

# Neutrino masses and CDM in a non-supersymmetric model

メタデータ	言語: eng 出版者: 公開日: 2022-01-27 キーワード (Ja): キーワード (En): 作成者: メールアドレス: 所属:
URL	<a href="https://doi.org/10.24517/00064614">https://doi.org/10.24517/00064614</a>

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 International License.



# Neutrino masses and CDM in a non-supersymmetric model

Jisuke Kubo, Daijiro Suematsu\*

*Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

Received 3 October 2006; received in revised form 24 October 2006; accepted 4 November 2006

Available online 13 November 2006

Editor: T. Yanagida

## Abstract

We propose a model for neutrino mass generation based on both the tree-level seesaw mechanism with a single right-handed neutrino and one-loop radiative effects in a non-supersymmetric framework. The generated mass matrix is composed of two parts which have the same texture and produce neutrino mass eigenvalues and mixing suitable for the explanation of neutrino oscillations. The model has a good CDM candidate which contributes to the radiative neutrino mass generation. The stability of the CDM candidate is ensured by  $Z_2$  which is the residual symmetry of a spontaneously broken  $U(1)'$ . We discuss the values of  $U_{e3}$  and also estimate the masses of the relevant fields to realize an appropriate abundance of the CDM.

© 2006 Elsevier B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

## 1. Introduction

Recent experimental and observational results on neutrino masses [1] and cold dark matter (CDM) [2] suggest that the standard model (SM) should be extended by introducing some neutral fields. A well studied candidate for the extension is the minimal supersymmetric SM (MSSM). Although the MSSM contains a good CDM candidate as the lightest superparticle (LSP) as long as the  $R$ -parity is conserved, the parameter regions preferable for the explanation of the WMAP data are found to be strictly restricted in certain types of the MSSM [3]. Confronting these situations, it seems to be interesting to consider models in which we can explain these new features from the same origin in a non-supersymmetric extension of the SM: a certain symmetry related to the smallness of neutrino masses can guarantee the stability of a CDM candidate. Backgrounds that forth coming collider experiments like LHC may find signatures of such extended models make this kind of trials worthy enough at present stage. Several recent works have been done along this line [4].

In this Letter we follow this line to propose an extension of a previously considered model by introducing a local  $U(1)'$  sym-

metry at TeV regions. As in the radiative mass generation models [5], we introduce an additional  $SU(2)$  doublet  $\eta^T \equiv (\eta^+, \eta^0)$  to the ordinary Higgs doublet  $H^T \equiv (H^+, H^0)$ . We also introduce a singlet  $\phi$  whose vacuum expectation value breaks  $U(1)'$  symmetry spontaneously down to  $Z_2$  which is responsible for the stability of the CDM candidate. This extension seems to remedy defects in the previous models that certain fine tunings are required for both the generation of small neutrino masses and the reconciliation between the CDM abundance and the constraints from lepton flavor violating processes. Based on such a model we calculate the value of the element  $U_{e3}$  of the MNS matrix and masses of the relevant fields which produce an appropriate abundance of the CDM.

## 2. A model

We consider a model with a similar symmetry to the model in [4,6]. We extend it by introducing a singlet Higgs scalar  $\phi$ . This extension makes the model able to contain an additional  $U(1)'$  symmetry. In this Letter we assume that this symmetry is leptophobic and then leptons do not have its charge for simplicity.<sup>1</sup> The  $U(1)'$  charge for the ingredients of the model is shown

\* Corresponding author.

*E-mail addresses:* [jik@hep.s.kanazawa-u.ac.jp](mailto:jik@hep.s.kanazawa-u.ac.jp) (J. Kubo), [suematsu@hep.s.kanazawa-u.ac.jp](mailto:suematsu@hep.s.kanazawa-u.ac.jp) (D. Suematsu).

<sup>1</sup> We need to introduce some additional fermions to cancel gauge anomaly. Since such extensions will be done without changing the following results, we do not go further into this problem here.

Table 1  
Field contents and their charges.  $Z_2$  is the residual symmetry of  $U(1)'$

	$Q_\alpha$	$\bar{U}_\alpha$	$\bar{D}_\alpha$	$L_\alpha$	$\bar{E}_\alpha$	$\bar{N}_1$	$\bar{N}_2$	$H$	$\eta$	$\phi$
$U(1)'$	$2q$	$-2q$	$-2q$	$0$	$0$	$0$	$q$	$0$	$-q$	$-2q$
$Z_2$	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	$-1$	$+1$	$-1$	$+1$

in Table 1, in which fermions are assumed to be left-handed. Note that we need only two right-handed neutrinos  $N_1$  and  $N_2$  to generate appropriate neutrino masses and mixings in a minimal case. Then the invariant Lagrangian relevant to the neutrino masses can be expressed as

$$\mathcal{L}_m = \sum_{\alpha=e,\mu,\tau} (h_{\alpha 1} L_\alpha H \bar{N}_1 + h_{\alpha 2} L_\alpha \eta \bar{N}_2) + \frac{1}{2} M_* \bar{N}_1^2 + \frac{1}{2} \lambda \phi \bar{N}_2^2 + \text{h.c.}, \quad (1)$$

where we assume that Yukawa couplings for charged leptons are diagonal. The most general invariant scalar potential up to dimension five may also be written as

$$V = \frac{1}{2} \lambda_1 (H^\dagger H)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \frac{1}{2} \lambda_3 (\phi^\dagger \phi)^2 + \lambda_4 (H^\dagger H) (\eta^\dagger \eta) + \lambda_5 (H^\dagger H) (\eta^\dagger H) + \frac{\lambda_6}{2 M_*} [\phi (\eta^\dagger H)^2 + \text{h.c.}] + (m_H^2 + \lambda_7 \phi^\dagger \phi) H^\dagger H + (m_\eta^2 + \lambda_8 \phi^\dagger \phi) \eta^\dagger \eta + m_\phi^2 \phi^\dagger \phi. \quad (2)$$

We add a non-renormalizable  $\lambda_6$  term and a bare mass term for  $N_1$ . The scalar potential (2) without the  $\lambda_6$  term has an accidental  $U(1)$  symmetry, which forbids the one-loop contribution of the  $\eta$  exchange diagram to neutrino masses. This symmetry is explicitly broken by the Yukawa interactions (1), so that terms like the  $\lambda_6$  term, i.e.  $(\phi^\dagger \phi)^n \phi (\eta^\dagger H)^2$ , can be generated in high orders in perturbation theory in general. All of them contribute to radiative neutrino masses.<sup>2</sup> Here we do not ask the origin of the  $\lambda_6$  term. They might be supposed to be effective terms generated through some dynamics at an intermediate scale  $M_*$ . We can check that there are no other dimension five operators invariant under the above mentioned symmetry in the scalar potential.

As the model discussed in [4],  $H$  plays the role of the ordinary doublet Higgs scalar in the SM but  $\eta$  is assumed to obtain no vacuum expectation value (VEV). A singlet scalar  $\phi$  is assumed to obtain a VEV, which breaks  $U(1)'$  down to  $Z_2$  (see Table 1). This VEV also gives the mass for  $N_2$  through  $M_{N_2} = \lambda \langle \phi \rangle$  and also yields an effective coupling for the  $\lambda_6$  term as  $\lambda_6 \langle \phi \rangle / M_*$ . It can be small enough as long as  $\langle \phi \rangle \ll M_*$  is satisfied. Thus, the masses of the real and imaginary parts of  $\eta^0$  are found to be almost degenerate. They are expressed as  $M_{\eta^0}^2 \simeq m_\eta^2 + (\lambda_4 + \lambda_5) \langle H^0 \rangle^2 + \lambda_8 \langle \phi \rangle^2$ . In the model discussed in [4], the coupling constant of the term corresponding to this

<sup>2</sup> It turns out that one-loop corrections generating the  $\lambda_6$  term, i.e.  $(\phi^\dagger \phi) \phi (\eta^\dagger H)^2$ , vanish if the condition (5) discussed later is satisfied.

$\lambda_6$  term is required to be extremely small to generate appropriate neutrino masses. This point is automatically improved by introducing the new  $U(1)'$  symmetry.

### 3. Masses and mixings of neutrinos

We find that there are two origins for the neutrino masses under these settings for the model. One is the ordinary seesaw mass induced by a right-handed neutrino  $N_1$  [7] and another is one-loop radiative mass mediated by the exchange of  $\eta^0$  and  $N_2$  [5,6]. These effects generate a mass matrix for three light neutrinos. It is expressed by

$$M_\nu = \frac{v^2}{M_*} \left[ \mu^{(1)} + \frac{\lambda_6}{8\pi^2 \lambda} I \left( \frac{M_{N_2}^2}{M_{\eta^0}^2} \right) \mu^{(2)} \right], \quad (3)$$

$$I(x) = \frac{x}{1-x} \left( 1 + \frac{x \ln x}{1-x} \right),$$

where  $v = \langle H^0 \rangle$  and  $\mu^{(a)}$  is defined by

$$\mu^{(a)} = \begin{pmatrix} h_{ea}^2 & h_{ea} h_{\mu a} & h_{ea} h_{\tau a} \\ h_{ea} h_{\mu a} & h_{\mu a}^2 & h_{\mu a} h_{\tau a} \\ h_{ea} h_{\tau a} & h_{\mu a} h_{\tau a} & h_{\tau a}^2 \end{pmatrix} \quad (a = 1, 2). \quad (4)$$

Although two terms of  $M_\nu$  may be characterized by different mass scales, the texture of both terms is the same as found in Eq. (4). This type of the texture for neutrino mass matrix has been studied in [7,8]. We neglect CP phases in the following discussion.

Now we study eigenvalues and the mixing matrix for the neutrino mass matrix (3). We consider to diagonalize  $M_\nu$  by using an orthogonal matrix  $U$  in such a way as  $U^T M_\nu U = \text{diag}(m_1, m_2, m_3)$ . If Yukawa couplings satisfy a condition

$$h_{e1} h_{e2} + h_{\mu 1} h_{\mu 2} + h_{\tau 1} h_{\tau 2} \propto [\mu^{(1)}, \mu^{(2)}] = 0, \quad (5)$$

$\mu^{(1)}$  and  $\mu^{(2)}$  can be simultaneously diagonalized. Since  $U$  can be analytically found in such cases, we confine ourselves to these interesting ones. We define a matrix  $\tilde{U}$  as

$$\tilde{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_2 & \sin \theta_2 \\ 0 & -\sin \theta_2 & \cos \theta_2 \end{pmatrix} \begin{pmatrix} \cos \theta_3 & 0 & \sin \theta_3 \\ 0 & 1 & 0 \\ -\sin \theta_1 & 0 & \cos \theta_3 \end{pmatrix}. \quad (6)$$

The first term of  $M_\nu$  can be diagonalized by this  $\tilde{U}$  if the following conditions is satisfied:

$$\tan \theta_2 = \frac{h_{\mu 1}}{h_{\tau 1}}, \quad \tan \theta_3 = \frac{h_{e1}}{\sqrt{h_{\mu 1}^2 + h_{\tau 1}^2}}. \quad (7)$$

Then the mass eigenvalues for the first term of  $M_\nu$  are obtained by using the following eigenvalues of  $\mu^{(1)}$ :

$$\mu_{\text{diag}}^{(1)} = \text{diag}(0, 0, h_{e1}^2 + h_{\mu 1}^2 + h_{\tau 1}^2). \quad (8)$$

We consider diagonalization of  $\mu^{(2)}$  next. At first, it should be noted that  $\mu^{(2)}$  is transformed by the same  $\tilde{U}$ . However, if the condition (5), which can be written as

$$h_{e2} \sin \theta_3 + (h_{\mu 2} \sin \theta_2 + h_{\tau 2} \cos \theta_2) \cos \theta_3 = 0 \quad (9)$$

is satisfied,  $\mu^{(2)}$  can be diagonalized by applying an orthogonal transformation  $\tilde{U}U_3$  supplemented by an additional one given by

$$U_3 = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

This additional transformation by  $U_3$  does not affect the diagonalization of  $\mu^{(1)}$ . Consequently, both terms of  $M_\nu$  can be simultaneously diagonalized by setting

$$\tan \theta_1 = -\frac{\tan \tilde{\theta}_2 \tan \theta_2 + 1}{(\tan \tilde{\theta}_2 - \tan \theta_2) \sin \theta_3}, \quad (11)$$

where we define  $\tilde{\theta}_2$  as  $\tan \tilde{\theta}_2 = h_{\mu 2}/h_{\tau 2}$ . Finally, we obtain non-zero mass eigenvalues of the light neutrinos as

$$m_2 = AB \frac{\tan^2 \theta_1 + 1}{\tan^2 \theta_2 + 1} (\tan \tilde{\theta}_2 - \tan \theta_2)^2, \\ m_3 = \frac{A}{2} (\tan^2 \theta_2 + 1) (\tan^2 \theta_3 + 1), \quad (12)$$

where  $A = 2h_{\tau 1}^2 v^2/M_*$  and  $B = (\lambda_6/16\pi^2\lambda)(h_{\tau 2}/h_{\tau 1})^2 \times I(M_{N_2}^2/M_{\eta^0}^2)$ .

Here we fix  $\tan \theta_2 = 1$  which is supported by the data of the atmospheric neutrino and K2K experiment. CHOOZ experiments give the constraint on  $\theta_3$  such as  $|\sin \theta_3| < 0.22$  [9]. If we use these conditions, the mixing matrix  $U = \tilde{U}U_3$  can be approximately written as

$$U = \begin{pmatrix} \cos \theta_1 & \sin \theta_1 & \frac{\sin \theta_3}{\sqrt{2}} \\ -\frac{\sin \theta_1}{\sqrt{2}} & \frac{\cos \theta_1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_1}{\sqrt{2}} & -\frac{\cos \theta_1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (13)$$

Only two mass eigenvalues  $M_\nu$  are non-zero and then we impose that squared mass differences required by the neutrino oscillation data satisfy  $m_3 = \sqrt{\Delta m_{\text{atm}}^2}$  and  $m_2 = \sqrt{\Delta m_{\text{sol}}^2}$ . Although there is a possibility that two non-zero eigenvalues have almost degenerate values such as  $\sqrt{\Delta m_{\text{atm}}^2}$  and their squared difference is given by  $\Delta m_{\text{sol}}^2$ , we do not consider it since  $\mu^{(1)}$  and  $\mu^{(2)}$  have independent origins. We suppose  $\theta_1 = \theta_{\text{sol}}$ , where  $\theta_{\text{sol}}$  is a mixing angle relevant to the solar neutrino. Then we can determine  $\theta_3$  through Eq. (11) by using  $\tan \theta_{\text{sol}}$ ,  $\sqrt{\Delta m_{\text{atm}}^2}$ ,  $\sqrt{\Delta m_{\text{sol}}^2}$  and  $B$ . If we use neutrino oscillation data for these, we can find allowed regions of  $\theta_3$  as a function of  $B$ . This is shown in Fig. 1, where we have used values of the measured neutrino oscillation parameters [10]

$$\Delta m_{\text{sol}}^2 = 8.0_{-0.4}^{+0.6} \times 10^{-5} \text{ eV}^2, \\ \Delta m_{\text{atm}}^2 = (1.9\text{--}3.6) \times 10^{-3} \text{ eV}^2, \\ \tan^2 \theta_{\text{sol}} = 0.45_{-0.07}^{+0.09}. \quad (14)$$

This figure shows that  $B$  is restricted in narrow regions such as  $0.03 < B < 0.1$ .

As an example, let us assume  $M_{\eta^0}/M_{N_2} = 0.3\text{--}0.7$  and then  $I(M_{N_2}^2/M_{\eta^0}^2) = 0.1\text{--}1.3$ . In such cases  $h_{\tau 2}/h_{\tau 1} \simeq 10(\lambda/\lambda_6)^{1/2}$

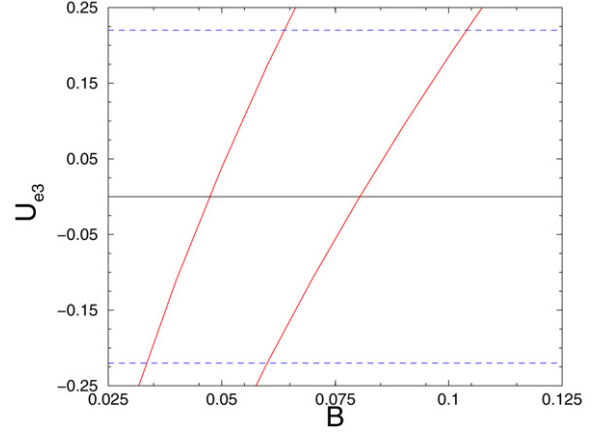


Fig. 1.  $U_{e3}$  as a function of  $B$ . Allowed regions are shown as the regions surrounded by red solid lines. Horizontal dashed lines stand for the present experimental upper bounds for  $|U_{e3}|$ .

should be satisfied. If we obtain more constraints on the relevant coupling constants, we may restrict the value of  $U_{e3}$  much more. Although  $U_{e3}$  takes a non-zero value for  $0.03 \lesssim B \lesssim 0.05$  and  $0.08 \lesssim B \lesssim 0.1$ ,  $U_{e3} = 0$  is also allowed for  $0.03 < B < 0.08$ . The condition for the coupling constants can be easily satisfied even if we assume that coupling constants are  $O(1)$ . Therefore, the model needs no fine tuning to be consistent with all the present experimental data for neutrino oscillations. The effective mass  $m_{ee}$  for the neutrinoless double beta decay takes the values in the range  $|m_{ee}| \lesssim 6.3 \times 10^{-3} \text{ eV}$ .

#### 4. Relic abundance of a CDM candidate

The lightest field with an odd  $Z_2$  charge can be stable since an even charge is assigned to each SM content. If both the mass and the annihilation cross section of such a field have appropriate values, it can be a good CDM candidate as long as it is neutral. As found from Table 1, such candidates are  $N_2$  and  $\eta^0$ . Since they have a new  $U(1)'$  gauge interaction, their annihilation to quarks is considered to be dominantly mediated by this interaction.<sup>3</sup> If their annihilation is mediated only by the exchange of  $\eta^0$  or  $N_2$  through Yukawa couplings as in the model discussed in [4], we cannot simultaneously explain, without fine tuning of coupling constants, both the observed value of the CDM abundance and the constraints coming from lepton flavor violating processes such as  $\mu \rightarrow e\gamma$ . Since  $U(1)'$  is supposed to be a generation independent gauge symmetry, we can easily escape this problem by assuming that the Yukawa couplings  $h_{\alpha 2}$  are small enough or both  $\eta^0$  and  $N_2$  are heavy enough. In the following study we consider the case that  $N_2$  is lighter than  $\eta^0$ . As seen in the last part of the previous section, this case is consistent with the present experimental bounds for  $U_{e3}$  without fine tuning.

Now we estimate the relic abundance of  $N_2$  and compare it with the CDM abundance obtained from the WMAP data. We

<sup>3</sup> A role of  $U(1)'$  in annihilation of the CDM in supersymmetric models has been studied in [13].

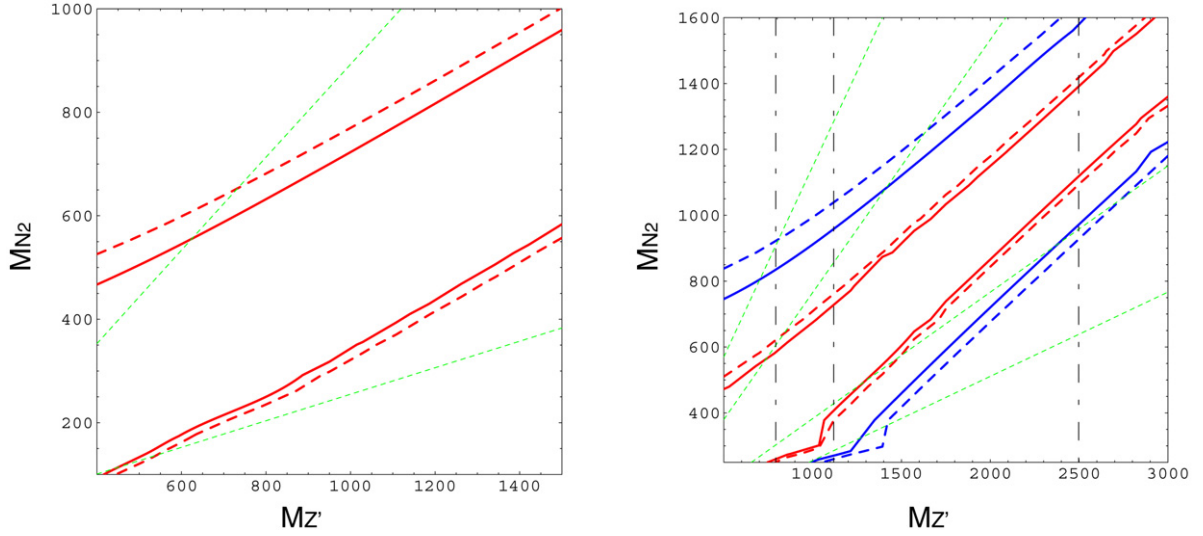


Fig. 2. Allowed regions by the WMAP data in the  $(M_{Z'}, M_{N_2})$  plane. In both figures solid and dotted lines represent contours for  $\Omega_{N_2} h^2 = 0.0945$  and  $\Omega_{N_2} h^2 = 0.1287$ . Green dotted lines stand for  $M_{N_2}$  for typical values of  $\lambda$  which are given in the text. In the right figure vertical dash-dotted lines represent lower bounds of  $M_{Z'}$  in case of  $|\theta| = 10^{-2}, 5 \times 10^{-3}, 10^{-3}$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

suppose that possible annihilation processes  $N_2 N_2 \rightarrow f \bar{f}$  are dominantly mediated by the  $U(1)'$  gauge field. If it is expanded by relative velocity  $v$  between annihilating  $N_2$ 's as  $\sigma v = a + b v^2$ , the coefficients  $a$  and  $b$  are expressed as

$$a = \sum_f c_f \frac{g'^4}{2\pi} Q_{fA}^2 q^2 \frac{m_f^2 \beta}{(s - M_{Z'}^2)^2},$$

$$b = \sum_f c_f \frac{g'^4}{6\pi} (Q_{fV}^2 + Q_{fA}^2) q^2 \frac{M_{N_2}^2 \beta}{(s - M_{Z'}^2)^2}, \quad (15)$$

where  $\beta = \sqrt{1 - m_f^2/M_{N_2}^2}$  and  $c_f = 3$  for quarks.  $s$  is the center of mass energy of collisions and  $q$  is the  $U(1)'$  charge of  $N_2$  given in Table 1. The charge of the final state fermion  $f$  is defined as

$$Q_{fV} = Q_{fR} + Q_{fL}, \quad Q_{fA} = Q_{fR} - Q_{fL}. \quad (16)$$

Using these quantities, the present relic abundance of  $N_2$  can be estimated as [11],

$$\Omega_{N_2} h^2 \Big|_0 = \frac{M_{N_2} n_{N_2}}{\rho_{\text{cr}}/h^2} \Big|_0 \simeq \frac{8.76 \times 10^{-11} g_*^{-1/2} x_F}{(a + 3b/x_F) \text{ GeV}^2}, \quad (17)$$

where  $g_*$  enumerates the degrees of freedom of relativistic fields at the freeze-out temperature  $T_F$  of  $N_2$ .  $T_F$  is determined through the equation for a dimensionless parameter  $x_F = M_{N_2}/T_F$

$$x_F = \ln \frac{0.0955 m_{\text{pl}} M_{N_2} (a + 6b/x_F)}{(g_* x_F)^{1/2}}, \quad (18)$$

where  $m_{\text{pl}}$  is the Planck mass. If we fix the  $U(1)'$  charge of fields and its coupling constant  $g'$ , we can estimate the present  $N_2$  abundance using these formulas. Assuming a GUT relation  $g' = \sqrt{5/3} g_1$  and  $q = 0.6$  as an example, we calculate  $\Omega_{N_2} h^2$ . The results are given in Fig. 2.

In the left figure of Fig. 2 we plot favorable regions in the  $(M_{Z'}, M_{N_2})$  plane, where  $\Omega_{N_2} h^2$  takes values in the range 0.0945–0.1285, which is required by the WMAP data.  $\Omega_{N_2} h^2$  has a valley in the parameter region of Fig. 2, and therefore the allowed regions appear as two narrow bands, each sandwiched by a solid line and a dashed line. Since  $M_{N_2}$  and  $M_{Z'}$  are induced by  $\langle \phi \rangle$  and written as

$$M_{N_2} = \lambda \langle \phi \rangle, \quad M_{Z'} = 2\sqrt{2} g' q \langle \phi \rangle, \quad (19)$$

$M_{N_2}$  is determined by  $M_{Z'}$ . We plot this  $M_{N_2}$  values by green dotted lines for  $\lambda = 0.2$  and  $0.7$ . The lower bounds of  $M_{Z'}$  come from constraints for  $ZZ'$  mixing and direct search of  $Z'$ .  $H$  is assumed to have no  $U(1)'$  charge and then its VEV induces no  $ZZ'$  mixing. Moreover, since it is leptophobic, the constraints on  $M_{Z'}$  obtained from its hadronic decay is rather weak. Thus, the lower bounds of  $M_{Z'}$  may be  $M_{Z'} \lesssim 450$  GeV in the present model [14]. Taking account of this, Fig. 1 shows that this model can well explain the CDM abundance. Since  $\lambda$  is included in the definition of  $B$ , values of  $\theta_3$  may be constrained by the mass of the CDM if we can obtain more informations on  $\lambda_6$ ,  $h_{\tau 2}/h_{\tau 1}$  and  $M_{Z'}$ .

Here we briefly discuss the relation to lepton flavor violating processes such as  $\mu \rightarrow e\gamma$ . As in the model of [15],  $\mu \rightarrow e\gamma$  is induced through the mediation of  $\eta^0$  and  $N_2$ . Its branching ratio can be given by

$$B(\mu \rightarrow e\gamma) = \frac{3\alpha}{64\pi (G_F M_{\eta^0}^2)^2} \left| h_{\mu 2} h_{e 2} F_2 \left( \frac{M_{N_2}^2}{M_{\eta^0}^2} \right) \right|^2,$$

$$F_2(x) = \frac{1}{6(1-x)^4} (1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x). \quad (20)$$

Taking account that  $1/12 < F_2(x) < 1/6$  is satisfied in case of  $M_{N_2} < M_{\eta^0}$  and imposing the present experimental upper bound  $B(\mu \rightarrow e\gamma) \lesssim 1.2 \times 10^{-11}$ , we find that  $M_{\eta^0}$  should sat-

isfy

$$M_{\eta^0} \gtrsim (360\text{--}500) \left( \frac{h_{\tau 2}}{0.1} \right) \text{ GeV.} \quad (21)$$

Here we use the results of the previous section. Constraints coming from  $\mu \rightarrow e\gamma$  and the CDM abundance can be consistent for reasonable values of  $h_{\tau 2}$ . Since  $N_2$  annihilation due to an  $\eta^0$  exchange is ineffective for these values of couplings and masses [4], the results of the  $N_2$  abundance given above is not affected by this process.

Finally, it may be useful to refer to the cases of general  $U(1)'$ . In these cases a crucial condition for the mass of the  $U(1)'$  gauge field comes from the constraint for  $ZZ'$  mixing. A mass matrix for neutral gauge bosons can be expressed as

$$\begin{pmatrix} \frac{1}{2}(g_1^2 + g_2^2)v^2 & -g'\sqrt{g_1^2 + g_2^2}q_H v^2 \\ -g'\sqrt{g_1^2 + g_2^2}q_H v^2 & 2g'^2 q_\phi^2 (4\langle\phi\rangle^2 + v^2) \end{pmatrix}, \quad (22)$$

where  $q_H$  and  $q_\phi$  stand for the  $U(1)'$  charge of  $H$  and  $\phi$ . Since a  $ZZ'$  mixing angle  $\theta$  is known to be strongly suppressed [12], the magnitude of  $\langle\phi\rangle$  should satisfy

$$\langle\phi\rangle \gtrsim \frac{v}{2} \frac{(g_1^2 + g_2^2)^{1/4}}{(2qg'|\theta|)^{1/2}}. \quad (23)$$

This condition gives a lower bound on both  $M_{Z'}$  and  $M_{N_2}$ . In the right panel of Fig. 2 we plot this bound in cases of  $|\theta| = 10^{-2}, 5 \times 10^{-3}, 10^{-3}$ , which are drawn by vertical dash-dotted lines. We also plot the values of  $M_{N_2}$  for  $\lambda = 0.2, 0.3, 0.6$  and  $0.9$ . They are drawn by green dotted lines. Although  $|\theta|$  should be less than  $10^{-3}$ , we may suppose larger values of  $|\theta|$  in Eq. (23) by extending the model without changing the results in the previous section. In fact, if the model has two Higgs doublets  $H_u$  and  $H_d$  which couple to up- and down-sectors respectively, off-diagonal elements of Eq. (22) is proportional to  $g'(q_{H_u}\langle H_u \rangle^2 - q_{H_d}\langle H_d \rangle^2)$  where  $q_{H_{u,d}}$  expresses the  $U(1)'$  charge. Cancellation between these two contributions can make the  $ZZ'$  mixing smaller for the same value of  $\langle\phi\rangle$ . In such cases we can apply this effect by using larger  $|\theta|$  values in Eq. (23). In this figure  $\theta$  values larger than  $10^{-3}$  should be understood based on this reasoning.

On the other hand, the introduction of additional Higgs doublets may require us to take account of new final states for the  $N_2$  annihilation induced by the  $Z'$  exchange. If  $N_2$  is heavier than  $W^\pm$ , the final states should include gauge bosons and Higgs scalars such as  $W^+W^-$ ,  $H_i^0 H_j^0$ ,  $W^\pm H^\mp$ ,  $H^+H^-$  and  $ZH_i^0$ , where  $H_i^0$  is a mass eigenstate of the neutral Higgs. Since the annihilation to  $W^+W^-$  is suppressed by the  $ZZ'$  mixing in the present model, important modes are expected to be  $H_i^0 H_j^0$  and they may give the same order of contributions as the annihilation to  $f\bar{f}$  [11]. In order to take such effects into account without practicing tedious estimation of such processes, we show in the right figure of Fig. 2 an additional  $\Omega_{N_2} h^2$  contour which is obtained by using  $5 \times (\sigma v)_{f\bar{f}}$  for cross section. It is drawn by blue lines. An original contour for the cross section

$(\sigma v)_{f\bar{f}}$  is drawn by red lines.<sup>4</sup> Since main parts of the cross section into these final states are expected to have the similar dependence on  $M_{Z'}$  and  $M_{N_2}$ , this is considered to give good references for these cases. This figure also suggests that this kind of models can explain the CDM abundance even under the constraint for  $Z'$  physics.

## 5. Summary

We have studied neutrino masses and CDM abundance in a non-supersymmetric, but  $U(1)'$  symmetric model which is obtained from the SM by adding certain neutral fields. Neutrino masses are generated through both the seesaw mechanism with a single right-handed neutrino and the one-loop radiative effects. They induce the same texture which can realize favorable mass eigenvalues and mixing angles. One of the introduced neutral fields is stable due to an unbroken  $Z_2$  symmetry which is the residual symmetry of the spontaneously broken  $U(1)'$ . Thus it can be a good CDM candidate. Since it has the  $U(1)'$  gauge interaction, the annihilation is dominantly mediated through this interaction. If this  $U(1)'$  symmetry is broken at a suitable scale, the present relic abundance of right-handed neutrinos can explain the WMAP result for the CDM abundance. This model suggests that two of the biggest questions in the SM, that is, neutrino masses and the CDM may be explained on the common basis of an extension of the SM. An interesting feature of the model is that the value of the third mixing angle  $\theta_3$  may be related to the mass of the CDM. The model may be examined through the search of the  $Z'$  and the additional Higgs doublet  $\eta$  at LHC.

## Acknowledgements

This work is partially supported by a Grant-in-Aid for Scientific Research (C) from Japan Society for Promotion of Science (Nos. 17540246 and 18540257). We would like to thank the referee of Physics Letters B for pointing out that the originally proposed  $Z_2$  is redundant because  $Z_2$  is nothing but the residual symmetry of  $U(1)'$ .

## References

- [1] Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562;  
Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Lett. B 436 (1998) 33;  
Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Lett. B 433 (1998) 9;  
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 87 (2001) 071301;  
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011302;  
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301;  
K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 90 (2003) 041801;  
K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 93 (2004) 051801;

<sup>4</sup> In this calculation we use the  $U(1)'$  charge assignment for Higgs doublets, quarks and leptons such as  $q_{H_u} = q_{H_d} = -2q$ ,  $q_Q = q_L = 2q$ .

- KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802;  
KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 92 (2004) 071301;  
T. Araki, et al., Phys. Rev. Lett. 94 (2005) 081801.
- [2] WMAP Collaboration, D.N. Spergel, et al., Astrophys. J. Suppl. 148 (2003) 175;  
SDSS Collaboration, M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501.
- [3] J. Ellis, K.A. Olive, Y. Santoso, V.C. Spanos, Phys. Lett. B 565 (2003) 176;  
A.B. Lahanas, D.V. Nanopoulos, Phys. Lett. B 568 (2003) 55;  
H. Baer, C. Balazs, JCAP 0305 (2003) 006;  
U. Chattopadhyay, A. Corsetti, P. Nath, Phys. Rev. D 68 (2003) 035005;  
R. Arnowitt, B. Dutta, B. Hu, hep-ph/0310103;  
S. Profumo, C.E. Yaguna, hep-ph/0407036;  
E.A. Baltz, P. Gondolo, hep-ph/0407039;  
G. Bélanger, F. Boudjema, A. Cottrant, A. Pukhov, A. Semenov, hep-ph/0407218.
- [4] L.M. Krauss, S. Nasri, M. Trodden, Phys. Rev. D 67 (2003) 085002;  
E. Ma, Phys. Rev. D 73 (2006) 077301;  
E. Ma, Mod. Phys. Lett. A 21 (2006) 1777;  
E. Ma, hep-ph/0607142;  
J. Kubo, E. Ma, D. Suematsu, Phys. Lett. B 642 (2006) 18;  
T. Hambye, K. Kannike, E. Ma, M. Raidal, hep-ph/0609228.
- [5] A. Zee, Phys. Lett. B 93 (1980) 339;  
A. Zee, Phys. Lett. B 161 (1985) 41;  
E. Ma, Phys. Rev. Lett. 81 (1998) 1171.
- [6] E. Ma, Phys. Lett. B 625 (2005) 76;  
R. Barbieri, L.J. Hall, V.S. Rychkov, hep-ph/0603188.
- [7] D. Suematsu, Phys. Lett. B 392 (1997) 413.
- [8] D. Suematsu, Prog. Theor. Phys. 99 (1998) 483;  
D. Suematsu, Int. J. Mod. Phys. A 15 (2000) 3967;  
D. Suematsu, Prog. Theor. Phys. 106 (2001) 587.
- [9] M. Apollonio, et al., Phys. Lett. B 466 (1999) 415.
- [10] W.-M. Yao, et al., J. Phys. G 33 (2006) 1.
- [11] K. Griest, M. Kamionkowski, M.S. Turner, Phys. Rev. D 41 (1990) 3565.
- [12] G. Altarelli, et al., Mod. Phys. Lett. A 5 (1990) 495;  
G. Altarelli, et al., Phys. Lett. B 263 (1991) 459.
- [13] B. de Carlos, J.R. Espinosa, Phys. Lett. B 407 (1997) 12;  
D. Suematsu, Phys. Rev. D 73 (2006) 035010;  
S. Nakamura, D. Suematsu, hep-ph/0609061.
- [14] CDF Collaboration, F. Abe, et al., Phys. Rev. Lett. 82 (1999) 2038;  
CDF Collaboration, T. Affolder, Phys. Rev. Lett. 85 (2000) 2062.
- [15] E. Ma, M. Raidal, Phys. Rev. Lett. 87 (2001) 011802.