Harmonic-Domain Analysis of Ac Machine with Magnetic Saturation by Finite Element Method

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HARMONIC-DOMAIN ANALYSIS OF AC MACHINE WITH MAGNETIC SATURATION BY FINITE ELEMENT METHOD

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ABSTRACT

This paper presents the 3-dimensinal finite element analysis to ac electromagnetic machines with nonlinear magnetic characteristics in a steady state. The method enables us to calculate the time-periodic (non-sinusoidal) characteristics in harmonic domain. Unlike the field calculations in time domain, we can obtain only the steady-state solution directly. The analyses of the inductor with shading coil and the model of flux-concentration type electromagnetic pump are presented.

INTRODUCTION

There are difficulties in the numerical calculation of the time-periodic electromagnetic field problems including nonlinear phenomena, for instant saturation with hysteresis and eddy currents. When the magnetic field analysis is applied to the time-periodic performance of ac electromagnetic devices, the procedure of the calculation includes both the space and time variation of magnetic field variables [1].

To calculate the non-sinusoidal steady-state performance directly, we have proposed the finite element method which is combined with the well-known harmonic balance method [2], [3]. We call the method 'Harmonic Balance FEM (HBFEM)'. The HBFEM for time-periodic dynamic problems has the calculation procedure like the nonlinear static analysis. The authors made the formulation for 3-D HBFEM and analyzed some electromagnetic devices.

3-D FORMULATION FOR HARMONIC BALANCE FEM

We consider the time-periodic dynamic eddy-current problems with nonlinear properties. When the A-ϕ method is employed, the time-periodic variables A(x,y,z,t) and ϕ(x,y,z,t) on the bounded interval t=[0, 2π/ω] may be given as

$$A\,(t) = A_S \sin\omega t + A_S \cos\omega t \quad (1) \qquad \phi\,(t) = \phi_S \sin\omega t + \phi_S \cos\omega t \quad (2)$$

where ω is the fundamental angular frequency and n is harmonic order. We may express the same variables as

$$A\;(x,y,z,t) = \sum_{n=-\infty(odd)}^{+\infty} A_n\, e^{jn\omega t} \quad (3) \qquad \phi\;(t) = \sum_{n=-\infty(odd)}^{+\infty} \phi_n\, e^{jn\omega t} \quad (4)$$

where A_{-n} is A_n^*(conjugate of A_n) [4]. The coefficients in Eqs.(1) −(4) satisfy the following relations;

$$\mathrm{Re}(A) = \mathrm{Re}(A^*) = \tfrac{1}{2}A_C, \quad \mathrm{Im}(A) = -\,\mathrm{Im}(A^*) = -\tfrac{1}{2}A_S \quad (5)$$

As the magnetic field strength H is the function of B, the magnetic reluctivity ν (t) is defined in Fourier expansion as

$$\nu(t) \equiv \frac{H(t)}{B(t)} = \sum_{n=-\infty(even)}^{+\infty} \nu_n e^{jn\omega t} \tag{6}$$

where n is an even integer including zero.

We employ the governing equations expressed by

$$\mathrm{rot}\,\{\nu\,(\mathrm{rot}\,\boldsymbol{A})\} = \boldsymbol{J}_0 + \sigma\left(-\frac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\phi\right) \tag{7}$$

$$\mathrm{div}\,\boldsymbol{J}_e = -\,\mathrm{div}\left\{\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\,\phi\right)\right\} = 0 \tag{8}$$

The formulation is made by use of the Galerkin method and the prism element with 6 nodes is used. After some formulation processes, we obtain the system equation for one prism element

$$\begin{bmatrix} \mathrm{D}\otimes\mathrm{S}_{yy}+\mathrm{D}\otimes\mathrm{S}_{zz}+\sigma\omega\mathrm{N}\otimes\mathrm{S} & -\mathrm{D}\otimes\mathrm{S}_{yx} & -\mathrm{D}\otimes\mathrm{S}_{zx} & \sigma\mathrm{I}\otimes\mathrm{S}_x \\ -\mathrm{D}\otimes\mathrm{S}_{yx}^T & \mathrm{D}\otimes\mathrm{S}_{xx}+\mathrm{D}\otimes\mathrm{S}_{zz}+\sigma\omega\mathrm{N}\otimes\mathrm{S} & -\mathrm{D}\otimes\mathrm{S}_{zy} & \sigma\mathrm{I}\otimes\mathrm{S}_y \\ -\mathrm{D}\otimes\mathrm{S}_{zx}^T & -\mathrm{D}\otimes\mathrm{S}_{zy}^T & \mathrm{D}\otimes\mathrm{S}_{xx}+\mathrm{D}\otimes\mathrm{S}_{yy}+\sigma\omega\mathrm{N}\otimes\mathrm{S} & \sigma\mathrm{I}\otimes\mathrm{S}_z \\ \sigma\omega\mathrm{N}\otimes\mathrm{S}_x^T & \sigma\omega\mathrm{N}\otimes\mathrm{S}_y^T & \sigma\omega\mathrm{N}\otimes\mathrm{S}_z^T & \sigma(\mathrm{I}\otimes\mathrm{S}_{xx}+\mathrm{I}\otimes\mathrm{S}_{yy}+\mathrm{I}\otimes\mathrm{S}_{zz}) \end{bmatrix} \begin{Bmatrix} \{A_x\} \\ \{A_y\} \\ \{A_z\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{K_x\} \\ \{K_y\} \\ \{K_z\} \\ \{0\} \end{Bmatrix} \tag{9}$$

where $\otimes$ is the Toeplitz operator. The column vectors $\{A_x\}$ and $\{\phi\}$ are the harmonic components of vector and scalar potentials as follows

$$\{A_x\} = \{\ \cdots\ A^1_{x,-3}\ A^1_{x,-1}\ A^1_{x,1}\ A^1_{x,3}\ \cdots\ \ \cdots\ A^6_{x,-3}\ A^6_{x,-1}\ A^6_{x,1}\ A^6_{x,3}\ \cdots\ \}^T \tag{10}$$

$$\{\phi\} = \{\ \cdots\ \phi^1_{-3}\ \phi^1_{-1}\ \phi^1_1\ \phi^1_3\ \cdots\ \ \cdots\ \phi^6_{-3}\ \phi^6_{-1}\ \phi^6_1\ \phi^6_3\ \cdots\ \}^T \tag{11}$$

The matrices S_{xx}, S_{xy}, S_x , S and others are defined only by the shape of a prism element. The matrices $\boldsymbol{D}$ and $\boldsymbol{N}$ have the key points of HBFEM denoting the magnetic nonlinear characteristics and the time derivative respectively and are given by

$$\boldsymbol{D} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & \nu_0 & \nu_{-2} & \nu_{-4} & \nu_{-6} & \nu_{-8} & \nu_{-10} & \cdots \\ \cdots & \nu_2 & \nu_0 & \nu_{-2} & \nu_{-4} & \nu_{-6} & \nu_{-8} & \cdots \\ \cdots & \nu_4 & \nu_2 & \nu_0 & \nu_{-2} & \nu_{-4} & \nu_{-6} & \cdots \\ \cdots & \nu_6 & \nu_4 & \nu_2 & \nu_0 & \nu_{-2} & \nu_{-4} & \cdots \\ \cdots & \nu_8 & \nu_6 & \nu_4 & \nu_2 & \nu_0 & \nu_{-2} & \cdots \\ \cdots & \nu_{10} & \nu_8 & \nu_6 & \nu_4 & \nu_2 & \nu_0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{12} \qquad \boldsymbol{N} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & -5j & & & & & 0 & \cdots \\ \cdots & & -3j & & & & & \cdots \\ \cdots & & & -j & & & & \cdots \\ \cdots & & & & j & & & \cdots \\ \cdots & & & & & 3j & & \cdots \\ \cdots & 0 & & & & & 5j & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{13}$$

The system equation for the entire region is obtained by the same procedure as the conventional FEM. Each harmonic component is connected with other harmonic ones because of the nonlinearity. If the relations are relatively weak, it is possible to separate the system equations for each harmonic component. Hence the system equation for the fundamental component is given by

$$[H_{11}]\{A_1\}^k = -\sum_{j=3,5,\ldots h} [H_{1j}]\{A_j\}^{k-1} + \{G_1\} \tag{14}$$

where the first term at the right-hand side is the effect of higher harmonic components. The equation for h-order harmonics is

$$[H_{hh}]\{A_h\}^k = -\sum_{\substack{j=1,3,\ldots h \\ j\neq h}} [H_{hj}]\{A_j\}^{k-1} + \{G_h\} \qquad (15)$$

It is note that the size of the matrix $[H_{hh}]$ is the same in the static FEM even if the iteration procedure is needed.

ANALYSES OF EDDY-CURRENT PROBLEMS

The HBFEM is applied to a reactor with saturated core and shading coil as shown in Fig.1. The volume for calculation is one fourth of the whole region as indicated in Fig.1. The magnetic characteristics are shown in Fig.2. When the sinusoidal exciting current density with $6*10^5 A/m^2$ is applied, the components of magnetic density up to 5-order are shown in Fig.3. The harmonic components generated can be indicated separately in the HBFEM analysis.

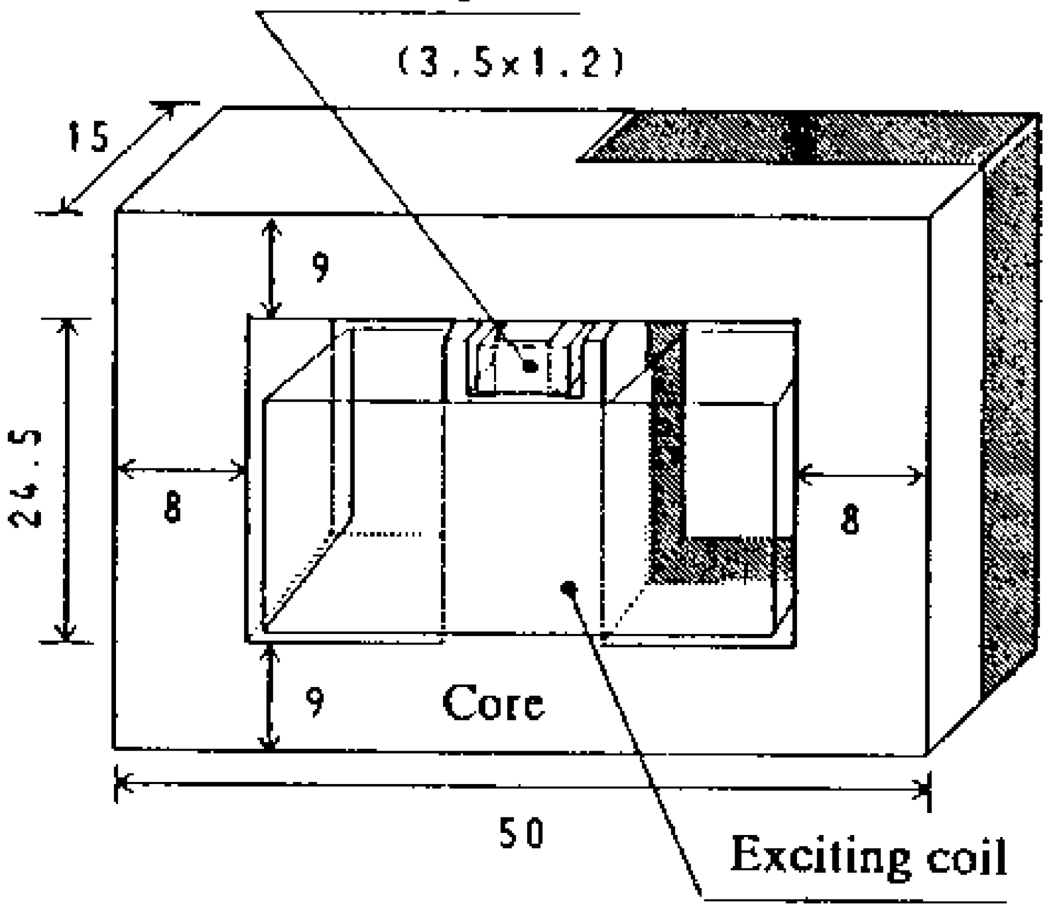


Fig.1 Saturated reactor with shading coil

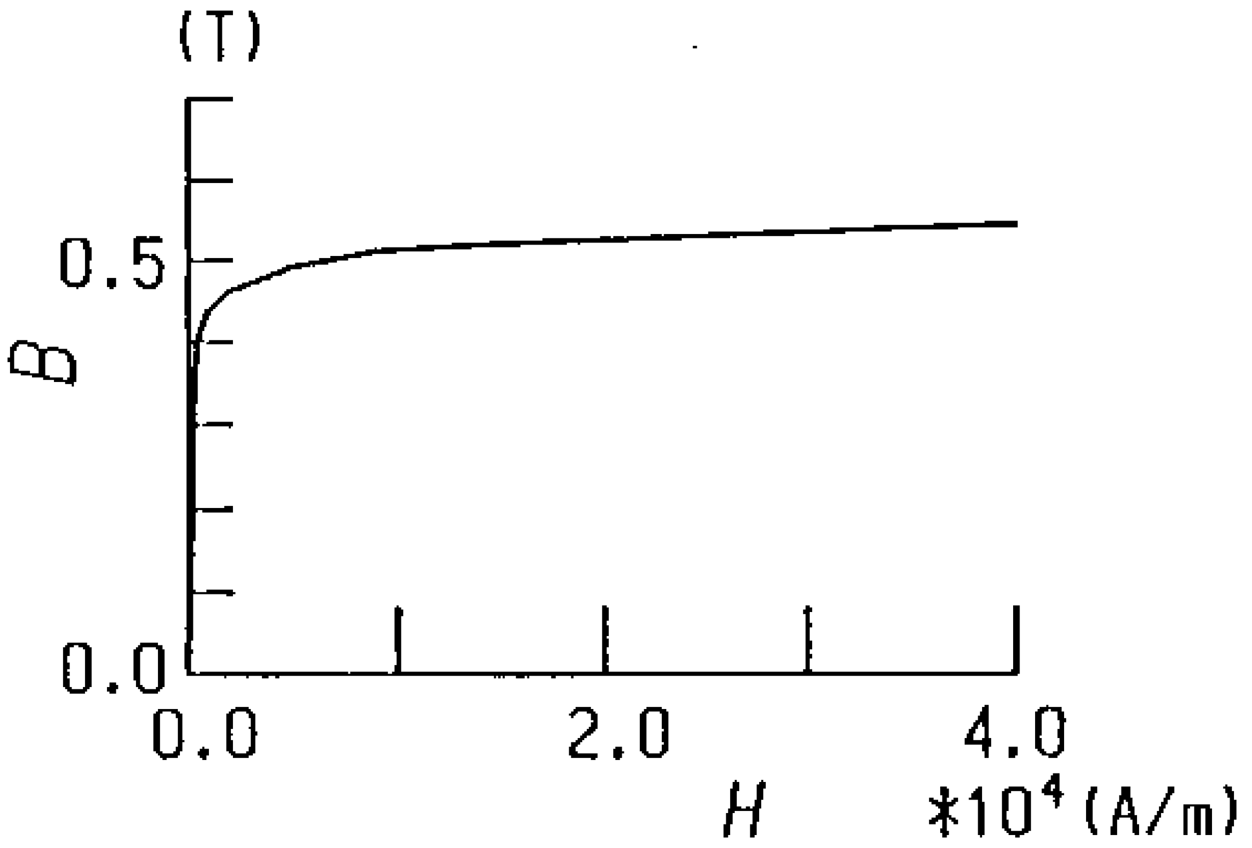
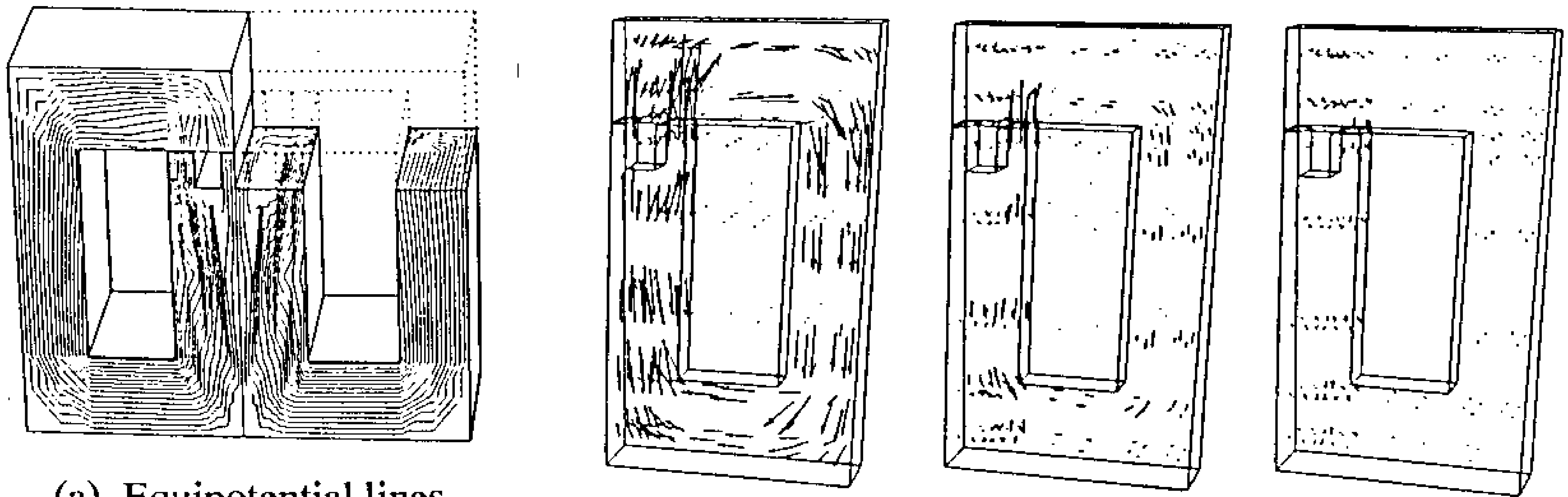


Fig.2 Magnetizing curve



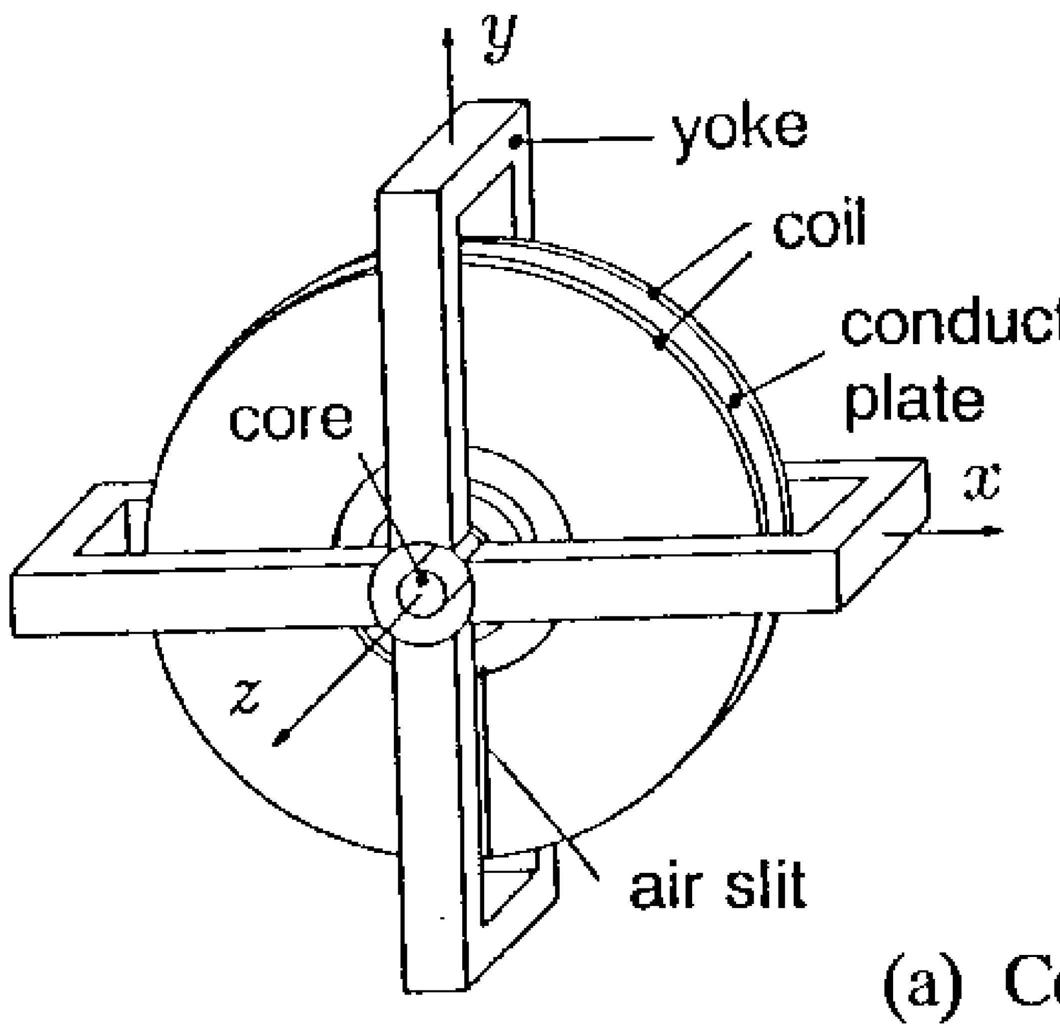
(a) Equipotential lines (Fundamental comp.)

Fundamental Comp. Third harmonics Fifth harmonics

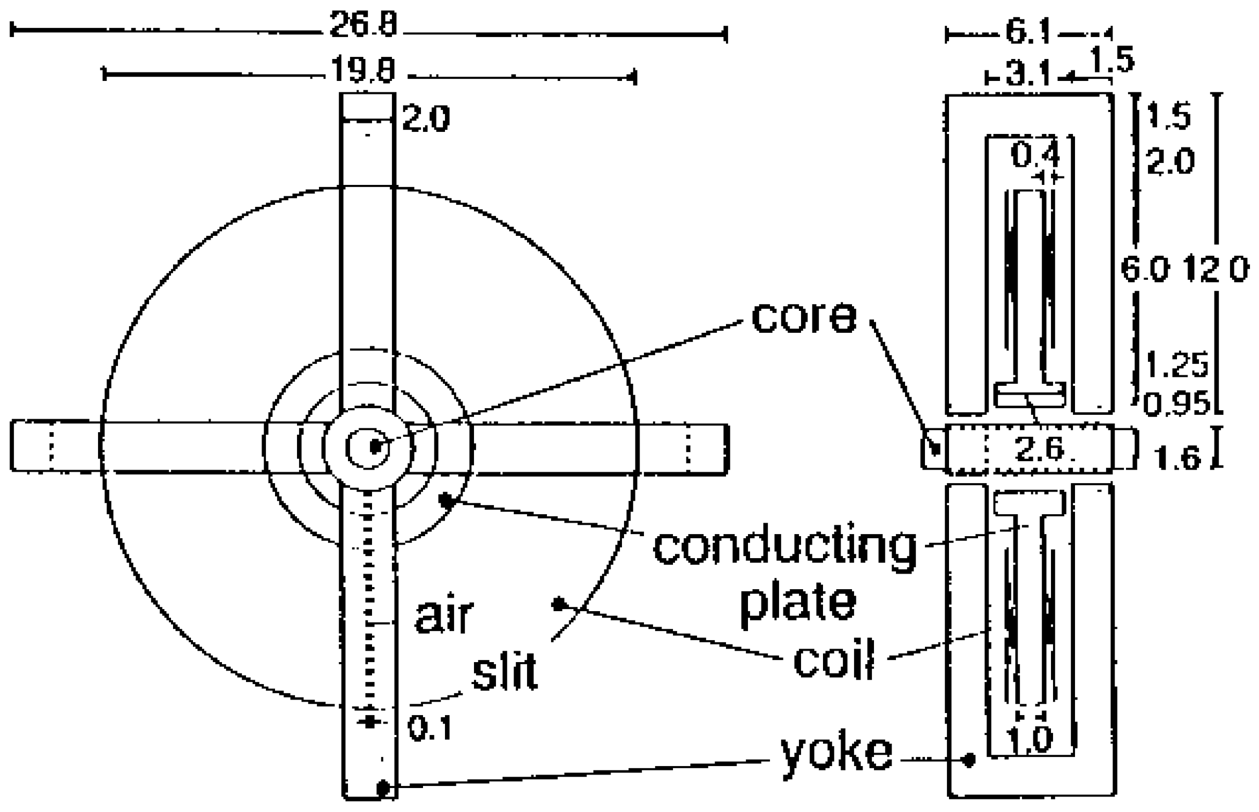
(b) Flux density vectors

Fig.3 Flux distribution

The device shown in Fig.4 is the unit model of the flux-concentration type electromagnetic pump we have developed [5]. The device employs the positive nature of eddy current , magnetic shielding. The key element of the device is the copper plate with an air slit The eddy currents in the conducting plate suppress the leakage flux between yokes.and the flux forces to concentrate into the center core. Figure 5 shows the flux distribution of the fundamental components.



(a) Configuration



(b) Shape

Fig.4 Unit model of the flux-concentration type electromagnetic pump

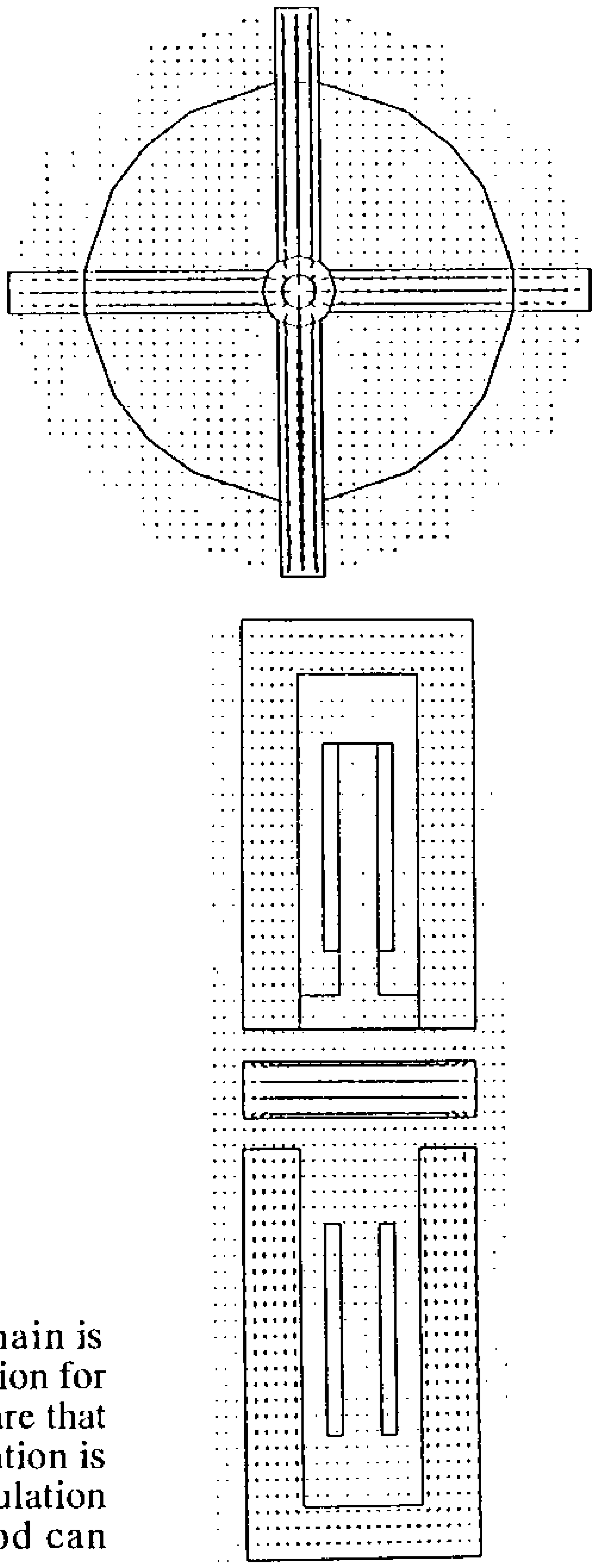


Fig.5 Flux density vectors

CONCLUSIONS

The finite element method in the harmonic domain is described. We can have directly the steady-state field solution for dynamic steady-state nonlinear problems. The advantages are that the time derivative is removed and the numerical calculation is similar to the non-linear static analysis. The separate calculation for each harmonic component and the iteration method can suppress the increase of the matrix size.

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