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Cold dark matter, radiative neutrino mass, $\mu \to e\gamma$, and neutrinoless double beta decay

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Abstract

Two of the most important and pressing questions in cosmology and particle physics are: (1) what is the nature of cold dark matter? and (2) will near-future experiments on neutrinoless double beta decay be able to ascertain that the neutrino is a Majorana particle, i.e. its own antiparticle? We show that these two seemingly unrelated issues are intimately connected if neutrinos acquire mass only because of their interactions with dark matter.

The existence of cold dark matter in the Universe is now well accepted [1]. From the viewpoint of particle physics, it should consist of a weakly interacting yet-to-be-discovered neutral stable fermion or boson. A prime candidate is the lightest supersymmetric particle (LSP) in the minimal supersymmetric standard model (MSSM). More generally, we need only an exactly conserved Z_2 symmetry [2,3] and some new particles which are odd under it, keeping all known particles even. In the MSSM, this Z_2 symmetry is called R parity, and the new particles are the squarks, sleptons, gauginos, and higgsinos.

Consider now the interactions of the neutrino with particles in this new class. To realize the well-known dimension-five operator for Majorana neutrino mass [4],

$$\mathcal{L}_{\mathrm{eff}} = \frac{f_{\alpha\beta}}{\Lambda}\left(\nu_\alpha\phi^0 - l_\alpha\phi^+\right)\left(\nu_\beta\phi^0 - l_\beta\phi^+\right) + \mathrm{h.c.}, \tag{1}$$

where (ν_α, l_α) and $\Phi = (\phi^+, \phi^0)$ are the usual lepton and Higgs doublets of the standard model (SM), it is clear that the new particles must form a loop with four external legs given by $\nu_\alpha\nu_\beta\phi^0\phi^0$. There are generically three such one-loop diagrams [5]. In the MSSM, this does not happen because this operator also requires lepton number to change by two units. However, if a neutral singlet superfield N is added, then Fig. 1 is generated as a radiative contribution to the neutrino mass.

We note that the particles in the loop all have odd R parity. Of course, in the context of supersymmetry, this also implies that ν couples to N directly through ϕ_u^0, so that a Dirac mass already appears, and together with the heavy Majorana mass of N, the famous seesaw mechanism [6] allows ν to obtain a tree-level Majorana mass. That is why Fig. 1 is always negligible in the MSSM with the addition of N. However, in a more general framework, the new particles in the loop may be the only source of neutrino mass [7,8], and in that case there will be interesting phenomenological implications on lepton flavor transitions and neutrinoless double beta decay, as shown below.

Basically, the argument goes as follows. In order for the new particles in the loop to be identified as cold dark matter with the correct value of their relic density in the Universe at present, their interactions with neutrinos and charged leptons must not be too

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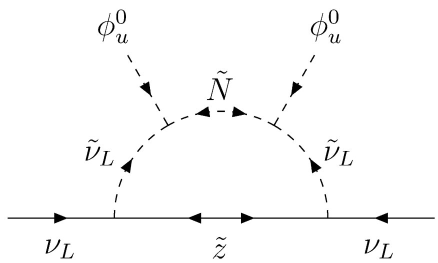

Fig. 1. One-loop radiative neutrino mass in supersymmetry.

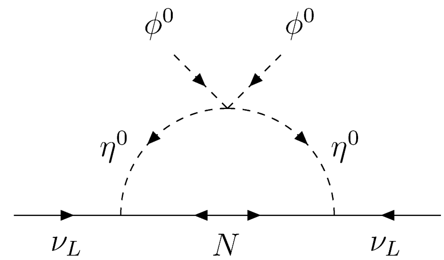

Fig. 2. One-loop radiative neutrino mass in the model of Ref. [8].

weak. On the other hand, they are also responsible for the masses of neutrinos and their observed mixing in neutrino oscillations. This implies necessarily flavor changing transitions such as $\mu \to e\gamma$. In order to suppress the latter, the parameter space of neutrino masses is limited, thereby enforcing a lower bound on neutrinoless double beta decay.

Of the three generic one-loop diagrams giving rise to a radiative neutrino mass, the simplest in terms of new particle content is given in Ref. [8]. The standard model is extended by adding three neutral singlet fermions N_i and a second scalar doublet (η^+, η^0), which are odd under an exactly conserved Z_2 symmetry, keeping all SM particles even. In that case, the analog of Fig. 1 is Fig. 2, as depicted below.

We note again that the particles in the loop are all odd, and that lepton number changes by two units as in Fig. 1. The Z_2 invariant Higgs potential is given by

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2}\lambda_5 [(\Phi^\dagger \eta)^2 + \text{h.c.}], \tag{2}$$

with $\langle \phi^0 \rangle = v$ and $\langle \eta^0 \rangle = 0$. Let us choose the bases where N_i and l_α, l_α^c are diagonal, and consider the interactions of (ν_α, l_α) with N_i and (η^+, η^0), i.e.

$$\mathcal{L}_N = h_{\alpha i} (\nu_\alpha \eta^0 - l_\alpha \eta^+) N_i + \text{h.c.} \tag{3}$$

Since $\langle \eta^0 \rangle = 0$ is required to preserve the exact Z_2 symmetry, there are no Dirac masses linking ν_α with N_i. In other words, even though N_i have heavy Majorana masses, the canonical seesaw mechanism is not operative. Further, the lightest among the new particles will be stable and becomes an excellent candidate for the cold dark matter of the Universe. We see thus that *neutrinos acquire mass here only because of their interactions with dark matter*.

We note that Fig. 2 depends on the existence of the λ_5 coupling of Eq. (2). If it were zero, we could assign the exactly conserved additive lepton number -1 to (η^+, η^0) and 0 to N_i, in which case the neutrinos would stay massless. This means that it is natural for λ_5 to be very small, which we will assume from here on. Without loss of generality, λ_5 may also be chosen to be real. We now define $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$ and obtain $m_R^2 - m_I^2 = 2\lambda_5 v^2$, where m_R (m_I) is the mass of η_R (η_I). Using $m_0^2 = (m_R^2 + m_I^2)/2$, the radiative neutrino mass matrix is then given by [8]

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} I(M_i^2/m_0^2)}{M_i}, \tag{4}$$

where

$$I(x) = \frac{\lambda_5 v^2}{8\pi^2} \left(\frac{x}{1-x} \right) \left[1 + \frac{x \ln x}{1-x} \right]. \tag{5}$$

Assuming that atmospheric neutrino mixing is maximal, the neutrino mixing matrix U which diagonalizes $\mathcal{M}_\nu$ can be written as $U = \hat{U} P$, where $\hat{U}$ is approximately given by

$$\hat{U} \simeq \begin{pmatrix} c_{12} & s_{12} & s_{13} e^{-i\delta} \\ -s_{12}/\sqrt{2} + c_{12} s_{13} e^{i\delta}/\sqrt{2} & c_{12}/\sqrt{2} + s_{12} s_{13} e^{i\delta}/\sqrt{2} & -1/\sqrt{2} \\ -s_{12}/\sqrt{2} - c_{12} s_{13} e^{i\delta}/\sqrt{2} & c_{12}/\sqrt{2} - s_{12} s_{13} e^{i\delta}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{6}$$

$$P = \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

and $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$, $s_{13} = \sin\theta_{13}$, with $\tan^2\theta_{12} \simeq 0.45$, and $s_{13} \lesssim 0.2$.

We now assume the lightest N to be dark matter. Call it N_k. We need to calculate its relic density as a function of its interaction strengths $h_{\alpha k}$, its mass M_k, and the masses of $\eta^\pm$, η_R, and η_I, which we take for simplicity to be all given by m_0, with $m_0 > M_k$. Our goal is to obtain the observed dark-matter relic density of $\Omega_d h^2 \simeq 0.12$ [9,10].

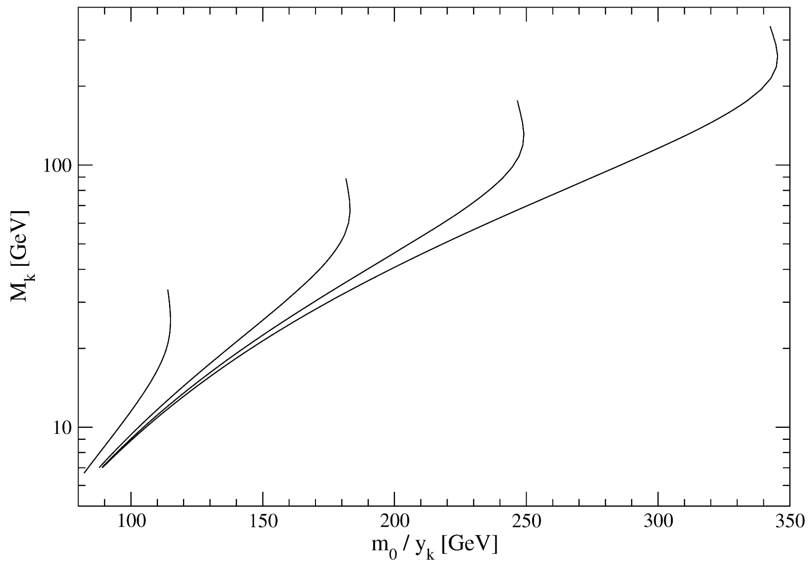

Fig. 3. M_k versus m_0/y_k for $y_k = 0.3, 0.5, 0.7, 1.0$ (left to right) for $\Omega_d h^2 = 0.12$, where y_k is defined in Eq. (9).

The thermally averaged cross section for the annihilation of two N_k's into two leptons is computed by expanding the corresponding relativistic cross section σ in powers of their relative velocity and keeping only the first two terms. Using the result of Ref. [11], and recognizing that lepton masses are very small, we have

$$\langle \sigma v\rangle = a + b_k v^2 + \cdots, \quad a = 0,\ b_k = \frac{y_k^4 r_k^2 (1 - 2r_k + 2r_k^2)}{24\pi M_k^2}, \tag{8}$$

where

$$r_k = M_k^2 / \left(m_0^2 + M_k^2\right), \qquad y_k^4 = \sum_{\alpha\beta} \left|h_{\alpha k} h_{\beta k}^*\right|^2. \tag{9}$$

Following Ref. [12], the relic density of N_k is then given by $\Omega_d h^2 = Y_\infty s_0 M_k h^2/\rho_c$, where Y_∞ is the asymptotic value of the ratio n_{N_k}/s, with $Y_\infty^{-1} = 0.264 g_*^{1/2} M_{\mathrm{Pl}} M_k (3b_k/x_f^2)$, $s_0 = 2970/\mathrm{cm}^3$ is the entropy density at present, $\rho_c = 3H^2/8\pi G = 1.05 \times 10^{-5} h^2\ \mathrm{GeV/cm}^3$ is the critical density, h is the dimensionless Hubble parameter, $M_{\mathrm{Pl}} = 1.22 \times 10^{19}$ GeV is the Planck energy, and g_* is the number of effectively massless degrees of freedom at the freeze-out temperature. Further, x_f is the ratio M_k/T at the freeze-out temperature and is given by

$$x_f = \ln \frac{0.0764 M_{\mathrm{Pl}} (6b_k/x_f) c(2+c) M_k}{(g_* x_f)^{1/2}}. \tag{10}$$

Using $g_*^{1/2} = 10$ and $c = 1/2$ as in Ref. [12], we obtain

$$\left[\frac{M_k}{\mathrm{GeV}}\right] = 5.86 \times 10^{-8} x_f^{-1/2} e^{x_f} \left[\frac{\Omega_d h^2}{0.12}\right], \tag{11}$$

$$\left[\frac{b_k}{(\mathrm{GeV})^{-2}}\right] = 2.44 \times 10^{-11} x_f^2 \left[\frac{0.12}{\Omega_d h^2}\right], \tag{12}$$

where b_k and y_k are given in Eqs. (8) and (9). Since b_k/y_k^4 is a function of M_k and m_0, Eqs. (11) and (12) allow us to calculate M_k and m_0 in units of GeV for a given set of y_k^2, x_f and $\Omega_d h^2$.

In Fig. 3, we plot M_k versus m_0/y_k for $y_k = 0.3, 0.5, 0.7, 1.0$. As we can see from the figure, the dark matter constraint requires that M_k increases as m_0 increases and for each value of m_0, it may be as large as m_0. We also see that m_0/y_k cannot exceed 350 GeV or so in the perturbative regime $y_k \lesssim 1$. This is a very strong constraint, because m_0/y_k sets the scale also for lepton flavor transitions such as $\mu \to e\gamma$ and the experimental upper bound of its branching fraction cannot be satisfied, unless some cancellation mechanism is at work.

The branching fraction of $\mu \to e\gamma$ is given in this model by [13]

$$B(\mu \to e\gamma) = \frac{3\alpha}{64\pi (G_F m_0^2)^2} C^4 \simeq \left(\frac{30\ \mathrm{GeV}}{m_0/C}\right)^4, \tag{13}$$

where

$$C^2 = \left|\sum_i h_{\mu i} h_{ei}^* F_2\left(M_i^2/m_0^2\right)\right|, \tag{14}$$

and

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}. \tag{15}$$

Since $M_k < m_0$ should be satisfied for N_k dark matter, the function $F_2(x_k)$ can vary only between $1/12(x_k = 1)$ and $1/6(x_k = 0)$. To suppress the branching fraction $B(\mu \to e\gamma)$ which is inversely proportional to the fourth power of m_0, we need a large value of m_0. On the other hand, the observed dark matter relic density $\Omega_d h^2 \simeq 0.12$ requires m_0 to be below 350 GeV for $y_k = (\sum_{\alpha\beta} |h_{\alpha k} h_{\beta k}^*|^2)^{1/4} \lesssim 1$. This means that if $|\sum_i h_{\mu i} h_{ei}^*|$ appearing in Eq. (14) is also of order 1, then $B(\mu \to e\gamma) \gtrsim 5 \times 10^{-7}$, which is several orders of magnitude above the experimental upper bound of 1.2×10^{-11}.

To satisfy the $\mu \to e\gamma$ constraint, we consider the possibility that $M_{1,2,3}$ are nearly degenerate. In that limit,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{I(M^2/m_0^2)}{M} \sum_i h_{\alpha i} h_{\beta i} = \hat{U}^* \begin{pmatrix} m_1 e^{-i\alpha_1} & 0 & 0 \\ 0 & m_2 e^{-i\alpha_2} & 0 \\ 0 & 0 & m_3 \end{pmatrix} \hat{U}^\dagger, \tag{16}$$

where $\hat{U}$ is given by Eq. (6). A simple solution is then

$$h_{\alpha i} = \left[\frac{M m_i}{I(M^2/m_0^2)}\right]^{1/2} e^{-i\alpha_i/2} \hat{U}_{\alpha i}^* \quad \text{with } \alpha_3 = 0. \tag{17}$$

Then we obtain

$$C^2 = \frac{F_2(M^2/m_0^2)M}{I(M^2/m_0^2)} \left|\sum_i \hat{U}_{\mu i} \hat{U}_{ei}^* m_i\right| = \frac{F_2(M^2/m_0^2)M}{I(M^2/m_0^2)} \left|\frac{s_{12}c_{12}}{\sqrt{2}}(m_2 - m_1) + \frac{s_{13}e^{-i\delta}}{\sqrt{2}}\left(c_{12}^2 m_1 + s_{12}^2 m_2 - m_3\right)\right|. \tag{18}$$

Thus the suppression of C^2 is possible because $m_2 - m_1$ is related to Δm_{sol}^2 and $c_{12}^2 m_1 + s_{12}^2 m_2 - m_3$ is related to Δm_{atm}^2 in neutrino oscillations [13,14].

Let us assume $\delta = 0$ and consider the normal ordering of neutrino masses, i.e. m_3 is the largest mass. We then set $h_3 = (\sum_\alpha |h_{\alpha 3}|^2)^{1/2} = 1$ which is equivalent to having $M m_3 / I(M^2/m_0^2) = 1$. Hence

$$C^2 \simeq \left(\frac{0.067}{m_3}\right) \left|c_{12}(s_{12} - s_{13}c_{12})(m_2 - m_1) - s_{13}(m_3 - m_2)\right| < 4.6 \times 10^{-4}, \tag{19}$$

where $F_2 = 0.0948$ (corresponding to $m_0 = 345$ GeV and $M = 290$ GeV). Using $\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 = 7.9 \times 10^{-5}\ \text{eV}^2$ and $\Delta m_{32}^2 = \Delta m_{\text{atm}}^2 = 2.3 \times 10^{-3}\ \text{eV}^2$, we plot C^2 versus m_1 in Fig. 4 for $s_{13} = 0.1, 0.05, 0.01$. The horizontal line is the experimental bound $C^2 = 4.6 \times 10^{-4}$ corresponding to $B(\mu \to e\gamma) = 1.2 \times 10^{-11}$. We find that for $s_{13} \gtrsim 0.26$, this constraint cannot be satisfied. For s_{13} less than its experimental bound of 0.2, there is a lower bound on m_1 according to the approximate empirical formula

$$\left[\frac{m_1}{\text{eV}}\right] \gtrsim 0.02 + 1.4|s_{13} - 0.02| - 2.9|s_{13} - 0.02|^2, \tag{20}$$

except for a tiny region near $m_1 = 0$ and $s_{13} = 0.09$, and a small region near $m_1 = 0.01$ eV and $s_{13} = 0.04$. In Fig. 4, we can see that the plot for $s_{13} = 0.1$ is getting close to the first region from its dip at $m_1 = 0$, and that the plot for $s_{13} = 0.05$ has a small allowed range near $m_1 = 0.01$ eV.

In the case of inverted ordering, i.e. m_2 is the largest mass, we are already guaranteed that $m_{1,2} > \sqrt{\Delta m_{32}^2} \simeq 0.048$ eV. For completeness, we set $h_2 = 1$ and plot C^2 versus m_3 in Fig. 5 for $s_{13} = 0.2, -0.05, 0.0$. Here, for $s_{13} \gtrsim 0.24$ and $s_{13} \lesssim -0.27$, the experimental constraint cannot be satisfied. In other words, the constraint on θ_{13} from $\mu \to e\gamma$ coincides roughly also with that from neutrino oscillations.

For the simple solution of Eq. (16), as we can see from Figs. 4 and 5, all the neutrino masses may be assumed to be degenerate to satisfy the $\mu \to e\gamma$ constraint; hence the effective mass $\langle m_{ee} \rangle$ in neutrinoless double beta decay is approximately given by

$$\langle m_{ee} \rangle \simeq m_1 \left|0.572 + 0.428 \cos(\alpha_1 - \alpha_2)\right|^{1/2}. \tag{21}$$

We also allow the heavy N_i masses $M_{1,2,3}$ to be slightly different, so that our approximation that only one of them is the candidate for dark matter remains valid. Note that $\Delta M/M$ only needs to be of order 10^{-3} for Eq. (18) to be valid. In analogy to $\mu \to e\gamma$, there are also dark-matter contributions to $\tau \to \mu\gamma$, $\tau \to e\gamma$, and the anomalous magnetic moment of the muon. However, they are at least one or more orders of magnitude below the present experimental bounds.

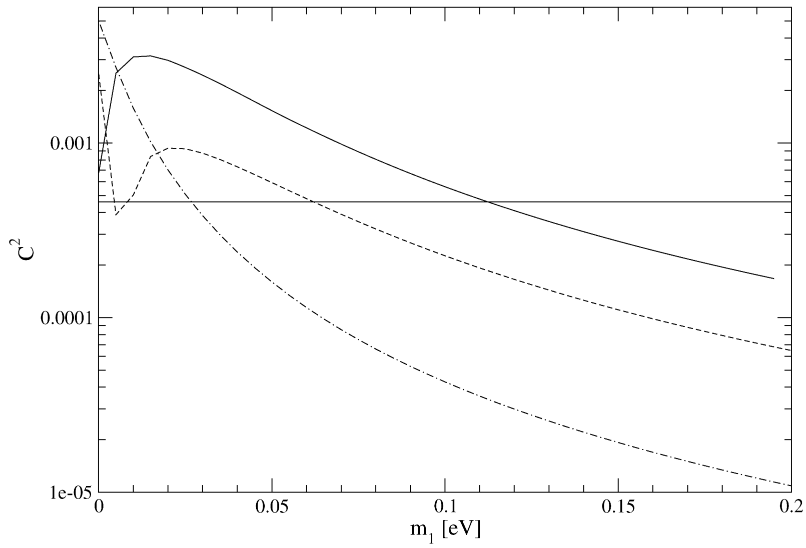

Fig. 4. C^2 versus m_1 in the case of normal ordering for $s_{13} = 0.1$ (solid), 0.05 (dash), 0.01 (dot–dash), where $C^2 = 4.6 \times 10^{-4}$ (horizontal line) corresponds to the experimental upper bound $B(\mu \to e\gamma) = 1.2 \times 10^{-11}$.

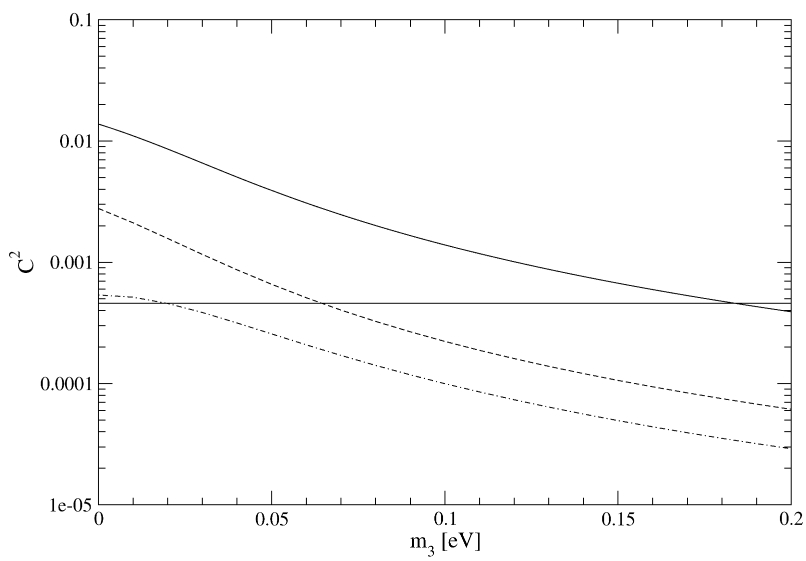

Fig. 5. C^2 versus m_3 in the case of inverted ordering for $s_{13} = 0.2$ (solid), -0.05 (dash), 0.0 (dot–dash).

In conclusion, we have shown how cold dark matter and neutrinoless double beta decay may be connected if neutrinos acquire mass only because of their interactions with dark matter. We repeat the basic argument presented earlier. The existence of dark matter requires a class of new particles which are odd with respect to an exactly conserved Z_2 symmetry. Their interactions with neutrinos and charged leptons must not be too weak to be identified as cold dark matter with the correct value of their relic density in the Universe at present. On the other hand, they are also responsible for the masses of neutrinos and their observed mixing in neutrino oscillations. This implies necessarily flavor changing transitions such as $\mu \to e\gamma$. In order to suppress the latter, the parameter space of neutrino masses is limited, thereby enforcing a lower bound on neutrinoless double beta decay. For N as dark matter, this is typically of order 0.05 eV, even though much lower values are still allowed from accidental cancellations. More importantly, this connection between cold dark matter and neutrinoless double beta decay can be tested in the near future at the Large Hadron Collider and complemented by a host of experiments on neutrino oscillations and neutrinoless double beta decay already under way and being planned.

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