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| メタデータ | 言語: eng 出版者: 公開日: 2022-01-07 キーワード (Ja): キーワード (En): 作成者: メールアドレス: 所属: |
| URL | https://doi.org/10.24517/00064634 |

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Dark matter stability and one-loop neutrino mass generation based on Peccei–Quinn symmetry

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Received: 18 November 2017 / Accepted: 3 January 2018 / Published online: 16 January 2018
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Abstract We propose a model which is a simple extension of the KSVZ invisible axion model with an inert doublet scalar. Peccei–Quinn symmetry forbids tree-level neutrino mass generation and its remnant Z_2 symmetry guarantees dark matter stability. The neutrino masses are generated by one-loop effects as a result of the breaking of Peccei–Quinn symmetry through a nonrenormalizable interaction. Although the low energy effective model coincides with an original scotogenic model which contains right-handed neutrinos with large masses, it is free from the strong CP problem.

1 Introduction

The standard model (SM) has been confirmed by the discovery of the Higgs scalar [1, 2]. However, it is now considered to be extended to explain several experimental and observational data such as neutrino masses and mixings [3–10], and dark matter (DM) [11, 12]. Strong CP problem is also one of such problems suggested by an experimental bound of the electric dipole moment of a neutron [13, 14]. Invisible axion models are known to give a simple and interesting solution to it [15–18]. The KSVZ model, which is one of such realizations, is an extension of the SM by a complex singlet scalar and a pair of colored fermions. It has a global $U(1)$ symmetry, which is violated only by the QCD anomaly and plays a role of Peccei–Quinn (PQ) symmetry [19, 20]. If the spontaneous breaking of this $U(1)_{PQ}$ symmetry occurs, a pseudo Nambu–Goldstone boson associated to this breaking called axion appears to solve the strong CP problem [21, 22]. If the axion decay constant f_a is large enough such as $10^9 \text{ GeV} < f_a < 10^{12} \text{ GeV}$ due to a vacuum expectation value (VEV) of the singlet scalar, the axion mass is very small and its coupling is extremely weak so as not to

cause any contradiction with experiments and astrophysical observations [23–25].

On the other hand, the $U(1)_{PQ}$ breaking is known to cause N degenerate minima for the axion potential due to the QCD anomaly depending on both the field contents and the PQ charge assignment for them. As a result, the model is generally annoyed by the dangerous production of topologically stable domain walls [26]. It can be escapable only for $N = 1$ unless one consider the domain wall free universe brought about by inflation. If a certain subgroup of $U(1)_{PQ}$ remains as a discrete symmetry broken only by the QCD anomaly in a model with $N = 1$, it could present an interesting scenario in relation to the DM physics at the low energy regions.¹

In this paper, we consider such a possibility in an extension of the KSVZ model, in which an inert doublet scalar and three right-handed neutrinos are added. The low energy effective model obtained from it after the breakdown of the $U(1)_{PQ}$ symmetry is reduced to the original scotogenic neutrino mass model with an effective Z_2 symmetry [30]. This Z_2 symmetry could guarantee the stability of a lightest neutral component of the inert doublet scalar to give a DM candidate. The neutrino masses are generated through a one-loop effect as a result of the $U(1)_{PQ}$ breaking. The relevant diagram is caused by both right-handed neutrinos and a nonrenormalizable interaction between the inert doublet scalar and the ordinary Higgs doublet. The model might be recognized as a well motivated simple framework at high energy regions for the original scotogenic model.

The remaining parts are organized as follows. In the next section, we introduce a model by fixing charge assignment of $U(1)_{PQ}$ to the field contents. We discuss basic features of the model such as remnant effective symmetry, scalar mass spectrum, vacuum stability and so on. In Sect. 3, phenomenological features such as neutrino mass generation, leptogenesis and DM abundance in this model are discussed. The

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¹ The similar idea has been discussed in several articles, recently [27–29]. However, the present model is different from them.

consistency of the scenario is also studied from a viewpoint of the vacuum stability and a cut-off scale of the model. We summarize the paper in Sect. 4.

2 An extension of the KSVZ model

The KSVZ model is constructed by introducing a singlet complex scalar S and a vector-like colored fermions (D_L, D_R) to the SM [15, 16]. We assume $D_{L,R}$ as triplets of the color $SU(3)$. Although they are $SU(2)_L$ singlets, they could have a suitable weak hypercharge Y , in general. This point is crucial for phenomenological consistency of the model as discussed below. The model has a global $U(1)_{PQ}$ symmetry and its charge is assigned to S and $D_{L,R}$, but it is not assigned to the SM contents. We assume the existence of a gauge invariant Yukawa coupling $y_D S \bar{D}_L D_R$ so that the PQ mechanism could work to solve the strong CP problem. This requires that the PQ charge X of these new ingredients should satisfy $X_S = X_{D_L} - X_{D_R}$. On the other hand, this symmetry should be chiral to have the QCD anomaly and $X_{D_L} \neq X_{D_R}$ is satisfied. Thus, this $U(1)_{PQ}$ is spontaneously broken through the VEV of S .

The $U(1)_{PQ}$ transformation $D_{L,R} \rightarrow e^{iX_{D_{L,R}}\alpha} D_{L,R}$ for the colored fermions $D_{L,R}$ shifts the QCD θ parameter through the anomaly as [13, 14, 26]

$$\theta_{\text{QCD}} \rightarrow \theta_{\text{QCD}} - \frac{1}{2}(X_{D_R} - X_{D_L})\alpha. \tag{1}$$

Since θ_{QCD} has a period 2π , the model is invariant for $\alpha = \frac{2\pi k}{N}$ where $N \equiv \frac{1}{2}|X_{D_R} - X_{D_L}|$ is an integer and $k = 0, 1, \dots, N - 1$. This means that the model could have a discrete symmetry Z_N after taking account of the QCD anomaly.² If we assign the $U(1)_{PQ}$ charge S as $X_S = 2$, the model has $N = 1$ and no degenerate minima in the axion potential. Thus, the model has no domain wall problem as is well known.³ Here, we note that an effective Z_2 symmetry could remain after the symmetry breaking due to $\langle S \rangle \neq 0$ although it is violated by the QCD anomaly. Since the SM contents are supposed to have no PQ charge, it could play an important role in the leptonic sector of the model to guarantee the stability of the lightest Z_2 odd field in that sector, which could be DM.

If both D_L and D_R cannot couple with quarks, which occurs in case $Y(D_{L,R}) = 0$ for example, they are stable and then its relic abundance has to be smaller than the DM

² The axion decay constant f_a is related with the PQ symmetry breaking scale $\langle S \rangle$ as $Nf_a = \langle S \rangle$ by using this N .

³ Although the model has domain walls bounded by the string caused from the spontaneous $U(1)_{PQ}$ breaking, it is not topologically stable and then it can shrink and decay. As a result, no cosmological difficulty appears [31].

Table 1 The hypercharge Y and the $U(1)_{PQ}$ charge X of new fields in the model. The SM contents are assumed to have no PQ charge

| | D_L | D_R | S | η | N_i |
|-------|----------------|----------------|-----|----------------|-------|
| Y | $-\frac{1}{3}$ | $-\frac{1}{3}$ | 0 | $-\frac{1}{2}$ | 0 |
| X | 2 | 0 | 2 | 1 | -1 |
| Z_2 | + | + | + | - | - |

Parity for the effective symmetry Z_2 which remains after the $U(1)_{PQ}$ breaking is also listed

abundance [32]. Even if its relic abundance satisfies such a condition, the existence of the fractionally charged D hadrons is generally forbidden by the present bound obtained from the search of fractionally charged states. On the other hand, if we assign $Y = -\frac{1}{3}$ or $\frac{2}{3}$ to $D_{L,R}$, all the D hadrons can have integer charge. In that case, the D relic abundance will restrict the D mass into a narrow range such as $m_D \gtrsim 1$ TeV [32]. Moreover, they are allowed to couple with quarks through a renormalizable Yukawa interaction as long as their PQ charge is zero. For example, using the left handed quark doublet q_L and the Higgs doublet ϕ or $\tilde{\phi} (\equiv i\tau_2 \phi^*)$, the coupling $\tilde{\phi} \bar{q}_L D_R$ is allowed for D_R with $X = 0$ and $Y = -\frac{1}{3}$ and also $\phi \bar{q}_L D_R$ for D_R with $X = 0$ and $Y = \frac{2}{3}$. In these cases, D_R decays to the SM fields through these couplings. D_L can also decay via the mass mixing with D_R induced by the coupling $y_D S \bar{D}_L D_R$ through $\langle S \rangle \neq 0$. As a result, the mass m_D has no constraint other than the bound obtained through the accelerator experiments. Anyway, in the model where the PQ charge is assigned as discussed above, the strong CP problem could be solved without inducing any cosmological and astrophysical difficulty, as long as the symmetry breaking scale satisfies $10^9 \text{ GeV} < \langle S \rangle < 10^{12} \text{ GeV}$.

Now, we consider a modification of this model by introducing an inert doublet scalar η and three right-handed neutrinos N_i . The PQ charge assignment of the fields contained in the model is shown in Table 1. Invariant terms under the assumed symmetry for the Yukawa couplings and the scalar potential of the relevant fields are summarized as

$$\begin{aligned}
 -\mathcal{L}_y &= y_D S \bar{D}_L D_R + h_D \bar{q}_L \tilde{\phi} D_R + y_i S \bar{N}_i^c N_i \\
 &\quad + h_{\alpha i} \bar{\ell}_\alpha \eta N_i + \text{h.c.}, \\
 V &= m_S^2 S^\dagger S + \kappa_1 (S^\dagger S)^2 + \kappa_2 (S^\dagger S)(\phi^\dagger \phi) + \kappa_3 (S^\dagger S)(\eta^\dagger \eta) \\
 &\quad + m_\eta^2 \eta^\dagger \eta + m_\phi^2 \phi^\dagger \phi + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 \\
 &\quad + \lambda_3 (\phi^\dagger \phi)(\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta)(\eta^\dagger \phi) \\
 &\quad + \frac{\lambda_5}{2} \left[\frac{S}{M_*} (\eta^\dagger \phi)^2 + \text{h.c.} \right], \tag{2}
 \end{aligned}$$

where λ_5 is taken to be real and M_* is a cut-off scale of the model. The quark generation index is abbreviated in the Yukawa coupling h_D . We find that V given in Eq. (2) is the most general scalar potential up to the dimension 5.

After the symmetry breaking due to $\langle S \rangle \neq 0$, $D_{L,R}, N_i$ and S are found to get masses such as $m_D = y_D \langle S \rangle$, $M_i = y_i \langle S \rangle$ and $M_S^2 = 4\kappa_1 \langle S \rangle^2$, respectively. Since $D_{L,R}$ can decay to the SM fields through the second term in \mathcal{L}_y as discussed above, there is no thermal relic of $D_{L,R}$ in the present Universe. The effective model at the scale below M_S could be obtained by integrating out S [33]. This can be done by using the equation of motion for S . As its result, we obtain the corresponding effective model whose scalar potential of the light scalars can be written as

$$V_{\text{eff}} = \tilde{m}_\phi^2 (\phi^\dagger \phi) + \tilde{m}_\eta^2 (\eta^\dagger \eta) + \tilde{\lambda}_1 (\phi^\dagger \phi)^2 + \tilde{\lambda}_2 (\eta^\dagger \eta)^2 + \tilde{\lambda}_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\tilde{\lambda}_5}{2} [(\phi^\dagger \eta)^2 + \text{h.c.}], \tag{3}$$

where we use the shifted parameters which are defined as

$$\begin{aligned} \tilde{\lambda}_1 &= \lambda_1 - \frac{\kappa_2^2}{4\kappa_1}, & \tilde{\lambda}_2 &= \lambda_2 - \frac{\kappa_3^2}{4\kappa_1}, \\ \tilde{\lambda}_3 &= \lambda_3 - \frac{\kappa_2 \kappa_3}{2\kappa_1}, & \tilde{\lambda}_5 &= \lambda_5 \frac{\langle S \rangle}{M_*}, \\ \tilde{m}_\phi^2 &= m_\phi^2 + \kappa_2 \langle S \rangle^2, & \tilde{m}_\eta^2 &= m_\eta^2 + \kappa_3 \langle S \rangle^2. \end{aligned} \tag{4}$$

We note that the model contains the neutrino Yukawa couplings between heavy right-handed neutrinos and the inert doublet scalar as shown in the above \mathcal{L}_y .

Vacuum stability condition for the scalar potential V_{eff} in Eq. (3) is known to be given as [34–36]

$$\begin{aligned} \tilde{\lambda}_1 > 0, & \quad \tilde{\lambda}_2 > 0, & \quad \tilde{\lambda}_3 > -2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}, \\ \tilde{\lambda}_3 + \lambda_4 - |\tilde{\lambda}_5| > -2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}, \end{aligned} \tag{5}$$

and these should be satisfied at the energy region $\mu < M_S$. On the other hand, at $M_S < \mu < M_*$, both the same conditions for $\lambda_{1,2,3}$ as Eq. (5) except for the last one and new conditions

$$\kappa_1 > 0, \quad \kappa_2 > -2\sqrt{\lambda_1 \kappa_1}, \quad \kappa_3 > -2\sqrt{\lambda_2 \kappa_1}, \tag{6}$$

should be satisfied. The couplings in both regions should be connected through Eq. (4). We can examine whether these conditions could be satisfied or not by using one-loop renormalization group equations (RGEs). This is the subject studied later.

This effective model obtained after the spontaneous breaking of $U(1)_{PQ}$ is just the original scotogenic model [30].⁴ This model connects the neutrino mass generation with the DM existence. It has been extensively studied from various phenomenological view points [37–52]. In the present case,

⁴ In the case of $Y(D_{L,R}) \neq 0$, $U(1)_{PQ}$ and then its subgroup Z_2 could be broken by the electroweak anomaly also. However, since this breaking does not induce the decay of the lightest Z_2 odd field, this Z_2 can be considered to be a good symmetry in the effective model.

the right-handed neutrinos do not have their masses in a TeV region but they are considered to be much heavier. The coupling $\tilde{\lambda}_5$ which is crucial for the one-loop neutrino mass generation is derived from a nonrenormalizable term as a result of the PQ symmetry breaking. The model contains the inert doublet scalar η which has odd parity of the remnant effective Z_2 . It has charged components η^\pm and two neutral components $\eta_{R,I}$. Their mass eigenvalues can be expressed as

$$M_{\eta^\pm}^2 = \tilde{m}_\eta^2 + \tilde{\lambda}_3 \langle \phi \rangle^2, \quad M_{\eta_{R,I}}^2 = \tilde{m}_\eta^2 + (\tilde{\lambda}_3 + \lambda_4 \pm \tilde{\lambda}_5) \langle \phi \rangle^2. \tag{7}$$

We suppose $\tilde{m}_\eta = O(1)$ TeV although it requires fine tuning because of $|\langle S \rangle| \gg |\langle \phi \rangle|$. As a result of the effective Z_2 symmetry, the lightest one among the components of η is stable to be a DM candidate if it is neutral. If it is supposed to be η_R , we find that this requires $\lambda_4 < 0$ and $\tilde{\lambda}_5 < 0$ as long as $|\tilde{\lambda}_5| \ll |\lambda_4|$ is satisfied. On the other hand, since $\tilde{m}_\eta^2 \gg \langle \phi \rangle^2$ is satisfied in Eq. (7), the mass eigenvalues of the components η are found to be degenerate enough so that the coannihilation processes among them are expected to be effective. This observation suggests that the abundance of η_R could be suitably suppressed and then it could be a good DM candidate as the ordinary inert doublet model [53–56]. The charged states with the mass of $O(1)$ TeV are also expected to be detected in the accelerator experiments.

3 Phenomenological features

3.1 Neutrino mass, leptogenesis and DM relic abundance

In this model, neutrino masses are forbidden at tree-level. However, since both the right-handed neutrino masses and the mass difference between η_R and η_I are induced after the $U(1)_{PQ}$ breaking, the small neutrino masses can be generated radiatively through one-loop diagrams in the same way as the original scotogenic model. Since $M_{\eta_{R,I}}^2 \gg |M_{\eta_R}^2 - M_{\eta_I}^2|$ is satisfied, the neutrino mass formula can be approximately written as

$$\mathcal{M}_{\alpha\beta} = \sum_i h_{\alpha i} h_{\beta i} \Lambda_i, \quad \Lambda_i \simeq \frac{\tilde{\lambda}_5 \langle \phi \rangle^2}{8\pi^2 M_i} \ln \frac{M_i^2}{\tilde{M}_\eta^2}, \tag{8}$$

where $\tilde{M}_\eta^2 = \tilde{m}_\eta^2 + (\tilde{\lambda}_3 + \lambda_4) \langle \phi \rangle^2$. In order to take account of the constraints from the neutrino oscillation data in the analysis, we may fix the flavor structure of neutrino Yukawa couplings $h_{\alpha i}$ at the one which induces the tri-bimaximal mixing [42,43]⁵

⁵ Although a certain modification is required to reproduce the observed mixing in the lepton sector, this simplified example could give a rather good approximation for the present purpose as found from [45,46].

$$\begin{aligned}
 h_{ej} &= 0, \quad h_{\mu j} = h_{\tau j} \equiv h_j \quad (j = 1, 2); \\
 h_{e3} &= h_{\mu 3} = -h_{\tau 3} \equiv h_3,
 \end{aligned}
 \tag{9}$$

where the charged lepton mass matrix is assumed to be diagonal. In that case, the mass eigenvalues are estimated as

$$\begin{aligned}
 m_1 &= 0, \quad m_2 = 3|h_3|^2\Lambda_3, \\
 m_3 &= 2\left[|h_1|^4\Lambda_1^2 + |h_2|^4\Lambda_2^2 \right. \\
 &\quad \left. + 2|h_1|^2|h_2|^2\Lambda_1\Lambda_2\cos 2(\theta_1 - \theta_2)\right]^{1/2},
 \end{aligned}
 \tag{10}$$

where $\theta_j = \arg(h_j)$.

As is known generally and found also from this mass formula, neutrino masses could be determined only by two right-handed neutrinos. It means that the mass and neutrino Yukawa couplings of a remaining right-handed neutrino could be free from the neutrino oscillation data as long as its contribution to the neutrino mass is negligible. In Eq. (10), such a situation can be realized for $|h_1|^2\Lambda_1 \ll |h_2|^2\Lambda_2$. This is good for the thermal leptogenesis [57] since a sufficiently small neutrino Yukawa coupling h_1 makes the out-of-equilibrium decay of the right-handed neutrino N_1 possible.⁶ We find that the squared mass differences required by the neutrino oscillation data could be explained if we fix the parameters relevant to the neutrino masses, for example, as

$$\begin{aligned}
 M_1 &= 10^8 \text{ GeV}, \quad M_2 = 4 \times 10^8 \text{ GeV}, \\
 M_3 &= 10^9 \text{ GeV}, \\
 |h_1| &= 10^{-4.5}, \quad |h_2| \simeq 7.2 \times 10^{-4}\tilde{\lambda}_5^{-0.5}, \\
 |h_3| &\simeq 3.1 \times 10^{-4}\tilde{\lambda}_5^{-0.5},
 \end{aligned}
 \tag{11}$$

for $\tilde{m}_\eta = 1 \text{ TeV}$. Using these values, we can estimate the expected baryon number asymmetry through the out-of-equilibrium decay of the thermal N_1 by solving the Boltzmann equation as done in [45,46]. The numerical analysis shows that the required baryon number asymmetry could be generated for $M_1 \gtrsim 10^8 \text{ GeV}$, which is somewhat smaller than the Davidson–Ibarra bound [61] in the ordinary thermal leptogenesis. In case of the parameter set given in (11), we find $Y_B (\equiv \frac{n_B}{s}) = 4.0 \times 10^{-10}$ if we assume $\tilde{\lambda}_5 = 2.5 \times 10^{-3}$ and a maximal CP phase in the CP violation parameter ε . In Fig. 1, we plot Y_B as a function of $\tilde{\lambda}_5$. Its feature can be easily understood by taking account of Eq. (11). If $\tilde{\lambda}_5$ takes larger values, the neutrino Yukawa couplings become smaller to make the CP violation ε in the N_1 decay smaller but also the washout of the generated lepton number asymmetry smaller. On the other hand, if $\tilde{\lambda}_5$ takes smaller values, the neutrino Yukawa couplings become larger to induce

⁶ If we consider the TeV scale right-handed neutrinos, leptogenesis requires fine degeneracy among the right-handed neutrinos for the resonance [58–60]. We need not consider such a possibility in the present case.

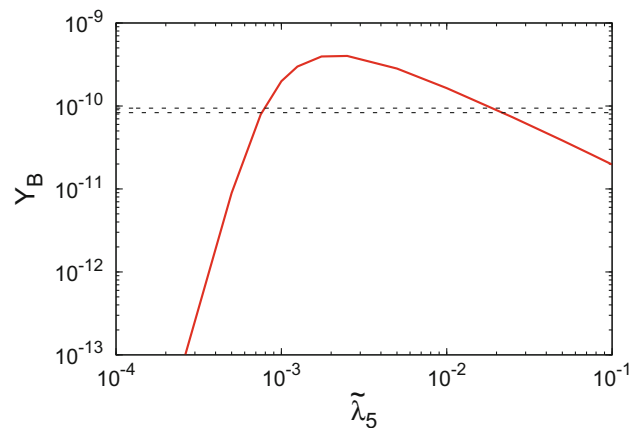


Fig. 1 Baryon number asymmetry Y_B generated through the out-of-equilibrium decay of N_1 . Y_B is plotted as a function of $\tilde{\lambda}_5$ for the parameter set shown in Eq. (11), which can explain the neutrino mass differences required by the neutrino oscillation data. Horizontal dotted lines show the required value for Y_B

the reverse effects. This makes the required baryon number asymmetry be generated only for the $\tilde{\lambda}_5$ in the limited regions as found in this figure.

The relic abundance of η_R is tuned to the observed value if the couplings $\tilde{\lambda}_3$ and λ_4 take suitable values. In fact, since \tilde{m}_η is assumed to be of $O(1) \text{ TeV}$ in this scenario, the mass of each component of η could be degenerate enough for wide range values of $\tilde{\lambda}_3$ and λ_4 as remarked at Eq. (7). This makes the coannihilation among them effective enough to reduce the η_R abundance [45,46]. We search the region of $\tilde{\lambda}_3$ and λ_4 , which realizes the required DM abundance as the η_R relic abundance by taking the values of \tilde{m}_η and $\tilde{\lambda}_5$ as the ones given below Eq. (11). They are suitable for the explanation of the neutrino oscillation data and the cosmological baryon number asymmetry. In the estimation of the DM relic abundance, we follow the procedure given in [62,63] where the coannihilation effects are taken into account.

We present a brief review of the procedure adopted here. The η_R relic abundance is estimated as

$$\Omega h^2 \simeq \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{J(x_F)g_*^{1/2}m_{\text{pl}}},
 \tag{12}$$

where g_* is the relativistic degrees of freedom. The freeze-out temperature $T_F (\equiv \frac{M_{\eta_R}}{x_F})$ of η_R and $J(x_F)$ are defined as

$$\begin{aligned}
 x_F &= \ln \frac{0.038m_{\text{pl}}g_{\text{eff}}M_{\eta_R}\langle\sigma_{\text{eff}}v\rangle}{(g_*x_F)^{1/2}}, \\
 J(x_F) &= \int_{x_F}^{\infty} \frac{\langle\sigma_{\text{eff}}v\rangle}{x^2} dx.
 \end{aligned}
 \tag{13}$$

In these formulas, the effective annihilation cross section $\langle\sigma_{\text{eff}}v\rangle$ and the effective degrees of freedom g_{eff} are expressed as⁷

$$\langle\sigma_{\text{eff}}v\rangle = \frac{1}{g_{\text{eff}}^2} \sum_{i,j=1}^4 \langle\sigma_{ij}v\rangle \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_1^{\text{eq}} n_1^{\text{eq}}}, \quad g_{\text{eff}} = \sum_{i=1}^4 \frac{n_i^{\text{eq}}}{n_1^{\text{eq}}}, \quad (14)$$

where $\langle\sigma_{ij}v\rangle$ is the thermally averaged (co)annihilation cross section and n_i^{eq} is the thermal equilibrium number density of η_i . If the former is expanded by the thermally averaged relative velocity $\langle v^2 \rangle$ as $\langle\sigma_{ij}v\rangle = a_{ij} + b_{ij}\langle v^2 \rangle$, it could be approximated only by a_{ij} since $\langle v^2 \rangle \ll 1$ is satisfied for the cold DM. Final states of the relevant (co)annihilation are composed only of the SM contents. The corresponding a_{eff} can be approximately calculated as [45,46,56]

$$\begin{aligned} a_{\text{eff}} = & \frac{(1 + 2c_w^4)g^4}{128\pi c_w^4 M_{\eta_1}^2} (N_{11} + N_{22} + 2N_{34}) \\ & + \frac{s_w^2 g^4}{32\pi c_w^2 M_{\eta_1}^2} (N_{13} + N_{14} + N_{23} + N_{24}) \\ & + \frac{1}{64\pi M_{\eta_1}^2} \left[(\tilde{\lambda}_+^2 + \tilde{\lambda}_-^2 + 2\tilde{\lambda}_3^2) (N_{11} + N_{22}) \right. \\ & + (\tilde{\lambda}_+ - \tilde{\lambda}_-)^2 (N_{33} + N_{44} + N_{12}) \\ & + \left. \left\{ (\tilde{\lambda}_+ - \tilde{\lambda}_3)^2 + (\tilde{\lambda}_- - \tilde{\lambda}_3)^2 \right\} \right. \\ & \times (N_{13} + N_{14} + N_{23} + N_{24}) \\ & + \left. \left\{ (\tilde{\lambda}_+ + \tilde{\lambda}_-)^2 + 4\tilde{\lambda}_3^2 \right\} N_{34} \right], \quad (15) \end{aligned}$$

where $\tilde{\lambda}_{\pm} = \tilde{\lambda}_3 + \lambda_4 \pm \tilde{\lambda}_5$ and N_{ij} is defined by using M_{η_i} given in Eq. (7) as

$$\begin{aligned} N_{ij} \equiv & \frac{1}{g_{\text{eff}}^2} \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_1^{\text{eq}} n_1^{\text{eq}}} \\ = & \frac{1}{g_{\text{eff}}^2} \left(\frac{M_{\eta_i} M_{\eta_j}}{M_{\eta_1}^2} \right)^{3/2} \exp \left[-\frac{M_{\eta_i} + M_{\eta_j} - 2M_{\eta_1}}{T} \right]. \quad (16) \end{aligned}$$

We use this procedure to find the points in the $(\tilde{\lambda}_3, \lambda_4)$ plane, where the required DM abundance $\Omega_{\text{DM}} h^2 = 0.12$ is realized by η_R . In Fig. 2, we plot such points by a red solid line for $\tilde{m}_{\eta} = 1$ TeV and $\tilde{\lambda}_5 = 2.5 \times 10^{-3}$ which are used in the previous part. In this figure, we take account of the condition $\lambda_4 < 0$ which has been already discussed in relation to Eq. (7). Moreover, if we use the Higgs mass formula $m_{h^0}^2 = 4\tilde{\lambda}_1 \langle\phi\rangle^2$, we find $\tilde{\lambda}_1 \simeq 0.13$ for $m_{h^0} = 125$ GeV and then the last condition in Eq. (5) can be also plotted for a fixed $\tilde{\lambda}_2$ in the same plane.⁸ An allowed points are con-

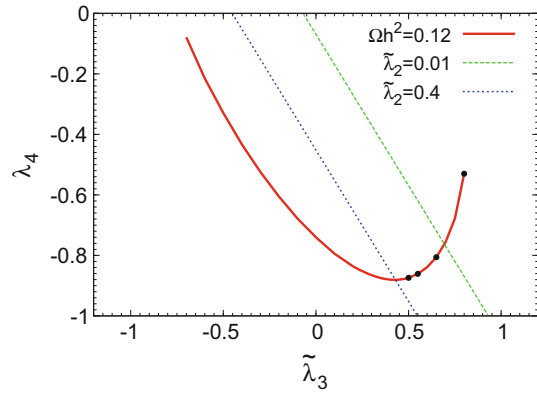


Fig. 2 Points plotted by a red solid line in the $(\tilde{\lambda}_3, \lambda_4)$ plane can realize the required DM relic abundance $\Omega h^2 = 0.12$ as the relic η_R abundance. The last condition in Eq. (5) is satisfied at a region above the straight line which represents $\tilde{\lambda}_3 + \lambda_4 = |\tilde{\lambda}_5| - 2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}$ for a fixed $\tilde{\lambda}_2$

tained in the region above a straight line which is fixed by an assumed value of $\tilde{\lambda}_2$. We give two examples here. Although the DM abundance can be satisfied for the negative value of $\tilde{\lambda}_3$, we find that such cases contradict with the vacuum stability condition for $\tilde{\lambda}_3$ given in Eq. (5). The figure shows that $\tilde{\lambda}_3$ and/or $|\lambda_4|$ are required to take rather large values for realization of the DM abundance. This suggests that the RG evolution of the scalar quartic couplings $\tilde{\lambda}_i$ could be largely affected if they are used as initial values at the weak scale. In that case, vacuum stability and perturbativity of the model could give constraints on the model. In the next part, we focus our study on this point.

Before proceeding to this subject, we comment on the contribution of the axion to the DM abundance and also a possible violation of $U(1)_{PQ}$ by the quantum gravity effect. In this model, the axion could also contribute to the DM abundance through the misalignment mechanism. If the initial misalignment of the axion is written as $\langle\theta_i\rangle$, the axion contribution to the present energy density is estimated as [13,14]

$$\Omega_a h^2 = 2 \times 10^4 \left(\frac{\langle S \rangle}{10^{16} \text{ GeV}} \right)^{7/6} \langle\theta_i^2\rangle. \quad (17)$$

The axion contribution to the DM abundance crucially depends on the scale of $\langle S \rangle$ and $\langle\theta_i\rangle$. This estimation shows that it could be too small to give the required value $\Omega_{\text{DM}} h^2 = 0.12$ for $\langle S \rangle < 10^{11}$ GeV even if we assume $\langle\theta_i\rangle = O(1)$.⁹ Thus, the axion contribution to the DM abundance is subdominant or negligible for $\langle S \rangle < 10^{11}$ GeV. In this region of

⁷ In this part, we label $(\eta_R, \eta_I, \eta^+, \eta^-)$ as $(\eta_1, \eta_2, \eta_3, \eta_4)$.

⁸ We note that the second condition in Eq. (5) is automatically satisfied if the last one is fulfilled.

⁹ The estimation of the relic axion abundance has to take account of the contribution from the decay of string and domain walls. Depending on it, the upper bound on the PQ breaking scale seems to be somewhat ambiguous. While one group finds that the axion production is more efficient than the misalignment case [64,65], the other group finds that it is less efficient than the misalignment case [66,67].

$\langle S \rangle$, the result obtained for $(\tilde{\lambda}_3, \lambda_4)$ through the above study can be still applicable even if the axion contribution to the DM abundance is taken into account.

Although we assume that $U(1)_{PQ}$ is exact in this study, continuous global symmetry is suggested to be violated by the quantum gravity. This possible effect on the PQ mechanism has been studied [68–70]. If the $U(1)_{PQ}$ symmetry is violated by the gravity induced effective interaction which is suppressed by the Planck scale such as

$$\frac{|S|^{n+3}}{M_{\text{pl}}^n} (gS + g^*S), \tag{18}$$

it has been shown that $n \geq 6$ should be satisfied for the PQ mechanism to give a solution to the strong CP problem in case that $|g|$ is of $O(1)$. If accidental appearance of global $U(1)$ happens due to some discrete or continuous gauge symmetry [71–78], it might protect the PQ symmetry up to sufficiently higher order operators. The same breaking effect could also affect the axion CDM abundance [68–70]. If the contribution to the axion mass due to the quantum gravity is small compared to the one due to the QCD anomaly, $\langle S \rangle \simeq 10^{11}$ GeV is required for saturating $\Omega h^2 = 0.12$ by the axion contribution. Even if its contribution to the axion mass is larger than the one from the QCD anomaly within the bound which is required so as not to disturb the PQ mechanism, $\langle S \rangle \simeq 10^{11}$ GeV is required again for saturating $\Omega h^2 = 0.12$. Thus, η_R could play a dominant role in the DM abundance as long as $\langle S \rangle$ is smaller than 10^{11} GeV.

The stability for η_R could be also violated through the same effect. The most effective processes for the η_R decay are induced by nonrenormalizable Yukawa couplings such as

$$\frac{S^n}{M_{\text{pl}}^n} (h_u \bar{q}_L u_R \eta + h_d \bar{q}_L d_R \tilde{\eta} + h_e \bar{\ell}_L e_R \tilde{\eta}). \tag{19}$$

If the allowed dimension for these kind of operators is the same as the one which guarantees the PQ mechanism to work, the lifetime of η_R could be longer than the age of our universe in case $m_\eta = O(1)$ TeV and $h_{u,d,e} = O(1)$ as long as we take $\langle S \rangle \simeq 10^{10}$ GeV. If the lower dimension operators such as $n < 6$ are allowed, its lifetime cannot be long enough to be the DM at the present universe.

3.2 Consistency of the scenario with a cut-off scale of the model

It is crucial to check what kind of values of the right-handed neutrino mass M_i and $\tilde{\lambda}_5$ could be consistent with a value of $\langle S \rangle$ which is restricted by the axion physics. In this model, DM is identified with η_R whose mass is of $O(1)$ TeV. In such a mass region, we find that its abundance is determined by the values of the scalar quartic couplings $\tilde{\lambda}_3$ and λ_4 . On the

other hand, these couplings could affect the vacuum stability and also the perturbativity of the model through the radiative effects on the scalar quartic couplings $\tilde{\lambda}_i$. Here, we examine the consistency of the values of $\tilde{\lambda}_3$ and λ_4 required to realize of the DM abundance with these issues.¹⁰ Since the breaking of the perturbativity is considered to be relevant to a scale for the applicability of the model, we could obtain an information for the cut-off scale M_* . It allows us to judge whether the required value for $\tilde{\lambda}_3$ by the neutrino masses and the leptogenesis could be induced through the VEV of S .

The one-loop β -functions for the scalar quartic couplings in the effective model at energy regions below M_S are given as follows [80, 81],

$$\begin{aligned} \beta_{\tilde{\lambda}_1} &= 24\tilde{\lambda}_1^2 + \tilde{\lambda}_3^2 + (\tilde{\lambda}_3 + \lambda_4)^2 + \tilde{\lambda}_5^2 \\ &\quad + \frac{3}{8} (3g^4 + g'^4 + 2g^2g'^2) \\ &\quad - 3\tilde{\lambda}_1 (3g^2 + g'^2 - 4h_t^2) - 6h_t^4, \\ \beta_{\tilde{\lambda}_2} &= 24\tilde{\lambda}_2^2 + \tilde{\lambda}_3^2 + (\tilde{\lambda}_3 + \lambda_4)^2 + \tilde{\lambda}_5^2 \\ &\quad + \frac{3}{8} (3g^4 + g'^4 + 2g^2g'^2) - 3\tilde{\lambda}_2 (3g^2 + g'^2), \\ \beta_{\tilde{\lambda}_3} &= 2(\tilde{\lambda}_1 + \tilde{\lambda}_2)(6\tilde{\lambda}_3 + 2\lambda_4) + 4\tilde{\lambda}_3^2 + 2\lambda_4^2 + 2\tilde{\lambda}_5^2 \\ &\quad + \frac{3}{4} (3g^4 + g'^4 - 2g^2g'^2) - 3\tilde{\lambda}_3 (3g^2 + g'^2 - 2h_t^2), \\ \beta_{\lambda_4} &= 4(\tilde{\lambda}_1 + \tilde{\lambda}_2)\lambda_4 + 8\tilde{\lambda}_3\lambda_4 + 4\lambda_4^2 + 8\tilde{\lambda}_5^2 + 3g^2g'^2 \\ &\quad - 3\lambda_4 (3g^2 + g'^2 - 2h_t^2), \\ \beta_{\tilde{\lambda}_5} &= 4(\tilde{\lambda}_1 + \tilde{\lambda}_2)\tilde{\lambda}_5 + 8\tilde{\lambda}_3\tilde{\lambda}_5 + 12\lambda_4\tilde{\lambda}_5 \\ &\quad - 3\tilde{\lambda}_5 (3g^2 + g'^2 - 2h_t^2), \end{aligned} \tag{20}$$

where β_λ is defined as $\beta_\lambda = 16\pi^2 \mu \frac{d\lambda}{d\mu}$. In these equations, we can expect that the positive contributions of $\tilde{\lambda}_3$ and λ_4 to the β -functions of $\tilde{\lambda}_{1,2}$ tend to save the model from violating the first two vacuum stability conditions in Eq. (5). On the other hand, the same contributions of $\tilde{\lambda}_3$ and λ_4 could induce the breaking of the perturbativity of the model at a rather low energy scale since they could give large positive contributions to $\beta_{\tilde{\lambda}_1}, \beta_{\tilde{\lambda}_2}$ and $\beta_{\tilde{\lambda}_3}$. Here, we identify a cut-off scale M_* of the model with a scale where any of the perturbativity conditions $\lambda_i(M_*) < 4\pi$ and $\kappa_i(M_*) < 4\pi$ is violated.¹¹ In this case, $M_* > |\langle S \rangle|$ should be satisfied. If M_* is smaller than $\langle S \rangle$, the consistency of the scenario is lost.

¹⁰ The constraint due to the vacuum stability and the perturbativity is taken into account in the DM study of the inert doublet model on the basis of a different viewpoint from the present one [53–56]. The consistency between fermionic DM and the vacuum stability is also studied in the scotogenic model [44, 79].

¹¹ Since the Landau pole appearing scale is expected to be near to this M_* , it seems to be natural to identify M_* with a cut-off scale of the model.

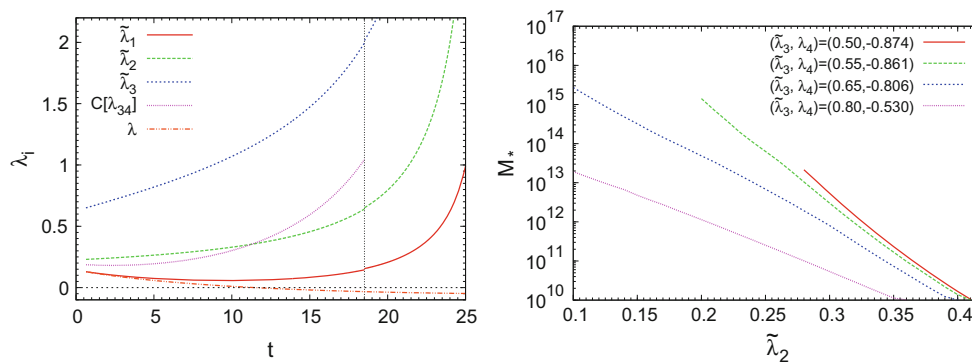


Fig. 3 Left panel: running of the scalar quartic couplings for $t = \ln \frac{\mu}{M_Z}$. $\tilde{\lambda}_2 = 0.23$, $\tilde{\lambda}_3 = 0.65$ and $\lambda_4 = -0.806$ are used as the initial values at $\mu = M_Z$. A vertical line corresponds to $t = \ln \left(\frac{M_S}{M_Z} \right)$.

The running of the SM Higgs quartic coupling λ is also plotted as a reference. Right panel: the cut-off scale M_* as a function of $\tilde{\lambda}_2$ which is fixed as a value at M_Z for four points marked by the black bulbs in Fig. 2 where $\Omega h^2 = 0.12$ is satisfied

We analyze this issue by solving the above one-loop RGEs at $\mu < M_S$ and also the ones at $\mu > M_S$, which are given in Appendix. The quartic couplings $\tilde{\lambda}_i$ in the tree-level potential at the energy scale $\mu < M_S$ are connected with the ones λ_i at $\mu > M_S$ through Eq. (4). Since the masses of the right-handed neutrinos N_i are considered to be heavy in the present model, they decouple at the scale $\mu < M_i \lesssim O(M_S)$ to be irrelevant to the RGEs there. On the other hand, the mass of the colored fields $D_{L,R}$ can take any values larger than 1 TeV as discussed before, they can contribute to the RGEs at larger scales than their mass. In the present study, we assume that $D_{L,R}$ is light of $O(1)$ TeV but its Yukawa coupling h_D with the ordinary quarks is small enough.¹² Thus, they are considered to contribute substantially only to the β -functions of the gauge couplings. In this study, we take its hypercharge as $Y = -\frac{1}{3}$ as shown in Table 1.

The free parameters in the scalar potential of the effective model (3) are $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$, λ_4 and $\tilde{\lambda}_5$ at M_Z as long as we assume $\tilde{m}_\eta = 1$ TeV.¹³ Among them, we should fix $\tilde{\lambda}_5$ at a value used in the discussion of the neutrino mass and the leptogenesis. Both $\tilde{\lambda}_3$ and λ_4 are fixed at values determined through the DM relic abundance as shown in Fig. 2. We also have $\tilde{\lambda}_1 \simeq 0.13$ from the Higgs mass. From this point of view, $\tilde{\lambda}_2$ is an only remaining parameter. Thus, if we solve the RGEs varying the value of $\tilde{\lambda}_2$ for other fixed parameters, we can find M_* checking the vacuum stability for each $\tilde{\lambda}_2$.

In the left panel of Fig. 3, as an example, we present the running of the scalar quartic couplings $\lambda_{1,2,3}$ for the initial

values $\tilde{\lambda}_2 = 0.23$, $\tilde{\lambda}_3 = 0.65$ and $\lambda_4 = -0.806$ at M_Z by assuming the $U(1)_{PQ}$ breaking scale as $\langle S \rangle = M_S = 10^{10}$ GeV. In the same panel, we also plot the value of $\tilde{\lambda}_3 + \lambda_4 - |\tilde{\lambda}_5| + 2\sqrt{\tilde{\lambda}_1 \tilde{\lambda}_2}$ as $C[\lambda_{34}]$, which corresponds to the last one in Eq. (5). In this example, we can see that the vacuum stability is kept until the cut-off scale $M_* \simeq 1.54 \times 10^{13}$ GeV. These values of $\langle S \rangle$ and M_* can naturally realize the assumed value for $\tilde{\lambda}_5$ through the relation given in Eq. (4) just by taking λ_5 as a value of $O(1)$. This feature can be verified for other allowed values of $\tilde{\lambda}_3$ and λ_4 . Here, we note that the axion contribution to the DM abundance can be neglected for a value such as $\langle S \rangle < 10^{11}$ GeV. In the right panel of Fig. 3, we plot M_* as a function of $\tilde{\lambda}_2$ for four sets of $(\tilde{\lambda}_3, \lambda_4)$ which are shown by black bulbs in Fig. 2. End points found in the two lines represent the value of $\tilde{\lambda}_2$ for which the vacuum stability is violated before reaching M_* . This figure shows that $\tilde{\lambda}_2$ which is restricted to a rather narrow region can make M_* appropriate values in order to realize a required value of $\tilde{\lambda}_5$ for $\langle S \rangle < 10^{11}$ GeV. This study suggests that the scenario could work well without strict tuning of the relevant parameters.

As found from the above study, the simultaneous explanation of the neutrino masses and the DM abundance could be preserved in this extended model in the same way as in the original scotogenic model. We should stress that no other additional constraint from the DM physics and the neutrino physics is brought about by taking the present scenario. The cosmological baryon number asymmetry is expected to be explained through the out-of-equilibrium decay of the lightest right-handed neutrino. The required right-handed neutrino mass could be smaller compared with the Davidson–Ibarra bound in the ordinary thermal leptogenesis [61]. This is consistent with the result in [45,46] where the mass bound of the right-handed neutrino for the successful leptogenesis is shown to be relaxed in the radiative neutrino mass model in comparison with the ordinary seesaw model.

¹² In the light D case, study of the bound for this Yukawa coupling is an interesting subject related to the search of mixing with the ordinary quarks. However, it is beyond the scope of the present study and we do not discuss it here.

¹³ Quartic couplings κ_i for S are fixed as $\kappa_1 = \frac{M_S^2}{4\langle S \rangle^2}$ and $\kappa_{2,3} = 0.1$ at M_S in the present study. As easily found from RGEs, larger values of $\kappa_{2,3}$ make M_* smaller.

Finally, we give brief comments on possible experimental signatures of the model. The present model might be examined through (i) the search of the η_R DM and the charged scalars η^\pm through the DM direct detection experiments and the accelerator experiments, (ii) the search of the mixing of D with the ordinary quarks although it could be observed only in the light D case, and (iii) the search of the axion whose coupling with photon is characterized by $g_{a\gamma\gamma} = \frac{m_a}{\text{eV}} \frac{2.0}{10^{10}\text{GeV}} (6Y^2 - 1.92)$, where Y is the hypercharge of D [32].

4 Summary

We have proposed an extension of the KSVZ invisible axion model so as to include a DM candidate and explain the small neutrino masses. An extra inert doublet scalar η and three right-handed neutrinos N_i are introduced as new ingredients. After the $U(1)_{PQ}$ symmetry breaking, its subgroup Z_2 could remain as a remnant effective symmetry, which is violated through the QCD anomaly but it can play the same role as the Z_2 in the scotogenic neutrino mass model. Since only the new ones η and N_i have its odd parity, the model reduces to the scotogenic model which has Z_2 in the leptonic sector. The neutrino masses are generated at one-loop level and the DM abundance can be explained by the thermal relics of the neutral component of η . The cosmological baryon number asymmetry could be generated through the out-of-equilibrium decay of a right-handed neutrino in the same way as the ordinary thermal leptogenesis in the tree-level seesaw model. However, the bound for the right-handed neutrino mass can be relaxed in this model. Since this simple extension can relate the strong CP problem to the origin of neutrino masses and DM, it may be a promising extension of both the KSVZ model and the scotogenic model.

Acknowledgements This work is partially supported by MEXT Grant-in-Aid for Scientific Research on Innovative Areas (Grant No. 26104009).

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Appendix

The β -function for the scalar quartic couplings at $\mu > M_S$ are given as

$$\begin{aligned}
 \beta_{\lambda_1} &= 24\lambda_1^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \kappa_2^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) \\
 &\quad - 3\lambda_1(3g^2 + g'^2 - 4h_t^2) - 6h_t^4, \\
 \beta_{\lambda_2} &= 24\lambda_2^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \kappa_3^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - 3\lambda_2(3g^2 + g'^2) \\
 &\quad + 4\lambda_2[2(h_1^2 + h_2^2) + 3h_3^2] - 8(h_1^2 + h_2^2)^2 - 18h_3^4, \\
 \beta_{\lambda_3} &= 2(\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\kappa_2\kappa_3 \\
 &\quad + \frac{3}{4}(3g^4 + g'^4 - 2g^2g'^2) \\
 &\quad - 3\lambda_3(3g^2 + g'^2 - 2h_t^2) + 2\lambda_3[2(h_1^2 + h_2^2) + 3h_3^2], \\
 \beta_{\lambda_4} &= 4(\lambda_1 + \lambda_2)\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 3g^2g'^2 \\
 &\quad - 3\lambda_4(3g^2 + g'^2 - 2h_t^2) \\
 &\quad + 2\lambda_4[2(h_1^2 + h_2^2) + 3h_3^2], \\
 \beta_{\kappa_1} &= 20\kappa_1^2 + 2\kappa_2^2 + 2\kappa_3^2 + 4\kappa_1\left(3y_D^2 + \sum_i y_i^2\right) \\
 &\quad - 2\left(3y_D^4 + \sum_i y_i^4\right), \\
 \beta_{\kappa_2} &= 4\kappa_2^2 + 2\kappa_2(6\lambda_1 + 4\kappa_1) + 2\kappa_3(2\lambda_3 + \lambda_4) \\
 &\quad + 2\kappa_2\left(3y_D^2 + \sum_i y_i^2\right) \\
 &\quad - \frac{3}{2}\kappa_2(3g^2 + g'^2 - 4h_t^2), \\
 \beta_{\kappa_3} &= 4\kappa_3^2 + 2\kappa_3(6\lambda_2 + 4\kappa_1) + 2\kappa_2(2\lambda_3 + \lambda_4) \\
 &\quad + 2\kappa_3\left(3y_D^2 + \sum_i y_i^2\right) \\
 &\quad - \frac{3}{2}\kappa_3\left[3g^2 + g'^2 - \frac{4}{3}(2(h_1^2 + h_2^2) + 3h_3^2)\right], \quad (21)
 \end{aligned}$$

where Eq. (9) is assumed for the flavor structure of neutrino Yukawa couplings. The β -functions for the gauge couplings and the Yukawa couplings for top, D and neutrinos are given as

$$\begin{aligned}
 \beta_{g_s} &= -11 + \frac{2}{3}(6 + \delta)g_s^3, \quad \beta_g = -3g^3, \\
 \beta_{g'} &= (7 + 4Y^2\delta)g'^3, \\
 \beta_{h_t} &= h_t\left(\frac{9}{2}h_t^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right), \\
 \beta_{y_k} &= y_k\left(y_k^2 + 3y_D^2 + \sum_i y_i^2\right), \\
 \beta_{y_D} &= y_D\left(-8g_s^3 - 6Y^2\delta g'^2 + 4y_D^2 + \sum_i y_i^2\right),
 \end{aligned}$$

$$\beta_{h_{1,2}} = h_{1,2} \left[-\frac{9}{4}g^2 - \frac{3}{4}g'^2 + 5(h_1^2 + h_2^2) + 3h_3^2 + \frac{1}{2} \sum_i y_i^2 \right],$$

$$\beta_{h_3} = h_3 \left[-\frac{9}{4}g^2 - \frac{3}{4}g'^2 + 2(h_1^2 + h_2^2) + 6h_3^2 + \frac{1}{2} \sum_i y_i^2 \right],$$
(22)

where δ stands for the number of extra color triplets $D_{L,R}$. Since $D_{L,R}$ is assumed to be light in this study, δ is treated as 1. The Yukawa coupling h_D with the ordinary quarks is assumed to be small enough and then its contribution is neglected in these equations.

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