Dissertation

Studies on Data-Driven Controller Tuning for Cascade Control Systems

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Abstract

Data-driven approach is an effective solution to achieve the optimal controllers in the control process. In this approach, a mathematical model of a plant is not required, only a set of data directly collected from the plant to be controller is required for designing the controller. It means that we do not have to implement the identification to know the dynamics of the plant, this is an advantage in compare with the conventional method. In addition, since the data obtained from the practical system includes the dynamics of the plant more explicitly and directly than mathematical models which is described in the form of the compressed formula, data-driven approach is expected to bring more desired controllers. Due to these reasons, there are many authors studying and developing data-driven approach to control systems such that, H.Hjalmarsson and F.De Bruyne in [6, 7, 8] with iterative feedback tuning (IFT), M.C.Campi, A.Lecchini, G.O.Guardabassi in [13, 14, 12] with virtual reference feedback tuning (VRFT) and S.Souma [9], O.Kaneko [10, 11, 15, 16, 17], H.T.Nguyen [18, 19, 20] with fictitious reference iterative tuning (FRIT).

Cascade control systems are developed and used for practical multiple-loop control systems, and are applied to many industrial processes such that, temperature, humidity control, pressure, level of fluids control, oil-gas industry and adjustment of DC motor speed e.g.,.Cascade control system consists of the inner loop and the outer loop, which are often referred as the secondary loop and the primary loop, respectively. Inner loop or secondary loop controller is designed to eliminate the effect of the disturbance, to achieve faster recovery from disturbance, to improve the dynamics for the outer loop. Outer loop or primary loop controller is designed to obtained the final purpose of the controlled systems. one of the main features on cascade control systems is to be possible to independently assign the characteristics

of both these two loops for two different purposes.

In this dissertation, data-driven approach to the cascade control system is presented. Here, I treat two representative methods on this issue, one is virtual reference feedback tuning (VRFT), the other is fictitious reference iterative tuning (FRIT). Main feature to be pointed out for these two methods is that only one-shot experimental data is required for obtaining the desired controllers. I apply these two methods (VRFT and FRIT methods) of data-driven approaches to cascade control systems to obtain the optimal parameters for both inner and outer controllers . Particularly, I focus on VRFT method and clarify the meaning of the cost function. Furthermore, the prefilter is originally derived for cascade control systems to assign the inner and the outer loop property independently. This is also effective strategy to overcome non-proper problem in the VRFT method to cascade system.

Finding out the original prefilter for cascade control systems is extremely important point, it enable us to have a new method of applying VRFT approach to achieve the optimal parameters for both inner and outer controllers in the cascade scheme. Also this point is a big difference from study of the results derived by F.Previdi et.al. in [5]. The simulation results of illustrated examples demonstrate the effectiveness and the validity of my proposal results.

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Chapter 1

Introduction

1.1 Cascade control system

A cascade control system (see Fig. 1.1) is a multiple-loop system where the output of the controller in the outer loop (the "primary" or "master") is the set point of a controller in the inner loop (the "secondary" or "slave"). The inner loop controller generates an intermediate process variable that can be used to obtain more effective control of the outer process variable. Cascade control systems are developed and used for practical multiple-loop control systems, and are applied to many industrial processes such that, temperature, humidity control, pressure, level of fluids control, oil-gas industry and adjustment of DC motor speed [1, 2, 3, 4] e.g.,.

In the configuration of the cascade control system, the process is divided into two parts (the inner process and the outer process) and therefore two controllers are used, but only one signal generated from inner controller is manipulated. These two processes can be affected by disturbances d_1 and d_2, respectively.

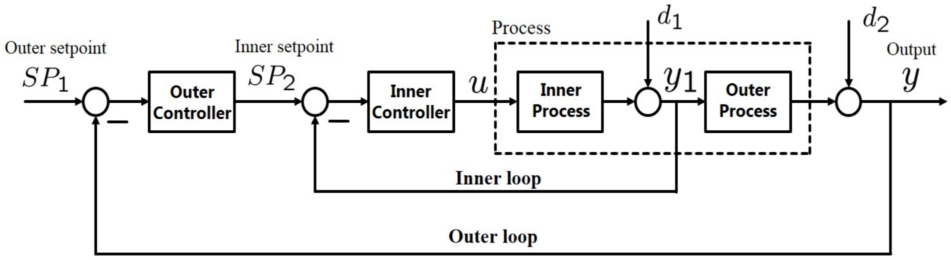

Fig. 1.1: Block diagram of a cascade control system

The closed-loop of cascade control system can be shown in a block diagram form (Fig. 1.1). Here, the outer (master) controller generates the set point SP_2 for the inner loop which includes the inner (slave) controller and the inner process. The controlled variable of the inner loop (y_1) also affects the outer process and therefore it also affects the primary controlled variable (y), which is also the output of cascade control system.

Cascade control system has several advantages on applications where the inner process has a large dead time or time lag . Cascade control system is also effective when the large disturbance occurs in the secondary loop. Because the disturbance applied to the inner loop is eliminated by the inner controller before they affect on the outer process.

The case where the cascade control system is not affected by disturbances then the configuration of cascade control system can be shown in the Fig. 1.2.

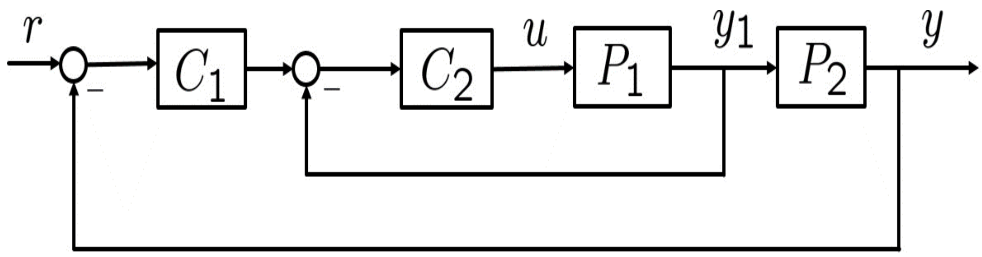

Fig. 1.2: Cascade Control System

As shown in Fig.1.2, the cascade control system consists of an inner loop where an inner controller C_2 operates as a feedback controller for plant P_1 and an outer loop where an outer controller C_1 performs the same function for inner closed loop $\frac{P_1C_2}{1+P_1C_2}$, which is serially connected to a plant P_2. The outer controller C_1 generates the set point signal for the inner loop, which includes the inner controller C_2. The controlled variable of the inner loop y_1 affects on the outer loop y. Though this is an ideal configuration, it is useful to analyze the mechanism of cascade control architecture.

1.2 Data-driven Approach

Similarly to most other control architectures, appropriate mathematical models are needed to design controllers. Practically speaking, many cases exist in which a controller has already been designed to achieve desired specifications based on mathematical models reflecting dynamics. It might thus be important to maintain the initial desired performance under aging changes, sudden changes, or intrinsic uncertainties. In these cases, it is desirable to develop update and tuning methods for parameters of the implemented controllers based on directly used measured data. This is a basic principle of " *data-driven approach*" to control.

Studies on data-driven controller tuning and updating have been developed such as, iterative feedback tuning (IFT, [6, 7, 8]), fictitious reference iterative tuning (FRIT, [9, 10]) and virtual reference feedback tuning (VRFT, [12, 13, 14]).

Specifically, FRIT and VRFT require only a one-shot experiment for the desirable parameter, so they have practical advantages over IFT, which requires many experiments in off-line nonlinear optimization.

In the structure of cascade control system, we want to design desired controllers C_1 and C_2, then the first important step is to obtain a mathematical model of plants P_1 and P_2 as exactly as possible. Controllers are designed by using conventional methods to meet a given specification based on mathematical models. Nevertheless, there are cases in where a desirable experiment to achieve mathematical model of the plants is too hard to be done. And it is also very difficult to take much time and cost to execute efficient experiments for identification P_1 and P_2. To overcome these problems, an effective solution should be used is to apply data-driven approaches to cascade control system.

In reference [5], Previdi et.al. studied VRFT design of cascade control systems with application to an electro-hydrostatic actuator. In this work the author gave two desired reference model (for the inner and the outer loop). The optimal parameters of the inner controller is achieved by adjusting the inner loop to obtain tracking properties of the inner loop so as to be approximated as desired inner reference model with respect to the outer loop. Also, the author gave the outer desired reference model, and use the same way to find the optimal parameters of the outer controller. In fact, in the reference [5] the authors minimized the cost function and used the prefilter introduced by Campi et.al. [13] to obtain the two desired controllers.

As another approach of application of VRFT to cascade control systems, in my proposed method, I applied and developed VRFT methods to cascade control systems in the cases in where plants are minimum and non-minimum phase systems. Here, I construct the original cost function for cascade control system, and simultaneously obtain optimal parameters for both inner and outer controllers by only minimizing this cost function. The most important point and different points compared with the Previdi's work, I clarified the meaning of the cost function in VRFT without the prefilter. As a result, it is clarified that the cost function of VRFT aims to optimize the open loop transfer function. Moreover, I also derived the original prefilter for cascade control system not only to avoid the problem of non-properness appearing in the cost function but also to obtain the matching between optimal parameters achieved from model reference criterion of VRFT and one yielded from original cost function. The above two points are major important different points compared the reference [5].

Besides, I also developed FRIT for cascade control systems in [18]. In the reference [18] , the authors have just applied FRIT method to cascade control system to deal with the case plants are minimum phases . FRIT for cascade control systems with non-minimum phase systems will be also implemented in this dissertation.

1.3 Outline of The Dissertation

The structure of dissertation consists of the following items

Chapter 2, VRFT approach to cascade control system is applied to simultaneously achieve the optimal parameters for both inner and outer controllers such that the output of outer cascade control system can be approximated well with a desired reference model of cascade control system. Besides, the meaning of the cost function is also explained clearly.

Chapter 3 presents the derivation of the prefilter used in the VRFT for the cascade control systems, which guarantees the optimality of the cost function to be minimized. The similar method to obtain the prefilter is also used in the FRIT case, as shown in Chapter 4.

Chapters 5 and 6 presented the extensions of VRFT and FRIT for cascade control systems to the non-minimum phase case. The strategy for the unstable zeros are presented.

Chapter 7 presented the concluding remarks and future works.

Chapter 2

Virtual Reference Feedback Tuning for Cascade Control Systems

Virtual Reference Feedback Tuning (VRFT), proposed by Campi et-al.,[13] is one of the effective data-driven tuning methods used in feedback controllers in the sense that desired parameters implemented in the controller are obtained by using only one-shot experiment data. In this chapter, I present a VRFT method for the cascade control system. I propose a direct tuning method for controllers in cascade control system to simultaneously obtain optimal parameters for both the inner and outer controllers by performing one-shot experiment to collect a set of initial data from the cascade system. I also clarify the meaning of the cost function in my proposed method and provide an example for demonstrating our proposal's effectiveness and validity.

2.1 Cascade control systems

In the introduction part of chapter 1, cascade control system was described as multiple-loop control system (see Fig. 1.2). It includes two loops such that inner loop and outer loop.

The inner loop the inner controller C_2 generates input control signal u to control the plant P_1, and y_1 is the output of the inner loop, it also affects on the plant P_2 of the outer loop and the output of the cascade control system y.

The outer loop cascade contains the outer controller C_1 operating as a feedback controller for the inner for inner closed loop $\frac{P_1C_2}{1+P_1C_2}$, which is serially connected to a plant P_2. The output of the outer loop y also is the output of the cascade control system.

Consider the cascade control system with tunable parameters in Fig. 2.1.

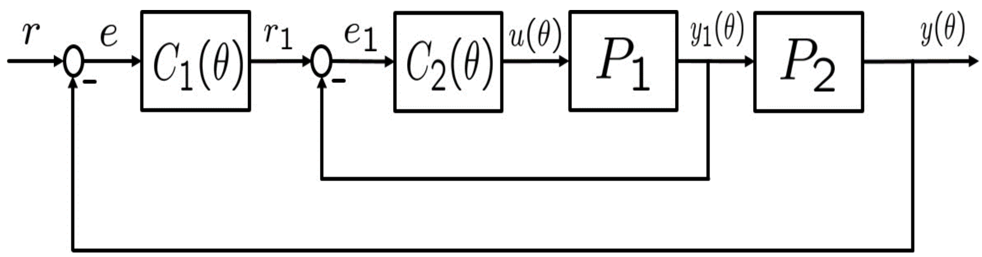

Fig. 2.1: Cascade control system with tunable parameters

In this case, we consider that P_1 and P_2 are linear, time-invariant, single-input single-output, strictly proper, stable and minimum phase. The two controllers of cascade control system are parameterized as

$$C_1(\theta) = \frac{\theta_{n+1}q^m + \cdots + \theta_{\nu-1}q + \theta_\nu}{q^n + \theta_1 q^{n-1} + \cdots + \theta_{n-1}q + \theta_n} \tag{2.1}$$

and

$$C_2(\theta) = \frac{\theta_{\nu+n'+1}q^{m'} + \cdots + \theta_{\mu-1}q + \theta_\mu}{q^{n'} + \theta_{\nu+1}q^{n'-1} + \cdots + \theta_{\nu+n'-1}q + \theta_{\nu+n'}} \tag{2.2}$$

by using a parameter vector

$$\theta = \begin{bmatrix} \theta_1 & \cdots & \theta_\nu & \theta_{\nu+1} & \cdots & \theta_\mu \end{bmatrix}. \tag{2.3}$$

A transfer function with a tunable parameter vector θ from r to y is defined by $T(\theta)$, it is shown as

$$T(\theta) = \frac{P_1P_2C_1(\theta)C_2(\theta)}{1 + P_1C_2(\theta) + P_1P_2C_1(\theta)C_2(\theta)} \tag{2.4}$$

Similarly, input, output, and inner output with parameter θ are denoted by $u(\theta)$, $y(\theta)$, and $y_1(\theta)$.

Using a parameter vector θ_{ini}, assume that the current or initial closed loop is stable. The desired tracking closed loop transfer function from r to y is given as M.

The initial output $y_{\text{ini}} := y(\theta_{\text{ini}})$ is different from desired output of cascade system $y_{\text{d}} := Mr$. Here, the purpose of tuning parameters is to find optimal parameter vector θ^* such that the output of cascade control system with these optimal parameters can approximated well with the desired output $y_{\text{d}} := Mr$.

To find the optimal parameters for both inner and outer controllers we have to minimize $\|y(\theta^*) - Mr\|_N^2$ and use the initial data $u_{\text{ini}} = u(\theta_{\text{ini}})$, y_{ini}, and $y_{1\text{ini}} := y_1(\theta_{\text{ini}})$.

2.2 Basic of VRFT method

In this section, I present some main points of an original VRFT method based on reference [13]. A diagram of conventional feedback system with the tunable controller is shown in Fig. 2.2.

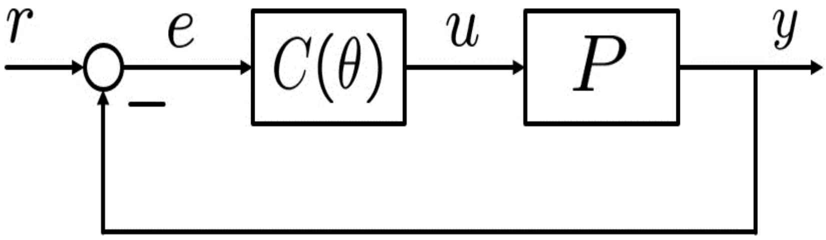

Fig. 2.2: A conventional feedback control system

Here, we consider that P is a linear, time-invariant, single-input, single-output dynamical system. We also assume that P is unknown. Controller $C(\theta)$ is parameterized by using tunable parameter vector $\theta := [\theta_1 \ \theta_2 \cdots \theta_\nu]$ as follow:

$$C(\theta) = \frac{\theta_{n+1} q^m + \cdots + \theta_{\nu-1} q + \theta_\nu}{q^n + \theta_1 q^{n^1} + \cdots + \theta_{n-1} q + \theta_n}. \tag{2.5}$$

The only set of data we use is the one-shot experimental data of input u and output y. We also give desired reference transfer function M from r to y.

With above conditions, virtual reference signal $\bar{r}$ is given to satisfy

$$y = M\bar{r} \tag{2.6}$$

by using actual (initial) output, and define virtual error

$$\bar{e} := \bar{r} - y. \tag{2.7}$$

Cost function

$$J_V(\theta) = \|u - C(\theta)\bar{e}\|_N^2 \tag{2.8}$$

is then minimized for θ. u is actual (initial) input and $C(\theta)\bar{e}$ is referred as virtual input.

Roughly speaking, the ideal minimization of $J_V(\theta)$ is equivalent to that of

$u - C(\theta)\bar{e} = 0$, i.e., $u = C(\theta)\bar{e} = C(\theta)(\bar{r} - Pu)$. The last relation is also equivalent to stating that

$$(1 + PC(\theta))\, u = C(\theta)\bar{r} \tag{2.9}$$

where the right side of (2.9) is equal to

$$C(\theta)\bar{r} = C(\theta)\frac{1}{M}y = \frac{C(\theta)P}{M}u. \tag{2.10}$$

From (2.9) and (2.10), we see that

$$Mu = \frac{PC(\theta)}{1 + PC(\theta)}u \tag{2.11}$$

holds.

To briefly explain the mechanism of VRFT in [13], the closed loop with optimal θ is thus close to desired model M under the influence of initial input data u.

The problem of the non-properness of $1/M$ that appears in the virtual reference is avoided by using prefilter $L = M(1 - M)$, which guarantees the optimality of J_V in cases where ideal minimization can not be achieved, applied to data. In [13], a more theoretical analysis of this prefilter is discussed for cases in which $C(\theta)$ is linearly parameterized, which is done by convex optimization. See [13] for details.

2.3 VRFT approach for cascade control systems

2.3.1 Construct an original cost function of cascade control systems

In presenting of VRFT approach to cascade control systems, we concentrate on the inner closed loop. Applying [13] to the inner loop, we introduce cost function

$$J_V(\theta) = \|u_{\mathrm{ini}} - C_2(\theta)\bar{e}_1\|_N^2. \tag{2.12}$$

$\bar{e}_1$ is the error between the initial output of inner closed-loop $y_{1\text{ini}}$ and virtual reference $\bar{r}_1$ for the inner loop, which is calculated as

$$\bar{e}_1 = \bar{r}_1 - y_{1\text{ini}} \tag{2.13}$$

Note that $\bar{r}_1$ is also virtual output of outer controller C_1.

Now let us focus on the outer loop of the cascade control system. Similar to the inner loop, we introduce virtual error $\bar{e}$ by using actual output y_{ini} as follows:

$$\bar{e} = \bar{r} - y_{\text{ini}} \tag{2.14}$$

$\bar{r}$ is a virtual reference such that

$$y_{\text{ini}} = M\bar{r} \tag{2.15}$$

for the outer loop.

As stated above, $\bar{r}_1$ is also the output of $C_1(\theta)$. We thus see

$$\bar{r}_1 = C_1(\theta)\bar{e} \tag{2.16}$$

By substituting $\bar{r}_1$ in (2.16) into (2.13) with (2.14) and (5.19), we obtain

$$\begin{aligned}
\bar{e}_1 &= C_1(\theta)\bar{e} - y_{1\text{ini}} \\
&= C_1(\theta)(\bar{r} - y_{\text{ini}}) - y_{1\text{ini}} \\
&= C_1(\theta)\left(\frac{1}{M} - 1\right)y_{\text{ini}} - y_{1\text{ini}}.
\end{aligned} \tag{2.17}$$

Last, we substitute $\bar{e}_1$ in (2.17) into (2.12) to obtain performance index $J_V(\theta)$ of cascade control system as

$$J_V(\theta) = \left\| u_{\text{ini}} + C_2(\theta)y_{1\text{ini}} - C_1(\theta)C_2(\theta)\left(\frac{1}{M} - 1\right)y_{\text{ini}} \right\|_N^2 \tag{2.18}$$

As seen trivially, J_V is minimized by using only initial one-shot experimental data y_{ini}, u_{ini} and $y_{1\text{ini}}$. This means that our proposed method has practical advantages in cascaded controller tuning.

To minimize the above cost function $J_V(\theta)$, we only perform one-shot experiment on cascade control system to collect a set of initial data $\{u_{ini}, y_{1ini}, y_{ini}\}$. After minimizing the cost function, we simultaneously obtain the optimal parameters for both inner and outer controllers of the cascade control system such that the output of cascade control system with these optimal parameters is almost the same with desired output $y_d = rM$. This important point is a big difference from F. Previdi's studies in [5].

In the reference [5], F. Previdi and co-authors gave a VRFT method design of cascade control systems with application to an electro-hydrostatic actuator. In the pages 628, 629 of reference [5], F. Previdi gave two desired reference models (the inner desired reference model M_v, the outer desired reference model M_x), and then he used cost functions introduced by Campi [13], $J_{VR}(\theta_v) = \frac{1}{N}\sum_{t=1}^{N}(u_{vL}(t) - C_v(\theta_v)e_{vL}(t))^2$ for the inner loop, and $J_{VR}(\theta_x) = \frac{1}{N}\sum_{t=1}^{N}(r_{vL}(t) - C_x(\theta_x)e_{xL}(t))^2$ for the outer loop.

To obtain the tracking properties of the inner loop, he minimized the cost function $J_{VR}(\theta_v)$ to achieve the optimal parameters of the inner controller C_v. Similarly, the optimal parameters of the outer controller C_x are obtained by minimizing outer cost function $J_{VR}(\theta_x)$.

From these points, we see that in my proposed method , I present an VRFT approach to cascade control system such that we achieve the tracking properties of the outer cascade control system , the output of cascade control system can approximate well with desired output. Also, I calculated to establish the original cost function (equation 2.18) of VRFT method for cascade systems. Only one-shot experiment to collect the initial data is necessary for minimizing the cost function (2.18). Minimizing this cost function simultaneously yields the optimal parameters for both inner and outer controllers. These important points are absolutely different from the study of authors in [5].

2.3.2 The meaning of the cost function with ideal case

First, we consider the meaning of $J_V(\theta)$ in the ideal case, i.e., the case in which θ^* exists such that $J_V(\theta^*) = 0$. In this case, note that

$$u_{\text{ini}} + C_2(\theta^*)y_{1\text{ini}} - C_1(\theta^*)C_2(\theta^*)\left(\frac{1}{M} - 1\right)y_{\text{ini}} = 0 \tag{2.19}$$

holds generically. Note that trivial relations

$$P_1 u_{\text{ini}} = y_{1\text{ini}} \tag{2.20}$$

$$P_2 y_{1\text{ini}} = y_{\text{ini}} \tag{2.21}$$

Also hold. Using them, we rewrite (2.19) as

$$\left(1 + P_1 C_2(\theta^*) - C_1(\theta^*) C_2(\theta^*) P_1 P_2 \left(\frac{1}{M} - 1\right)\right) u_{\text{ini}} = 0$$

which is equivalent to

$$\frac{1 + P_1 C_2(\theta^*)}{C_1(\theta^*) C_2(\theta^*) P_1 P_2} u_{\text{ini}} = \left(\frac{1}{M} - 1\right) u_{\text{ini}}. \tag{2.22}$$

Simple algebraic manipulation of (2.22) then yields

$$T(\theta^*) u_{\text{ini}} = M u_{\text{ini}} \tag{2.23}$$

which means that the optimal closed loop is achieved over initial input.

2.3.3 The meaning of the cost function with ordinary case

It is difficult to achieve $J_V(\theta) = 0$. Even in this case, we rationally interpret the minimization of $J_V(\theta)$. First, note that

$$\frac{M}{1 - M} =: H_M \tag{2.24}$$

is interpreted as the desired open loop transfer function for M. This is because $1 - M$ is the sensitivity function of desired closed loop M and the open loop transfer function of the feedback loop is represented by the ratio of sensitivity function $1 - M$ and the complementary sensitivity function where the later function is equal to closed loop transfer function M from the reference signal to output .

Similarly, an open loop transfer function for $T(\theta)$ with parameter θ is interpreted as

$$\frac{T(\theta)}{1 - T(\theta)} =: H_T(\theta) \tag{2.25}$$

From the Fig. 2.1, after some simple calculations yield

$$H_T(\theta) = \frac{P_1 P_2 C_1(\theta) C_2(\theta)}{1 + P_1 C_2(\theta)} \tag{2.26}$$

The cost function (2.18) is also rewritten as follows:

$$\begin{aligned} J_V(\theta) &= \left\| \left(1 + P_1C_2(\theta) - P_1P_2C_1(\theta)C_2(\theta)\left(\frac{1}{M} - 1\right)\right) u_{\text{ini}} \right\|_N^2 \\ &= \left\| (1 + P_1C_2(\theta))\left(1 - \frac{P_1P_2C_1(\theta)C_2(\theta)}{1 + P_1C_2(\theta)}\frac{1}{H_M}\right) u_{\text{ini}} \right\|_N^2 \\ &= \left\| (1 + P_1C_2(\theta))\left(1 - \frac{H_T(\theta)}{H_M}\right) u_{\text{ini}} \right\|_N^2 \end{aligned} \tag{2.27}$$

The part $\frac{1}{1+P_1C_2(\theta)}$ is regarded as the sensitivity function of the inner loop.

Above equation in (2.27) shows that the minimization of $J_V(\theta)$ in (2.18) corresponds to that of the relative error between open loop transfer function $H_T(\theta)$ and H_M under the influence of the inverse sensitivity function of the inner loop and initial input data u_{ini}.

2.3.4 Algorithm

The proposed method is summarized in the following algorithm:

1. Set initial parameter vector θ_{ini}

2. Execute a one-shot experiment to achieve a set of data $\{u_{\text{ini}}, y_{1\text{ini}}, y_{\text{ini}}\}$. With θ_{ini}, controllers are assumed to stabilize the closed-loop cascade control system so that these data are bounded.

3. Calculate the virtual reference signal as $\bar{r}(\theta) = \frac{1}{M}y(\theta_{ini})$ and construct performance index $J_V(\theta)$ as equation (2.18).

4. Minimize performance index $J_V(\theta)$ using nonlinear optimization such as the Least Squares, Gauss-Newton, Gradient methods, or CMA-ES[21].

5. Obtain optimal parameter vector $\theta^* := \arg\min_\theta J_V(\theta)$, which yields optimal controllers and desired output of the cascade control system.

2.3.5 Remarks

To overcome the problem non-properness of $\frac{1}{M}$ appearing in the cost function of VRFT to cascade control system, we have to use prefilter introduced by Campi [13], this prefilter also guarantees the optimality of the cost function in VRFT method. This prefilter is applied for convention system, and it can be used to avoid problem non-properness of $\frac{1}{M}$ in cost function (2.18). Of course, finding out an original prefilter for cascade control systems in VRFT method is an important issue which allows us to obtain better results. We will focus on this work in the next chapter.

In this method, we conducted one-shot experiment on closed loop cascade control system to collect the initial data. However, we can also obtain good results in the open loop cascade control systems case.

The meaning of cost function in VRFT method for cascade control system given by equation (2.27) shows that when we minimize it, we consider the relative error between the open loop desired transfer function and the open loop transfer function cascade system. In the case of FRIT to cascade control system in [19], the meaning of the cost function is considered by the relative error between the closed loop desired transfer function and the closed loop transfer function cascade system. This point is an important difference between two methods.

2.4 Numerical Example

To demonstrate the validity of the proposed method, we give an illustrative example of a cascade control system in a continuous-time domain.

Unknown plants of cascade control system are described as follows:

$$P_1 = \frac{s+8}{s^2+3s+2} \tag{2.28}$$

and

$$P_2 = \frac{s+9}{s^2+7s+5} \tag{2.29}$$

The outer and inner controllers are parameterized as

$$C_1(\theta) = \frac{\theta_1 s^2 + \theta_2 s + \theta_3}{\theta_4 s^2 + \theta_5 s + \theta_6} \tag{2.30}$$

and

$$C_2(\theta) = \frac{\theta_7 s + \theta_8}{\theta_9 s + \theta_{10}} \tag{2.31}$$

with $\theta := [\theta_1 \ \ \theta_2 \ \ \theta_3 \ \ \theta_4 \ \ \theta_5 \ \ \theta_6 \ \ \theta_7 \ \ \theta_8 \ \ \theta_9 \ \ \theta_{10}]^T$.

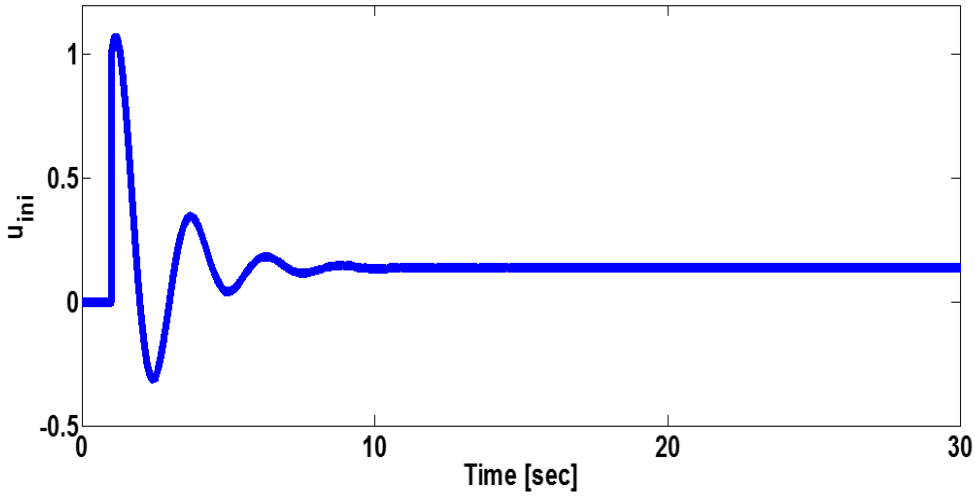

Fig. 2.3: Initial input $u_{\rm ini}$

The desired reference model of the cascade control systems M is given by

$$M = \frac{1}{s+1}. \tag{2.32}$$

Initial parameter vectors are set as

$\theta_{\rm ini} = [0.0 \ \ 1.0 \ \ 1.0 \ \ 0.0 \ \ 1.0 \ \ 0.0 \ \ 1.0 \ \ 1.0 \ \ 1.0 \ \ 0.0]^T$

We then conduct a one-shot experiment on a cascade control system to obtain initial data $u_{\rm ini}$, $y_{1\rm ini}$ and $y_{\rm ini}$.

The initial input $u_{\rm ini}$ and the initial output of the inner loop $y_{1\rm ini}$ are shown as in Fig. 2.3 and Fig. 2.4.

In Fig. 2.5, initial output of cascade control system $y_{\rm ini}$ is drawn as a solid line, reference signal r as a dot-dash line, and desired output $y_{\rm d} := Mr$ as a dotted line.

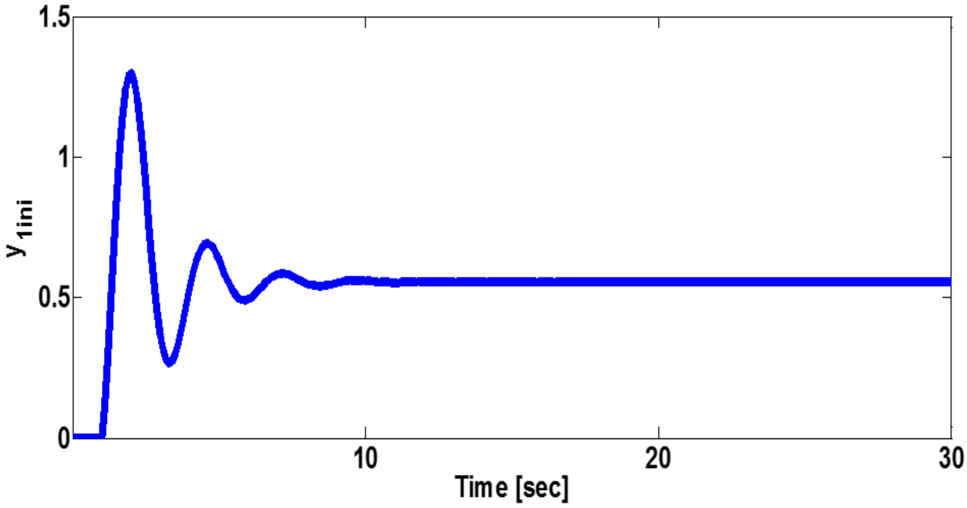

Fig. 2.4: Initial output of the inner loop $y_{1\text{ini}}$

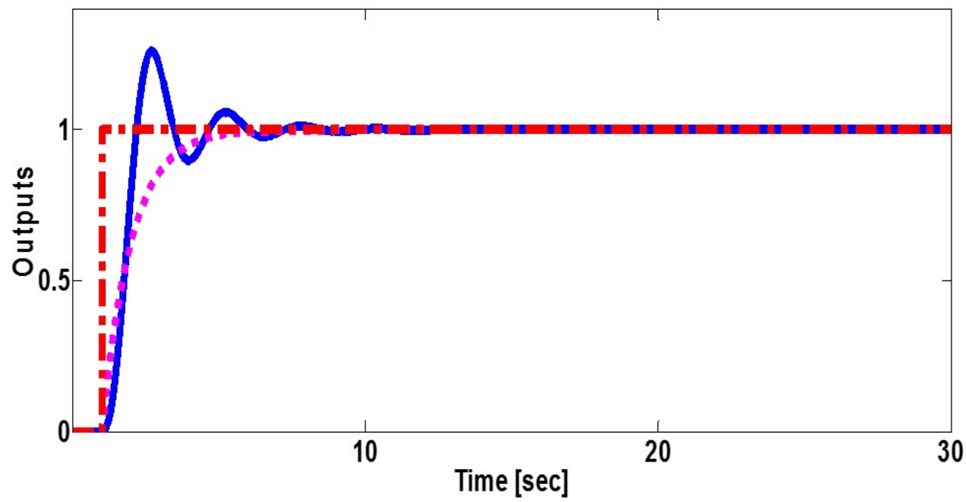

Fig. 2.5: Initial cascade control system output y_{ini} (solid line), reference signal r (dot-dash line), and desired output y_{d} (dotted line).

By applying our proposed algorithm with VRFT and using the prefilter introduced by Campi [13] $L = M(1 - M)$, the performance index $J_V(\theta)$ minimization problem is solved by using covariance matrix adaptation evolution strategy CMA-ES in [21].

In this study, we programmed the CMA-ES algorithm in MATLAB and ran it on a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 3000$.

We obtained the optimal parameter vector as

$$\theta^* = [0.4664\ \ 1.7533\ \ 1.3209\ \ 0.0283\ \ 1.2162\ \ -0.0010\ \ 0.8008\ \ 0.2504\ \ 0.0494\ \ 1.0844]^T.$$

We then conducted the experiment by using optimal parameter vectors θ^*, obtaining the results in Fig. 2.6.

In Fig. 2.6, the actual output of cascade control system with optimal parameter vectors $y(\theta^*)$ is shown as a solid line, reference signal r as a dot-dash line, the desired output y_d as a dotted line.

Input with optimal parameters $u(\theta^*)$ is shown in Fig. 2.7 and inner loop output with optimal parameters in Fig. 2.8.

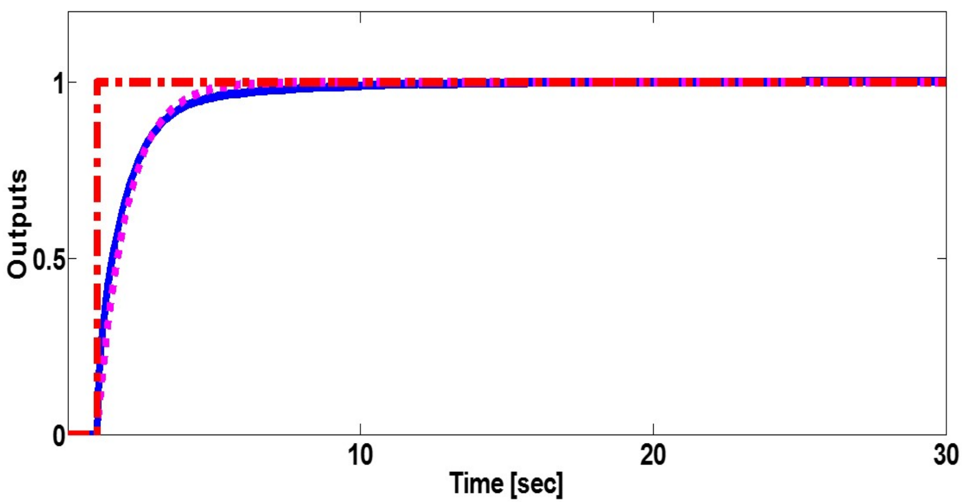

Fig. 2.6: Cascade control system output with optimal parameters $y(\theta^*)$ (solid line), the reference signal r (dot-dash line), and desired output y_d (dotted line)

Results in Fig. 2.6 show that actual output $y(\theta^*)$ and desired output y_d are almost the same, implying that we can achieve the desired output of the cascade control system by using optimal parameter vectors θ^*.

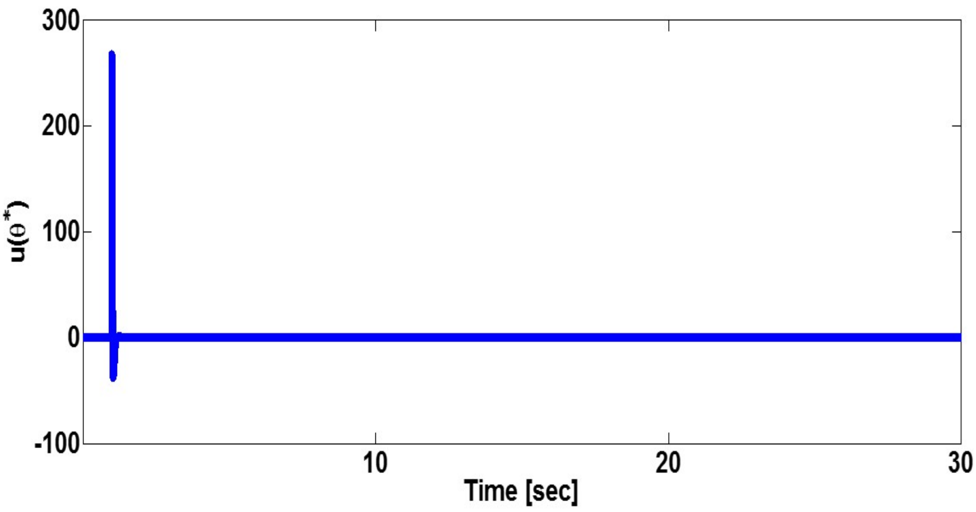

Fig. 2.7: Input with optimal parameters $u(\theta^{*})$

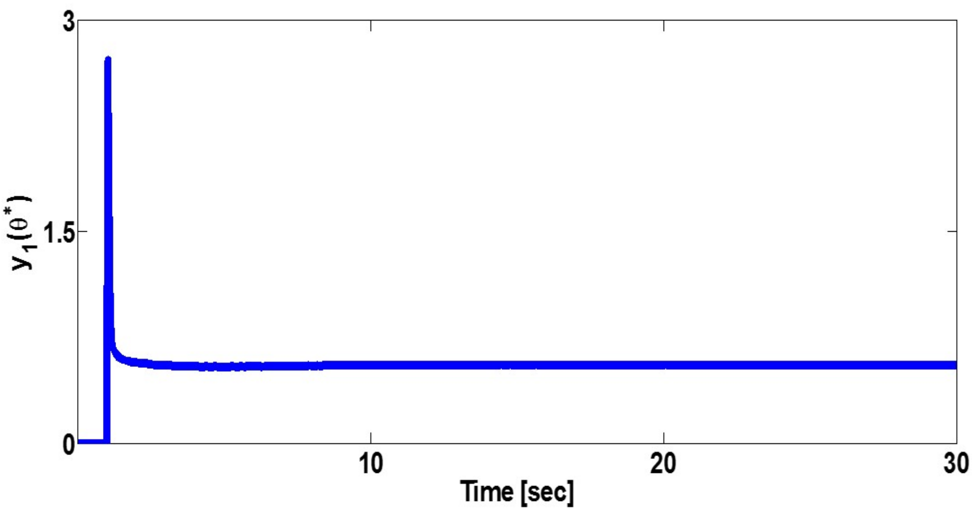

Fig. 2.8: Inner loop output with optimal parameters $y_1(\theta^{*})$

2.5 Comparing my proposed method with F.Previdi's method in the reference [5]

In this section, I use the method of F. Previdi in the reference [5] to apply for the same example given in the section 2.4. I also show clearly the advantage of my proposed method over than F. Previdi's method by comparing the results of two methods.

The structure of the cascade control system which F. Previdi used in the reference [5] is given as in Fig. 2.9

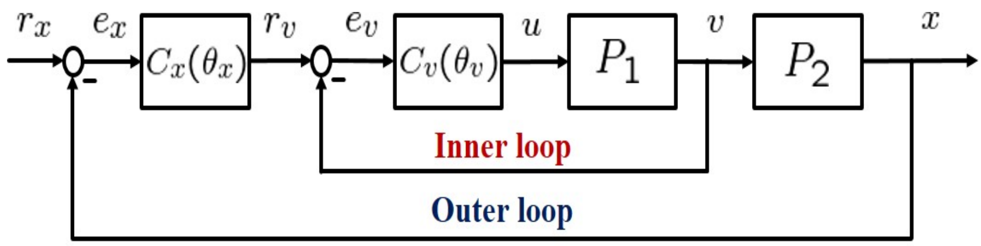

Fig. 2.9: Closed loop cascade control architecture

Where the inner and outer controllers are given such as C_v and C_x. F. Previdi used the cost functions introduced by Campi [13] for the inner and outer loops

The cost function of the inner loop is shown such as

$$J_{VR}(\theta_v) = \left\| u^0 - C_v(\theta_v)\left(\frac{1}{M_v} - 1\right)v^0 \right\|_N^2 \tag{2.33}$$

And the cost function of the outer loop is

$$J_{VR}(\theta_x) = \left\| r_v^0 - C_x(\theta_x)\left(\frac{1}{M_x} - 1\right)x^0 \right\|_N^2 \tag{2.34}$$

Where M_v and M_x are desired reference models of the inner and outer loops. The initial data $\{u^0, v^0, x^0\}$ are obtained from the cascade control system Fig. 2.9 by conducting a single experiment.

We give the desire reference model of the inner loop such as $M_v = \frac{1}{2s+1}$, and choose the same desired reference model like in the section (2.4) for the outer loop such as $M_x = \frac{1}{s+1}$

Similarly, the structures of the inner and outer controllers are selected as

$$C_v(\theta_v) = \frac{\theta'_7 s + \theta'_8}{\theta'_9 s + \theta'_{10}} \tag{2.35}$$

and

$$C_x(\theta) = \frac{\theta'_1 s^2 + \theta'_2 s + \theta'_3}{\theta'_4 s^2 + \theta'_5 s + \theta'_6} \tag{2.36}$$

Where $\theta_v = [\theta'_7 \;\; \theta'_8 \;\; \theta'_9 \;\; \theta'_{10}]^T$, $\theta_x = [\theta'_1 \;\; \theta'_2 \;\; \theta'_3 \;\; \theta'_4 \;\; \theta'_5 \;\; \theta'_6]^T$

By choosing the initial data such as

$\theta_v^0 = [1.0\ \ 1.0\ \ 1.0\ \ 0.0]^T$, $\theta_x^0 = [0.0\ \ 1.0\ \ 1.0\ \ 0.0\ \ 1.0\ \ 0.0\ \ 1.0]^T$,

we obtain the same initial output of the cascade control system as in Fig.2.5. The non-properness problems of $\frac{1}{M_v}$ and $\frac{1}{M_x}$ appeared in the cost functions(2.33), (2.34) are avoided by using prefilters introduced by Campi [13] such as $L_v = M_v(1 - M_v)$ and $L_x = M_x(1 - M_x)$.

We minimize the cost functions (2.33) and (2.34) by using a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 3000$ to run the CMA-ES algorithm in MATLAB.

This brings out the optimal parameter vectors for the inner and outer controllers as

$\theta_v^* = [0.5062\ \ 0.2354\ \ 2.4708\ \ -0.0118]^T$

$\theta_x^* = [0.4188\ \ 0.8970\ \ 0.7839\ \ 0.0046\ \ 1.3773\ \ 0.0061]^T$

Using these optimal parameter vectors to implement again on the cascade control system Fig.2.9, we obtain the outputs of the cascade control system as in Fig.2.10

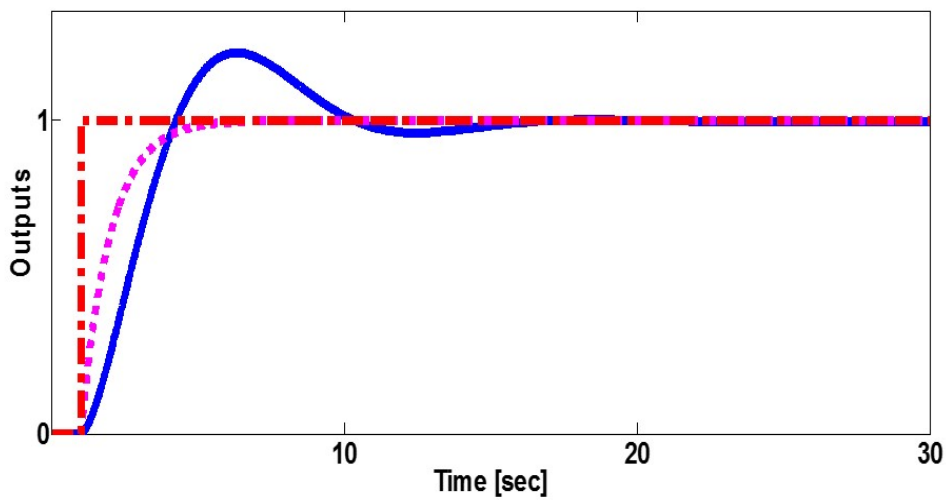

Fig. 2.10: Cascade control system outputs with optimal parameter vectors

In the Fig.2.10, the solid line, dotted line and dot-dash line indicate the actual output of the cascade control system with optimal parameters, desired output and reference signal, respectively.

Obviously, when comparing result obtained by my proposed method as in the

Fig. 2.6 with the achieved result by using F. Previdi's method as in Fig. 2.10, we see that the result in Fig. 2.10 does not satisfy the tracking property as in my result. From that observation, we can conclude that my proposed method works much better than F. Previdi's one.

2.6 Summary

In this chapter, I presented VRFT method to the cascade control systems. Only a set data of input/output experiments collected from a closed cascade control system loop is required to simultaneously achieve the optimal parameters for both inner and outer controllers in my approach. Using optimal parameters, we are able to bring out the results showing that the output of cascade control systems is almost the same with a desired output.

Also, I have shown that VRFT method effectively yields both the optimal controllers in the cascade systems. Moreover, the meaning of the cost function is theoretically analyzed by using my proposed method.

Chapter 3

Prefilter Approach to Virtual Reference Feedback Tuning for Cascade Control Systems

In this chapter, I develop the work of chapter 2 with cascade control systems by providing a more effective data-driven approach. It is based on finding out an original prefilter of Virtual Reference Feedback Tuning (VRFT) method to cascade control systems.

Deriving an original prefilter of cascade control system is a very important point, it not only overcomes the problem of non-properness existed in the cost function of VRFT method but also ensures whether the optimal parameters obtained from model reference criterion in VRFT method are closed to the optimal parameters achieved from VRFT original cost function. In addition, it enables us to have a new VRFT approach to cascade control systems. Using the original prefilter is also the main difference in comparison with the study by authors in reference [5].

The proposed approach allows us to obtain the optimal parameters of the inner and outer controllers in the structure of cascade control systems. A numerical example is given to demonstrate my proposal's effectiveness and validity.

3.1 Problem Formulation

Consider again a cascade control system with the tunable controller as in Fig. 3.1. We assume that P_1 and P_2 are linear, time-invariant, single-input single-output, strictly proper, stable and minimum phase. And we also do not mention the effect of the disturbance in the explanation.

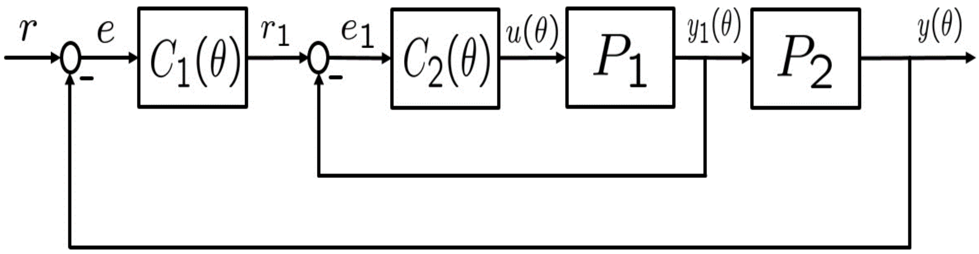

Fig. 3.1: Cascade control system with tunable parameters

The two controllers are parameterized as

$$C_1(\theta) = \frac{\theta_{n+1}q^m + \cdots + \theta_{\nu-1}q + \theta_\nu}{q^n + \theta_1 q^{n-1} + \cdots + \theta_{n-1}q + \theta_n} \tag{3.1}$$

and

$$C_2(\theta) = \frac{\theta_{\nu+n'+1}q^{m'} + \cdots + \theta_{\mu-1}q + \theta_\mu}{q^{n'} + \theta_{\nu+1}q^{n'-1} + \cdots + \theta_{\nu+n'-1}q + \theta_{\nu+n'}} \tag{3.2}$$

by using a parameter vector

$$\theta = \begin{bmatrix} \theta_1 & \cdots & \theta_\nu & \theta_{\nu+1} & \cdots & \theta_\mu \end{bmatrix}. \tag{3.3}$$

A transfer function with a tunable parameter vector θ from r to y is denoted by $G_{ry}(\theta)$, which is represented as

$$G_{ry}(\theta) = \frac{P_1P_2C_1(\theta)C_2(\theta)}{1 + P_1C_2(\theta) + P_1P_2C_1(\theta)C_2(\theta)} \tag{3.4}$$

Similarly, input, output, and inner output of cascade system with parameter θ are denoted by $u(\theta)$, $y(\theta)$, and $y_1(\theta)$.

Using a parameter vector θ^0, we assume that with this parameter the initial output of the closed loop cascade control system is bounded.

The desired reference model of a closed loop cascade system is given by M. Initial output $y^0 := y(\theta^0)$ is different from the desired output $y_\mathrm{d} := Mr$. Here, the optimal parameter vector θ^* is found by minimizing $\|y(\theta^*) - Mr\|_N^2$ and using the initial data $u^0 := u(\theta^0)$, y^0, and $y_1^0 := y_1(\theta^0)$.

3.2 Prefilter for Virtual Reference Feedback Tuning in the structure of cascade control systems

3.2.1 VRFT approach to cascade control systems

Standard of VRFT method for a convention closed-loop control system (Fig. 2.2) is shown by M. Campi [13]. It is based on minimizing the criterion index to obtain the optimal parameters for the controller.

$$J_{VR}^{N}(\theta) = \frac{1}{N}\sum_{t=1}^{N}[u(t) - C(\theta)e(t)]^2 \tag{3.5}$$

In the chapter 2, I have applied VRFT method to cascade control systems. As shown in the chapter 2 and reference [27], a virtual reference signal $\bar{r}$ of cascade control system is introduced to satisfy

$$y = M\bar{r} \tag{3.6}$$

And the criterion index of cascade control system $J_{VR}^{K}(\theta)$ is described as

$$J_{VR}^{K}(\theta) = \frac{1}{K}\sum_{t=1}^{K}\left[u(t) + C_2(\theta)y_1(t) - C_1(\theta)C_2(\theta)\left(\frac{1}{M} - 1\right)y(t)\right]^2 \tag{3.7}$$

In here, we note that the output of the inner loop $y_1(t)$ and the output of the outer loop cascade control $y(t)$ are expressed through the input $u(t)$ such that

$y_1(t) = u(t)P_1$ and $y(t) = y_1(t)P_2 = u(t)P_1P_2$

Hence, the criterion index of cascade control system $J_{VR}^{K}(\theta)$ can be described as

$$J_{VR}^{K}(\theta) = \frac{1}{K}\sum_{t=1}^{K}\left[\left(1 + C_2(\theta)P_1 - C_1(\theta)C_2(\theta)P_1P_2\left(\frac{1}{M} - 1\right)\right)u(t)\right]^2 \tag{3.8}$$

By using a compatible prefilter L_c, the signals $u(t), y_1(t)$ and $y(t)$ are filtered into $L_cu(t), L_cy_1(t)$ and $L_cy(t)$, respectively.

When the number of data increases ($K \to \infty$), using the discrete Parseval theorem [28, 29], the criterion $J_{VR}^{K}(\theta)$ is rewritten in the frequency domain as

$$J_{VR}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|L_c\left[1 + C_2(\theta)P_1 - C_1(\theta)C_2(\theta)P_1P_2\left(\frac{1}{M} - 1\right)\right]\right|^2 \Phi_u d\omega \tag{3.9}$$

Where Φ_u is the power spectrum density of $u(t)$.

3.2.2 Derivation of original prefilter for cascade control systems

As stated above, the purpose of control in VRFT approach to cascade control system scheme is to achieve the optimal parameter vector θ^* by minimizing the following model reference criterion

$$\begin{aligned} J_{MR}(\theta) &= \left\| G_{ry}(\theta) - M \right\|_2^2 \\ &= \left\| \frac{P_1 P_2 C_1(\theta) C_2(\theta)}{1 + P_1 C_2(\theta) + P_1 P_2 C_1(\theta) C_2(\theta)} - M \right\|_N^2 \end{aligned} \tag{3.10}$$

With a reference signal r is used as a virtual input of the cascade control system, we expect that it yields the desired output of cascade system.

Model reference criterion of cascade control system $J_{MR}(\theta)$ can be described as

$$J_{MR}^N(\theta) = \frac{1}{N} \sum_{t=1}^{N} \left[Mr - G_{ry}(\theta) r \right]^2 \tag{3.11}$$

Similarly to section 3.1, when $N \to \infty$ using the discrete Parseval theorem [28, 29] we obtain

$$J_{MR}(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| G_{ry}(\theta) - M \right|^2 \Phi_r d\omega \tag{3.12}$$

Where Φ_r is the power spectrum density of the signal r.

We introduce two ideal controllers C_1^d and C_2^d such that

$$\frac{P_1 P_2 C_1^d C_2^d}{1 + P_2 C_2^d + P_1 P_2 C_1^d C_2^d} = M \tag{3.13}$$

In the scheme of cascade control system, C_1^d, C_2^d are chosen to satisfy the problem model-matching such that the output of closed loop cascade control system is equal to the desired output Mr.

By substituting the desired reference model of the cascade control system M in (3.13) to equation (3.12), and after some simple calculations, the model reference criterion $J_{MR}(\theta)$ is rewritten as

$$\begin{aligned} J_{MR}(\theta) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{P_1 P_2 C_1 C_2}{1 + P_1 C_2 + P_1 P_2 C_1 C_2} - \frac{P_1 P_2 C_1^d C_2^d}{1 + P_1 C_2^d + P_1 P_2 C_1^d C_2^d} \right|^2 \Phi_r d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{|P_1 P_2|^2 |C_1 C_2 (1 + P_1 C_2^d) - C_1^d C_2^d (1 + P_1 C_2)|^2}{|1 + P_1 C_2 + P_1 P_2 C_1 C_2|^2 |1 + P_1 C_2^d + P_1 P_2 C_1^d C_2^d|^2} \Phi_r d\omega \end{aligned} \tag{3.14}$$

We consider again the cost function $J_{VR}(\theta)$ in (3.9) used in the VRFT framework for cascade control system. By using prefilter L_c and substituting M in (3.13) to equation (3.9) and after some calculations, we obtain the representation for cost function $J_{VR}(\theta)$ of cascade control system as follows

$$J_{VR}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{|L_c|^2|P_1P_2|^2|C_1C_2(1+P_1C_2^d) - C_1^dC_2^d(1+P_1C_2)|^2}{|M|^2|1+P_1C_2^d+P_1P_2C_1^dC_2^d|^2}\Phi_u d\omega \tag{3.15}$$

By comparing equation (3.14) with equation (3.15), the prefilter L_c is chosen as

$$|L_c|^2 = \frac{|M|^2}{|1+P_1C_2+P_1P_2C_1C_2|^2}\frac{\Phi_r}{\Phi_u} \tag{3.16}$$

to guarantee that $J_{VR}(\theta) = J_{MR}(\theta)$ and hence minimizing $J_{VR}(\theta)$ is the same as minimizing $J_{MR}(\theta)$.

In addition, from the equation (3.13) we acquire

$$\frac{1}{|1+P_1C_2^d+P_1P_2C_1^dC_2^d|^2} = \frac{|1-M|^2}{|1+P_1C_2^d|^2} \tag{3.17}$$

In here, the purpose of control is to obtain the model-matching problem such as the output of cascade control system equals to the desired reference model output $y = Mr$. So, we expect that $|1+P_1C_2+P_1P_2C_1C_2|^2 \approx |1+P_1C_2^d+P_1P_2C_1^dC_2^d|^2$ and $|1+P_1C_2|^2 \approx |1+P_1C_2^d|^2$ for

$$argminJ_{MR}(\theta) = argminJ_{VR}(\theta) \tag{3.18}$$

With above respect, the prefilter L_c need to satisfy

$$|L_c|^2 = |M|^2|1-M|^2\frac{1}{|1+P_1C_2|^2}\frac{\Phi_r}{\Phi_u} \tag{3.19}$$

In the equation (3.19), the prefilter L_c is still unclear since the term $\frac{1}{|1+P_1C_2|^2}$ remains unknown. However, we can overcome this difficulty by a strategy given in the next explanation.

In the diagram of cascade control system Fig. 3.1, we concentrate on the inner loop where the sensitivity function of the inner loop is given by

$$S_i = 1 - \frac{P_1C_2}{1+P_1C_2} = \frac{1}{1+P_1C_2} \tag{3.20}$$

If M_1 is a desired reference model of the inner loop, we expect that it is possible to obtain the model-matching of the inner loop such that the closed loop transfer function of the inner loop is equal to the desired reference model M_1

$$\frac{P_1 C_2}{1 + P_1 C_2} = M_1 \tag{3.21}$$

If we hold the above condition, then the sensitivity function of the inner loop S_i is rewritten as

$$S_i = \frac{1}{1 + P_1 C_2} = 1 - M_1 \tag{3.22}$$

Finally, we obtain the original prefilter L_c of the cascade control systems by substituting equation (3.22) to equation (3.19) to get

$$|L_c|^2 = |M|^2 |1 - M|^2 |1 - M_1|^2 \frac{\Phi_r}{\Phi_u} \tag{3.23}$$

The derivation of above original prefilter L_c enables us to have a new strategy in applying VRFT approach to cascade control systems. This is an important different point from the work of the chapter 2 and [27].

Besides, finding out the original prefilter of VRFT method for cascade control system in equation (3.23) is also the main difference in comparison with work of F. Previdi and co-authors in [5]. In the pages 628, 629 of the reference [5], F. Previdi only used the cost function and prefilter introduced by Campi [13] for using VRFT method to apply to an electro-hydrostatic actuator.

By constructing the cost function of outer loop cascade system as equation (2.18) with original prifilter of cascade system in (3.23), we can achieve the tracking properties problem of cascade control system such that the output of cascade control system $y(\theta^*)$ with optimal parameters can be almost the same with desired output $y_d = rM$.

Identify prefilter L_c:

The original prefilter of cascade control system L_c in (3.23) can be calculated exactly if we know the power spectrum density of r, u signals as Φ_r, Φ_u. In this case, before implementing an experiment on structure of cascade control system, the designer should decide the kind of excited input signal and author also should consider to estimate Φ_r and Φ_u.

In another case, the original prefilter of cascade control system L_c can be considered to identify as follows:

From equation (3.23), under the effect of the initial signals r^0, u^0, the original prefilter of cascade systems can be selected as

$$L_c u^0 = M(1-M)(1-M_1)r^0 \tag{3.24}$$

If we give a structure of the original prefilter L_c and parameterize it by vector $\eta = [\eta_1 \;\; \eta_2 ... \eta_n]^T$, then we establish the cost function of L_c as

$$J_{L_c}(\eta) = \|L_c(\eta)u^0 - M(1-M)(1-M_1)r^0\|_N^2 \tag{3.25}$$

Where the initial data r^0, u^0 are collected from cascade control system by one-shot experiment, M_1 and M are desired reference models of the inner loop and outer loop.

The optimal parameters of the original prefilter cascade control system L_c can be obtained by minimizing the cost function (3.25) as

$$\eta^* = argmin_\eta J_{L_c}(\eta) \tag{3.26}$$

3.2.3 Algorithm

We summarize the proposed method by the following algorithm:

In the scheme of cascade control system Fig. 3.1 we implement

1. Tuning inner loop of the cascade control system to obtain the optimal inner controller $C_2(\theta_{II}^*)$:

   - Given a reference model M_1 of the inner loop cascade control systems
   - As presented in [13], we construct the cost function of the inner loop such as

$$J_{VR_{in}}(\theta_{II}) = \left\| u_{ini} - C_2(\theta_{II})\left(\frac{1}{M_1} - 1\right) y_{1ini} \right\|_N^2 \tag{3.27}$$

Where: $\{u_{ini}, y_{1ini}\}$ are initial data of the inner loop. We also can use prefilter of Campi in [13] $L_1 = M_1(1 - M_1)$ to avoid the problem of the non-properness of $\frac{1}{M_1}$ that appears in the above cost function.

- Minimizing the cost function (3.27) to achieve the optimal parameters of the inner controller $C_2(\theta_{II}^*)$.

2. Tuning outer loop of cascade control system, which includes the outer controller $C_1(\theta_I)$ and the optimal inner controller $C_2(\theta_{II}^*)$ to achieve the optimal outer controller $C_1(\theta_I^*)$:

   - A desired reference model of the outer loop cascade control system is given by M. Conducting the second experiment on outer cascade control systems to obtain a set of initial data $\{u^0, y_1^0, y^0\}$ from cascade control system. With these initial data, the outer controller is assumed to stabilize the closed-loop cascade control system so that these data are bounded.

   - As shown in the chapter 2 and reference [27], using the original prefilter L_c derived in the equation (3.23), the cost function of the outer loop cascade control system is described as

$$J_{VR_{out}}(\theta_I) \quad = \quad \left\| L_c u^0 + C_2(\theta_{II}^*) L_c y_1^0 - C_1(\theta_I) C_2(\theta_{II}^*) \left(\frac{1}{M} - 1 \right) L_c y^0 \right\|_N^2 \tag{3.28}$$

   - Minimize the cost function (3.28) using nonlinear optimization methods such as the Least Squares, Gauss-Newton, Gradient methods, or CMA-ES [21] to obtain the optimal parameters of outer controller $C_1(\theta_I^*)$.

3.3 Numerical Example

In this section we demonstrate the validity of the proposed method by giving an illustrative example of a cascade control system in a continuous-time domain and assume that it is not affected by disturbance.

Two unknown plants of cascade control system are described as follows:

$$P_1 = \frac{s+1}{s^2+5s+6} \tag{3.29}$$

and

$$P_2 = \frac{s+5}{s^2+2.5s+1.5} \tag{3.30}$$

Firstly, we will turn the closed- loop inner of cascade control system to guarantee that the sensitivity function of the inner loop is approximate to the desired reference model of the inner loop M_1.

A desired reference model of the inner loop is given by:

$$M_1 = \frac{1}{2s+1} \tag{3.31}$$

The inner controller is parameterized as:

$$C_2(\theta_{II}) = \frac{\theta_1 s^2 + \theta_2 s + \theta_3}{\theta_4 s^2 + \theta_5 s + \theta_6} \tag{3.32}$$

with $\theta_{II} := [\theta_1\ \ \theta_2\ \ \theta_3\ \ \theta_4\ \ \theta_5\ \ \theta_6]^T$.

We use an initial parameter vector $\theta_{II\text{ini}} = [0.0\ \ 1.0\ \ 1.0\ \ 0.0\ \ 1.0\ \ 0.0]^T$, and then conduct an one-shot experiment on the inner closed-loop cascade control system to obtain initial data $\{u_{\text{ini}}, y_{1\text{ini}}\}$. The initial input of the inner loop is shown in Fig. 3.2.

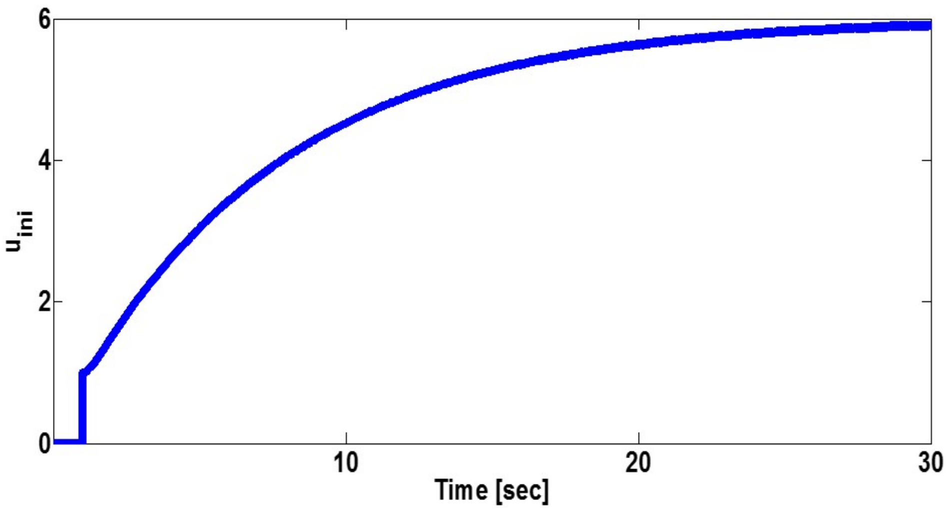

Fig. 3.2: The initial input of the inner loop.

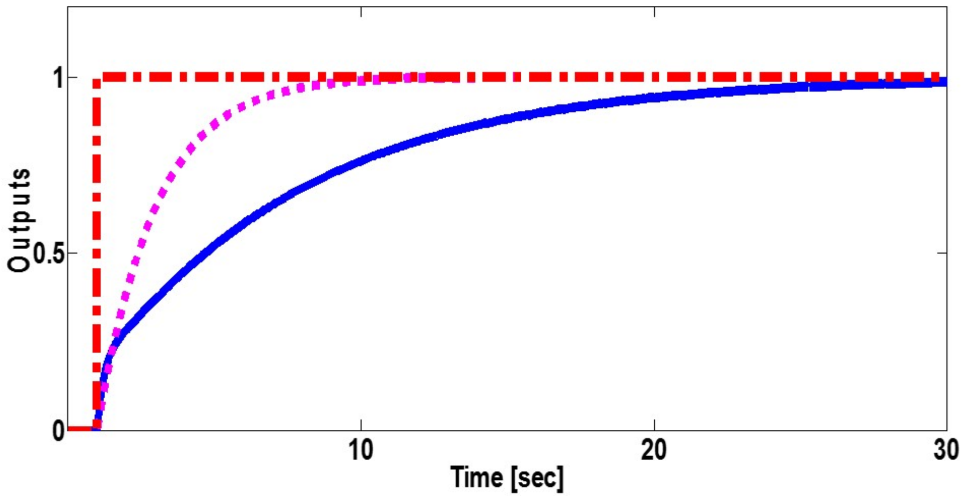

Fig. 3.3: Initial inner loop cascade control system output $y_{1\text{ini}}$ (solid line), reference signal r (dot-dash line), and desired inner loop output y_{1d} (dotted line).

In Fig. 3.3, the initial output of the inner loop cascade control system $y_{1\text{ini}}$ is the solid line, reference signal r is the dot-dash line and the desired inner loop output $y_{1\text{d}} := M_1 r$ is the dotted line.

We can obtain the cost function of the inner loop by using equation (3.27) and the prefilter introduced by Campi [13] $L_1 = M_1(1 - M_1)$. Then the cost function $J_{VR_{in}}(\theta_{II})$ minimization problem is solved by using covariance matrix adaptation evolution strategy CMA-ES in [21].

Here, a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 1000$ are used to run CMA-ES algorithm program in MATLAB.

The optimal parameter vector of the inner controller is achieved as

$$\theta^*_{II} = [0.3338 \;\; 1.3079 \;\; 1.5522 \;\; 0.5180 \;\; 0.5176 \;\; 0.0]^T.$$

We implement the experiment again on the inner loop cascade control system by using the optimal parameter vector θ^*_{II} to obtain the results in Fig. 3.4, in this figure the actual output of the inner loop cascade control system with optimal parameter vector $y_1(\theta^*_{II})$, the reference signal r and the inner desired output $y_{1\text{d}}$ are shown in solid line, dot-dash line and dotted line respectively.

We also achieve the input of the inner loop with the optimal parameters $u(\theta^*_{II})$ as in Fig. 3.5.

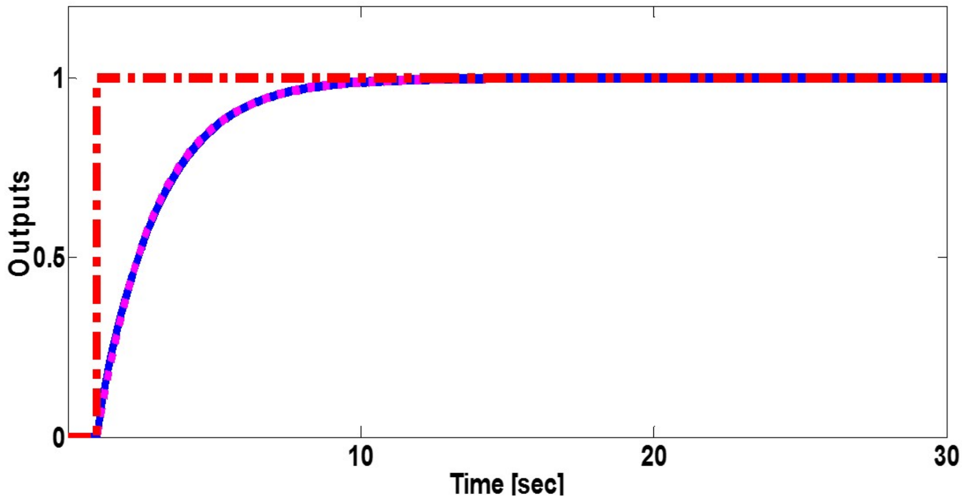

Fig. 3.4: Inner loop cascade control system output with optimal parameters $y_1(\theta^*)$(solid line), the reference signal r (dot-dash line), and desired inner loop output y_{1d} (dotted line).

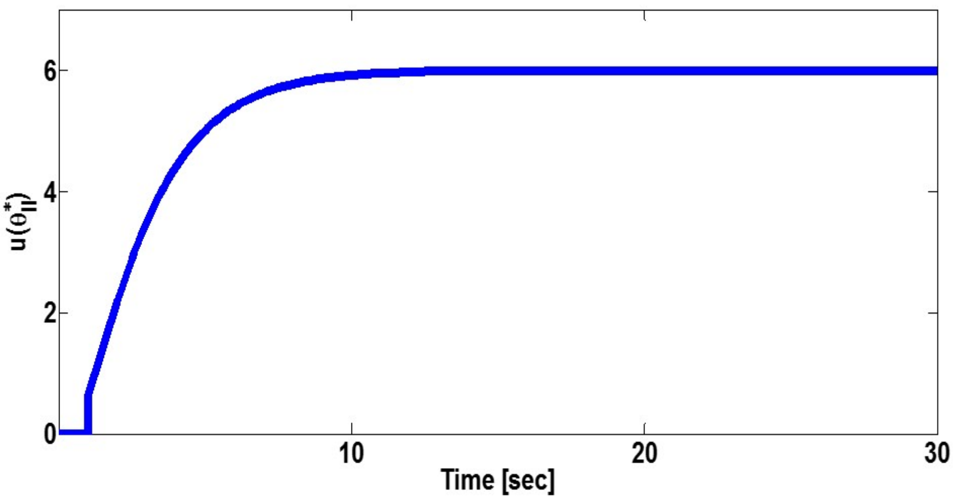

Fig. 3.5: The input of inner loop with the optimal parameters $u(\theta^*_{II})$.

Results in Fig. 3.4 show that we achieved the tracking property of the inner loop such that actual output of the inner loop cascade control system with optimal parameters $y_1(\theta^*_{II})$ and desired output $y_{1\mathrm{d}}$ are truly the same.

Next, we implement tuning the outer loop cascade control system to get the optimal outer controller $C_1(\theta^*_I)$ after obtaining the optimal inner controller $C_2(\theta^*_{II})$.

The outer controller cascade control system should be parameterized as

$$C_1(\theta_I) = \frac{\theta'_1 s^3 + \theta'_2 s^2 + \theta'_3 s + \theta'_4}{\theta'_5 s^3 + \theta'_6 s^2 + \theta'_7 s + \theta'_8} \tag{3.33}$$

Where $\theta_I := [\theta'_1 \ \ \theta'_2 \ \ \theta'_3 \ \ \theta'_4 \ \ \theta'_5 \ \ \theta'_6 \ \ \theta'_7 \ \ \theta'_8]^T$.

Desired reference model of the outer loop cascade system M is given by

$$M = \frac{1}{s+1} \tag{3.34}$$

Initial parameter vector is chosen as $\theta_I^0 = [0.0 \ \ 1.0 \ \ 1.0 \ \ 1.0 \ \ 0.0 \ \ 1.0 \ \ 1.0 \ \ 2.0]^T$, then we conduct one -shot experiment on the outer loop cascade control systems to achieve the set of the initial data $\{u^0, y_1^0, y^0\}$.

In Figs. 3.6 and 3.7, I show the first two signals (initial input u^0 and initial output of inner loop y_1^0 of the second tunning). The initial output of the outer loop cascade control system y^0 (solid line), the reference signal r (dot-dash line) and the desired output $y_d = Mr$ (dotted line) are shown respectively in Fig. 3.8.

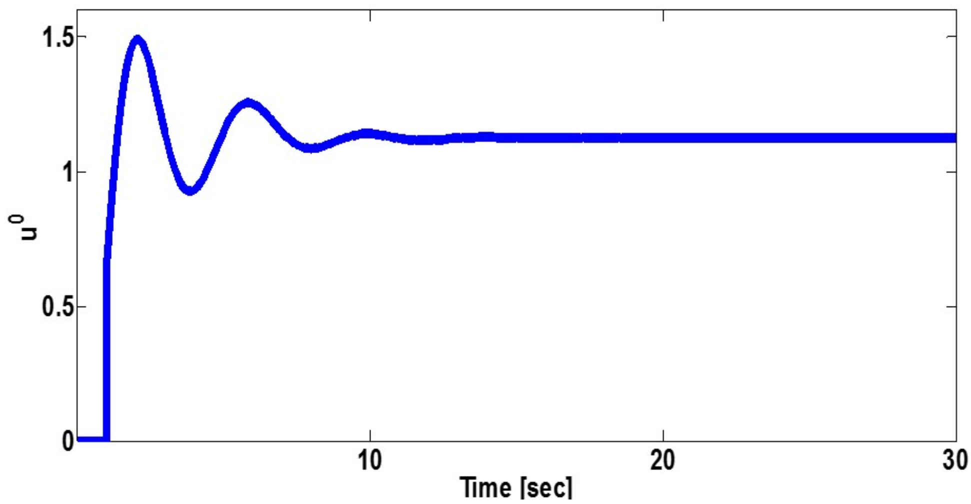

Fig. 3.6: Initial input of the outer loop cascade control system u^0.

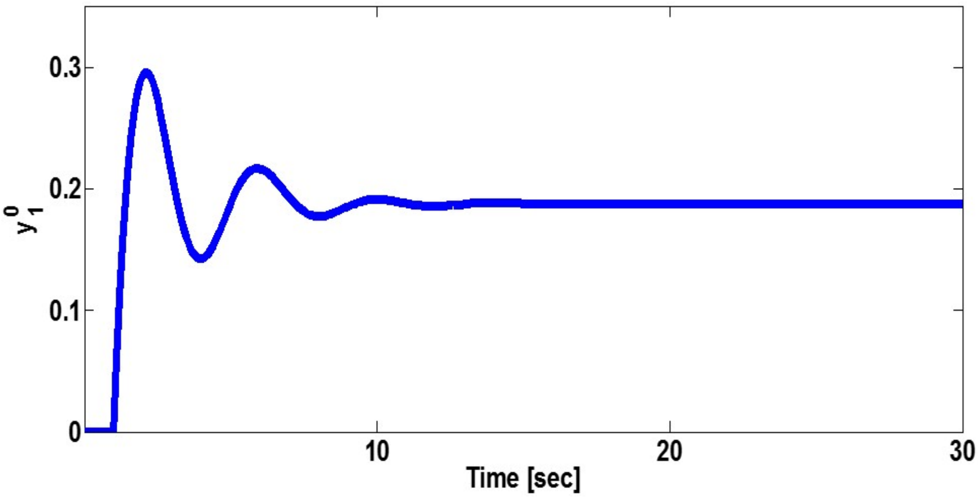

Fig. 3.7: Initial output of the inner loop y_1^0.

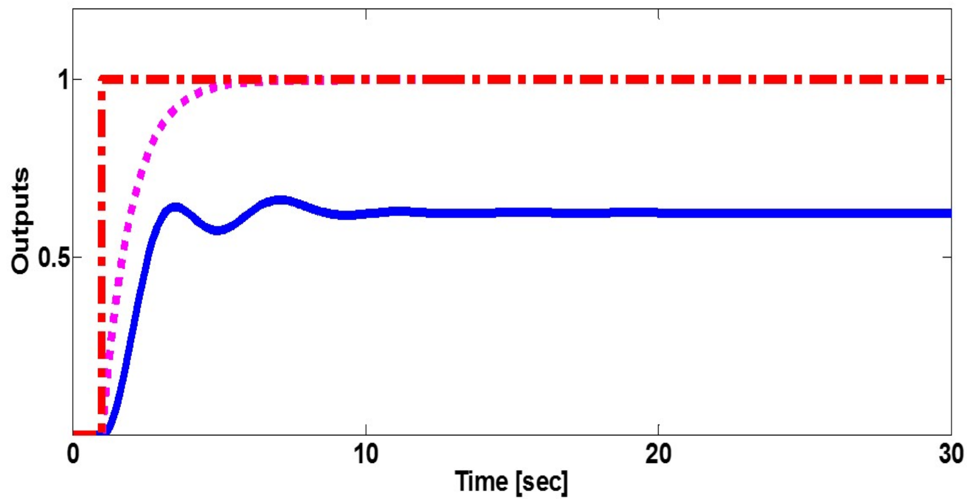

Fig. 3.8: Initial outer loop cascade control system output y^0 (solid line), reference signal r (dot-dash line), and desired output y_d (dotted line).

We use original prefilter L_c in (3.23) to establish the cost function of the outer loop cascade control system as shown in equation (3.28).

Identify original prefilter L_c:

The original prefilter L_c is identified as follows

We give the structure of the original prefilter L_c as

$$L_c = \frac{\eta_1 s^2 + \eta_2 s + \eta_3}{\eta_4 s^3 + \eta_5 s^2 + \eta_6 s + \eta_7} \tag{3.35}$$

With the parameter vector $\eta = [\eta_1\ \eta_2\ \eta_3\ \eta_4\ \eta_5\ \eta_6\ \eta_7]^T$. Then, the cost function of the original prefilter L_c is established as (3.25)

$$J_{L_c}(\eta) = \|L_c(\eta)u^0 - M(1 - M)(1 - M_1)r^0\|_N^2 \tag{3.36}$$

Where the initial data $\{u^0, r^0\}$ are obtained from cascade control system at the second experiment (tuning outer loop cascade system).

To minimize the cost function (3.36), we use CMA-ES program. Here, a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 5000$ are used to run CMA-ES algorithm program in MATLAB.

The optimal parameter vector of the original prefilter L_c is achieved as

$$\eta^* = [1.5998\ 0.2602\ \ -0.0100\ 0.9939\ 7.0649\ 3.5228\ 1.9913]^T \tag{3.37}$$

Therefore, we can obtain the original prefilter L_c for cascade control system as

$$L_c = \frac{1.5998s^2 + 0.2602s - 0.0100}{0.9939s^3 + 7.0649s^2 + 3.5228s + 1.9913} \tag{3.38}$$

This original prefilter L_c is used in the cost function of the outer loop cascade control system $J_{VR_{out}}(\theta_I)$ (3.28).

To minimize the cost function $J_{VR_{out}}(\theta_I)$, we use the same method (iterative step $N = 3000$) and tool as in the first tuning of the inner loop. The optimal parameter vector of the outer controller is obtained as:

$$\theta_I^* = [1.4036\ \ 3.5248\ \ 3.2346\ \ 0.8723\ \ 0.0082\ \ 0.4025\ \ 2.9443\ \ -0.0011]^T \tag{3.39}$$

By using this optimal parameter vector to perform again experiment on the outer loop cascade control system, we achieve results as in Figs. 3.9, 3.10 and 3.11. The actual output of the outer loop cascade control system with the optimal parameter vector $y(\theta_I^*)$ (solid line), the reference signal r (dot-dash line), and the desired output y_d (dotted line) are shown in Fig. 3.9. The input and the inner loop output with the optimal parameters $u(\theta_I^*), y_1(\theta_I^*)$ are also shown in Fig. 3.10 and Fig. 3.11.

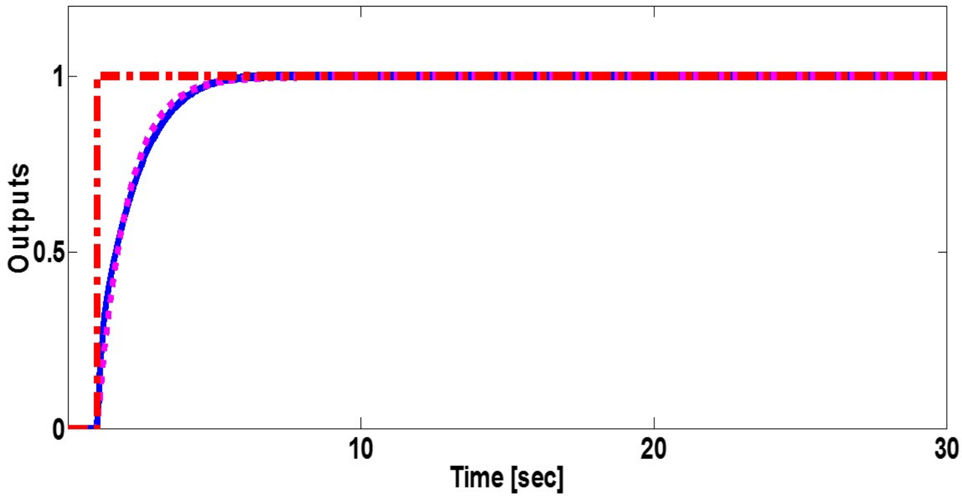

Fig. 3.9: Cascade control system output with optimal parameters $y(\theta_I^*)$ (solid line), reference signal r (dot-dash line), and desired output y_d (dotted line).

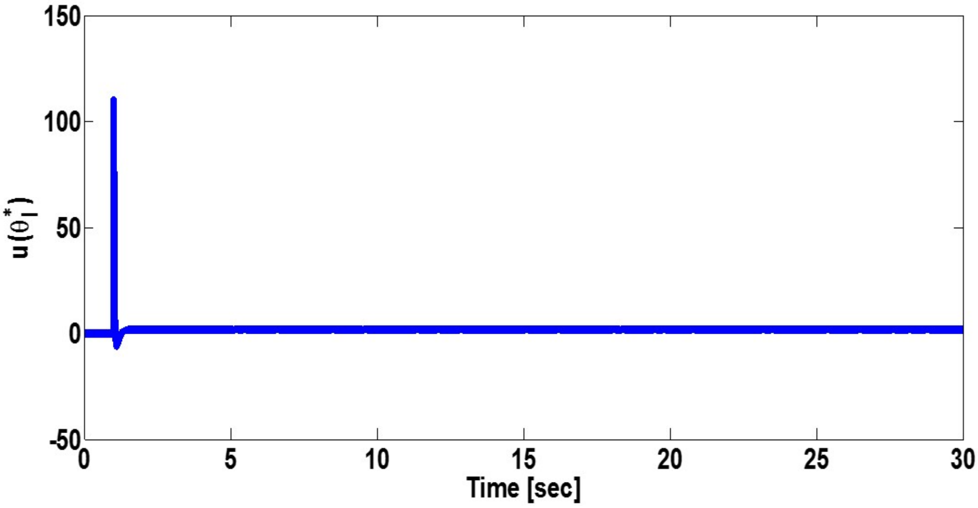

Fig. 3.10: Input with optimal parameters $u(\theta_I^*)$.

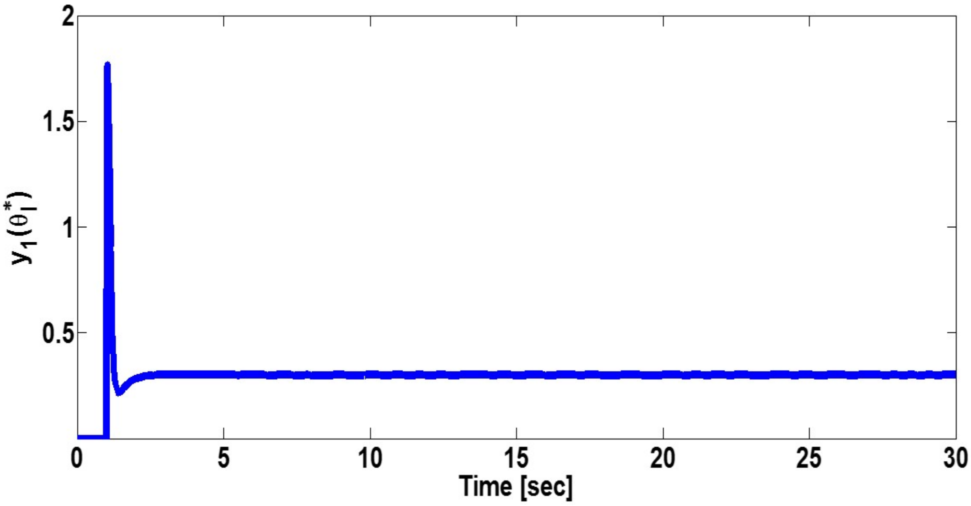

Fig. 3.11: Inner loop output with optimal parameters $y_1(\theta_I^*)$.

Besides, we consider the case the output of cascade control systems is affected by measurement noise. The results are shown in Fig 3.12, we see that the output of cascade system with optimal parameters still can be approximated well with desired output in the measurement noise case.

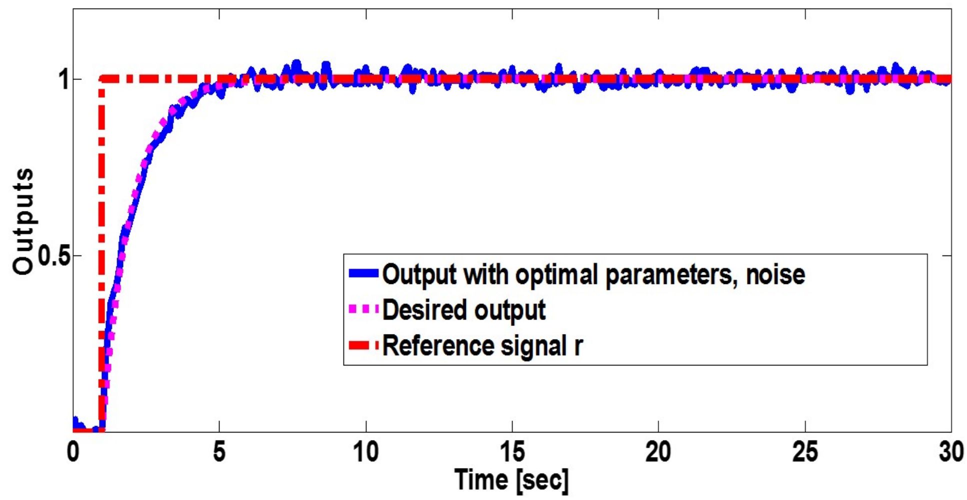

Fig. 3.12: Cascade control system output with optimal parameters $y(\theta_I^*)$ (solid line) affected by measurement noise, reference signal r (dot-dash line), and desired output y_d (dotted line).

Positively, results in Fig. 3.9 indicate that the output of cascade control systems

$y(\theta_I^*)$ achieved by using the optimal outer controller $C_1(\theta_I^*)$ is almost the same with the desired output y_d.

3.4 Comparing with F.Previdi's method in the reference [5]

In this section, To show the usefulness of my proposed method over than the method of F. Previdi and co-authors in the reference [5], we will apply F. Previdi's method for the same plants as in the section 3.3, $P_1 = \frac{s+1}{s^2+5s+6}$ and $P_2 = \frac{s+5}{s^2+2.5s+1.5}$

Similar to the section 2.5 of chapter 2, the cost functions of the inner and outer loops are shown as

$$J_{VR}(\theta_v) = \left\| u^0 - C_v(\theta_v)\left(\frac{1}{M_v} - 1\right)v^0 \right\|_N^2 \tag{3.40}$$

and

$$J_{VR}(\theta_x) = \left\| r_v^0 - C_x(\theta_x)\left(\frac{1}{M_x} - 1\right)x^0 \right\|_N^2 \tag{3.41}$$

We also use the same desired reference models of the inner and outer loops as $M_v = \frac{1}{2s+1}$ and $M_x = \frac{1}{s+1}$.

The inner and outer controllers are parameterized as

$$C_v(\theta_v) = \frac{\theta_1' s^2 + \theta_2' s + \theta_3'}{\theta_4' s^2 + \theta_5' s + \theta_6'} \tag{3.42}$$

and

$$C_x(\theta_x) = \frac{\theta_1'' s^3 + \theta_2'' s^2 + \theta_3'' s + \theta_4''}{\theta_5'' s^3 + \theta_6'' s^2 + \theta_7'' s + \theta_8''} \tag{3.43}$$

Where , $\theta_v = [\theta_1' \ \ \theta_2' \ \ \theta_3' \ \ \theta_4' \ \ \theta_5' \ \ \theta_6']^T$, $\theta_x = [\theta_1'' \ \ \theta_2'' \ \ \theta_3'' \ \ \theta_4'' \ \ \theta_5'' \ \ \theta_6'' \ \ \theta_7'' \ \ \theta_8'']^T$.

We choose the same initial parameter vectors as in the section 3.3

$\theta_v^0 = [0.0 \ \ 1.0 \ \ 1.0 \ \ 0.0 \ \ 1.0 \ \ 0.0]^T$

$\theta_x^0 = [0.0 \ \ 1.0 \ \ 1.0 \ \ 1.0 \ \ 0.0 \ \ 1.0 \ \ 1.0 \ \ 2.0]^T$

to obtain the initial outputs of the cascade control system like in Fig. 3.13. In this figure, the output of outer cascade control system with initial parameter vectors, the desired output and the reference signal are demonstrated such as solid line, dotted line and dot-dash line, respectively.

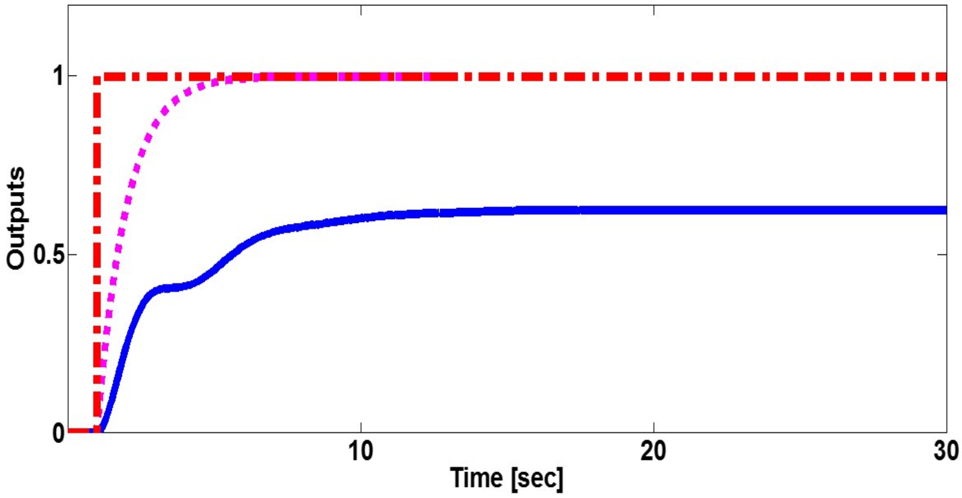

Fig. 3.13: Initial outputs of outer cascade control system

Here, we use the prefilters introduced by Campi [13] for the inner and outer loops as $L_v = M_v(1 - M_v)$, $L_x = M_x(1 - M_x)$ to overcome the non-properness problems of $\frac{1}{M_v}$ and $\frac{1}{M_x}$ appeared in the cost functions (3.40), (3.41).

To minimize the cost functions (3.40) and (3.41), a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 3000$ are used to run CMA-ES algorithm program in MATLAB.

This yields the optimal parameter vectors of the inner and outer controllers as

$\theta_v^* = [0.2304 \ \ 1.1356 \ \ 1.8772 \ \ 0.4418 \ \ 0.6320 \ \ -0.0003]^T$,

$\theta_x^* = [0.2980 \ \ 2.0303 \ \ 4.2835 \ \ 0.5666 \ \ 0.0027 \ \ 0.0969 \ \ 1.8198 \ \ 0.0026]^T$

Using these optimal parameter vectors to conduct again the experiment on the cascade control system Fig.2.9, we obtain the outputs of the cascade control system as in Fig.3.14

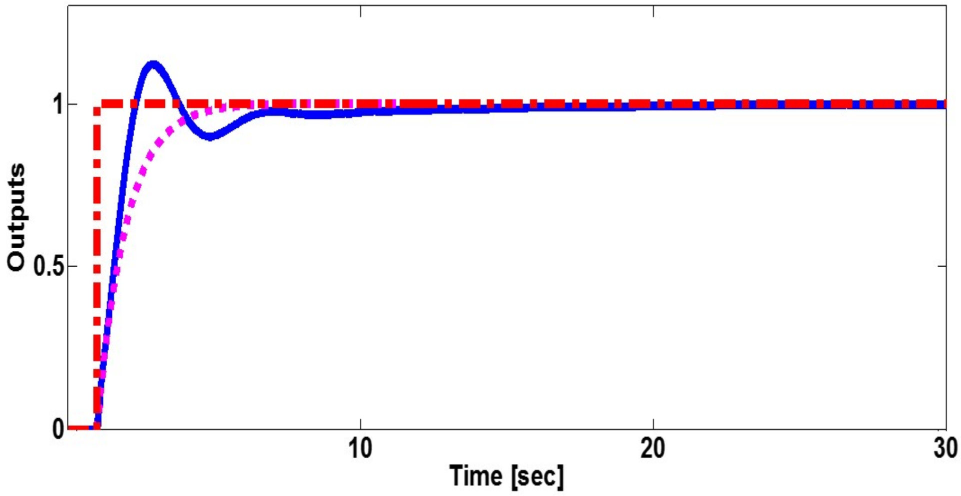

Fig. 3.14: Cascade control system outputs with optimal parameter vectors

In the Fig.3.14, the solid line, dotted line and dot-dash line present the actual output of the cascade control system with optimal parameters, desired output and reference signal, respectively.

By comparing Fig. 3.9 and Fig. 3.14, we can easily deduce that the obtained result of my method is better than achieved result of F. Previdi's method.

3.5 Summary

In this chapter, I have presented a new VRFT approach for cascade control systems by deriving an original prefilter for cascade control systems . This method allows us to obtain the optimal parameters for both inner and outer controllers. The optimal parameters for both inner and outer controllers guarantee that the output of cascade systems can approximate well with desired output.

Through an illustrative example, I have shown that the optimal controllers of cascade control systems are absolutely achieved. This brings out a precise evidence of the proposed method. Moreover, finding out the original prefilter of VRFT method for cascade control systems is a crucial point which distinguishes my works from F. Previdi's works in [5].

Chapter 4

Prefilter of FRIT Approach to Cascade Control Systems

In chapter 3 , I derived the original prefilter for cascade control system in VRFT approach. This original prefilter guarantees that the optimal parameters obtained from criterion reference model is closed to the optimal parameters achieved from the original cost function of cascade control systems.

Similarly, in this chapter, the original prefilter of cascade control structure is derived in FRIT method. This original prefilter allows us to achieve optimal parameters for both controllers in the cascade control systems. A numerical example is given to demonstrate the effectiveness and validity of this study.

4.1 Basic of FRIT approach to cascade control systems

4.1.1 Problem formulation

Consider a cascade control system with the tunable controller as in Fig. 4.1. We assume that P_1 and P_2 are linear, time-invariant, single-input single-output, strictly proper, stable and minimum phase. In this case, the effect of the disturbance to cascade control system is omitted. Two controllers of cascade control system are parameterized as

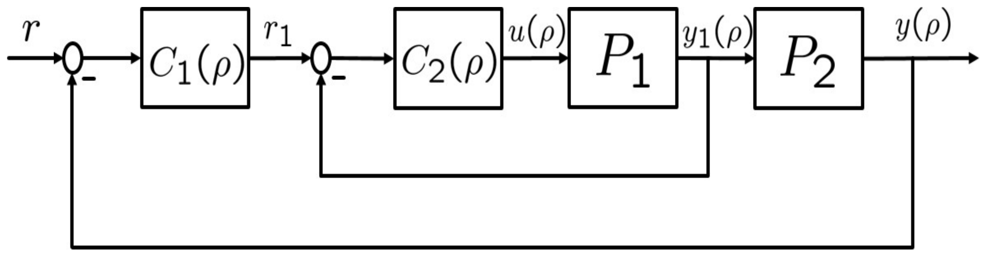

Fig. 4.1: Cascade control system with tunable parameters

$$C_1(\rho) = \frac{\rho_{n+1}q^m + \cdots + \rho_{\nu-1}q + \theta_\nu}{q^n + \rho_1 q^{n-1} + \cdots + \rho_{n-1}q + \rho_n} \tag{4.1}$$

and

$$C_2(\rho) = \frac{\rho_{\nu+n'+1}q^{m'} + \cdots + \rho_{\mu-1}q + \rho_\mu}{q^{n'} + \rho_{\nu+1}q^{n'-1} + \cdots + \rho_{\nu+n'-1}q + \rho_{\nu+n'}} \tag{4.2}$$

by using a parameter vector

$$\rho = \begin{bmatrix} \rho_1 & \cdots & \rho_\nu & \rho_{\nu+1} & \cdots & \rho_\mu \end{bmatrix}. \tag{4.3}$$

We denote a transfer function with a tunable parameter vector ρ from r to y by $G_{ry}(\rho)$, which is shown as

$$G_{ry}(\rho) = \frac{P_1 P_2 C_1(\rho) C_2(\rho)}{1 + P_1 C_2(\rho) + P_1 P_2 C_1(\rho) C_2(\rho)} \tag{4.4}$$

Similarly, Let $u(\rho)$, $y(\rho)$, and $y_1(\rho)$ denote input, output, and inner output of cascade system with parameter ρ, respectively.

Using a parameter vector ρ^0, assume that with this parameter the initial output of the closed loop cascade control system is bounded.

The desired reference model of a closed loop cascade system is given as T_d. Initial output $y^0 := y(\rho^0)$ is different from desired output $y_\mathrm{d} := T_d r$.

Here, the purpose of tuning parameters is to find the optimal parameter vector ρ^* that minimizes $\|y(\rho^*) - T_d r\|_N^2$ by using the initial data $u^0 := u(\rho^0)$, $y^0 := y(\rho^0)$, and $y_1^0 := y_1(\rho^0)$.

We use a suitable L_F for signals u^0, y_1^0 and y^0 of cascade control system respectively, which are $L_F u^0$, $L_F y_1^0$ and $L_F y^0$. It is reasonable to expect that by using this suitable prefilter, the optimal parameters obtained in FRIT method is closed to the optimal parameters for the original cost function.

4.1.2 FRIT method for cascade control system[18]

In the reference [18], the authors have developed FRIT method to cascade control systems. In the following, I show a brief review along the reference [18].

First, we introduce the fictitious reference signal of cascade control systems as

$$\widetilde{r} = C_1(\rho)^{-1}C_2(\rho)^{-1}u^0 + C_1(\rho)^{-1}y_1^0 + y^0 \tag{4.5}$$

The cost function in FRIT for cascade control systems is described by

$$J_F(\rho) = \|y^0 - T_d\widetilde{r}\|_N^2 \tag{4.6}$$

By substituting $\widetilde{r_1}$ in (4.5) into (4.6), the cost function in FRIT for cascade control systems is shown by

$$J_F(\rho) = \|(1 - T_d)y^0 - T_dC_1(\rho)^{-1}C_2(\rho)^{-1}u^0 - T_dC_1(\rho)^{-1}y_1^0\|_N^2 \tag{4.7}$$

As seen trivially, $J_F(\rho)$ is minimized by using only initial one-shot experimental data y^0, u^0 and y_1^0 from cascade system loop.

4.2 Original prefilter of Fictitious Reference Iterative Tuning to cascade control systems

The purpose of control in cascade control structure is to achieve the optimal parameter ρ^* by minimizing of the following model reference criterion

$$\begin{aligned} J(\rho) &= \left\|G_{ry}(\rho)r - T_dr\right\|_N^2 \\ &= \left\|\frac{P_1P_2C_1(\rho)C_2(\rho)}{1 + P_1C_2(\rho) + P_1P_2C_1(\rho)C_2(\rho)}r - T_dr\right\|_N^2 \end{aligned} \tag{4.8}$$

Similarly to [13], as $N \to \infty$ by using the discrete Parseval theorem [28] under the Ergode assumptions, we obtain

$$J(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|G_{ry}(\rho) - T_d\right|^2 \Phi_r d\omega \tag{4.9}$$

where Φ_r is the power spectrum density of r.

We then introduce the ideal controllers C_1^{d} and C_2^{d} such that the closed loop transfer function cascade control system with these controllers is equivalent to the desired closed loop T_d as

$$\frac{P_1P_2C_1^{\mathrm{d}}C_2^{\mathrm{d}}}{1+P_2C_2^{\mathrm{d}}+P_1P_2C_1^{d}C_2^{\mathrm{d}}}=T_d \tag{4.10}$$

By substituting the desired reference model described by (4.10) to the equation (4.8) and after some calculations, the model reference criterion $J(\rho)$ is rewritten as

$$\begin{aligned} J(\rho) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{P_1P_2C_1C_2}{1+P_1C_2+P_1P_2C_1C_2}-\frac{P_1P_2C_1^{\mathrm{d}}C_2^{\mathrm{d}}}{1+P_1C_2^{\mathrm{d}}+P_1P_2C_1^{\mathrm{d}}C_2^{\mathrm{d}}}\right|^2\Phi_r d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|P_1P_2|^2}{|1+P_1C_2+P_1P_2C_1C_2|^2}\frac{|C_1C_2(1+P_1C_2^{\mathrm{d}})-C_1^{\mathrm{d}}C_2^{\mathrm{d}}(1+P_1C_2)|^2}{|1+P_1C_2^{\mathrm{d}}+P_1P_2C_1^{\mathrm{d}}C_2^{\mathrm{d}}|^2}\Phi_r d\omega \end{aligned} \tag{4.11}$$

We focus on $J_F(\rho)$ in (4.7). Here, we apply prefilter L_F to the initial data of cascade control loop as L_Fy^0, $L_Fy_1^0$ and L_Fu^0. By applying L_F, the cost function in FRIT to cascade control systems can be modified as

$$J_F(\rho)=\|(1-T_d)L_Fy^0-T_dC_1(\rho)^{-1}C_2(\rho)^{-1}L_Fu^0-T_dC_1(\rho)^{-1}L_Fy_1^0\|_N^2 \tag{4.12}$$

We use trivial relationships $P_2y_1^0=y^0$ and $P_1P_2u^0=y^0$ to rewritten the cost function $J_F(\rho)$ as

$$\begin{aligned} J_F(\rho) &= \left\|\left(1-T_dC_1^{-1}C_2^{-1}\frac{1}{P_1P_2}-T_dC_1^{-1}\frac{1}{P_2}-T_d\right)L_Fy^0\right\|_N^2 \\ &= \left\|\left[1-T_d\left(\frac{1+P_1C_2+P_1P_2C_1C_2}{P_1P_2C_1C_2}\right)\right]L_Fy^0\right\|_N^2 \end{aligned} \tag{4.13}$$

Next, we substitute T_d in (4.10) to equation (4.13), do some calculations again and use the discrete Parseval theorem [28] under the Ergode assumptions to obtain the cost function in FRIT to cascade control systems as

$$J_F(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|L_F|^2|C_1C_2(1+P_1C_2^d)-C_1^dC_2^d(1+P_1C_2)|^2}{|C_1C_2|^2|1+P_1C_2^d+P_1P_2C_1^dC_2^d|^2}\Phi_{y^0}d\omega \tag{4.14}$$

Where Φ_{y^0} is the power spectrum density of y^0.

By comparing equation (4.14) with equation (4.11), the original prefilter L_F should be chosen as

$$|L_F|^2 = \frac{|P_1P_2|^2|C_1C_2|^2}{|1 + P_1C_2 + P_1P_2C_1C_2|^2}\frac{\Phi_r}{\Phi_{y^0}} \tag{4.15}$$

so that $J_F(\rho) = J(\rho)$ and minimizing $J_F(\rho)$ is the same as minimizing $J(\rho)$.

As shown in [13] and [10], we might expect that $|C_1^dC_2^d|^2 \approx |C_1C_2|^2$ and $|1 + P_1C_2^d + P_1P_2C_1^dC_2^d|^2 \approx |1 + P_1C_2 + P_1P_2C_1C_2|^2$ for $argminJ_F(\rho) = argminJ(\rho)$

Finally, with above respect, the original prefilter L_F in FRIT method to cascade control systems is given as

$$\begin{aligned} |L_F|^2 &= \frac{|P_1P_2|^2|C_1^dC_2^d|^2}{|1 + P_1C_2^d + P_1P_2C_1^dC_2^d|^2}\frac{\Phi_r}{\Phi_{y^0}} \\ &= |T_d|^2\frac{\Phi_r}{\Phi_{y^0}} \end{aligned} \tag{4.16}$$

Finding out the above original prefilter L_F enables us to have an effective strategy in applying FRIT method to cascade control systems which allows us to achieve the optimal parameters for both inner and outer controllers.

4.3 Algorithm

We summarize the proposed method by the following algorithm:

In the diagram of cascade control systems Fig. 4.1 we implement

1. Given a desired reference model of cascade control systems T_d
2. Set an initial parameter vector ρ^0
3. Conduct a one-shot experiment to achieve a set of data u^0, y_1^0, y^0. Using this parameter vector, the controllers is assumed to stabilize the closed-loop cascade control system so that these data are bounded.
4. Calculate the fictitious reference signal $\widetilde{r}$ as in (4.5), use original prefilter L_F in (4.16) to construct the cost function of cascade control system $J_F(\rho)$ as in (4.12).

5. Minimize the cost function $J_F(\rho)$ using a nonlinear optimization such as CMA-ES [21].

6. Obtain the optimal parameter vector $\rho^* := argmin J_F(\rho)$, which yields the optimal controllers and the desired output of the cascade control system.

4.4 Numerical Example

In this section, I demonstrate the validity of the proposed method by giving an illustrative example of a cascade control system in a continuous-time domain and assume that it is not affected by disturbance.

Two unknown plants of cascade control system are described as follows:

$$P_1 = \frac{s + 1.2}{s^2 + 2.7s + 1.8} \tag{4.17}$$

and

$$P_2 = \frac{s + 2.5}{s^2 + 6.5s + 10.5} \tag{4.18}$$

A desired reference model of cascade control system is given by:

$$T_d = \frac{1}{2s + 1} \tag{4.19}$$

The outer and inner controllers are parameterized as

$$C_1(\rho) = \frac{\rho_1 s^2 + \rho_2 s + \rho_3}{\rho_4 s^2 + \rho_5 s + \rho_6} \tag{4.20}$$

and

$$C_2(\rho) = \frac{\rho_7 s + \rho_8}{\rho_9 s + \rho_{10}} \tag{4.21}$$

with $\rho := [\rho_1 \ \ \rho_2 \ \ \rho_3 \ \ \rho_4 \ \ \rho_5 \ \ \rho_6 \ \ \rho_7 \ \ \rho_8 \ \ \rho_9 \ \ \rho_{10}]^T$

Initial parameter vectors are set as $\rho^0 = [1.0 \ 2.0 \ 7.0 \ 3.0 \ 1.0 \ 3.0 \ 17.0 \ 0.0 \ 2.0 \ 0.0]^T$. We then conduct one-shot experiment on a cascade control system to obtain initial data u^0, y_1^0 and y^0. The first two signals are shown in Fig. 4.2 and Fig. 4.3.

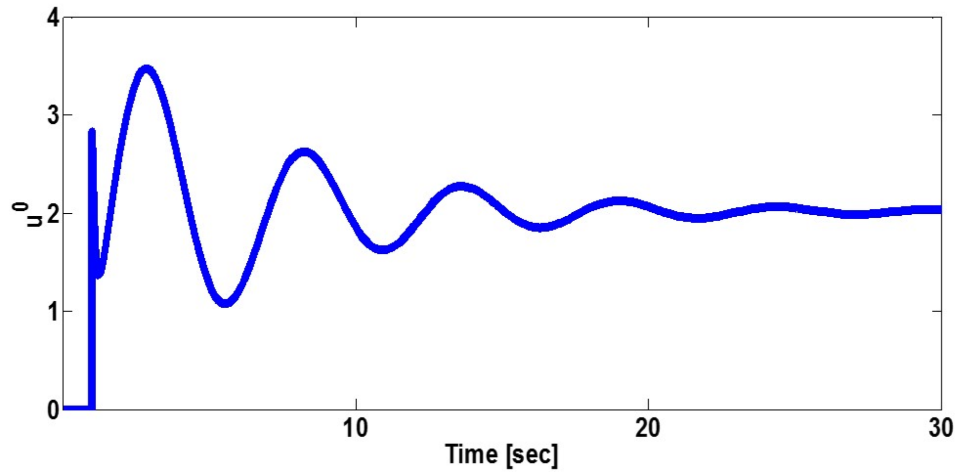

Fig. 4.2: Initial input u^0

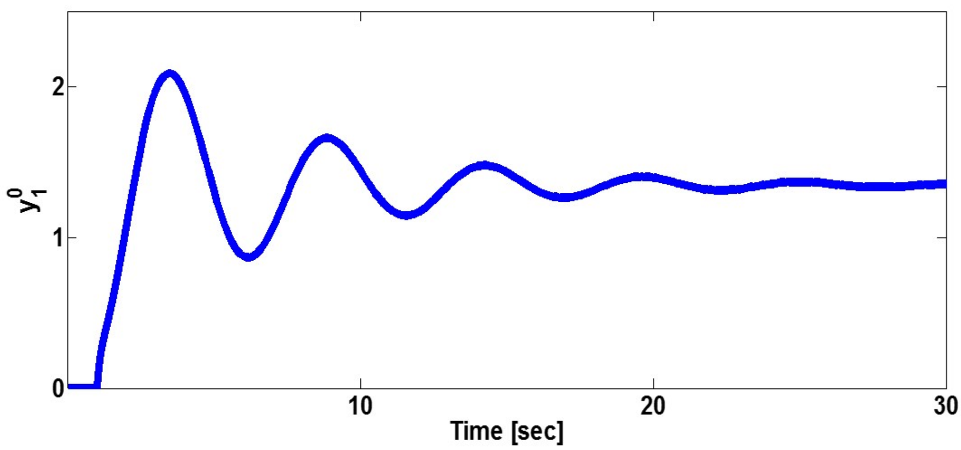

Fig. 4.3: Initial output of the inner loop y_1^0

In Fig. 4.4, initial output of cascade control system y^0 is drawn as a solid line, reference signal r as a dot-dash line, and desired output $y_\mathrm{d} := T_d r$ as a dotted line.

By applying our proposed algorithm with FRIT and using the original prefilter L_F as in (4.16), the cost function $J_F(\rho)$ minimization problem is solved by using covariance matrix adaptation evolution strategy CMA-ES in [21].

In this study, we programmed the CMA-ES algorithm in MATLAB and ran it on a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and iterative step $N = 3000$.

This yielded optimal parameter vector as

$$\rho^* = [0.5218\ 4.2790\ 9.8957\ 0.9734\ 4.6909\ 0.0001\ 16.9954\ 5.1102\ 0.0712\ 0.0136]^T.$$

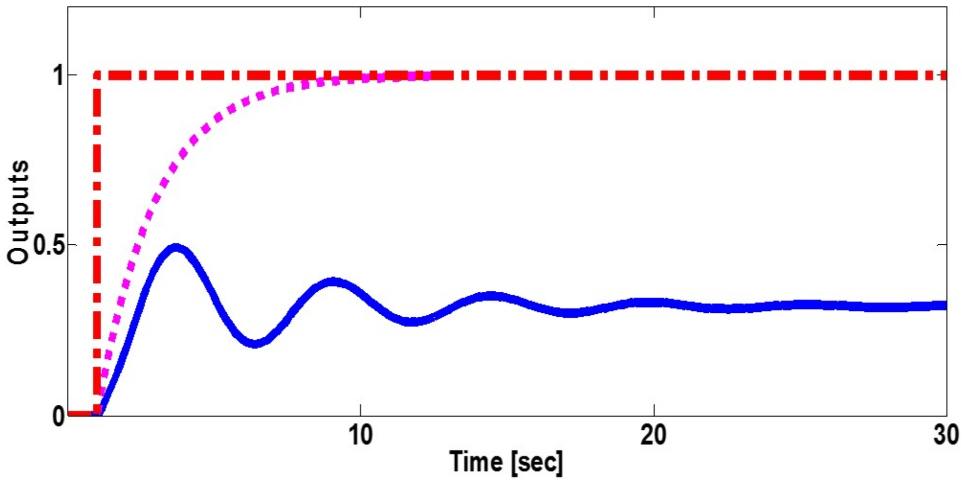

Fig. 4.4: Initial cascade control system output y^0 (solid line), reference signal r (dot-dash line), and desired output y_{d} (dotted line)

We then implement again the experiment by using optimal parameter vectors ρ^* to obtain the results in Fig. 4.5, in this figure the actual output of cascade control system with optimal parameter vectors $y(\rho^*)$ is shown as a solid line, reference signal r as a dot-dash line, the desired output y_{d} as a dotted line.

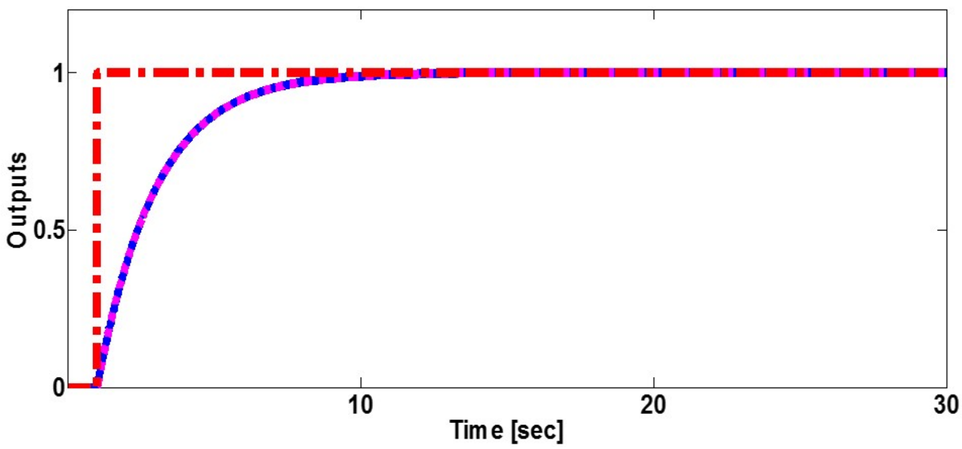

Fig. 4.5: Cascade control system output with optimal parameters $y(\rho^*)$ (solid line), the reference signal r (dot-dash line), and desired output y_{d} (dotted line)

Besides, input with optimal parameters $u(\rho^*)$ and inner loop output with optimal parameters $y_1(\rho^*)$ are shown in Fig. 4.6 and Fig. 4.7.

Results in Fig. 4.5 show that the output of cascade system using optimal parameters $y(\rho^*)$ and desired output y_{d} are almost the same. It indicates that we can achieve the desired output of the cascade control system by using optimal parameter vectors ρ^*.

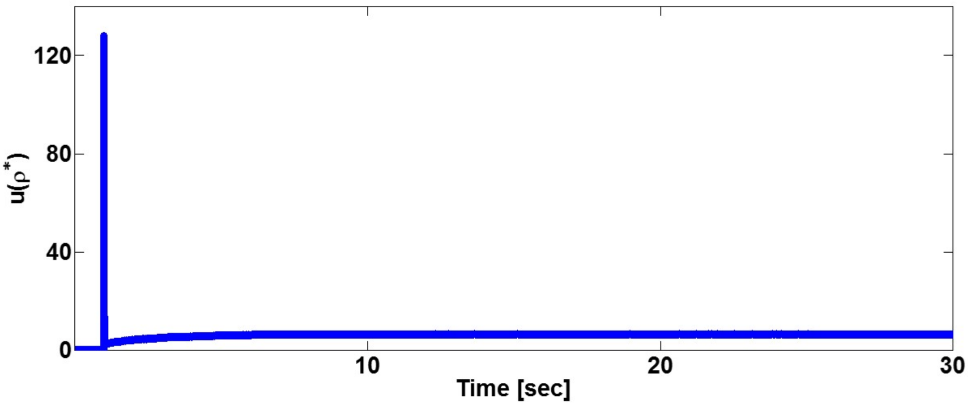

Fig. 4.6: Input with optimal parameters $u(\rho^*)$

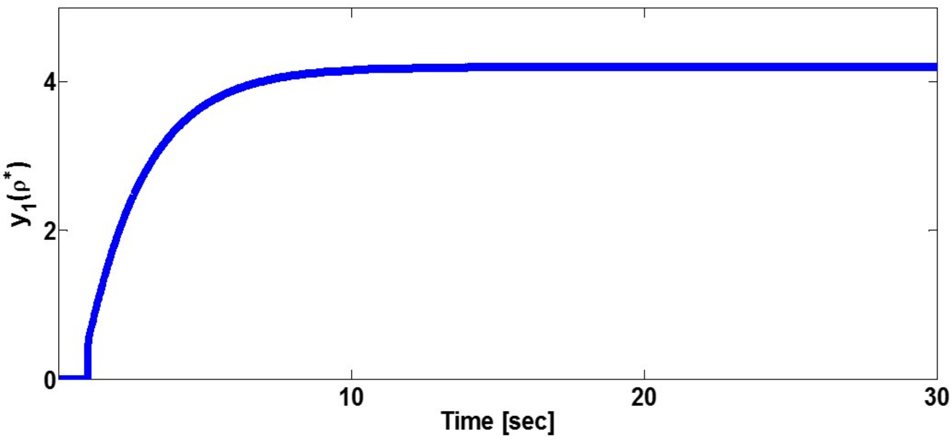

Fig. 4.7: Inner loop output with optimal parameters $y_1(\rho^*)$

4.5 Concluding Remarks

In this chapter, I have presented a new FRIT method for cascade control systems by deriving original prefilter for cascade control systems. This original prefilter guarantees the optimality of the cost function in FRIT method to cascade control systems. Through an illustrative example, it is possible to see that the optimal controllers of cascade control systems are absolutely achieved by using proposed method.

Chapter 5

Extension of VRFT Approach to Cascade Control System for Non-Minimum Phase Systems

In chapter 2 , I presented a VRFT method to cascade control systems. This method allows us to obtain the optimal parameters for both the inner and outer controllers by directly using data collected from the cascade system. However, we have just considered the case in which plants are minimum phase systems.

Practically, there are many industrial systems which include unstable zeros as in [22, 23]. In these systems, the unstable zeros cause an undershot in the initial step response [24, 25], and they also concerned with an overshot [26]. Thus, overcoming the problem of unstable zeros in the non-minimum phase systems is a crucial issue not only for conventional method but also for data-driven approach.

In this chapter, I extend VRFT method to cascade control systems in the case the plants are non-minimum phases to estimate the unstable zeros of plants and obtain the optimal parameters of both the inner controller and the outer controller. These optimal controllers guarantee that the output of cascade control systems has a good approximation to the desired output. Also, I give an illustrative example to demonstrate the effectiveness and validity of the proposed method.

5.1 Parameterize the plants in cascade control systems

We consider the cascade control system with tunable parameters as in Fig. 5.1.

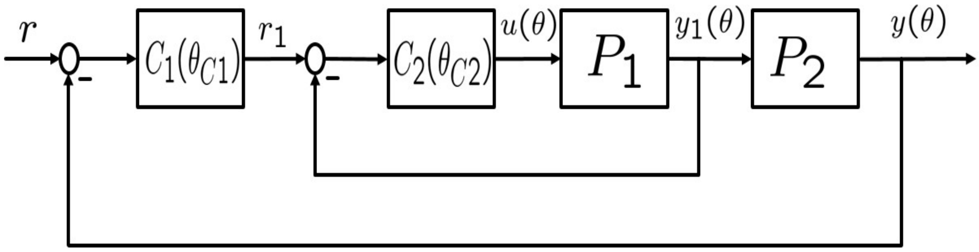

Fig. 5.1: Cascade control system with tunable parameters

In here, we address $P_1(s)$ and $P_2(s)$ in the cascade control system are linear, time-invariant, single-input single-output, strictly proper, stable, and non-minimum phase plants.

Let $D_1(s)$ and $D_2(s)$ denote the denominators of $P_1(s)$ and $P_2(s)$. $N_{1m}(s)$ and $N_{1n}(s)$ denote the polynomials whose roots are stable- and unstable zeros of $P_1(s)$ and $N_{2m}(s)$, $N_{2n}(s)$ denote the polynomials whose roots are stable- and unstable zeros of $P_2(s)$, respectively. We assume that $P_1(s), P_2(s)$ are unknown except the degrees of the numerator and denominator of $P_1(s), P_2(s)$. And the number of unstable zeros of $P_1(s), P_2(s)$ are also known.

By using factorization technique to the unstable zeros which has already been shown in the reference [11], $P_1(s)$ and $P_2(s)$ can be described as follows:

$$P_1(s) = P_{1m}(s)P_{1n}(s) = \underbrace{\frac{N_{1m}(s)N_{1n}^*(s)}{D_1(s)}}_{P_{1m}(s)} \underbrace{\frac{N_{1n}(s)}{N_{1n}^*(s)}}_{P_{1n}(s)} \tag{5.1}$$

$$P_2(s) = P_{2m}(s)P_{2n}(s) = \underbrace{\frac{N_{2m}(s)N_{2n}^*(s)}{D_2(s)}}_{P_{2m}(s)} \underbrace{\frac{N_{2n}(s)}{N_{2n}^*(s)}}_{P_{2n}(s)} \tag{5.2}$$

As above mention, we parameterize the plants $P_1(s)$ and $P_2(s)$ of the cascade control system as:

$$P_1(\theta_{1m}, \theta_{1n}, s) = P_{1m}(\theta_{1m}, \theta_{1n}, s)P_{1n}(\theta_{1n}, s) \tag{5.3}$$

Where

$$P_{1m}(\theta_{1m}, \theta_{1n}, s) = \frac{(\sum_{i=0}^{\mu} b_i s^i)(\sum_{i=0}^{d} c_{d-i} s^i)}{\sum_{i=1}^{\nu} a_i s^i + 1} \tag{5.4}$$

$$P_{1n}(\theta_{1n}, s) = \frac{\sum_{i=0}^{d} c_i s^i}{\sum_{i=0}^{d} c_{d-i} s^i} \tag{5.5}$$

with unknown parameter vectors

$\theta_{1m} := [a_1 \ldots a_\nu \; b_0 \ldots b_\mu]^T \in \mathbb{R}^{\nu+\mu+1}$

$\theta_{1n} := [c_0 \ldots c_d]^T \in \mathbb{R}^{d+1}$

And

$$P_2(\theta_{2m}, \theta_{2n}, s) = P_{2m}(\theta_{2m}, \theta_{2n}, s) P_{2n}(\theta_{2n}, s) \tag{5.6}$$

Where

$$P_{2m}(\theta_{2m}, \theta_{2n}, s) = \frac{(\sum_{j=0}^{\delta} b'_j s^j)(\sum_{j=0}^{d'} c'_{d'-j} s^j)}{\sum_{j=1}^{\gamma} a'_j s^j + 1} \tag{5.7}$$

$$P_{2n}(\theta_{2n}, s) = \frac{\sum_{j=0}^{d'} c'_j s^j}{\sum_{j=0}^{d'} c'_{d'-j} s^j} \tag{5.8}$$

with unknown parameter vectors

$\theta_{2m} := [a'_1 \ldots a'_\gamma \; b'_0 \ldots b'_\delta]^T \in \mathbb{R}^{\gamma+\delta+1}$

$\theta_{2n} := [c'_0 \ldots c'_{d'}]^T \in \mathbb{R}^{d'+1}$

$P_{1m}(\theta_{1m}, \theta_{1n})$, $P_{2m}(\theta_{2m}, \theta_{2n})$ and $P_{1n}(\theta_{1n})$, $P_{2n}(\theta_{2n})$ are parameterized as minimum- and non-minimum phase parts of the plants, respectively.

The inner and the outer controllers of the cascade control systems are parameterized as follows

$$C_1(\theta_{C1}) = \frac{\theta_{C1,g} s^g + \ldots + \theta_{C1,1} s + \theta_{C1,0}}{\theta_{C1,g+f} s^f + \ldots + \theta_{C1,g+1} s + 1} \tag{5.9}$$

with a tunable vector $\theta_{C1} := [\theta_{C1,0} \; \theta_{C1,1} \ldots \theta_{C1,g+f}]^T \in \mathbb{R}^{g+f+1}$

$$C_2(\theta_{C2}) = \frac{\theta_{C2,g'} s^{g'} + \ldots + \theta_{C2,1} s + \theta_{C2,0}}{\theta_{C2,g'+f'} s^{f'} + \ldots + \theta_{C2,g'+1} s + 1} \tag{5.10}$$

with a tunable vector $\theta_{C2} := [\theta_{C2,0}\ \theta_{C2,1}\ \ldots\ \theta_{C2,g'+f'}]^T \in \mathbb{R}^{g'+f'+1}$

Thus, we get the unknown parameter vectors as

$\theta := [\theta_{1m}^T\ \ \theta_{1n}^T\ \ \theta_{2m}^T\ \ \theta_{2n}^T\ \ \theta_{C1}^T\ \ \theta_{C2}^T]^T$

Under the influence of these unknown parameter vectors, the input and the outputs of the cascade control systems are defined as $u(\theta, s)$, $y_1(\theta, s)$ and $y(\theta, s)$.

Then, the closed loop cascade control system architecture with a tunable parameter θ is shown in Fig. 5.1. And $T_{ry}(\theta, s)$ denotes a transfer function from the reference signal $r(s)$ to the output $y(\theta, s)$.

5.2 Problem statement

5.2.1 Modification of the desired reference model

The main idea of applying VRFT method to cascade control systems is to obtain the optimal parameter vectors for both the inner and the outer controllers by using the direct utilization of the data collected from the closed loop cascade. In this case, when the plants are non-minimum systems and we have no information of the plants, so a desired reference model included the unstable zeros of the plants should be given. In the reference [11] an useful strategy has already been shown to overcome this problem.

According to the reference [11] , the desire reference model should be given as

$$M_d(\theta_n, s) = M_{dm}(s)P_{1n}(\theta_{1n}, s)P_{2n}(\theta_{2n}, s) \tag{5.11}$$

with unknown parameter vector $\theta_n := [\theta_{1n}^T\ \ \theta_{2n}^T]^T$

Where $M_{dm}(s)$ is the minimum phase part of the desired reference model, which is strictly proper and given by the designer. Hence, the desired output of the cascade control system is defined as follow

$$y_d(\theta_n, s) := M_d(\theta_n, s)r(s) \tag{5.12}$$

Here, the setting problem is to find an optimal parameter θ^* such that the actual output of the cascade control systems $y(\theta^*, s) = T_{ry}(\theta^*, s)r(s)$ approximates the desired output $y_d(\theta_n, s)$ by using the initial data $u_{ini} = u(\theta_{ini})$, $y_{ini} = y(\theta_{ini})$ and

$y_{1ini} = y_1(\theta_{ini})$ collected from the closed loop cascade with only one-shot experiment.

5.2.2 Standard of VRFT method to cascade control system [27]

We briefly present about the main results of VRFT method to cascade control system, which is based on chapter 2 and reference [27].

Consider the cascade control system in which the controllers are parameterized by vector θ as in Fig. 5.2.

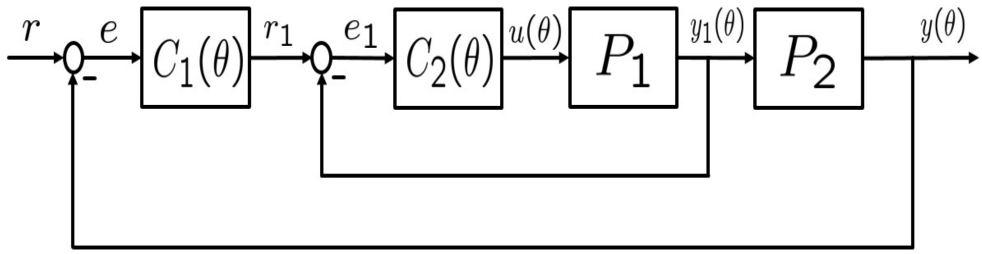

Fig. 5.2: A cascade control system with parameterized controllers θ

A desired reference model of cascade control system is given by M. The cost function $J_V(\theta)$ is described as

$$J_V(\theta) = \left\| u_{\text{ini}} + C_2(\theta) y_{1\text{ini}} - C_1(\theta) C_2(\theta) \left(\frac{1}{M} - 1 \right) y_{\text{ini}} \right\|_N^2 \tag{5.13}$$

We minimize $J_V(\theta)$ to achieve the optimal parameters for the inner and the outer controllers. J_V is minimized by using only initial one-shot experimental data y_{ini}, u_{ini} and $y_{1\text{ini}}$.

In the chapter 2 and reference [27], the authors showed the meaning of the cost function (5.13) such as

$$\begin{aligned} J_V(\theta) &= \left\| \left(1 + P_1 C_2(\theta) - P_1 P_2 C_1(\theta) C_2(\theta) \left(\frac{1}{M} - 1 \right) \right) u_{\text{ini}} \right\|_N^2 \\ &= \left\| (1 + P_1 C_2(\theta)) \left(1 - \frac{P_1 P_2 C_1(\theta) C_2(\theta)}{1 + P_1 C_2(\theta)} \frac{1}{H_M} \right) u_{\text{ini}} \right\|_N^2 \\ &= \left\| (1 + P_1 C_2(\theta)) \left(1 - \frac{H_T(\theta)}{H_M} \right) u_{\text{ini}} \right\|_N^2 \end{aligned} \tag{5.14}$$

Above equation in (5.14) shows that the minimization of $J_V(\theta)$ in (5.13) corresponds to that of the relative error between open loop transfer function $H_T(\theta)$ and H_M under the influence of the inverse sensitivity function of the inner loop and initial input data u_{ini}.

Where

$$\frac{M}{1 - M} =: H_M \tag{5.15}$$

is interpreted as the desired open loop transfer function for M. And

$$\frac{T(\theta)}{1 - T(\theta)} =: H_T(\theta) \tag{5.16}$$

is interpreted as an open loop transfer function for $T(\theta)$ with parameter θ.

$$H_T(\theta) = \frac{P_1 P_2 C_1(\theta) C_2(\theta)}{1 + P_1 C_2(\theta)} \tag{5.17}$$

A transfer function with a tunable parameter vector θ from r to y is denoted by $T(\theta)$, which is represented as

$$T(\theta) = \frac{P_1 P_2 C_1(\theta) C_2(\theta)}{1 + P_1 C_2(\theta) + P_1 P_2 C_1(\theta) C_2(\theta)} \tag{5.18}$$

5.3 VRFT method for non-minimum phase systems in Cascade Control Systems

5.3.1 Establishing the cost function of cascade control systems

Assume that a set of data $\{u_{ini}, y_{1ini}, y_{ini}\}$ is collected from the closed loop cascade system (5.1), as shown in the reference [27] , the virtual reference signal $\bar{r}$ of the cascade control system is calculated as

$$y_{\text{ini}} = M_d(\theta_n, s)\bar{r} \tag{5.19}$$

As the presentation in the section 5.2.1, we modify the desired reference model of cascade control system as

$$M_d(\theta_n, s) = M_{dm}(s) P_{1n}(\theta_{1n}, s) P_{2n}(\theta_{2n}, s) \tag{5.20}$$

Hence, cost function of cascade control system for non-minimum phase systems is introduced as

$$J_{V_{cas}}(\theta) = \left\| u_{\text{ini}} + C_2(\theta_{C2})y_{1\text{ini}} - C_1(\theta_{C1})C_2(\theta_{C2})\left(\frac{1}{M_d} - 1\right)y_{\text{ini}} \right\|_N^2 \tag{5.21}$$

The non-properness problem of $1/M_d$ appearing in the equation (5.21) is avoided by using prefilter of Campi [13], $L = M_d(1 - M_d)$, which guarantees the optimality of $J_{V_{cas}}$ in cases where ideal minimization can not be achieved.

When using prefilter $L = M_d(1 - M_d)$, the cost function of cascade control system for non-minimum phase systems is rewritten as

$$J_{V_{cas}}(\theta) = \left\| Lu_{\text{ini}} + C_2(\theta_{C2})Ly_{1\text{ini}} - C_1(\theta_{C1})C_2(\theta_{C2})\left(\frac{1}{M_d} - 1\right)Ly_{\text{ini}} \right\|_N^2 \tag{5.22}$$

The optimal parameters of the inner and outer controllers are achieved by minimizing the cost function (5.22).

5.3.2 Algorithm

We summarize the proposed method as follows.

1. Prepare a set of initial parameter vector θ_{ini} as $\theta_{ini} := [\theta^T_{1n_{ini}}\ \ \theta^T_{2n_{ini}}\ \ \theta^T_{C1_{ini}}\ \ \theta^T_{C2_{ini}}]^T$ and give the minimum phase part of the desired reference model M_{dm}.

2. Conduct only one-shot experiment on the cascade control systems as in Fig.5.1 to achieve a set of data $\{u_{ini},\ y_{1ini},\ y_{ini}\}$. With θ_{ini}, the controllers are assumed to stabilize the closed-loop cascade control system such that these data are bounded.

3. Calculate the virtual reference signal $\bar{r}(\theta)$ by using (5.19), construct cost function $J_{V_{cas}}(\theta)$ as in (5.22).

4. Minimize the cost function $J_{V_{cas}}(\theta)$ using the optimal minimization for the nonlinear system such as Least Square, Gauss-Newton, Gradient methods or CMA-ES program [21].

5. Obtain the optimal parameter vector $\theta^* := arg\ min_\theta J_{V_{cas}}(\theta)$ which yields the optimal controllers and the desired output of cascade control systems.

5.4 Example

To demonstrate the validity of proposed method, I give an illustrative example of a cascade control system with non-minimum phase plants in continuous -time domain.

The unknown non-minimum phase plants of cascade control system are described as

$$P_1 = \frac{s-1}{s^2+6s+8.75} \tag{5.23}$$

and

$$P_2 = \frac{s-1.5}{s^2+3.7s+3.4} \tag{5.24}$$

Two unknown non-minimum phase plants can be factorized and parameterized as

$$P_1 = \underbrace{\frac{s+\theta_4'}{\theta_1' s^2+\theta_2' s+\theta_3'}}_{P_{1m}} \underbrace{\frac{s-\theta_4'}{s+\theta_4'}}_{P_{1n}} \tag{5.25}$$

and

$$P_2 = \underbrace{\frac{s+\theta_8'}{\theta_5' s^2+\theta_6' s+\theta_7'}}_{P_{2m}} \underbrace{\frac{s-\theta_8'}{s+\theta_8'}}_{P_{2n}} \tag{5.26}$$

We use controllers for outer and inner controllers and they are parameterized as

$$C_1(\theta_{C1}) = \frac{\theta_1 s^2+\theta_2 s+\theta_3}{\theta_4 s^2+\theta_5 s+\theta_6} \tag{5.27}$$

and

$$C_2(\theta_{C2}) = \frac{\theta_7 s+\theta_8}{\theta_9 s+\theta_{10}} \tag{5.28}$$

Where $\theta_C = [\theta_{C1}^T \ \ \theta_{C2}^T]^T$ and $\theta_{C1} = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T$, $\theta_{C2} = [\theta_7\ \theta_8\ \theta_9\ \theta_{10}]^T$

We give the desired reference model which includes the unknown non-minimum phase parts $P_{1n}(\theta_{1n})$, $P_{2n}(\theta_{2n})$

$$\begin{aligned} M_d(\theta_n) &= M_{dm}P_{1n}(\theta_{1n})P_{2n}(\theta_{2n}) \\ &= \frac{1}{2s+1}\frac{s-\theta'_4}{s+\theta'_4}\frac{s-\theta'_8}{s+\theta'_8} \end{aligned} \tag{5.29}$$

Where $\theta_n = [\theta_{1n} \;\; \theta_{2n}]^T = [\theta'_4 \;\; \theta'_8]^T$

With the above setting, we set the initial parameter vector as

$\theta_{Cini} = [1.0 \;\; 2.0 \;\; 4.0 \;\; 1.0 \;\; 1.0 \;\; 3.0 \;\; 17.0 \;\; 0.0 \;\; 2.0 \;\; 0.0]^T$ and $\theta_{nini} = [0.6 \;\; 0.7]^T$

Then , we conduct one-shot experiment in the cascade control system diagram as in Fig. 5.1 to obtain the initial data u_{ini}, y_{1ini} and y_{ini}. The first two signals are shown in Fig. 5.3 and Fig. 5.4.

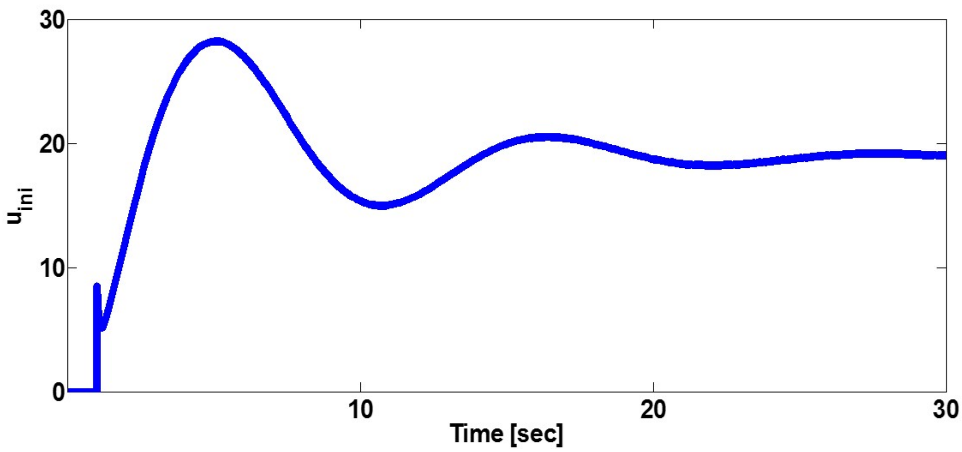

Fig. 5.3: The initial input u_{ini}

In Fig. 5.5, the initial output of cascade control system y_{ini} is drawn as a solid line, reference signal r as a dot-dash line, and the desired output $y_d = M_d r$ as a dotted line.

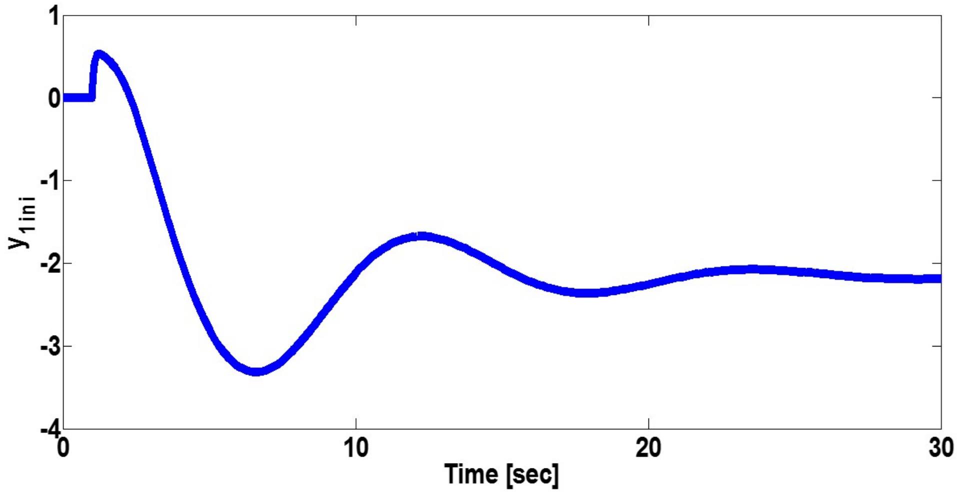

Fig. 5.4: The initial output of the inner loop y_{1ini}

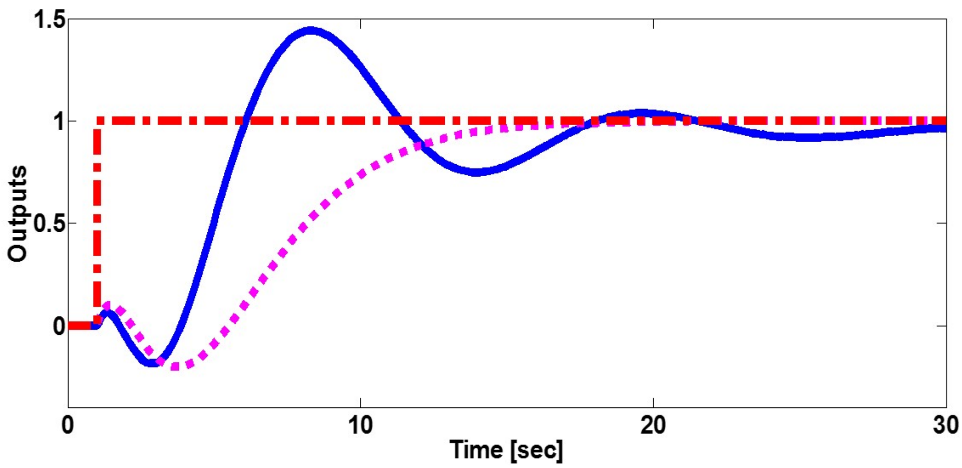

Fig. 5.5: The initial output of cascade control system y_{ini} (the solid line), the reference signal r (the dot-dash line), and desired output y_d (the dotted line)

By applying the proposed algorithm with VRFT method, the minimization problem of the performance index $J_{V_{cas}}(\theta)$ is solved by using the covariance matrix adaptation evolution strategy CMA-ES algorithm [21].

In this study, I programmed the CMA-ES algorithm in MATLAB and ran it on a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and the iterative step $N = 5000$.

This yielded optimal parameter vectors as

$\theta_C^* = [0.4023\ 2.4377\ 2.3755\ 0.4633\ 1.6874\ 3.3192\ 16.8500\ 0.1822\ 1.9234\ 0.0209]^T$

and $\theta_n^* = [1.0073 \ \ 1.4885]^T$.

I then conduct the experiment by using the optimal parameter vectors θ_C^* and θ_n^*. The obtained results are shown in Fig. 5.6, in this figure the actual output of cascade control system with the optimal parameter vectors $y(\theta^*)$, the reference signal r, and the desired output y_d are drawn as the solid line, the dot-dash line, and the dotted line, respectively.

Besides, the input with the optimal parameters $u(\theta^*)$ is shown in the Fig. 5.7, and the output of the inner loop with the optimal parameters $y_1(\theta^*)$ is also drawn in the Fig. 5.8.

From the result shown in Fig. 5.6, we see that the actual output $y(\theta^*)$ and the desired output y_d of the cascade control system are almost the same, which implies that we can achieve the desired output of cascade control system in the non-minimum phase plants case by using the optimal parameter vectors θ^*.

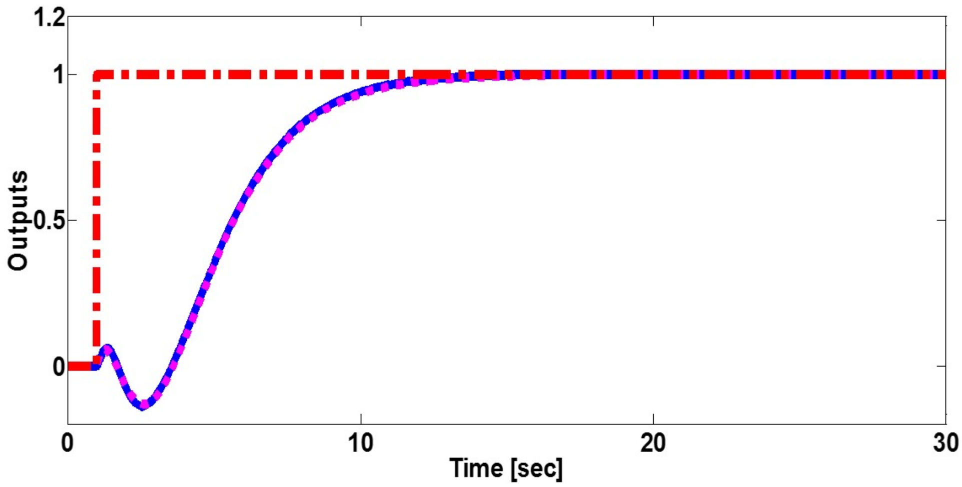

Fig. 5.6: Cascade control system outputs with optimal parameters $y(\theta^*)$ (the solid line), the reference signal r (the dot-dash line), and the desired output y_d (the dotted line)

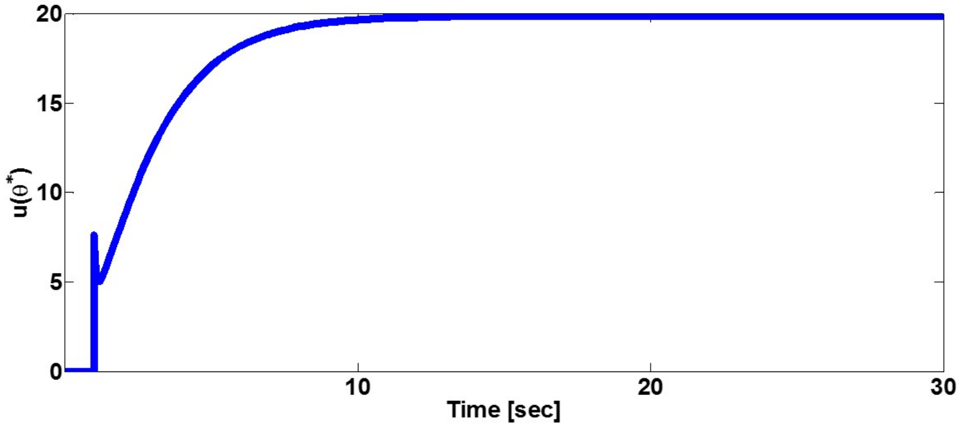

Fig. 5.7: Input with the optimal parameters $u(\theta^*)$

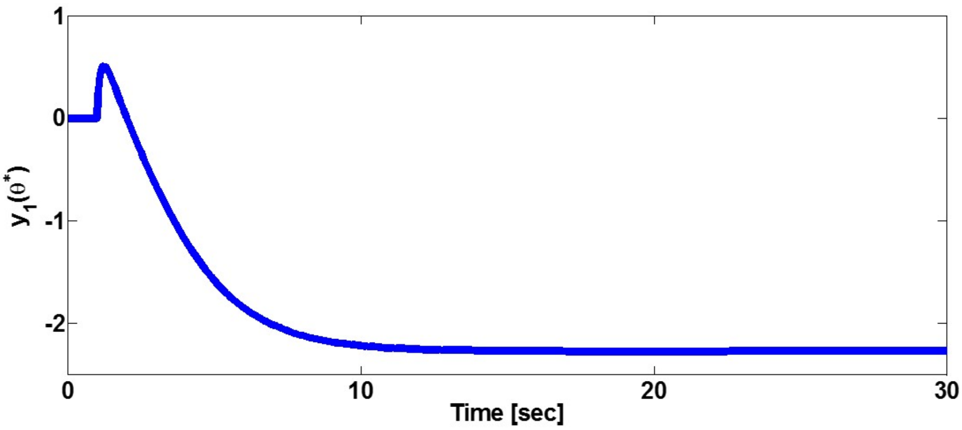

Fig. 5.8: Inner loop output with the optimal parameters $y_1(\theta^*)$

5.5 Summary

In this chapter, I have extended VRFT method to the cascade control systems in the case the plants are non-minimum phases. This method does not require the mathematical model of the plants but only a set of initial data collected from closed loop of cascade control systems. Two optimal controllers of cascade control architecture are absolutely achieved by using this method. Moreover, we can obtain the unstable zeros of the unknown plants in cascade control systems.

It has also been shown that VRFT method is an effective method to simultaneously obtain both optimal controllers in the cascade systems.

Chapter 6

Fictitious Reference Iterative Tuning of Cascade Control Systems for Non-minimum Phase Systems

In the reference [18], the authors presented FRIT method for cascade control system with minimum phase and stable system case. As in the references [22, 23, 24, 25, 26], we can see the importance of solving the problem of the unstable zeros in the non-minimum phase systems which create an undershot and overshot phenomena in the initial step response. In chapter 5, I have introduced the VRFT method for cascade control systems. It seems reasonable that FRIT method can also be applied here.

Thus, I applied FRIT method to cascade control systems in the case the plants are non-minimum phases to obtain the optimal parameters of both inner controller and outer controller in this chapter. Also we can obtain the unstable zeros of the unknown plants in the cascade system.

Besides, I also explain the meaning of the cost function in eliminating the unstable zeros. An illustrative example is given to demonstrate the effectiveness and validity of the proposed method.

6.1 Standard of FRIT method

In this section, I briefly present about the main idea of FRIT, which is based on reference [9, 10].

Consider a convention feedback system with the tunable controller as in Fig. 6.1. Assume that the plant P is unknown and the controller C is parameterized by tunable parameter vector ρ.

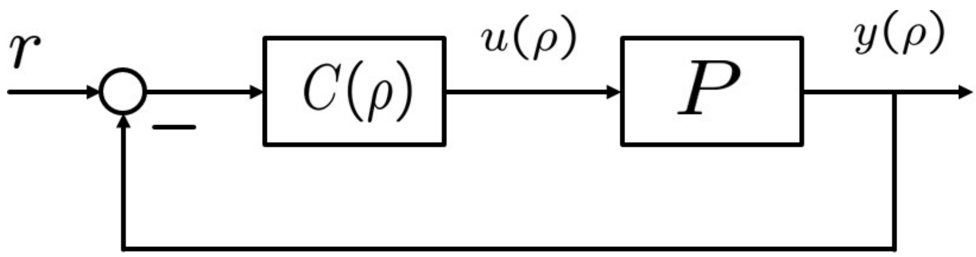

Fig. 6.1: A diagram of convention closed system with a tunable vector

The main idea of the FRIT scheme is to construct the model-reference criterion in the fictitious domain. By using only one-shot experiment on the closed loop system we obtain the initial data $u(\rho^0)$ and $y(\rho^0)$, the controller $C(\rho^0)$ is assumed to stabilize the closed loop system such that $u(\rho^0)$ and $y(\rho^0)$ are bounded.

The fictitious reference signal $\widetilde{r}(\rho)$ is computed by using the initial data $u(\rho^0)$ and $y(\rho^0)$ as follow

$$\widetilde{r}(\rho) = C^{-1}(\rho)u(\rho^0) + y(\rho^0) \tag{6.1}$$

Notice that when the closed loop system with the transfer function $\frac{PC(\rho)}{1+PC(\rho)}$ is excited by $\widetilde{r}$ then its output always equals to the initial output $y(\rho^0)$ for any ρ. For a given reference model T_d, we then introduce the cost function $J_F(\rho)$ as

$$J_F(\rho) := \|y(\rho^0) - T_d\widetilde{r}(\rho)\|_N^2 \tag{6.2}$$

By substituting $\widetilde{r}(\rho)$ in (6.1) into (6.2) with respect to $u(\rho^0) = \frac{y(\rho^0)}{P}$ the cost function can be computed as

$$J_F(\rho) := \left\| \left(1 - \frac{T_d}{G_{r-y}(\rho)}\right) y(\rho^0) \right\|_N^2 \tag{6.3}$$

Where $G_{r-y}(\rho) = \frac{PC(\rho)}{1+PC(\rho)}$ is the transfer function of the closed loop system.

We see that the cost function (6.2) with $\widetilde{r}(\rho)$ in (6.1) requires only the initial data $u(\rho^0)$ and $y(\rho^0)$. This means that the minimization of (6.2) can be conducted *off-line*. The meaning of the cost function is shown in the equation (6.3), that is the minimization of the relative error of the closed loop $G_{r-y}(\rho)$ and the desired reference model T_d under the effect of the initial output $y(\rho^0)$.

6.2 FRIT method for non-minimum phase systems in the Cascade Control Systems

6.2.1 Parameterize the non-minimum phase plants

We give a diagram of cascade control system with tunable parameters as in Fig. 6.2.

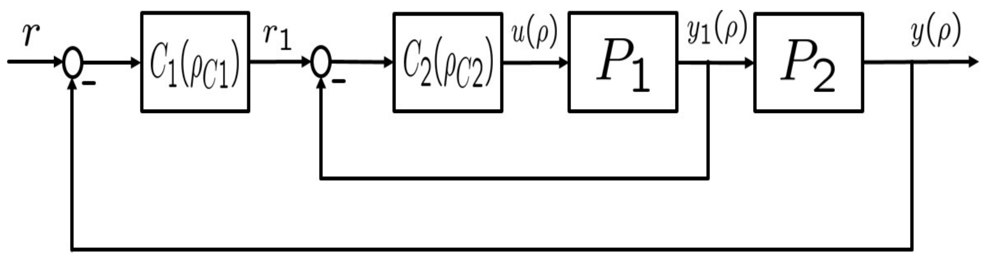

Fig. 6.2: A diagram of cascade control system with tunable parameters

In here, we assume that $P_1(s)$ and $P_2(s)$ in the cascade control system are linear, time-invariant, single-input single-output, strictly proper, stable, and non-minimum phase plants.

Let $D_1(s)$ and $D_2(s)$ denote the denominators of $P_1(s)$ and $P_2(s)$. $N_{1m}(s)$ and $N_{1n}(s)$ denote the polynomials whose roots are stable- and unstable zeros of $P_1(s)$ and $N_{2m}(s)$, $N_{2n}(s)$ denote the polynomials whose roots are stable- and unstable zeros of $P_2(s)$, respectively.

We use a factorization technique for the unstable zeros which has already been introduced in the reference [11] .

By using this technique, $P_1(s)$ and $P_2(s)$ can be described as follow:

$$P_1(s) = P_{1m}(s)P_{1n}(s) = \underbrace{\frac{N_{1m}(s)N_{1n}^*(s)}{D_1(s)}}_{P_{1m}(s)} \underbrace{\frac{N_{1n}(s)}{N_{1n}^*(s)}}_{P_{1n}(s)} \tag{6.4}$$

$$P_2(s) = P_{2m}(s)P_{2n}(s) = \underbrace{\frac{N_{2m}(s)N_{2n}^*(s)}{D_2(s)}}_{P_{2m}(s)} \underbrace{\frac{N_{2n}(s)}{N_{2n}^*(s)}}_{P_{2n}(s)} \tag{6.5}$$

As mentioned above, we parameterize the plants $P_1(s)$ and $P_2(s)$ of the cascade control system as:

$$P_1(\rho_{1m}, \rho_{1n}, s) = P_{1m}(\rho_{1m}, \rho_{1n}, s)P_{1n}(\rho_{1n}, s) \tag{6.6}$$

Where

$$P_{1m}(\rho_{1m}, \rho_{1n}, s) = \frac{(\sum_{i=0}^{\mu} b_i s^i)(\sum_{i=0}^{d} c_{d-i} s^i)}{\sum_{i=1}^{\nu} a_i s^i + 1} \tag{6.7}$$

$$P_{1n}(\rho_{1n}, s) = \frac{\sum_{i=0}^{d} c_i s^i}{\sum_{i=0}^{d} c_{d-i} s^i} \tag{6.8}$$

with unknown parameter vectors

$\rho_{1m} := [a_1 \ldots a_\nu \; b_0 \ldots b_\mu]^T \in \mathbb{R}^{\nu+\mu+1}$

$\rho_{1n} := [c_0 \ldots c_d]^T \in \mathbb{R}^{d+1}$

And

$$P_2(\rho_{2m}, \rho_{2n}, s) = P_{2m}(\rho_{2m}, \rho_{2n}, s)P_{2n}(\rho_{2n}, s) \tag{6.9}$$

Where

$$P_{2m}(\rho_{2m}, \rho_{2n}, s) = \frac{(\sum_{j=0}^{\delta} b'_j s^j)(\sum_{j=0}^{d'} c'_{d'-j} s^j)}{\sum_{j=1}^{\gamma} a'_j s^j + 1} \tag{6.10}$$

$$P_{2n}(\rho_{2n}, s) = \frac{\sum_{j=0}^{d'} c'_j s^j}{\sum_{j=0}^{d'} c'_{d'-j} s^j} \tag{6.11}$$

with unknown parameter vectors

$\rho_{2m} := [a'_1 \ldots a'_\gamma\ b'_0 \ldots b'_\delta]^T \in \mathbb{R}^{\gamma+\delta+1}$

$\rho_{2n} := [c'_0 \ldots c'_{d'}]^T \in \mathbb{R}^{d'+1}$

$P_{1m}(\rho_{1m}, \rho_{1n})$, $P_{2m}(\rho_{2m}, \rho_{2n})$ and $P_{1n}(\rho_{1n})$, $P_{2n}(\rho_{2n})$ are parameterized as minimum- and non-minimum phase parts of the plants, respectively.

The inner and outer controllers of the cascade control system are parameterized as follows

$$C_1(\rho_{C1}) = \frac{\rho_{C1,g}s^g + \ \ldots\ + \rho_{C1,1}s + \rho_{C1,0}}{\rho_{C1,g+f}s^f + \ \ldots\ + \rho_{C1,g+1}s + 1} \tag{6.12}$$

with a tunable vector $\rho_{C1} := [\rho_{C1,0}\ \rho_{C1,1} \ldots \rho_{C1,g+f}]^T \in \mathbb{R}^{g+f+1}$

$$C_2(\rho_{C2}) = \frac{\rho_{C2,g'}s^{g'} + \ \ldots\ + \rho_{C2,1}s + \rho_{C2,0}}{\rho_{C2,g'+f'}s^{f'} + \ \ldots\ + \rho_{C2,g'+1}s + 1} \tag{6.13}$$

with a tunable vector $\rho_{C2} := [\rho_{C2,0}\ \rho_{C2,1} \ldots \rho_{C2,g'+f'}]^T \in \mathbb{R}^{g'+f'+1}$

We get the unknown parameter vectors as

$\rho := [\rho_{1m}^T\ \ \rho_{1n}^T\ \ \rho_{2m}^T\ \ \rho_{2n}^T\ \ \rho_{C1}^T\ \ \rho_{C2}^T]^T$

The input and outputs of the cascade control systems are denoted as $u(\rho, s)$, $y_1(\rho, s)$ and $y(\rho, s)$.

Then, the closed loop in the structure of the cascade control systems with a tunable parameter ρ is shown in Fig. 6.2. And $G_{ry}(\rho, s)$ denotes a closed transfer function from the reference signal $r(s)$ to the output $y(\rho, s)$.

In this case, we consider that $P_1(s)$ and $P_2(s)$ are unknown except the degrees of the nominators and denominators of the non-minimum phase parts of $P_1(s)$ and $P_2(s)$.

6.2.2 Modification of the desired reference model

Similar to chapter 5, we modify the desired reference model of cascade control system as

$$T_d(\rho_n, s) = T_{dm}(s)P_{1n}(\rho_{1n}, s)P_{2n}(\rho_{2n}, s) \tag{6.14}$$

with unknown parameter vector $\rho_n := [\rho_{1n}^T\ \ \rho_{2n}^T]^T$

Where $T_{dm}(s)$ is the minimum phase part of the reference model, which is strictly proper and given by the designer. Hence, the desired output of the cascade control system is defined as follow

$$y_d(\rho_n, s) := T_d(\rho_n, s)r(s) \tag{6.15}$$

Here, the purpose of control is to find an optimal parameter ρ^* such that the output of the cascade control system $y(\rho^*, s) = G_{ry}(\rho^*, s)r(s)$ approximates the desired output $y_d(\rho_n, s)$ by using the initial data $u_{ini} = u(\rho_{ini})$, y_{ini} and $y_{1ini} = y_1(\rho_{ini})$ collected from closed loop cascade by only one-shot experiment.

6.2.3 Cost function of cascade control systems for non-minimum phase systems

In this section, I give a FRIT method to apply for cascade control systems in the case the plants are non-minimum phase systems, we consider a cascade control system shown in Fig. 6.3.

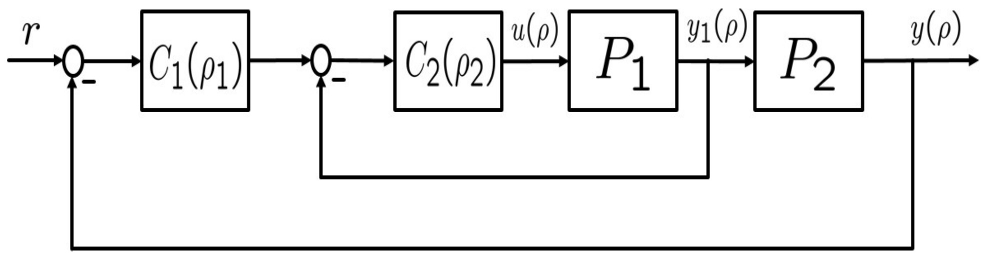

Fig. 6.3: A cascade control system with parameterized controllers ρ

Assume that a set of data $\{u_{ini}, y_{1ini}, y_{ini}\}$ is collected from the closed loop cascade system, as shown in the reference [18] , the fictitious reference signal $\widetilde{r}(\rho)$ of the cascade control system is calculated as

$$\widetilde{r}(\rho) = C_1(\rho_1)^{-1}C_2(\rho_2)^{-1}u_{ini} + C_1(\rho_1)^{-1}y_{1ini} + y_{ini} \tag{6.16}$$

As the presentation in the section 6.2.2, we introduce the desired reference model $T_d = T_d(\rho_n, s)$. Hence, the cost function of the cascade control system for non-minimum phase is introduced as

$$J_{F_{cas}}(\rho) = \|y_{ini} - T_d\widetilde{r}(\rho)\|_N^2 \tag{6.17}$$

By substituting $\widetilde{r}(\rho)$ in (6.16) into (6.17) the cost function of cascade control systems for non-minimum phase systems can be described as

$$J_{F_{cas}}(\rho) = \|(1 - T_d)y_{ini} - T_d C1(\rho_1)^{-1}C_2(\rho_2)^{-1}u_{ini} - T_d C_1(\rho_1)^{-1}y_{1ini}\|_N^2 \tag{6.18}$$

The above cost function, $J_{F_{cas}}$ is minimized by using only initial one-shot experiment data u_{ini}, y_{1ini} and y_{ini}, which means that the minimization of (6.18) can be conducted *off-line* by using a set of experimental data.

6.2.4 Analysis of the meaning of the cost function

We analyze the meaning of the cost function $J_{F_{cas}}$, let $G_{ry}(\rho)$ denote a closed-loop transfer function from r to $y(\rho)$ in the diagram of cascade control systems Fig. 6.3.

$$G_{ry}(\rho) = \frac{P_1C_2(\rho_2)P_2C_1(\rho_1)}{1 + P_1C_2(\rho_2) + P_1C_2(\rho_2)P_2C_1(\rho_1)} \tag{6.19}$$

It is shown in the [18] that the cost function of the cascade control systems can be rewritten as

$$J_{F_{cas}} = \left\|\left(1 - \frac{T_d}{G_{ry}(\rho)}\right)y_{ini}\right\|_N^2 \tag{6.20}$$

Which means that the minimization of $J_{F_{cas}}$ in (6.18) corresponds to that of the relative error between closed loop $G_{ry}(\rho)$ and desired transfer function T_d under the influence of the initial output data y_{ini}.

In the case the plants P_1 and P_2 of cascade control systems are non-minimum plants. By parameterizing P_1 and P_2 as in section 6.2.1, the transfer function of the closed loop cascade can be rewritten as

$$G_{ry}(\rho) = \frac{P_{1m}(s)P_{1n}(s)C_2(\rho_2)P_{2m}(s)P_{2n}(s)C_1(\rho_1)}{DN} \tag{6.21}$$

where

$$DN = 1 + P_{1m}(s)P_{1n}(s)C_2(\rho_2) + P_{1m}(s)P_{1n}(s)C_2(\rho_2)P_{2m}(s)P_{2n}(s)C_1(\rho_1) \quad (6.22)$$

In the section 6.2.2 we gave the desired reference model as

$$T_d(\rho_n, s) = T_{dm}(s)P_{1n}(s)P_{2n}(s) \quad (6.23)$$

From the equation (6.18), after some simple calculations we obtain the cost function of cascade control system in the non-minimum phase case such as

$$J_{F_{cas}}(\rho) = \left\| \left(1 - \frac{T_d}{G_{ry}}\right) y_{ini} \right\|_N^2 \quad (6.24)$$

We see that in the above cost function $J_{F_{cas}}(\rho)$, the minimization of $J_{F_{cas}}(\rho)$ corresponds to that of the relative error between closed loop $G_{ry}(\rho)$ and desired transfer function T_d under the influence of the initial output data y_{ini}.

6.2.5 Algorithm

We summarize the proposed method as follows.

1. Prepare a set of initial parameter vector ρ_{ini} as $\rho_{ini} := [\rho^T_{1n_{ini}} \ \rho^T_{2n_{ini}} \ \rho^T_{C1_{ini}} \ \rho^T_{C2_{ini}}]^T$ and give the minimum phase part of the desired reference model T_{dm}.

2. Using the system as in Fig.6.3, conduct one-shot experiment to achieve a set of data $\{u_{ini}, \ y_{1ini}, \ y_{ini}\}$. With ρ_{ini}, the controllers are assumed to stabilize the closed-loop cascade control system such that these data are bounded.

3. Calculate the fictitious reference signal $\widetilde{r}(\rho)$ by using (6.16), construct cost function $J_{F_{cas}}(\rho)$ as in (6.18).

4. Minimize the cost function $J_{F_{cas}}(\rho)$ using the optimal minimizing for the non-linear system such as Least Square, Gauss-Newton, Gradient,...methods or CMA-ES program [21].

5. Obtain the optimal parameter vector $\rho^* := arg\ min_\rho J_{F_{cas}}(\rho)$ which yields the optimal controllers and the desired output of cascade control system.

6.3 Example

To demonstrate the validity of proposed method, I give an illustrative example of a cascade control systems with non-minimum phase plants in continuous -time domain.

The unknown non-minimum phase plants of cascade control systems are described as

$$P_1 = \frac{s-1}{s^2+3s+2} \tag{6.25}$$

and

$$P_2 = \frac{s-2}{s^2+1.5s+0.5} \tag{6.26}$$

Two unknown non-minimum phase plants can be factorized and parameterized as

$$P_1 = \underbrace{\frac{s+\rho_4'}{\rho_1' s^2+\rho_2' s+\rho_3'}}_{P_{1m}} \underbrace{\frac{s-\rho_4'}{s+\rho_4'}}_{P_{1n}} \tag{6.27}$$

and

$$P_2 = \underbrace{\frac{s+\rho_8'}{\rho_5' s^2+\rho_6' s+\rho_7'}}_{P_{2m}} \underbrace{\frac{s-\rho_8'}{s+\rho_8'}}_{P_{2n}} \tag{6.28}$$

We use the outer and inner controllers which are parameterized as

$$C_1(\rho_{C1}) = \frac{\rho_1 s^2+\rho_2 s+\rho_3}{\rho_4 s^2+\rho_5 s+\rho_6} \tag{6.29}$$

and

$$C_2(\rho_{C2}) = \frac{\rho_7 s+\rho 8}{\rho_9 s+\rho_{10}} \tag{6.30}$$

Where $\rho_C = [\rho_{C1}^T \ \rho_{C2}^T]^T$ and $\rho_{C1} = [\rho_1\, \rho_2\, \rho_3\, \rho_4\, \rho_5\, \rho_6]^T$, $\rho_{C2} = [\rho_7\, \rho_8\, \rho_9\, \rho_{10}]^T$

We give the desired reference model which includes the unknown non-minimum phase parts $P_{1n}(\rho_{1n})$, $P_{2n}(\rho_{2n})$

$$\begin{aligned} T_d(\rho_n) &= T_{dm} P_{1n}(\rho_{1n}) P_{2n}(\rho_{2n}) \\ &= \frac{1}{2s+1}\frac{s-\rho_4'}{s+\rho_4'}\frac{s-\rho_8'}{s+\rho_8'} \end{aligned} \tag{6.31}$$

Where $\rho_n = [\rho_{1n} \;\; \rho_{2n}]^T = [\rho'_4 \;\; \rho'_8]^T$

With the above setting, we set the initial parameter $\rho_{Cini} = [1\ 1\ 1\ 1\ 1\ 1\ 3\ 0\ 5\ 0]^T$ and $\rho_{nini} = [0.5 \;\; 0.6]^T$

Then , we conduct one-shot experiment in the cascade control systems as in Fig. 6.1 to obtain the initial data u_{ini}, y_{1ini} and y_{ini}. The first two signals are shown in Fig. 6.4 and Fig. 6.5.

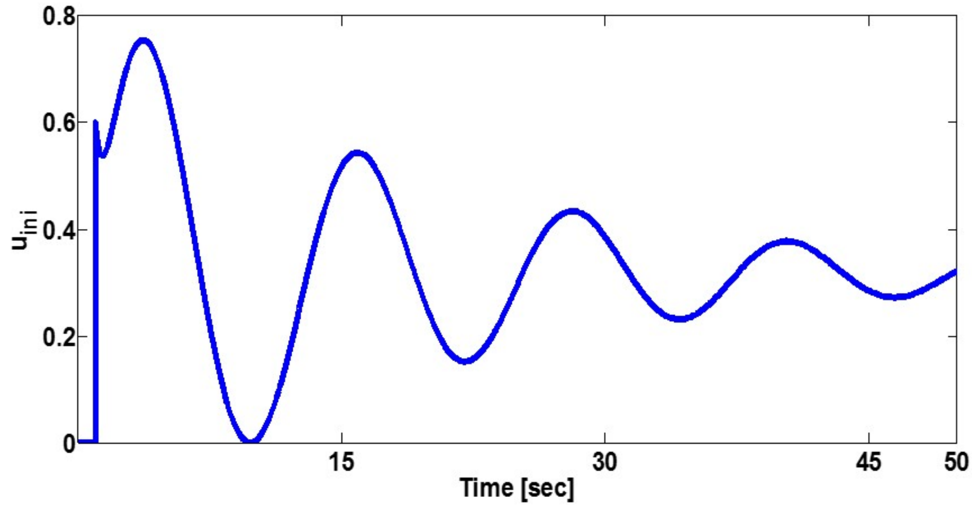

Fig. 6.4: The initial input u_{ini}

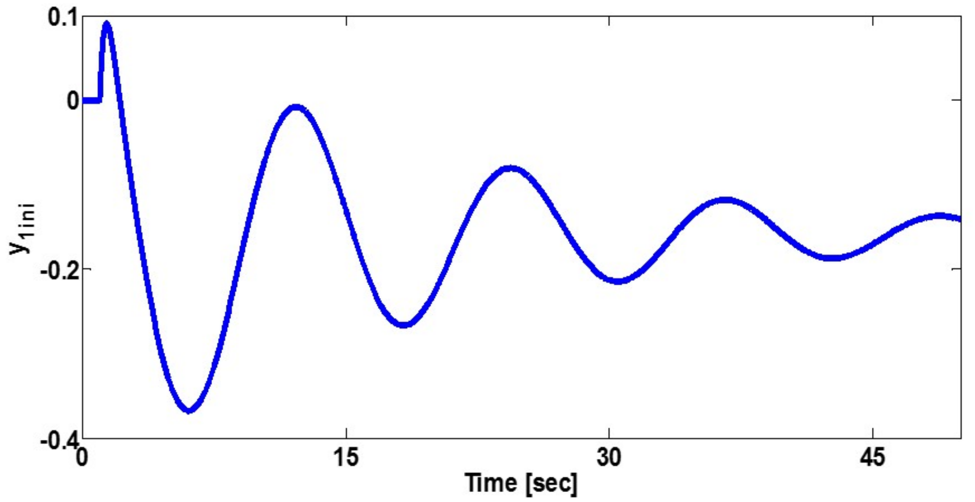

Fig. 6.5: The initial output of the inner loop y_{1ini}

In Fig. 6.6, the initial output of cascade control system y_{ini} is drawn as a solid line, reference signal r as a dot-dash line, and the desired output $y_d = T_d r$ as a dotted line.

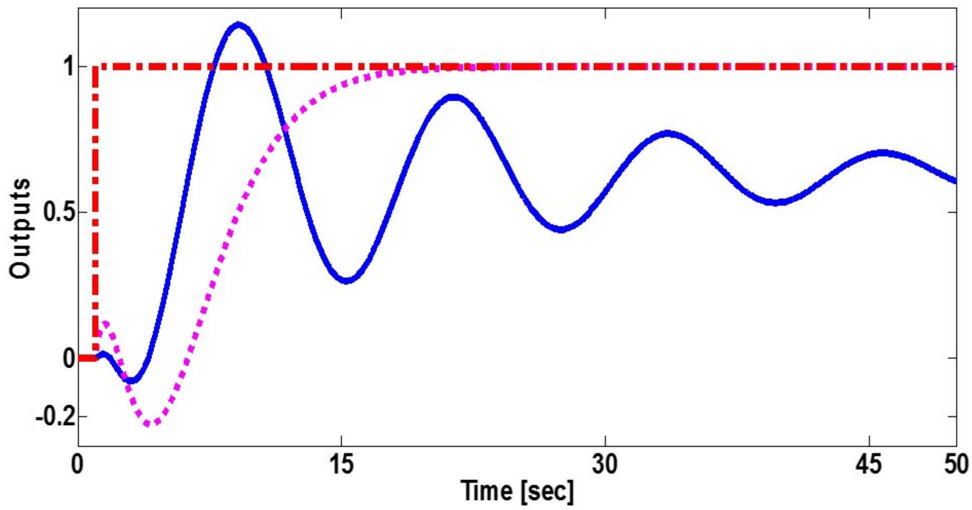

Fig. 6.6: The initial output of cascade control system y_{ini} (the solid line), the reference signal r (the dot-dash line), and desired output y_d (the dotted line)

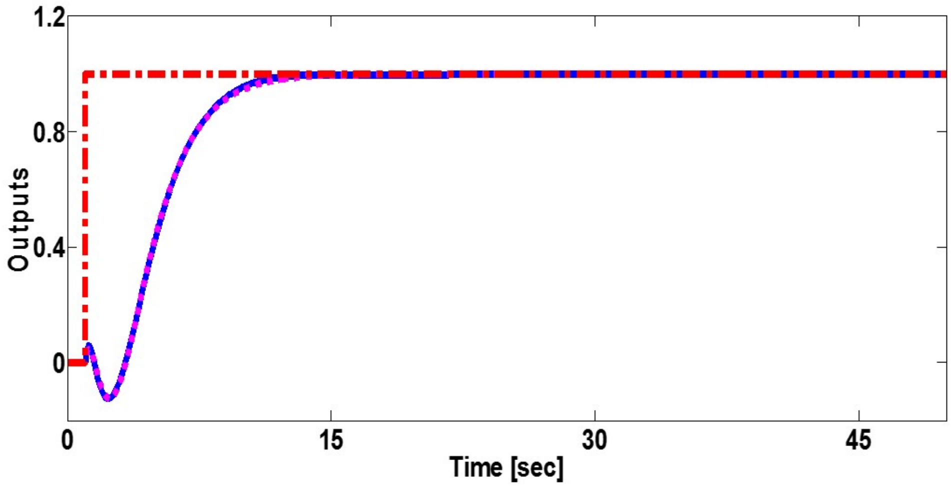

Fig. 6.7: Cascade control system outputs with optimal parameters $y(\rho^*)$ (the solid line), the reference signal r (the dot-dash line), and the desired output y_d (the dotted line)

We apply the proposed algorithm with FRIT, in which the minimization problem of the performance index $J_{F_{cas}}(\rho)$ is solved by using the covariance matrix adaptation evolution strategy CMA-ES algorithm [21].

In this study, I programmed the CMA-ES algorithm in MATLAB and ran it on a calculator with a 3.6 GHz Core i7-4790 CPU, 8GB RAM, and the iterative step $N = 3000$.

This yielded optimal parameter vectors as

$\rho_C^* = [1.1701\ 1.4631\ 0.6384\ 0.0321\ 0.9124\ 1.4983\ 2.6627\ 0.9982\ 4.3120\ 0.4989]^T$

and $\rho_n^* = [1.0053\ \ 2.0185]^T$.

I then conduct again the experiment by using the optimal parameter vectors ρ_C^* and ρ_n^*. The obtained results are shown in Fig. 6.7, in this figure the actual output of cascade control system with the optimal parameter vectors $y(\rho^*)$, the reference signal r, and the desired output y_d are drawn by the solid line, the dot-dash line, and the dotted line, respectively.

Besides, the input with the optimal parameters $u(\rho^*)$ is shown in the Fig. 6.8, and the output of the inner loop with the optimal parameters $y_1(\rho^*)$ is also drawn in the Fig. 6.9.

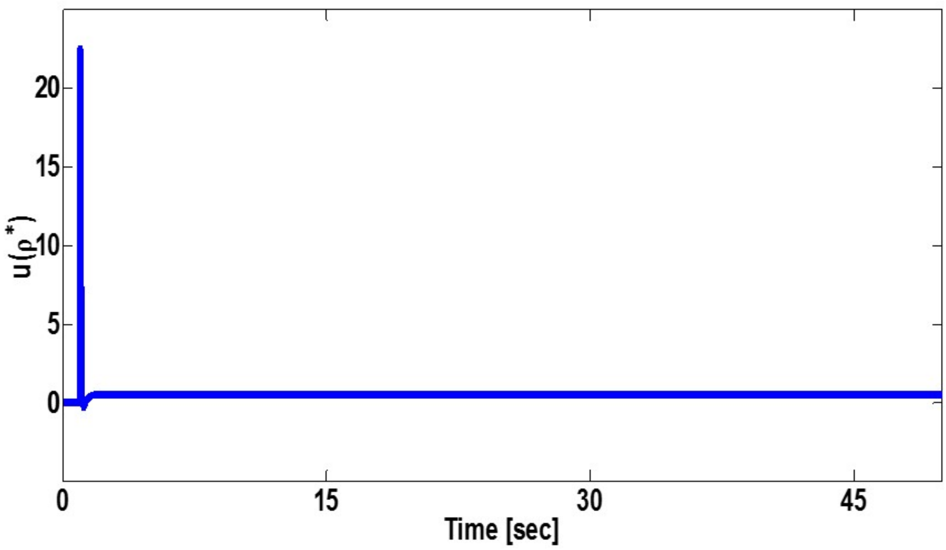

Fig. 6.8: Input with the optimal parameters $u(\rho^*)$

From the result shown in Fig. 6.7, we see that the actual output $y(\rho^*)$ and the desired output y_d of the cascade control systems are almost the same, which implies that we can achieve the desired output of the cascade control systems in the case non-minimum phase plants by using the optimal parameter vectors ρ^*.

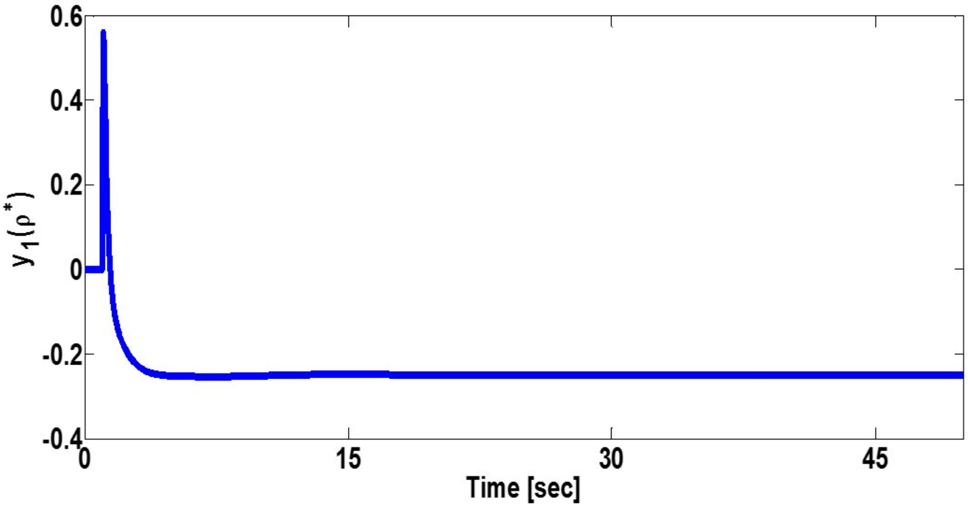

Fig. 6.9: Inner loop output with the optimal parameters $y_1(\rho^*)$

Also, we consider the case in which the output of the cascade systems is affected by measurement noise. The results are given in Fig. 6.10, we see that the output of the cascade control systems using optimal parameters still can approximate well with desired output in measurement noise case.

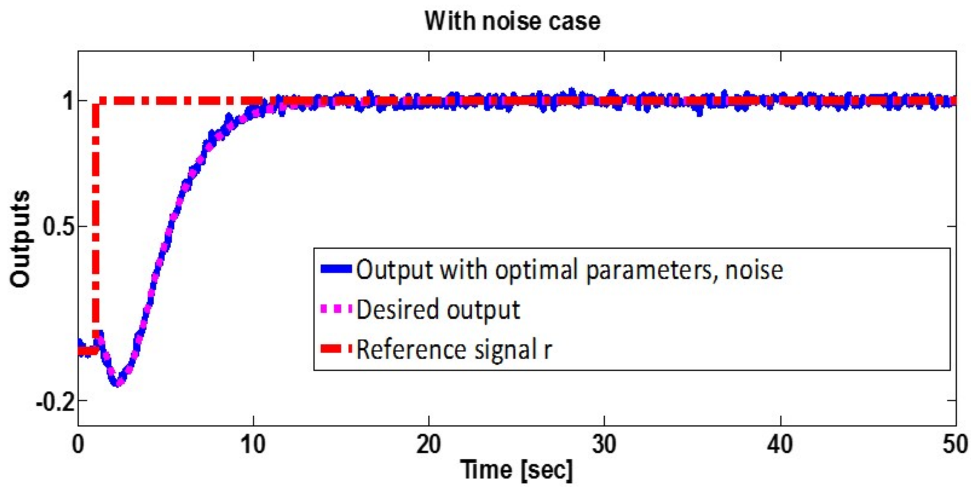

Fig. 6.10: Cascade control system outputs with optimal parameters $y(\rho^*)$ affected by measurement noise (the solid line), the reference signal r (the dot-dash line), and the desired output y_d (the dotted line)

6.4 Summary

In this chapter, I have developed FRIT method to the cascade control systems in the case the plants are non-minimum phases. This method allows us to obtain the optimal parameters for both the inner and outer controllers. These optimal controllers ensures that the output of cascade control systems is almost the same with the desired output. In addition, we can achieve the unstable zeros of the unknown plants in the cascade control systems.

I also analyzed the meaning of the cost function in my proposed method theoretically. It has also been shown that FRIT is an effective method to simultaneously obtain both optimal controllers in the cascade systems.

Chapter 7

Conclusions and Future Works

7.1 Conclusions

In this dissertation, I presented two methods (FRIT and VRFT) of data-driven approaches to cascade control system. Through out my studies, the mathematical models of the plants are not required, the only thing we need to achieve the optimal parameters for both inner and outer controllers is a set of initial data directly collected from a closed cascade control system loop by one-shot experiment.

In chapter 2, I have presented VRFT method for the class of minimum phase plants of cascade control systems. Also I constructed the original cost function for VRFT method to cascade systems and show that VRFT method simultaneously yields both optimal controllers in the cascade systems. In addition, I analyzed clearly the meaning of the cost function in two cases to show the strong effectiveness of my proposed method.

Moreover, I derived original prefilters of VRFT and FRIT methods for cascade control systems, using these original prefilters not only avoids the problem of non-properness appearing in the cost function of VRFT case but also obtains the matching between optimal parameters achieved from model reference criteria and one yielded from original cost functions . It also ensures the optimality of the cost functions in two methods. These original prefilters are remarkable additions to literature of data-driven approach. Also, these are very important different points when comparing with previous study of authors in reference [5]. Deriving original prefilters for cascade control systems in case the controllers are linearly parameterized enables us to have a new strategy in applying VRFT and FRIT methods to cascade control systems. These works are done in chapters 3 and 4.

In chapters 5 and 6, I have extended VRFT and FRIT methods to the cascade control systems in the case the plants are non-minimum phases systems. The results show that these methods yield the optimal parameters for both the inner and outer controller of cascade control systems. The optimal controllers guarantee that the output of cascade system is matching the desired output.

Consequently, the above achieved results convince that data-driven approach is a very effective method to design optimal controllers for not only cascade control systems but also other kind of control systems.

7.2 Future Works

In future works, I plan to expand data-driven of cascade control systems to unstable plants. Analysis of the effect of disturbance and giving the solution for eliminating disturbance are also important problems for consideration. In addition, I would like to apply data-driven approach to various practical cascade control systems.

Publications

Journal papers

[1] Huy Quang Nguyen, Osamu Kaneko and Yoshihiko Kitazaki: Virtual reference feedback tuning for cascade control systems, *Journal of Robotics and Mechatronics*, Vol.28, No. 5, pp. 739–744, 2016.

Conference papers

[1] Nguyen Quang, Huy and Kaneko, Osamu: Fictitious Reference Iterative Tuning of Cascade Control Systems for Non-Minimum Phase Systems, 2017 6^{th} *International Symposium on Advanced Control of Industrial Processes,* (AdCONIP), pp.517-522, May 2017.

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