## Anomaly induced dark matter decay and PAMELA／ATIC experiments

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# Anomaly induced dark matter decay and PAMELA/ATIC experiments 

Hiroki Fukuoka, Jisuke Kubo, Daijiro Suematsu*<br>Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

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#### Abstract

The cosmic ray data of PAMELA/ATIC may be explained by dark matter decay with a decay rate $\tau_{\mathrm{DM}}^{-1} \sim$ $10^{-26} \mathrm{~s}^{-1} \sim 10^{-45} \mathrm{eV}$, an energy scale which could not be understood within the framework of the standard model or its simple supersymmetric extension. We propose anomaly induced dark matter decay to exponentially suppress the decay rate, and apply to a supersymmetric extension of the Ma's inert Higgs model of the radiative seesaw mechanism for neutrino masses. In this model the lightest right-handed neutrino $\psi_{N}$ and the lightest neutralino $\chi$ can fill the observed necessary dark matter relic, and we find that $\psi_{N}$ can decay into $\chi$ through anomaly with a right order of decay rate, emitting only leptons. All the emitted positrons are right-handed.


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## 1. Introduction

Recent astrophysical observations [1] and neutrino oscillation experiments [2] require an extension of the standard model (SM) so as to include dark matter as well as to incorporate a generation mechanism for small neutrino masses. At the moment, however, we know about dark matter only a little; the constraint on its mass and abundance, but nothing about its detailed feature is known. Consequently, there are many consistent models for dark matter. Representative possibilities may be summarized as follows:
(i) Dark matter is a stable thermal relic, so that its relic abundance and annihilation cross section are strongly related to each other. The lightest neutralino in supersymmetric models with the conserved $R$-parity is a well studied example of this category [3]. Another well motivated example may be a stable neutral particle in the radiative seesaw scenario [4], which is an alternative model of the seesaw mechanism to generate small neutrino masses [5]. In fact, there exist similar models and the nature of the DM candidates in these models has been studied [6-10].
(ii) Dark matter consists of multiple components [11]. If some of them are unstable, dark matter can contain thermal components as well as non-thermal ones which can be produced by the decay of unstable components. In this case the dark matter relic abundance at present and the annihilation cross section of the dominant component need not to be related.

[^0](iii) Dark matter is not stable and is decaying with a very long lifetime [12-20].

In any case it will be crucial for the study for going beyond the SM to know which class the true dark matter model belongs to. Since the above mentioned dark matter models predict different signals for the annihilation and/or decay of dark matter in the Galaxy, it may be possible to use the data from these observations to distinguish the dark matter models [21]. The positron excess in the recent PAMELA observation [22] and ATIC data of $e^{+}+e^{-}$ flux [23] are such examples. PAMELA data show a hard positron excess compared with the background but no antiproton excess, while ATIC data show the excess of $e^{+}+e^{-}$flux at regions of 300800 GeV . A lot of works have been done to explain these data within the framework of dark matter models (see, for instance, [26] and references therein). However, the observed positron flux requires much larger annihilation cross section or much larger dark matter density than the ones needed for the explanation of WMAP data. The required large factor in the latter feature is parametrized as a boost factor in the references. ${ }^{1}$ So, it seems very difficult to give a natural explanation for the boost factor for the type (i) dark matter.

In this Letter, following [27], we consider a supersymmetric extension of the radiative seesaw model for the neutrino mass to understand the data obtained by the PAMELA and ATIC experiments. The radiative seesaw model is attractive in two respects: (a) The non-vanishing small neutrino mass and the presence of a dark matter candidate are closely related through a discrete sym-

[^1]Table 1
The matter content and the quantum number. $Z_{2 L}$ is a discrete lepton number. $Z_{2}$ is a subgroup of $Z_{4}$, which is assumed to be anomalous and spontaneously broken by VEV of $\Sigma$ and $\sigma$ down to $Z_{2}$.

|  | $L$ | $E^{c}$ | $N^{c}$ | $H^{u}$ | $H^{d}$ | $\eta^{u}$ | $\eta^{d}$ | $\phi$ | $\sigma$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $R \times Z_{2}$ | $(-,+)$ | $(-,+)$ | $(+,-)$ | $(+,+)$ | $(+,+)$ | $(-,-)$ | $(-,-)$ | $(-,-)$ | $(+,+)$ | $(+,+)$ |
| $Z_{4}$ | 0 | 0 | -1 | 0 | 0 | 1 | 1 | +1 |  |  |
| $Z_{2}^{L}$ | - | - | - | + | + | + | + | + |  |  |

metry $Z_{2}$. (b) The dark matter candidate in this model couples only with leptons but not quarks. This feature is favorable for the above mentioned PAMELA and ATIC data. However, a large boost factor still has to be introduced to explain the observed positron flux in this model [28-30]. In our supersymmetric extension of the model this problem is overcome as follows. There are two kinds of stable neutral particles corresponding to two discrete symmetries, $R$ and $Z_{2}$, where $R$ is the $R$ parity in supersymmetric theories, and $Z_{2}$ is mentioned above. If one of these discrete symmetries is broken, the heavier one can decay to the lighter one. We propose that this breaking can be induced by anomaly [31-33] to realize an exponentially suppressed decay rate of the heavier dark matter. It should be noted that the smallness of this decay rate is a crucial ingredient for the explanation of the observed $e^{+}+e^{-}$flux. Moreover, due to the very nature of the model, only lepton pairs can be produced through the dark matter decay. We show that both data of PAMELA and ATIC can be described well simultaneously in this scenario. The model for dark matter proposed in this Letter gives a concrete realistic example of type (ii).

## 2. Anomaly induced dark matter decay

The stability of the dark matter is usually ensured by an unbroken discrete symmetry $Z$. If the discrete symmetry is broken, the dark matter can decay. The preferable decay modes depend on how $Z$ is broken. However, its life time will be too short $\tau_{\mathrm{DM}} \sim(8 / \pi) m_{\mathrm{NDM}} \simeq 10^{-24} \mathrm{~s}$ for $m_{\mathrm{DM}} \simeq 1 \mathrm{TeV}$, unless the $Z$ breaking is extremely weak [12-20]. Such suppression may occur if $Z$ is broken by GUT or Planck scale physics [16-20].

Here we would like to suggest an alternative suppression mechanism which is based on the observation that if a symmetry, continuous or discrete, is anomalous, then non-perturbative effects can generally induce non-invariant terms, like quark masses in QCD. Although the discrete symmetry $Z$ in question can be anomaly free with respect to the SM gauge group, it can be anomalous at high energy when imbedded into a larger discrete group, because heavy particles can contribute to discrete anomalies [31,34]. If the discrete symmetry is anomalous at high energy, non-perturbative effects can produce $e^{-b S} \Phi^{n}$ in the superpotential [31-33], which is $Z$ invariant. This is because the dilaton superfield $S$ transforms inhomogeneously under the anomalous $Z$, where $\Phi$ is a generic chiral super field, and $b$ is a certain real number (see also $[37,38]$ ). Below the Planck scale, where the dilaton is assumed to be stabilized at a vacuum expectation value of $O$ (1), the factor $e^{-b\langle S\rangle}$ can work as a suppression factor for the noninvariant product $\Phi^{n}$.

Let us estimate the size of the suppression. To this end, consider a (chiral) $Z_{N}$ symmetry in a gauge theory based on the gauge group $G$ and assume $Z_{N}$ is anomalous. Then the Jacobian $J$ of the path integral measure corresponding to the $Z_{N}[G]^{2}$ anomaly can be written as $[35,37,38$ ]
$J=\exp \left(-\frac{2 \pi i}{N} \Delta Q \int d^{4} \times 2 \mathcal{A}(x)\right)$,
$\mathcal{A}(x)=\frac{1}{64 \pi^{2}} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}^{a} F_{\rho \sigma}^{a}$,
where $F_{\mu \nu}^{a}$ is the field strength for $G$. Since the Pontryagin index $\int d^{4} x \mathcal{A}(x)$ is an integer, $\Delta Q=0 \bmod N / 2$ means anomaly freedom of $Z_{N}$. In the anomalous case, we have $\Delta Q=k / 2$ with an integer $k<N$. (So, $\Delta Q / N=1 / 4$ for an anomalous $Z_{2}$, for instance.) This anomaly can be cancelled by the Green-Schwarz mechanism [36], which defines the transformation property of the dilaton supermultiplet $S=\left(\varphi+i a, \psi_{S}, F_{S}\right)$, where $\varphi(a)$ is the dilaton (axion) field, and they couple to the gauge field as
$\mathcal{L}_{F}=-\frac{\varphi}{4} F_{\mu \nu}^{a} F^{a \mu \nu}-\frac{a}{8} \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}^{a} F_{\rho \sigma}^{a}$.
To cancel the anomaly (1), the axion $a$ has to transform according to $a \rightarrow a-(1 / 2 \pi)(\Delta Q / N)$. Therefore, the $Z_{N}$ charge of $\exp (-b S)$ becomes $C$ if $b=4 \pi^{2} C / \Delta Q$. Since $\langle\varphi\rangle=1 / g^{2} \simeq O(1)$ at the Planck scale, the expression $\exp (-b S)$ would then yield a suppression factor $S F$ such as
$S F \simeq \exp \left(-4 \pi^{2} C / \Delta Q\right)$,
$(S F)^{2} \simeq 10^{-55}, 10^{-69}, 10^{-86}$ for $C / \Delta Q=8 / 5,2,10 / 4$,
where $C$ and $2 \Delta Q$ are defined modulo $N .{ }^{2}$
Do we need such a big suppression? According to [13,14], to explain the PAMELA/ATIC data, the decaying dark matter should decay with a life time of $\sim 10^{26}$ sec, which corresponds to a decay width $\Gamma_{\mathrm{NDM}} \sim 10^{-54} \times(1 \mathrm{TeV}) \sim 10^{-70} \times(1 \mathrm{TeV})\left(M_{\mathrm{PL}} / 1 \mathrm{TeV}\right) \sim$ $10^{-86} \times(1 \mathrm{TeV})\left(M_{\mathrm{PL}} / 1 \mathrm{TeV}\right)^{2}$, where we have assumed that the decay is induced by dimension four (three) operators for the first (third) expression. (The second one could appear accidentally.) The precise suppression needed depends, of course, on the details of the model. But it is clear that one needs a huge suppression factor for the decaying dark matter, if one would like to explain the PAMELA/ATIC data within the framework of particle physics [ $16,18,20]$. It is also clear that the existence of such a small number cannot be explained in a low energy theory.

## 3. The model: Radiative seesaw and dark matter candidates

Here we would like to supersymmetrize the model of [4]. (An first attempt has been made in [27].) We assume the $R$ parity invariance as usual. So, we have $R \times Z_{2}$ discrete symmetry at low energy. The matter content of the model with their quantum number is given in Table 1. $L, H^{u}, H^{d}$ and $\eta^{u}, \eta^{d}$ stand for $S U(2)_{L}$ doublets supermultiplets of the leptons, the MSSM Higgses and the inert Higgses, respectively. Similarly, $S U(2)_{L}$ singlet supermultiplets of the charged leptons and right-handed neutrinos are denoted by $E^{c}$ and $N^{c} . \phi$ is an additional neutral Higgs supermultiplet which is needed to generate neutrino masses radiatively. $\Sigma$ and $\sigma$ are also additional neutral Higgs supermultiplets which are needed to derive the superpotential (4) from a $Z_{4}$ invariant one.

We first consider a $R \times Z_{2}$ invariant superpotential below. Later on, using $\Sigma$ and $\sigma$, we will describe a possibility to obtain it from a $R \times Z_{4}$ invariant one:

[^2]$W=W_{4}+W_{2}$,
where
\[

$$
\begin{align*}
W_{4}= & Y_{i}^{e} L_{i} E_{i}^{c} H^{d}+Y_{i j}^{v} L_{i} N_{j}^{c} \eta^{u}+\lambda_{u} \eta^{u} H^{d} \phi+\lambda_{d} \eta^{d} H^{u} \phi \\
& +\mu_{H} H^{u} H^{d}  \tag{5}\\
W_{2}= & \frac{\left(M_{N}\right)_{i j}}{2} N_{i}^{c} N_{j}^{c}+\mu_{\eta} \eta^{u} \eta^{d}+\frac{1}{2} \mu_{\phi} \phi^{2} \tag{6}
\end{align*}
$$
\]

The Yukawa couplings of the charged leptons $Y_{i}^{e}$ can be assumed to be diagonal without loss of generality.

Soft-supersymmetry breaking terms are necessary to generate neutrino masses radiatively. For the relevant Higgs sector they are given by

$$
\begin{align*}
\mathcal{L}_{S B}= & -m_{\eta^{u}}^{2} \hat{\eta}^{u \dagger} \hat{\eta}^{u}-m_{\eta^{d}}^{2} \hat{\eta}^{d \dagger} \hat{\eta}^{d}-m_{\phi}^{2} \hat{\phi}^{\dagger} \hat{\phi} \\
& -\left(B_{\eta} \hat{\eta}^{u} \hat{\eta}^{d}+\text { h.c. }\right)-\left(\frac{1}{2} B_{\phi} \hat{\phi}^{2}+\text { h.c. }\right) \\
& +\left(A_{u} \lambda_{u} \hat{\eta}^{u} \hat{H}^{d} \hat{\phi}+A_{d} \lambda_{d} \hat{\eta}^{d} \hat{H}^{u} \hat{\phi}+\text { h.c. }\right) \tag{7}
\end{align*}
$$

where the hatted field is the scalar component of the corresponding superfield. The $B$ and $A$ soft terms are responsible for the radiative generation of the neutrino masses. We assume that $\hat{\eta}^{u}, \hat{\eta}^{d}$ and $\hat{\phi}$ do not acquire vacuum expectation values.

To calculate the one-loop neutrino mass matrix, we treat the $B$ terms as insertions. Then we find a one-loop diagram with one insertion of $B_{\eta} \hat{\eta}^{u} \hat{\eta}^{d}$, which mixes $\hat{\eta}^{u}$ and $\hat{\eta}^{d}$. Correspondingly, we define the approximate mass eigenstates $\eta_{0}^{ \pm}$(the neutral component of $\eta$ ) as
$\binom{\eta_{0}^{u}}{\eta_{0}^{d}}=\left(\begin{array}{cc}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right)\binom{\eta_{0}^{+}}{\eta_{0}^{-}}$,
where
$\tan 2 \theta=-\frac{2 m_{u d}^{2}}{m_{u u}^{2}-m_{d d}^{2}}$,
$m_{ \pm}^{2}=\frac{1}{2}\left\{m_{u u}^{2}+m_{d d}^{2} \pm\left[\left(m_{u u}^{2}-m_{d d}^{2}\right)^{2}+4 m_{u d}^{4}\right]^{1 / 2}\right\}$,
with
$m_{u u}^{2}=\mu_{\eta}^{2}+m_{\eta^{u}}^{2}-\frac{1}{2} M_{z}^{2} \cos 2 \beta+\frac{1}{2} \lambda_{u}^{2} v^{2} \cos ^{2} \beta$,
$m_{d d}^{2}=\mu_{\eta}^{2}+m_{\eta^{d}}^{2}+\frac{1}{2} M_{z}^{2} \cos 2 \beta+\frac{1}{2} \lambda_{d}^{2} v^{2} \sin ^{2} \beta$,
$m_{u d}^{2}=\frac{1}{2} \lambda_{u} \lambda_{d} v^{2} \cos \beta \sin \beta$,
and $\tan \beta=v_{u} / v_{d}, v^{2}=v_{u}^{2}+v_{d}^{2}, M_{z}=\left(g_{2}^{2}+g^{\prime 2}\right) v^{2} / 4$. Neglecting higher order insertions we obtain the neutrino mass matrix at one loop:
$\left(\mathbf{M}_{\nu}\right)_{i j}$

$$
\begin{align*}
= & \frac{1}{16 \pi^{2}} Y_{i l}^{v} U_{l k} M_{k} U_{k m}^{T} Y_{j m}^{v} B_{\eta} \sin 2 \theta\left[-\cos ^{2} \theta I\left(m_{+}, m_{+}, M_{k}\right)\right. \\
& \left.+\sin ^{2} \theta I\left(m_{-}, m_{-}, M_{k}\right)+\cos 2 \theta I\left(m_{+}, m_{-}, M_{k}\right)\right] \tag{13}
\end{align*}
$$

where $U$ is a unitary matrix defined by $\left(U^{T} M_{N} U\right)_{i k}=M_{k} \delta_{i k},{ }^{3}$ and

[^3]Table 2
The dark matter candidates. The $(+,+,-)$ candidates are dropped, because they are the left-handed neutrinos.

| $R \times Z_{2} \times Z_{2}^{L}$ | Bosons | Fermions |
| :--- | :--- | :--- |
| $(-,+,+)$ |  | $\psi_{h^{u}}, \psi_{h^{d}}, \tilde{Z}, \tilde{\gamma}$ |
| $(+,-,+)$ | $\psi_{\eta^{u}}, \psi_{\eta^{d}}, \psi_{\phi}$ |  |
| $(-,-,+)$ | $\hat{\eta}_{0}^{u}, \hat{\eta}_{0}^{d}, \hat{\phi}$ |  |
| $(+,-,-)$ | $\hat{N}$ 's | $\psi_{N^{\prime}} \mathrm{s}$ |
| $(-,-,-)$ | $\hat{v}_{L}$ 's |  |
| $(-,+,-)$ |  |  |

$$
\begin{align*}
& I\left(m_{a}, m_{b}, m_{c}\right) \\
& \quad=\int_{0}^{1} d x \int_{0}^{1-x} d y\left[m_{a}^{2} x+m_{b}^{2} y+m_{c}^{2}(1-x-y)\right]^{-1} \\
& \quad=\frac{m_{a}^{2} m_{c}^{2} \ln \left(m_{a}^{2} / m_{c}^{2}\right)+m_{b}^{2} m_{c}^{2} \ln \left(m_{b}^{2} / m_{c}^{2}\right)+m_{b}^{2} m_{a}^{2} \ln \left(m_{b}^{2} / m_{a}^{2}\right)}{\left(m_{a}^{2}-m_{b}^{2}\right)\left(m_{b}^{2}-m_{c}^{2}\right)\left(m_{c}^{2}-m_{a}^{2}\right)} \tag{14}
\end{align*}
$$

As we can see from (9), (12) and (13), the neutrino masses are proportional to $B_{\eta}$ and $\lambda_{u} \lambda_{d}$ at the lowest order, because $\sin 2 \theta \propto$ $\lambda_{u} \lambda_{d}$. So, the neutrino masses can be controlled by these parameters along with the Yukawa couplings $Y_{i l}^{v}$, the masses of the inert Higgses and right-handed neutrinos.

There are many candidates for the dark matter in this model [27]. The lightest combination of each row in Table 2 could be a dark matter. But there can exist only three types of dark matter including the left-handed neutrinos depending on which discrete symmetry guarantees their stability. We assume that the first right-handed neutrino $\psi_{N_{1}}$ is the lightest one among $\psi_{N}$ 's and denote it by $\psi_{N}$ (its mass is denoted by $m_{\text {NDM }}$ ). So, $\psi_{N}$ and the lightest neutralino $\chi$ (its mass is denoted by $m_{\chi \mathrm{DM}}$ ) are the dark matter candidates. Both have an odd $R$ parity, so that one of them can be the stable dark matter, while the other one is the decaying dark matter, if $Z_{2}$ is broken.

Following $[39,40]$, we have computed the thermally averaged cross section for the annihilation of two $\psi_{N}$ 's and that of two $\chi$ 's by expanding the corresponding relativistic cross section $\sigma$ in powers of their relative velocity, and we have then computed the relic densities $\Omega_{\mathrm{NDM}}$ and $\Omega_{\chi \mathrm{DM}}$. We have assumed that the SM particles are the only ones which are lighter than $\psi_{N}$ and $\chi$, so that we have used the SM degrees of freedom at the decoupling, i.e. $g_{*}=106.75$. We have found that, for the given interval of the dark matter masses, i.e., $1 \mathrm{TeV} \lesssim m_{\text {NDM }} \lesssim 3 \mathrm{TeV}$ and $0.2 \mathrm{TeV} \lesssim m_{\chi \mathrm{DM}} \lesssim 0.5 \mathrm{TeV}$, there is an enough parameter space in which $\left(\Omega_{\mathrm{NDM}}+\Omega_{\chi \mathrm{DM}}\right) h^{2} \simeq 0.11$ is satisfied. In the next section we let $\psi_{N}$ decay into $\chi$, while emitting high energy positrons. As it is clear from the superpotential (5), $\psi_{N}$ cannot decay into the quarks, because $\eta$ 's do not couple to the quarks.

## 4. Decaying right-handed neutrino dark matter and PAMELA/ATIC data

As long as the discrete symmetry $R \times Z_{2}$ is unbroken, there are two CDM particles in the present model. One finds that the $R \times Z_{2}\left[S U(3)_{C}\right]^{2}$ and $R \times Z_{2}\left[S U(2)_{L}\right]^{2}$ anomalies are cancelled with the matter content given Table $1 .{ }^{4}$ Our assumption is that $Z_{4}$ is anomalous and spontaneously broken to its subgroup $Z_{2}$. Note that $Z_{4}$ forbids $W_{2}$ in (6) while $W_{4}$ is allowed. Therefore, we have to

[^4]produce it from an additional sector. This situation can be realized as follows. Consider the $Z_{4}$ invariant superpotential including the SM singlet $\Sigma$ and $\sigma$ given in Table 1:
\[

$$
\begin{align*}
W_{\sigma}= & \xi \sigma+m_{\sigma} \sigma^{2}+\lambda_{\sigma} \sigma^{3}+\lambda_{\Sigma} \sigma \Sigma^{2}+\lambda_{\mu} \sigma H^{u} H^{d}+m_{\Sigma} \Sigma^{2} \\
& +\left(\frac{\left(\lambda_{N}\right)_{i j}}{2} N_{i}^{c} N_{j}^{c}+\lambda_{\eta} \eta^{u} \eta^{d}+\frac{1}{2} \lambda_{\phi} \phi^{2}\right) \Sigma . \tag{15}
\end{align*}
$$
\]

The superpotential (15) serves for $\Sigma$ and $\sigma$ to develop VEVs, and consequently, $Z_{4}$ is spontaneously broken to $Z_{2}$, producing effectively the superpotential (6). The true stable dark matter is the lightest one which has an odd parity of $R$. In the following discussion we assume that $\psi_{N}$ is heavier than $\chi$. Since the ATIC data are indicating that the mass of the decaying dark matter particle is preferably heavier than $O$ (1) TeV , all the superpartners should be heavier than $O(1) \mathrm{TeV}$ if $m_{\chi \mathrm{DM}}>m_{\mathrm{NDM}}$. It is, therefore, more welcome for $\psi_{N}$ to be the decaying dark matter, because a heavy $\psi_{N}$ means a heavy $\eta$ Higgs, which is desirable to suppress FCNC processes such as $\mu \rightarrow e+\gamma$.

As one can find, $Z_{4}$ is anomalous: $\Delta Q=1 \bmod N / 2(=2) .{ }^{5}$ Consequently, the suppression coefficient $b$ of (3) can take values
$b=\frac{4 \pi^{2} C}{\Delta Q}=4 \pi^{2} \times \frac{C(\bmod 4)}{1(\bmod 2)}$,
where $C$ is the charge of $\exp (-b S)$. We assume that the nonperturbative effect can generate $R$ invariant, but $Z_{4}$ violating operators. At $d=3$ there is only one operator $\eta^{u} L$ which is even under $R$, and has the $Z_{4}$ charge one. So we focus on $\eta^{u} L$ :
$W_{b}=\mu_{b i} \eta^{u} L_{i} \quad$ with $\mu_{b i}=\rho_{i} M_{P L} e^{-b\langle S\rangle}$,
where $\rho_{i}$ are dimensionless couplings. Since $\left\langle F_{S} / \varphi\right\rangle \sim m_{3 / 2}$ and $\langle\varphi\rangle \sim O(1)$, the superpotential $W_{b}$ induces a soft-supersymmetry breaking term
$\mathcal{L}_{b}=B_{b i} \hat{\eta}^{u} \hat{L}_{i} \quad$ with $B_{b i}=w \rho_{i} M_{\mathrm{PL}} m_{3 / 2} e^{-b}$
at the Planck scale, where is $w$ a dimensionless constant. Since the $Z_{4}$ charge of $\eta$ is 1 , the charge of $\exp (-b S)$ has to be $-1 \bmod 4$, and then
$b=4 \pi^{2} \times(\ldots, 7 / 3,11 / 5,11 / 7,7 / 5,1, \ldots)$,
which could give a huge suppression factor.
With this observation we proceed with our discussion. The tree diagrams contributing to the $\psi_{N}$ decay are shown in Fig. 1, where we have assumed that $\chi$ is the pure bino. We do not take into account the tree diagrams which exist due to the mixing of $\psi_{\eta^{u}}$ and $\psi_{e_{f}^{c}}$, because these diagrams are suppressed by $m_{f} / m_{\eta}$, where $m_{f}$ is the lepton mass. So, in the lowest order approximation only dimension two operators in the $B$ soft-breaking sector exist. At the lowest order, $\psi_{N}$ can decay only into the leptons along with a $\chi$. ( $R$ parity violating operator $L H^{u}$ allows the decay into the quarks, too.) The differential decay width is given by

$$
\begin{align*}
\frac{d \Gamma_{e^{+}}}{d E}= & \frac{m_{\mathrm{NDM}}^{4}}{768 \pi^{3}} x^{2}\left(1-\frac{z^{2}}{1-2 x}\right)^{2}\left[A_{1}\left(1-2 x-z^{2}\right)\right. \\
& \left.+2 A_{1}(1-x)\left(1+2 \frac{z^{2}}{1-2 x}\right)+6 A_{2} z+6 A_{3}(1-2 x)\right] \tag{19}
\end{align*}
$$

and the total decay width is

[^5]

Fig. 1. The diagrams contributing to the $\psi_{N}$ decay. We assume that $\chi$ is the pure bino. The emitted positrons are all right-handed. Similar diagrams exist because of the mixing of $\eta^{u}$ with $E^{c}$. The amplitude is suppressed by $e^{-b} m_{f} / m_{\eta}$, so that we do not consider them.

$$
\begin{align*}
\Gamma_{e^{+} T}= & \tau_{\mathrm{NDM}}^{-1} \\
= & \frac{m_{\mathrm{NDM}}^{5}}{12288 \pi^{3}}\left\{( 1 - z ^ { 2 } ) \left[\left(A_{1}+A_{3}\right)\left(1-7 z^{2}-7 z^{4}+z^{6}\right)\right.\right. \\
& \left.+4 A_{2} z\left(1+10 z^{2}+z^{4}\right)\right]+24 z^{2}\left[-\left(A_{1}+A_{3}\right) z\right. \\
& \left.\left.+2 A_{2}\left(1+z^{2}\right) \ln z\right]\right\} \tag{20}
\end{align*}
$$

where
$z=\frac{m_{\chi \mathrm{DM}}}{m_{\mathrm{NDM}}}<1, \quad x=\frac{E}{m_{\mathrm{NDM}}}<\left(1-z^{2}\right) / 2$,
$A_{1}=2 g^{\prime 2} \sum_{i, j}\left|Y_{j 1}^{*} B_{i}\right|^{2} \frac{1}{\tilde{m}_{L}^{4} m_{\eta}^{4}}$,
$A_{3}=2 g^{\prime 2} \sum_{i, j}\left|Y_{i 1}^{*} B_{j}\right|^{2} \frac{1}{\tilde{m}_{L}^{4} m_{\eta}^{4}}$,
$A_{2}=-g^{\prime 2} \sum_{i, j}\left[\left(Y_{j 1} B_{i}^{*}\right)\left(Y_{i 1}^{*} B_{j}\right)+\right.$ h.c. $] \frac{1}{\tilde{m}_{L}^{4} \tilde{m}_{\eta}^{4}}$,
where $g^{\prime}(\simeq 0.345)$ is the $U(1)_{Y}$ gauge coupling constant (the bino is assumed to be $\chi$ ). $j$ runs over the negatively charged leptons, and $i$ stands for a positively charged lepton in (22) and (23). We have assumed that all the scalar partners of the left-handed superpartners $\hat{l}_{L}$ have the same mass $\tilde{m}_{L}$. The positron can come from the decay of the anti-muons and anti-taus. In the following calculations, however, we assume that the energy spectrum of the positron coming from the anti-muon and tau does not differ very much from that of the direct production of the positron. So, we also sum over $i=e^{+}, \mu^{+}, \tau^{+}$in (22) and (23) to obtain $d \Gamma_{e^{+}} / d E$. At this order, all the emitted positrons are right-handed, as one can see from Fig. 1.

Before we calculate the positron spectrum, we briefly consider the suppression factor we need for our case. Assuming that $Y_{i j} \sim 1$ and that all the $\rho_{i}$ in $B$ 's in (18) are of the same size, we obtain

$$
\begin{align*}
\tau_{\mathrm{NDM}} \sim & \left(\frac{\mathrm{TeV}}{m_{\mathrm{NDM}}}\right)\left(\frac{m_{\eta}^{2} \tilde{m}_{L}^{2}}{m_{\mathrm{NDM}}^{3} m_{3 / 2}}\right)^{2}\left(\frac{m_{\mathrm{NDM}}}{M_{\mathrm{PL}} / 10^{16}}\right)^{2} \\
& \times\left(\rho_{\tau} \omega\right)^{-2}\left(10^{-79} e^{2 b}\right) \times 10^{26} \mathrm{~s} . \tag{24}
\end{align*}
$$

So, we have a right order of $\tau_{\text {NDM }}$ with $b=4 \pi^{2}(7 / 3)$ which gives a suppression factor of $10^{-80}$ (see (16)).

Now we come to compute the positron spectrum:
$f_{e^{+}}(E)=\int_{E}^{E_{\max }} d E^{\prime} G_{e^{+}}\left(E, E^{\prime}\right) \frac{d \Gamma_{e^{+}}\left(E^{\prime}\right)}{d E^{\prime}}$,
where $E_{\max }=\left(m_{\mathrm{NDM}}^{2}-m_{\chi \mathrm{DM}}^{2}\right) / 2 m_{\mathrm{NDM}}, d \Gamma_{e^{+}}(E) / d E=\left(\tau_{\mathrm{NDM}}\right)^{-1} \times$ $d n_{e^{+}}(E) / d E$, and we vary $\tau_{\mathrm{NDM}}$ freely to fit the data. The positron Green's function $G_{e^{+}}$of [41] can be approximately written as [13]

$$
\begin{align*}
& G_{e^{+}}\left(E, E^{\prime}\right) \\
& \quad \simeq\left(\frac{\Omega_{\mathrm{NDM}} h^{2}}{0.11}\right) \frac{10^{16}}{E^{2}} \exp \left[a+b\left(E^{\delta-1}-E^{\prime \delta-1}\right)\right] \mathrm{cm}^{-3} \mathrm{~s}, \tag{26}
\end{align*}
$$

where $a, b, \delta$ depend on the diffusion model $[13,42,43]$. Here we use those of the MED model [43]: $a=-1.0203, b=-1.4493$, $\delta=0.70$, and we have assumed that except for the normalization


Fig. 2. $e^{+} /\left(e^{+}+e^{-}\right)$versus the positron energy $E$. The blue lines are the predictions of the model, where we have used: $z=1 / 5$ (dashed), $1 / 6$ (dot-dashed), $1 / 5$ (dotted), $\tau_{\mathrm{NDM}}\left(0.11 / \omega_{\mathrm{NDM}} h^{2}\right)=1.4$ (dashed), 2.0 (dot-dashed), 3.0 (dotted) $\times 10^{-26} \mathrm{~s}$, $m_{\text {NDM }}=2.0$ (dashed), 1.5 (dot-dashed), 1.0 (dotted) TeV. The red points are the PAMELA data [22], where the predictions are written over the figure 4 of the PAMELA paper [22]. The solid line is the background published in [22], and it agrees with the one calculated from (27)-(29) without the primary source of the positron. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)
factor $\Omega_{\mathrm{NDM}} h^{2} / 0.11$ the decaying dark matter $\psi_{N}$ has the same density profile in our galaxy as the NFW profile [44]. The background differential flux for each species are [45]

$$
\begin{align*}
& \Phi_{e^{-}}^{\text {prim.bkg }}(E)=N_{\phi} 0.16 E^{-1.1}\left[1+11 E^{0.9}+3.2 E^{2.15}\right]^{-1},  \tag{27}\\
& \Phi_{e^{-}}^{\text {sec.bkg }}(E) \\
& \quad=N_{\phi} 0.7 E^{0.7}\left[1+110 E^{1.5}+600 E^{2.9}+580 E^{4.2}\right]^{-1},  \tag{28}\\
& \Phi_{e^{+}}^{\text {sec.bkg }}(E)=N_{\phi} 4.5 E^{0.7}\left[1+650 E^{2.3}+1500 E^{4.2}\right]^{-1} \tag{29}
\end{align*}
$$

in the units in $\left[\mathrm{GeV} \mathrm{cm}^{2} \mathrm{ssr}\right]^{-1}$, where the energy $E$ is in the units in GeV , and the normalization factor $N_{\phi}$ is so chosen that the prediction and the Fermi LAT agree around 70 GeV . The primary positron differential flux $\Phi_{e^{+}}^{\text {prim. }}$ is $(c / 4 \pi) f_{e^{+}}$, where $f_{e^{+}}$is given in (25). Using (25)-(29), we then calculate appropriate quantities for PAMELA and ATIC:

$$
\begin{equation*}
\frac{e^{+}}{e^{+}+e^{-}}=\frac{\Phi_{e^{+}}^{\text {prim. }}+\Phi_{e^{+} . b k g}^{\text {sec.bkg }}}{\Phi_{e^{+}}^{\text {prim. }}+\Phi_{e^{+}}^{\text {sec.bkg }}+\Phi_{e^{-}}^{\text {prim.bkg }}+\Phi_{e^{-}}^{\text {sec.bkg }}} \tag{30}
\end{equation*}
$$

for PAMELA,
$E^{3} \frac{d N}{d E}=E^{3}\left(\Phi_{e^{+}}^{\text {prim. }}+\Phi_{e^{+}}^{\text {sec.bkg }}+\Phi_{e^{-}}^{\text {prim.bkg }}+\Phi_{e^{-}}^{\text {sec.bkg }}\right)$

The results are shown in Figs. 2 and 3, where we have assumed $A_{1}=A_{3}=A_{2}$ in (22) and (23). The blue lines are the predictions of the model, and we have used: $z=1 / 5$ (dashed), $1 / 6$ (dot-dashed), $1 / 5$ (dotted), $\tau_{\mathrm{NDM}}\left(0.11 / \omega_{\mathrm{NDM}} h^{2}\right)=1.44$ (dashed), 2.0 (dot-dashed), 3.0 (dotted) $\times 10^{-26} \mathrm{~s}, m_{\text {NDM }}=2.0$ (dashed), 1.5 (dot-dashed), 1.0 (dotted) TeV, where $z$ is defined in (21). The predictions are written over the data of PAMELA [22] in Fig. 2, and those of ATIC [23], HESS [47] and Fermi LAT [48] in Fig. 3. We see


Fig. 3. The differential energy spectrum scaled by $E^{3}$. The solid, dashed, dot-dashed and dotted blue lines are the predictions of the model. The parameter values used here are the same as for Fig. 2. The solid blue line is calculated with $z=1 / 10, \tau_{\text {NDM }}\left(0.11 / \omega_{\text {NDM }} h^{2}\right)=0.94 \times 10^{26} \mathrm{~s}$ and $m_{\text {NDM }}=3 \mathrm{TeV}$. The predictions are plotted together with the data of ATIC [23], PPB-BETS [46], HESS [47] and Fermi LAT [48]. The black dashed line is the background presented in [23]. The normalization factor $N_{\phi}$ in (27)-(29) is so chosen that the prediction and the Fermi LAT data agree around 70 GeV . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)
from Fig. 3 that $m_{\text {NDM }}$ should be heavier than $O$ (1) TeV in this model, too.

If the two out-going charged particles in the tree-diagram 1 are connected, and the internal line so obtained emits a gamma, then the one-loop diagram describes the radiative decay of the decaying dark matter $N$ into the stable dark matter $\chi$. This decay defines the endpoint of the gamma-ray spectrum, which depends on $m_{\text {NDM }}$. The gamma ray can be observed by Fermi LAT, which is designed for the study of the gamma ray sky in the energy range between 30 and 300 GeV . The detailed analysis will be our next project and will be published elsewhere.

## 5. Conclusion

We have studied a dark matter model, in which one decaying and one stable dark matter particles coexist. We have assumed that one of the discrete symmetries ensuring the stability of the dark matter particles, when imbedded into a larger group, is anomalous, and the heavier dark matter can decay non-perturbatively. The huge suppression factor for the decay of dark matter to be needed can be obtained in this way. The concrete model we have considered is a supersymmetric extension of the Ma's inert Higgs model, so that the decaying dark matter (the lightest right-handed neutrino) can decay only into leptons along with the stable dark matter (LSP). We have shown that this scenario can explain the data of [22,23]. It is clear that if the recent and future data coming from the cosmic ray observations are intimately related to the nature of dark matter, its explanation may open the window to new physics beyond the SM. The radiative dark matter decay and high energy neutrino productions will be our next projects.

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[^0]:    * Corresponding author.

    E-mail address: suematsu@hep.s.kanazawa-u.ac.jp (D. Suematsu).

[^1]:    ${ }^{1}$ The necessary enhancement for the s-wave annihilation can be partly covered by the Sommerfeld effects [24] or others [25].

[^2]:    2 Since $b>0, C+N$ should appear instead of $C$ for a negative $C$.

[^3]:    ${ }^{3} M_{1}<M_{2,3}$ is assumed, and we denote $M_{1}$ by $m_{\text {NDM }}$ later on.

[^4]:    ${ }^{4}$ We do not consider the $R \times Z_{2}\left[U(1)_{Y}\right]^{2}$ and mixed gravitational anomalies, because they do not give us useful informations.

[^5]:    ${ }^{5}$ For the Green-Schwarz cancellation to work, the $Z_{4}\left[S U(3)_{C}\right]^{2}$ anomaly has to be matched to $Z_{4}\left[S U(2)_{L}\right]^{2}$ anomaly. To realize this we introduce, for instance, a pair of $\mathbf{3}$ and $\overline{\mathbf{3}}$ of $S U(3)_{C}$ with the $Z_{4}$ charge one. Their mass can be obtained from $\langle\Sigma\rangle \mathbf{3} \times \overline{\mathbf{3}}$.

