classified as the t-spin (ordinary isobaric spin), u-spin and v-spin multiplets respectively[2]. There are relations between the electric charge Q, hypercharge Y and t_3, u_3, v_3 as follows

$$t_3=Q-\frac{Y}{2},\quad u_3=Y-\frac{Q}{2},\quad v_3=-\frac{Q}{2}-\frac{Y}{2}. \qquad (4)$$

The SU (3) symmetry for the strong interactions leads to the conservation laws of three spin angular momenta,

$$\triangle t_3'=\triangle u_3'=\triangle v_3'=0,$$

and

$$\triangle t'=\triangle u'=\triangle v'=0; \qquad (5)$$

where

$$t_3'+u_3'+v_3'=0.$$

and a prime stands for an eigenvalue.

Next, the corresponding selection rules for nonleptonic weak interactions[1),2)] are

$$|\triangle t_3'|=\frac{1}{2},\quad |\triangle u_3'|=1,\quad |\triangle v_3'|=\frac{1}{2}. \qquad (6)$$

* *Department of Physics*
** *Department of Physics, Faculty of Science*

We assume further that the more restrictive selection rules

$$|\triangle t'| = \frac{1}{2}, \quad |\triangle u'| = 1, \quad |\triangle v'| = \frac{1}{2} \tag{7}$$

do hold for the t, u, v spin angular momenta. Note that these rules are in agreement with the octet K°-spurion formalism[1),2),3)].

Selection rules for the strangeness conserving leptonic decays are

$$|\triangle t'_3| = 1, \quad |\triangle u'_3| = \frac{1}{2}, \quad |\triangle v'_3| = \frac{1}{2}, \tag{8}$$

and from these,

$$|\triangle t'| = 1, \quad |\triangle u'| = \frac{1}{2}, \quad |\triangle v'| = \frac{1}{2} \tag{9}$$

are expected. These rules can be described by the octet $\pi^{\pm}$-spurion formalism.

Finally selection rules for the strangeness non-conserving leptonic decays are

$$|\triangle t'_3| = \frac{1}{2}, \quad |\triangle u'_3| = \frac{1}{2}, \quad |\triangle v'_3| = 1, \tag{10}$$

and then

$$|\triangle t'| = \frac{1}{2}, \quad |\triangle u'| = \frac{1}{2}, \quad |\triangle v'| = 1 \tag{11}$$

are expected to be valid. These rules are expressed by the octet $K^{\pm}$-spurion formalism analogously.

The selection rules (7) and (11) exclude $\triangle S/\triangle Q = -1$ currents. However, recently the necessity to include $\triangle S/\triangle Q = -1$ interactions seems to have become weaker than before[4)]. In accordance with the selection rules (7) and (11), we introduce a kind of a spurion,

$$y = \begin{pmatrix} 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{12}$$

where $\tan\varepsilon$ is of the order of $\frac{1}{4}$, and define the leptonic current as

$$j_\lambda = y[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)]. \tag{13}$$

We consider that a leptonic weak interaction must contain a j_λ or j_λ^+ and must be formally unitary invariant.

Let B be the baryon octet. Then, for example, we have the following leptonic weak interactions;

$$fT_r[\bar{B}\,0_\lambda B\,y][(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{14a}$$

$$fT_r[\bar{B}\,0_\lambda y\,B][(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{14b}$$

$$fT_r[\bar{B}\,0_\lambda\,By^+][(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)e) + (\bar{\nu}_\mu\,\gamma_\lambda(1+\gamma_5)\mu], \tag{14c}$$

$$fT_r[B\,0_\lambda\,y^+B][(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)e) + (\bar{\nu}_\mu\,\gamma_\lambda(1+\gamma_5)\mu)], \tag{14d}$$

where f is the weak coupling constant and $0_\lambda = (\alpha + \beta \gamma_5)$ with real constants α and β.

In the above expeessions, (14c) and (14d) are Hermitian conjugates of (14a) and (14b), respectively. Explicilty, they are

$\triangle S = 0$

F-type :
$$\mathrm{f}\cos\varepsilon\,[\sqrt{2}\,\bar{\mathrm{p}}\,0_\lambda \mathrm{n} - \sqrt{2}\,\bar{\Xi}^0 0_\lambda \Xi^- - 2\bar{\Sigma}^+ 0_\lambda \Sigma^0 + 2\bar{\Sigma}^0 0_\lambda \Sigma^-]$$
$$\times[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{15a}$$

D-type :
$$\mathrm{f}'\cos\varepsilon\,[\sqrt{2}\,\bar{\mathrm{p}}\,0'_\lambda \mathrm{n} + \frac{2}{\sqrt{3}}\bar{\Sigma}^+ 0'_\lambda \Lambda + \frac{2}{\sqrt{3}}\bar{\Lambda}\,0'_\lambda \Sigma^- + \sqrt{2}\,\bar{\Xi}^0 0'_\lambda \Xi^-]$$
$$\times[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{15b}$$

and

$\triangle S = 1$

F-type :
$$\mathrm{f}\sin\varepsilon\,[-\sqrt{3}\,\bar{\mathrm{p}}\,0_\lambda \Lambda + \sqrt{3}\,\bar{\Lambda}\,0_\lambda \Xi^- - \bar{\mathrm{p}}\,0_\lambda \Sigma^0 - \sqrt{2}\,\bar{\mathrm{n}} 0_\lambda \Sigma^- + \bar{\Sigma}^0 0_\lambda \Xi^- + \sqrt{2}\,\bar{\Sigma}^+ 0_\lambda \Xi^0]$$
$$\times[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{15c}$$

D-type :
$$\mathrm{f}'\sin\varepsilon[-\frac{1}{\sqrt{3}}\bar{\mathrm{p}}\,0'_\lambda \Lambda + \bar{\mathrm{p}} 0'_\lambda \Sigma^0 + \sqrt{2}\,\bar{\mathrm{n}}\,0'_\lambda \Sigma^- - \frac{1}{\sqrt{3}}\bar{\Lambda}\,0'_\lambda \Xi^- + \bar{\Sigma}^0 0'_\lambda \Xi^- + \sqrt{2}\,\bar{\Sigma}^+ 0'_\lambda \Xi^0]$$
$$\times[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)]. \tag{15d}$$

Likewise we obtain for the pseudoscalar boson octet and the vector boson octet,

$$\mathrm{f}\cos\varepsilon\,[2\pi^0 \overleftrightarrow{\partial}_\lambda \pi^+ - \sqrt{2}\,\mathrm{K}^\circ \overleftrightarrow{\partial}_\lambda \mathrm{K}^+]\,[\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu)], \tag{16a}$$

$$\mathrm{f}\sin\varepsilon\,[\sqrt{3}\,\eta\,\overleftrightarrow{\partial}_\lambda \mathrm{K}^+ + \pi^\circ \overleftrightarrow{\partial}_\lambda \mathrm{K}^+ + \sqrt{2}\,\pi^+ \overleftrightarrow{\partial}_\lambda \mathrm{K}^\circ]$$
$$\times[(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu)]; \tag{16b}$$

and

$$\mathrm{f}\cos\varepsilon\,\partial_\lambda \pi^+[(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu)], \tag{17a}$$
$$\mathrm{f}\cos\varepsilon\,\rho^+_\lambda[\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\mu)]; \tag{17b}$$
$$\mathrm{f}\sin\varepsilon\,\partial_\lambda \mathrm{K}^+[(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu], \tag{17c}$$
$$\mathrm{f}\sin\varepsilon\,\mathrm{K}^{*+}_\lambda[(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu)]. \tag{17d}$$

The equality of the coupling constant f in eqs. (15a, c) and (16) is assured by the hypothesis of CVC. (If we assume CVC, $0_\lambda = \gamma_\lambda(1+\gamma_5)$ and $0'_\lambda = \gamma_\lambda \gamma_5$.) We assume also that a weak interaction responsible for the μ-e decay is given by

$$\mathrm{f}\mathrm{Tr}\,[j^+_\lambda j_\lambda] = \mathrm{f}[(\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\mathrm{e}) + (\bar{\nu}_\mu \gamma_\lambda(1+\gamma_5)\mu)]$$
$$\times[(\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e) + (\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu)], \tag{18}$$

with the same coupling constant f.

Strangeness conserving and non-conserving leptonic decays of baryons are given by Hamiltonian (15), and Hamitonians (16) and (17) are responsible for leptonic decays of bosons, e. g. $\pi^+\to\pi^0 e^+\nu$, $K\to\pi l\nu$, $\pi\to l\nu$ and $K\to l\nu$ etc. Experimental data of $(K^+\to\mu^+\nu)/(\pi^+\to\mu^+\nu)$ and $(K^+\to\pi^0 e^+\nu)/(\pi^+\to\pi^0 e^+\nu)$ are in good agreement with the above choice of $\tan\varepsilon\approx\frac{1}{4}$. The factor $\cos\varepsilon$ in the Hamiltonian (15a) is favorable to the observed small difference between β-decay and μ-decay coupling constants. Final points worth noting are : (i) $\Sigma^-\to\Sigma^0 e^-\nu$ and $\Sigma^0\to\Sigma^+ e^-\nu$ take place only in F-type interaction, (ii) $\Sigma^-\to\Lambda e^-\nu$ and $\Sigma^+\to\Lambda e^+\nu$ only in D-type interaction, and (iii) $\Lambda\to p l^-\nu$, $\Sigma^-\to n l^-\nu$, and $\Xi^-\to\Lambda l^-\nu$ depend on a D-F mixing patameter[5].

One of the authors (A. Wakasa) would like express his sincere thanks to Prof. M. Takemura for his continual encouragement.

References

1) S. Hori, Prog. Theor. Phys. 29 (1963), 612.
K. Fujii and D. Ito, Preprint, H. Sugawara, Preprint.
2) S. Meshkov, C. A. Levinson and H. J. Lipkin, Phys. Rev. Letters 10 (1963), 361.
T. Tati, Prog. Theor. Phys. (to be published.)
3) Y. Ne'eman, Nucl. Phys. 26 (1961), 222.
A. Salam and J. C. Ward, Nuovo Cimento 19 (1961), 165.
4) Reports on Brookhaven conference (1963).
5) After this work was completed, we found that N. Cabibbo (Phys. Rev. Letters 10 (1963), 531) independently arrived at the same consequences.