# Pati-Salam unification with a spontaneous *CP* violation

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Recent neutrino oscillation experiments suggest that the Pontecorvo-Maki-Nakagawa-Sakata matrix in the lepton sector has a *CP* violating phase like the Cabibbo-Kobayashi-Maskawa matrix in the quark sector. However, the origins of these phases in both matrices are not clarified by now. Although complex Yukawa couplings could induce these phases, the phases remain as free parameters of the model even in that case. If the *CP* symmetry is considered to be spontaneously broken, they are expected to be determined by some physics at a much lower energy scale than the Planck scale. We study such a possibility in a framework of Pati-Salam-type unification. We also discuss other phenomenological issues in it.

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## I. INTRODUCTION

A *CP* violation in a quark sector has been confirmed to be explained by a Cabibbo-Kobayashi-Maskawa (CKM) phase through experiments of the $B$ meson system. However, its origin is still not known now. Although the CKM phase can be derived from complex Yukawa couplings of quarks [1], the *CP* symmetry is considered to be explicitly broken in such a case, and then the CKM phase remains as a free parameter of the model. Even if its origin could be explained in some physics at the Planck scale, it seems to be difficult to confirm it through experiments. As another problem related to the *CP* violation, we have a strong *CP* problem [2]. The experimental bound of the electric dipole moment of a neutron suggests that $\bar{\theta} \lesssim 10^{-10}$ should be satisfied [3], where $\bar{\theta}$ is defined as $\bar{\theta} \equiv \theta_{\rm QCD} + \arg(\det \mathcal{M}_u \mathcal{M}_d)$ for up- and down-type quark mass matrices $\mathcal{M}_u$ and $\mathcal{M}_d$. Since a QCD parameter $\theta_{\rm QCD}$ and the second term caused from the quark masses are irrelevant to each other, the required smallness of $\bar{\theta}$ seems to be unnatural, which is called the strong *CP* problem in the standard model (SM).

One of the solutions for this problem is known to be presented by the Peccei-Qiunn (PQ) mechanism [4]. Since its validity could be examined through the existence of a light pseudoscalar called axion [5–7], an axion search is now performed in various experiments [8]. As another solution for the strong *CP* problem, the Nelson-Barr (NB) model is known [9]. In this scenario, the *CP* symmetry is assumed to be an exact symmetry and then $\theta_{\rm QCD} = 0$ is satisfied. If quark mass matrices take a special form based on some symmetry to satisfy $\arg(\det \mathcal{M}_u \mathcal{M}_d) = 0$, $\bar{\theta} = 0$ could be realized at least at a tree level even after the spontaneous *CP* violation. On the other hand, this spontaneous *CP* violation could give a *CP* phase in the CKM matrix. In this point, the scenario is interesting since it could explain an origin of the *CP* violation at a much lower energy scale than the Planck scale. Moreover, if a *CP* breaking sector couples also with leptons, a *CP* phase in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [10], whose existence is suggested through the long baseline neutrino oscillation experiments such as NOvA and T2K, might be explained by the same source as the quark sector.

A minimal simple example of the NB-type model has been proposed by Bento, Branco and Parada (BBP) [11]. In this model, extra heavy vectorlike down-type quarks are introduced, and a $Z_2$ symmetry is imposed on the model which controls the down-type quark mass matrix so as not to bring about a contribution to $\bar{\theta}$ through $\arg(\det \mathcal{M}_u \mathcal{M}_d)$ after the spontaneous *CP* violation. If we impose a global $U(1)$ symmetry instead of the $Z_2$ symmetry and assign its charge to these extra heavy quarks, it is easy to find that the required form of the mass matrix could be realized in the same way. In such a case, interestingly enough, the model has a similar structure to an invisible axion model by Kim-Shifman-Vainstein-Zakharov [6], which solves the strong *CP* problem through the PQ mechanism. If the introduced global $U(1)$ works as the PQ symmetry, the contribution to $\bar{\theta}$ through radiative corrections to $\arg(\det \mathcal{M}_u \mathcal{M}_d)$ could be erased out. In that case, one of the problems in the NB model which is pointed out in [12] could disappear. In this paper, we study this scenario in a Pati-Salam-type unified model, in which the *CP* phases in the CKM matrix and the PMNS matrix could be related.

The remaining part of the paper is organized as follows. In Sec. II, we introduce our model and discuss a possible

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origin of $CP$ phases in both the CKM and PMNS matrices. The generation of small neutrino masses is also addressed. We additionally examine a possible spontaneous $CP$ violation in the model. In Sec. III, we discuss several phenomenological issues in the model. Section IV is devoted to the summary of the paper.

## II. ORIGIN OF $CP$ VIOLATION

### A. A Pati-Salam-type unified model

We consider a unification model of quarks and leptons via Pati and Salam [13]. The gauge symmetry is taken to be $SU(4)\times SU(2)\times U(1)_X$, in which the forth color is identified with a lepton. Fermion contents and their representations under this gauge group are assumed to be

$$f_{L_i}(4,2,0),\quad h_{R_i}(4,1,1/2),\quad k_{R_i}(4,1,-1/2),\tag{1}$$

where $i$ is the generation index ($i=1$, 2, 3). As they are easily found, these contain all ordinary quarks and leptons. We also introduce additional vectorlike colored fermions $F_{L,R}(4,1,-1/2)$, and $n$ triplet fermions $\Sigma_{R_\alpha}(1,3,0)$ where $\alpha=1-n$, and they are defined as

$$\Sigma_{R_\alpha}\equiv\sum_{a=1}^{3}\frac{\tau^a}{2}\Sigma^a_{R_\alpha}=\frac{1}{2}\begin{pmatrix}\Sigma^0_{R_\alpha} & \sqrt{2}\Sigma^+_{R_\alpha}\\ \sqrt{2}\Sigma^-_{R_\alpha} & -\Sigma^0_{R_\alpha}\end{pmatrix}.\tag{2}$$

On the other hand, scalar contents and their representations are taken to be

$$\begin{aligned}&\Phi(4,1,1/2),\qquad \Psi(4,1,1/2),\qquad \phi(1,2,-1/2),\qquad \eta(1,2,-1/2),\\ &\sigma(1,1,0),\qquad S(1,1,0),\qquad s(1,1,0).\end{aligned}\tag{3}$$

In addition to this structure, we impose a global $U(1)\times Z_8$ symmetry. Its charge is assigned to these fields as follows:

$$\begin{aligned}&f_{L_i},\quad h_{R_i},\quad k_{R_i}\Rightarrow(0,1),\quad F_L,\Rightarrow(0,7),\quad F_R\Rightarrow(2,1),\quad \Sigma_{R_\alpha}\Rightarrow(1,1),\quad S\Rightarrow(0,6),\\ &\sigma\Rightarrow(2,2),\quad \eta\Rightarrow(-1,1),\quad \Phi\Rightarrow(0,4),\quad \Psi,\phi\Rightarrow(0,0),\quad s\Rightarrow(0,1).\end{aligned}\tag{4}$$

We also assume that $CP$ is an exact symmetry of the model. Although $\Sigma_{R_\alpha}$ and $\eta$ might be considered needless in the model for the explanation of features shown through several experiments which cannot be explained in the SM framework,[1] we start our discussion in these field contents.

If we adopt these field contents, Yukawa couplings invariant under the imposed symmetry are written as

$$-\mathcal{L}_y=y^h_{ij}\bar{f}_{L_i}\phi h_{R_j}+y^k_{ij}\bar{f}_{L_i}\tilde{\phi}k_{R_j}+y_iS\bar{F}_Lk_{R_i}+x\sigma^*\bar{F}_LF_R+\gamma_{\Sigma_\alpha}\sigma^*\bar{\Sigma}^c_{R_\alpha}\Sigma_{R_\alpha}+\text{H.c.},\tag{5}$$

where $\tilde{\phi}=i\tau_2\phi^*$. On the other hand, scalar potential is expressed as

$$\begin{aligned}V=&\tilde{m}_S^2(S^\dagger S)+\tilde{m}_\sigma^2(\sigma^\dagger\sigma)+\tilde{m}_s^2(s^\dagger s)+\kappa_S(S^\dagger S)^2+\kappa_\sigma(\sigma^\dagger\sigma)^2+\kappa_s(s^\dagger s)^2+\kappa_{S\sigma}(S^\dagger S)(\sigma^\dagger\sigma)+\kappa_{s\sigma}(s^\dagger s)(\sigma^\dagger\sigma)\\ &+\kappa_{Ss}(S^\dagger S)(s^\dagger s)+\kappa_{\sigma\phi}(\sigma^\dagger\sigma)(\phi^\dagger\phi)+\kappa_{S\phi}(S^\dagger S)(\phi^\dagger\phi)+\kappa_{s\phi}(s^\dagger s)(\phi^\dagger\phi)+\kappa_{\sigma\eta}(\sigma^\dagger\sigma)(\eta^\dagger\eta)+\kappa_{S\eta}(S^\dagger S)(\eta^\dagger\eta)\\ &+\kappa_{s\eta}(s^\dagger s)(\eta^\dagger\eta)+\tilde{m}_\phi^2(\phi^\dagger\phi)+\tilde{m}_\eta^2(\eta^\dagger\eta)+\lambda_1(\phi^\dagger\phi)^2+\lambda_2(\eta^\dagger\eta)^2+\lambda_3(\phi^\dagger\phi)(\eta^\dagger\eta)+\lambda_4(\phi^\dagger\eta)(\eta^\dagger\phi)\\ &+m_\Phi^2(\Phi^\dagger\Phi)+m_\Psi^2(\Psi^\dagger\Psi)+\zeta_1(\Phi^\dagger\Phi)^2+\zeta_2(\Psi^\dagger\Psi)^2+\zeta_3(\Phi^\dagger\Phi)(\Psi^\dagger\Psi)+\zeta_4(\Phi^\dagger\Psi)(\Psi^\dagger\Phi)\\ &+(\zeta_\sigma\sigma^\dagger\sigma+\zeta_SS^\dagger S+\zeta_ss^\dagger s+\zeta_\phi\phi^\dagger\phi+\zeta_\eta\eta^\dagger\eta)(\Phi^\dagger\Phi+\Psi^\dagger\Psi)+V_b(S,S^\dagger,\sigma^\dagger\sigma,s^\dagger s,\Phi^\dagger\Psi,\Psi^\dagger\Phi,\phi^\dagger\phi,\eta^\dagger\eta),\end{aligned}\tag{6}$$

where $V_b$ contains potential terms which are invariant under the symmetry mentioned above, but it violates the $S$ number conservation. Since $CP$ is assumed to be exact, all coupling constants are real. If $\Phi$ and $\Psi$ get vacuum expectation values (VEVs) such as $\langle\Phi\rangle=\langle\Psi\rangle=(0,0,0,\Lambda)^T$ for example,[2] the gauge symmetry is broken to the one of the SM:

$$SU(4)\times SU(2)\times U(1)_X\xrightarrow{\langle\Phi\rangle,\langle\Psi\rangle}SU(3)_C\times SU(2)_L\times U(1)_Y.\tag{7}$$

[1] If the axion is identified with the dark matter, they might be needless. However, we would like to consider much wider possibilities because of reasons which are addressed later.

[2] $\langle\Phi\rangle=\langle\Psi\rangle$ is assumed just for simplicity.

The weak hypercharge $U(1)_Y$, whose charge is normalized as $Q_{\rm EM} = \frac{\tau_3}{2} + Y$, is obtained as a linear combination of a diagonal generator $T_{15}$ of $SU(4)$ and a charge $X$ of $U(1)_X$ as

$$Y = \frac{2}{\sqrt{6}} T_{15} + X, \tag{8}$$

where $T_{15} = \frac{1}{2\sqrt{6}} {\rm diag}(1,1,1,-3)$. We note that the imposed global $U(1)$ symmetry remains unbroken but $Z_8$ is broken to $Z_4$ at this stage. All fermions remain massless since they have no Yukawa couplings only with $\Phi$ and $\Psi$.

After this symmetry breaking, each fermion is decomposed to the contents of the SM, such as

$$f_{L_i} = (q_{L_i}, \ell_{L_i}), \quad h_{R_i} = (u_{R_i}, N_{R_i}), \quad k_{R_i} = (d_{R_i}, e_{R_i}), \tag{9}$$

where $q_{L_i}$ and $\ell_{L_i}$ are $SU(2)_L$ doublet quarks and leptons; and $u_{R_i}$, $d_{R_i}$, and $e_{R_i}$ are singlet quarks and charged leptons, respectively. The vectorlike fermions $F_{L,R}$ are decomposed as ($D_{L,R}$, $E_{L,R}$). If we use these decomposed fermions, Yukawa couplings in Eq. (5) are expressed as[3]

$$\begin{aligned} -\mathcal{L}_y = {} & y^u_{ij} \bar{q}_{L_i} \phi u_{R_j} + y^d_{ij} \bar{q}_{L_i} \tilde{\phi} d_{R_j} + (y^D_i S + \tilde{y}^D_i S^*) \bar{D}_L d_{R_i} \\ & + x_D \sigma^* \bar{D}_L D_R + y^\nu_{ij} \bar{\ell}_{L_i} \phi N_{R_j} + y^e_{ij} \bar{\ell}_{L_i} \tilde{\phi} e_{R_j} \\ & + (y^E_i S + \tilde{y}^E_i S^*) \bar{E}_L e_{R_i} + x_E \sigma^* \bar{E}_L E_R \\ & + \gamma_{\Sigma_\alpha} \sigma^* \bar{\Sigma}^c_{R_\alpha} \Sigma_{R_\alpha} + {\rm H.c.}, \end{aligned} \tag{10}$$

where the Yukawa coupling constants are expected to satisfy the conditions

$$\begin{aligned} & y^h_{ij} = y^u_{ij} = y^\nu_{ij}, \qquad y^k_{ij} = y^d_{ij} = y^e_{ij}, \qquad y_i = y^D_i = y^E_i, \\ & \tilde{y}_i = \tilde{y}^D_i = \tilde{y}^E_i, \qquad x = x_D = x_E, \end{aligned} \tag{11}$$

at a unification scale $\Lambda$. After the spontaneous breaking of $SU(4)$ via $\langle\Phi\rangle$ and $\langle\Psi\rangle$, new Yukawa couplings are expected to be induced effectively as invariant ones under the remaining symmetry,[4]

$$\begin{aligned} -\mathcal{L}'_y = {} & \left( y^N_i S + \tilde{y}^N_i S^* + a_i \frac{s^2}{\Lambda} + \tilde{a}_i \frac{s^{*2}}{\Lambda} \right) \bar{N}^c_{R_i} N_{R_i} \\ & + \tilde{h}_{i\alpha} \frac{s^*}{\Lambda} \bar{\ell}_{L_i} \Sigma_{R_\alpha} \eta + \left( b_i \frac{s^2}{\Lambda} + b_i \frac{s^{*2}}{\Lambda} \right) \bar{D}_L d_{R_i} \\ & + \left( c_i \frac{s^2}{\Lambda} + c_i \frac{s^{*2}}{\Lambda} \right) \bar{E}_L e_{R_i} + {\rm H.c.}, \end{aligned} \tag{12}$$

where we list the terms up to dimension five. The couplings $y^N_i$ and $\tilde{y}^N_i$ are assumed to be diagonal. We also note that there is a nonrenormalizable dimension five operator $\tilde{\lambda}_5 \frac{\sigma}{\Lambda} (\phi^\dagger \eta)^2$ as an invariant one. It plays a crucial role in the small neutrino mass generation, as seen later.

In this effective model, we consider symmetry breaking due to VEVs of the singlet scalars $\sigma$, $S$, and $s$, such as[5]

$$\langle\sigma\rangle = w e^{i\chi}, \qquad \langle S\rangle = u e^{i\rho}, \qquad \langle s\rangle = v e^{i\psi}. \tag{13}$$

They could also break the $CP$ symmetry spontaneously. Although we will discuss whether this spontaneous $CP$ violation could be realistic or not in the present model later, we assume it for a while. Here we note that for $\bar{D}_L d_{R_i}$, $\bar{E}_L e_{R_i}$, and $\bar{N}^c N_{R_i}$, in Eqs. (10) and (12) there are contributions from the dimension four and five operators. We can expect that the former ones give the dominant contribution as long as $v \lesssim u$ is satisfied at least. We suppose such a situation and take account of these contributions only in the following study.

After this symmetry breaking, the potential for the remaining scalars $\phi$ and $\eta$ can be written as

$$\begin{aligned} V = {} & m^2_\phi (\phi^\dagger \phi) + m^2_\eta (\eta^\dagger \eta) + \lambda_1 (\phi^\dagger_1 \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 \\ & + \lambda_3 (\phi^\dagger \phi)(\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta)(\eta^\dagger \phi) + \frac{\lambda_5}{2} [(\phi^\dagger \eta)^2 + {\rm H.c.}], \end{aligned} \tag{14}$$

where $\lambda_5$ is defined as $\lambda_5 = \tilde{\lambda}_5 \frac{w}{\Lambda}$ and it is real.[6] The scalar masses are shifted through the symmetry breaking effect as

$$\begin{aligned} & m^2_\phi = \tilde{m}^2_\phi + \kappa_{\sigma\phi} w^2 + \kappa_{S\phi} u^2 + \kappa_{s\phi} v^2 + 2\zeta_\phi \Lambda^2, \\ & m^2_\eta = \tilde{m}^2_\eta + \kappa_{\sigma\eta} w^2 + \kappa_{S\eta} u^2 + \kappa_{s\eta} v^2 + 2\zeta_\eta \Lambda^2. \end{aligned} \tag{15}$$

Since $m_\phi$ and $m_\eta$ are supposed to take much smaller values than $\Lambda$, serious fine tunings are required. However, we do not treat this hierarchy problem in the present study and just assume that both $m_\phi$ and $m_\eta$ are of $O(1)$ TeV. The coupling constants $\lambda_i$ are also related to the ones at high energy regions through threshold corrections at each symmetry breaking scale [14].

An interesting feature of the present model is that the spontaneous $CP$ violation through Eq. (13) could derive both $CP$ phases in the CKM matrix and the PMNS matrix, keeping $\bar{\theta} = 0$. In the next part, we discuss how the $CP$ phases in both CKM and PMNS matrices are induced.

[3] We note that a nonrenormalizable operator such as $S^*(\Psi \bar{F}_L)(\Phi^\dagger k_{R_i})$, which is invariant under the imposed symmetry, induces the Yukawa terms $S^* \bar{D}_L d_{R_i}$ and $S^* \bar{E}_L e_{R_i}$.

[4] It should be noted that the Yukawa term $S^* \bar{N}^c_{R_i} N_{R_i}$ can be induced by a nonrenormalizable operator $S^*(\Phi^\dagger h_{R_i})(\Psi h_{R_i})$ invariant under the imposed symmetry, for example.

[5] The global symmetry $U(1)$ is broken to $Z_2$ by these VEVs. The $Z_2$ guarantees the stability of DM as discussed later.

[6] The $CP$ phase $\chi$ can be removed by the field redefinition of $\eta$. It changes $h_{i\alpha}$ in Eq. (12) to $h_{i\alpha} e^{-i\frac{\chi}{2}}$.

### B. A *CP* phase in the CKM matrix

The *CP* symmetry is assumed to be exact in the model and then all the coupling constants in the Lagrangian are real. Thus, we cannot expect any origin of *CP* violation in the up-type quark sector, which has no extended structure compared with the SM. Since the up-sector mass matrix $m^u_{ij} = y^u_{ij}\langle\phi\rangle$ is real, they can be diagonalized by orthogonal transformations $u'_L = O^L u_L$ and $u'_R = O^R u_R$. In the present effective model, on the other hand, we find that the down-type quark sector has the same structure as the BBP model [11]. The BBP model is an extension of the SM by extra colored vectorlike down-type heavy quarks $(D_L, D_R)$ and a singlet complex scalar $S$. We can apply their discussion to the present model to show how the *CP* phase could be induced in the CKM matrix. Although the $Z_2$ symmetry is imposed to control the mass matrix in their model, the global $U(1)$ symmetry in Eq. (4) could play the same role as it in the present model. Moreover, since this $U(1)$ is chiral and has a color anomaly, it can play a role as the PQ symmetry, which has a domain wall number one as in the Kim-Shifman-Vainstein-Zakharov model [6]. As a result, a Nambu-Goldstone boson produced as a result of its spontaneous breaking through the VEV $\langle\sigma\rangle$ could work as an axion to solve the strong *CP* problem without inducing the domain wall problem [15]. On the other hand, since the axion phenomenology constrains a breaking scale of this symmetry, we have to fix the scale $w$ to be [16]

$$10^9\ \text{GeV} < w < 10^{12}\ \text{GeV}. \tag{16}$$

The Yukawa couplings of the down-type quarks shown in Eq. (10) derive a $4\times 4$ mass matrix $\mathcal{M}_d$ as

$$(\bar{d}_{Li}, \bar{D}_L)\begin{pmatrix} m^d_{ij} & 0 \\ \mathcal{F}^d_j & \mu_D \end{pmatrix}\begin{pmatrix} d_{R_j} \\ D_R \end{pmatrix}, \tag{17}$$

where $m^d_{ij} = y^d_{ij}\langle\tilde{\phi}\rangle$, $\mathcal{F}^d_j = (y^D_j u e^{i\rho} + \tilde{y}^D_j u e^{-i\rho})$, and $\mu_D = x_D w e^{i\chi}$. Due to the PQ mechanism, $\bar{\theta} = \theta_{\rm QCD} + \arg(\det \mathcal{M}_u\mathcal{M}_d) = 0$ is satisfied via the axion even after we take account of radiative corrections including the phases caused by the spontaneous *CP* violation. Next we see that this phase can generate the CKM phase following the BBP model.

We consider the diagonalization of a matrix $\mathcal{M}_d\mathcal{M}_d^\dagger$ by a unitary matrix such as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} m^d m^{d\dagger} & m^d\mathcal{F}^{d\dagger} \\ \mathcal{F}^d m^{d\dagger} & \mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger} \end{pmatrix}\begin{pmatrix} A^\dagger & C^\dagger \\ B^\dagger & D^\dagger \end{pmatrix} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix}, \tag{18}$$

where a $3\times 3$ matrix $m^2$ is diagonal in which the generation indices are abbreviated. Equation (18) requires

$$\begin{aligned} m^d m^{d\dagger} &= A^\dagger m^2 A + C^\dagger M^2 C, \\ \mathcal{F}^d m^{d\dagger} &= B^\dagger m^2 A + D^\dagger M^2 C, \\ \mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger} &= B^\dagger m^2 B + D^\dagger M^2 D. \end{aligned} \tag{19}$$

If $\mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger}$ is much larger than each component of $\mathcal{F}^d m^{d\dagger}$, which means $u, w \gg \langle\tilde{\phi}\rangle$, we find that $B$, $C$, and $D$ can be approximated as

$$B \simeq -\frac{A m^d\mathcal{F}^{d\dagger}}{\mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger}}, \quad C \simeq \frac{\mathcal{F}^d m^{d\dagger}}{\mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger}}, \quad D \simeq 1, \tag{20}$$

which guarantee the approximate unitarity of the matrix $A$. In such a case, it is also easy to find that

$$A^{-1}m^2 A = m^d m^{d\dagger} - \frac{1}{\mu_D\mu_D^\dagger + \mathcal{F}^d\mathcal{F}^{d\dagger}}(m^d\mathcal{F}^{d\dagger})(\mathcal{F}^d m^{d\dagger}). \tag{21}$$

The right-hand side is an effective mass matrix of the ordinary down-type quarks which are derived through mixing with the extra heavy quarks. Since the second term can have complex phases in off-diagonal components as long as $y^D_i \neq \tilde{y}^D_i$ is satisfied, the matrix $A$ could be complex. Moreover, if $\mu_D\mu_D^\dagger < \mathcal{F}^d\mathcal{F}^{d\dagger}$ is satisfied, the complex phase in $A$ could have a substantial magnitude since the second term is comparable with the first term. Since the CKM matrix is determined as $V_{\rm CKM} = O^{LT}A$, the *CP* phase of $V_{\rm CKM}$ is caused by the phase of $A$. Here, we have to note that whether such phases could be physical or not is dependent on the flavor structure of Yukawa couplings $y^d$, $y^D$, and $\tilde{y}^D$. It should also be noted that the matrix $A$ needs to take an almost diagonal form as long as there is no correlation between $A$ and $O^L$ since $V_{\rm CKM}$ has a nearly diagonal form. It may be instructive to show how the physical phase could be induced through this mechanism using a concrete example. We give such an example in the Appendix.

### C. Neutrino masses and the PMNS matrix

In the lepton sector, we can treat the charged lepton sector in the same way as the down-type quark sector. In fact, the Yukawa couplings in Eq. (10) induce the charged lepton mass matrix as follows,

$$(\bar{e}_{Li}, \bar{E}_L)\begin{pmatrix} m^e_{ij} & 0 \\ \mathcal{F}^e_j & \mu_E \end{pmatrix}\begin{pmatrix} e_{R_j} \\ E_R \end{pmatrix}, \tag{22}$$

where $m^e_{ij} = y^e_{ij}\langle\tilde{\phi}\rangle$, $\mathcal{F}^e_j = (y^E_j u e^{i\rho} + \tilde{y}^E_j u e^{-i\rho})$, and $\mu_E = x_E w e^{i\chi}$. Since the mass matrix takes the same form as the one of the down-type quarks (18), the diagonalization

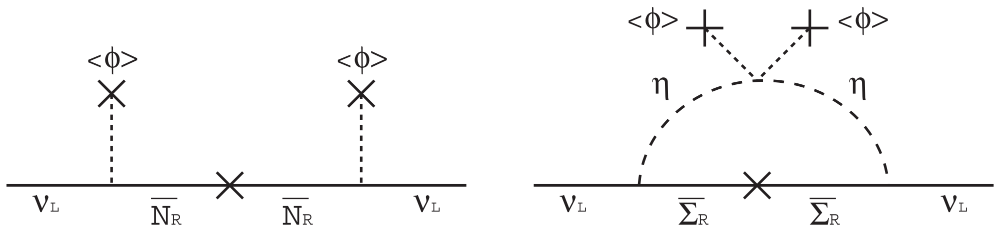


FIG. 1. Left: A diagram for the neutrino mass generation due to the type I seesaw in the minimal model. Right: A one-loop diagram for the neutrino mass generation due to the scotogenic type III seesaw in the extended model.

matrix $\tilde{A}$ for the above charged lepton mass matrix could be complex, and it should satisfy the relation

$$\tilde{A}^{-1}\tilde{m}^2\tilde{A} = m^e m^{e\dagger} - \frac{1}{\mu_E\mu_E^\dagger + \mathcal{F}^e\mathcal{F}^{e\dagger}}(m^e\mathcal{F}^{e\dagger})(\mathcal{F}^e m^{e\dagger}), \tag{23}$$

where $\tilde{m}^2$ corresponds to the diagonalized mass matrix $m^2$ in Eq. (18). As long as $\mu_E\mu_E^\dagger < \mathcal{F}^e\mathcal{F}^{e\dagger}$ is satisfied, non-negligible $CP$ phases could be expected in $\tilde{A}$ in the same way as the down-type quark sector.

On the other hand, small neutrino masses are expected to be produced not only by the type I seesaw [17] but also by the scotogenic type III seesaw [18] in this model. In fact, the lepton sector of the model has the structure in which the scotogenic type III seesaw mechanism could work, as found from the terms contained in Eqs. (10) and (12). Diagrams which contribute to the neutrino mass generation are shown in Fig. 1.

### 1. Neutrino masses due to the type I seesaw

The singlet fermions $N_{R_i}$ get Majorana mass via the VEV $\langle S\rangle$. On the other hand, they have Yukawa couplings with the doublet leptons and the ordinary Higgs doublet scalar $\phi$. Thus, the ordinary type I seesaw makes neutrinos $\nu_{L_i}$ massive through the diagram shown in the left of Fig. 1. The neutrino mass matrix caused by this can be written as

$$(\bar{\nu}_L^c, \bar{N}_R^c)\begin{pmatrix} 0 & y^\nu\langle\phi\rangle \\ y^{\nu T}\langle\phi\rangle & y^N u e^{i\rho} + \tilde{y}^N u e^{-i\rho} \end{pmatrix}\begin{pmatrix} \nu_L \\ N_R \end{pmatrix}. \tag{24}$$

Since $u \gg \langle\phi\rangle$ is supposed in this model, the contribution to neutrino masses from this diagram is estimated as

$$\mathcal{M}_{ij}^{(a)} = \sum_{k=1}^{3} y_{ik}^\nu y_{jk}^\nu \frac{\langle\phi\rangle^2}{y_k^N u e^{i\rho} + \tilde{y}_k^N u e^{-i\rho}}. \tag{25}$$

The neutrino Yukawa couplings $y_{ik}^\nu$ satisfy the same relation as the Yukawa couplings of the up-type quarks as found in Eq. (11). Since $\tilde{A}$ is expected to take an almost diagonal form as $A$, the PMNS matrix is considered to have a similar form to the CKM matrix. This means that other contributions to the neutrino masses are indispensable for the explanation of large flavor mixing required by the neutrino oscillation data. This is one of the reasons why we consider the extended structure with $\eta$ and $\Sigma_{R_\alpha}$. These fields could give additional contributions to the neutrino masses in the following way.

### 2. Neutrino masses due to the scotogenic type III seesaw

As found in Eq. (12), $\Sigma_{R_\alpha}$ has Yukawa couplings with $\nu_{L_i}$. However, since $\phi$ has no coupling with these and $\eta$ is assumed to have no VEV, neutrino masses via $\Sigma_{R_\alpha}$ are not generated at a tree level but are generated at a one-loop level. The coupling $\frac{\lambda_5}{2}(\eta^\dagger\phi)^2 + \mathrm{H.c.}$ brings about a small mass difference between the real and imaginary components of $\eta^0$. As its result, the one-loop diagram shown in the right of Fig. 1 gives a contribution to the neutrino masses. It can be estimated as

$$\begin{aligned}\mathcal{M}_{ij}^{(b)} &= \sum_{\alpha=1}^{n_\Sigma} \frac{h_{i\alpha}h_{j\alpha}\lambda_5\langle\phi\rangle^2 e^{-i\rho}}{32\pi^2 M_{\Sigma_\alpha}} \\ &\quad\times\left[\frac{M_{\Sigma_\alpha}^2}{M_\eta^2 - M_{\Sigma_\alpha}^2}\left(1 + \frac{M_{\Sigma_\alpha}^2}{M_\eta^2 - M_{\Sigma_\alpha}^2}\ln\frac{M_{\Sigma_\alpha}^2}{M_\eta^2}\right)\right] \\ &\simeq \sum_{\alpha=1}^{n_\Sigma} \frac{h_{i\alpha}h_{j\alpha}\lambda_5\langle\phi\rangle^2 e^{-i\rho}}{32\pi^2 M_{\Sigma_\alpha}}\ln\frac{M_{\Sigma_\alpha}^2}{M_\eta^2},\end{aligned} \tag{26}$$

where $M_{\Sigma_\alpha} = \gamma_{\Sigma_\alpha} w$ and $M_\eta^2 = m_\eta^2 + (\lambda_3 + \lambda_4)\langle\phi\rangle^2$. The second similarity is satisfied for $M_\eta = O(1)$ TeV since $w$ is much larger than a TeV scale as discussed in the previous part. Although neutrino mass eigenvalues are determined through $\mathcal{M}_{ij}^\nu = \mathcal{M}_{ij}^{(a)} + \mathcal{M}_{ij}^{(b)}$, $\mathcal{M}_{ij}^{(a)}$ should be sufficiently small compared with $\mathcal{M}_{ij}^{(b)}$ for large flavor mixings. If we consider that this matrix is diagonalized by a unitary matrix $U$ as $U^T\mathcal{M}^\nu U = \mathcal{M}^{\mathrm{diag}}$, the PMNS matrix is obtained as $V_{\mathrm{PMNS}} = \tilde{A}^\dagger U$ which could have a Dirac phase and two Majorana phases. An example of $V_{\mathrm{PMNS}}$ obtained through this framework in a simple model is given in the Appendix.

Next, we address the constraint on the relevant parameters caused by the neutrino oscillation data. Since $\mathcal{M}^{(a)}$ should be a subdominant contribution to the neutrino masses, we have to extend the model at least with two triplet fermions ($n_\Sigma = 2$) for the explanation of the neutrino oscillation data. In order to estimate the required magnitude of the neutrino Yukawa couplings in such a case, we suppose, for simplicity and definiteness, the tribimaximal flavor structure for $h_{i\alpha}$ as [19]

$$h_{e1} = 0, \quad h_{\mu 1} = h_{\tau 1} \equiv h_1; \quad h_{e2} = h_{\mu 2} = -h_{\tau 2} \equiv h_2, \tag{27}$$

and also diagonal $y^\nu_{ij}$ such as $y^\nu_{ij} = y^\nu_i \delta_{ij}$ with $y^\nu_1 \ll y^\nu_2 \ll y^\nu_3$.[7] We also assume $y^N_{1,2} = 0$ and $\tilde{y}^N_3 = 0$, for simplicity. Under this assumption, if the normal hierarchy for the neutrino masses is assumed, squared mass differences required by the neutrino oscillation data suggest [20]

$$\begin{aligned}
h_1^2 &\simeq 9.3 \times 10^{-3} \left(\frac{10^{-2}}{\lambda_5}\right)\left(\frac{M_{\Sigma_1}}{10^{10}\ \mathrm{GeV}}\right) \\
&\quad \times \left[1 - \left(\frac{y^\nu_3}{4.4 \times 10^{-3}}\right)^2 \left(\frac{10^{10}\ \mathrm{GeV}}{M_{N_3}}\right)\right], \\
h_2^2 &\simeq 9.3 \times 10^{-4} \left(\frac{10^{-2}}{\lambda_5}\right)\left(\frac{M_{\Sigma_2}}{10^{10}\ \mathrm{GeV}}\right), \\
y^{\nu 2}_1 &< 9.2 \times 10^{-7} \left(\frac{M_{N_1}}{10^9\ \mathrm{GeV}}\right), \\
y^{\nu 2}_2 &< 9.2 \times 10^{-7} \left(\frac{M_{N_2}}{10^9\ \mathrm{GeV}}\right), \\
y^{\nu 2}_3 &< 1.0 \times 10^{-5} \left(\frac{M_{N_3}}{10^{10}\ \mathrm{GeV}}\right),
\end{aligned} \tag{28}$$

where $M_{N_{1,2}} = \tilde{y}^N_{1,2} u$ and $M_{N_3} = y^N_3 u$, and $M_\eta = 1$ TeV is also assumed.

Finally, it may be useful to present a remark on the extension by the vectorlike fermions. Although these fermions are introduced to the down sector in the above discussion, the CKM phase could be derived in the same way even if we introduce them to the up sector. However, the situation could be largely changed for the $CP$ phases in the PMNS matrix and the small neutrino mass generation. The present choice seems to be crucial for the present scenario. It could also play an important role when we consider an embedding of the model into a fundamental model at the Planck scale region.[8]

### D. Spontaneous $CP$ violation

In the previous part, we just assume that Eq. (13) is realized as a potential minimum. Here, we discuss in what situation the spontaneous $CP$ violation could occur in a realistic way in the present model. The condition required for the spontaneous $CP$ violation has been studied in detail in [22]. If we follow their results, the VEVs of $\sigma$ is found not to break the $CP$ symmetry spontaneously, and then $\chi = 0$. The reason is that the spurions for it cannot be introduced since the imposed global $U(1)$ symmetry is assumed to be exact except for the color anomaly effect. On the other hand, we can introduce the spurions for $S$ which has no global $U(1)$ charge. In fact, if we introduce the terms such as $S^4$ and $S^2$ which break a $U(1)$ symmetry corresponding to the $S$ number, a nonzero $\rho$ could appear as a potential minimum.[9]

The relevant potential is found from Eq. (6) to be

$$\begin{aligned}
V_{CP} &= \bar{m}_S^2 (S^\dagger S) + \bar{m}_\sigma^2 (\sigma^\dagger \sigma) + \kappa_S (S^\dagger S)^2 + \kappa_\sigma (\sigma^\dagger \sigma)^2 \\
&\quad + \kappa_{S\sigma} (S^\dagger S)(\sigma^\dagger \sigma) + V_b,
\end{aligned} \tag{29}$$

where $\bar{m}_a^2 = \tilde{m}_a^2 + \zeta_a \Lambda^2$ ($a = S, \sigma$) and $\tilde{m}_a^2 > 0$ and $\zeta_a < 0$ are assumed since we suppose that the potential minimum is fixed as a result of the $SU(4)$ breaking. As examples, we consider two cases for $V_b$ in (29) such as[10]

$$\begin{aligned}
&\text{(i)}\ V_b = \alpha (S^4 + S^{\dagger 4}) + \mu^2 (S^2 + S^{\dagger 2}), \\
&\text{(ii)}\ V_b = \alpha (S^4 + S^{\dagger 4}) + \beta (S^2 + S^{\dagger 2})(\sigma^\dagger \sigma).
\end{aligned} \tag{30}$$

Here, we confine our study to the situation where the VEVs $u$ and $w$ are determined by a part of $V_{CP}$ except for $V_b$. It could be realized for $\kappa_S \gg \alpha$ and $|\bar{m}_S^2| \gg |\mu^2|$ in the case (i) and also for $\kappa_S \gg \alpha$ and $|\beta| \ll 1$ in the case (ii). The potential minimum could be found for sufficiently small $|\kappa_{S\sigma}|$ in both cases:

$$u^2 = -\frac{\bar{m}_S^2}{2\kappa_S}, \qquad w^2 = -\frac{\bar{m}_\sigma^2}{2\kappa_\sigma}, \tag{31}$$

and also the $CP$ phase is determined as

$$\text{(i)}\ \cos 2\rho = -\frac{\mu^2}{4\alpha u^2}, \quad \text{(ii)}\ \cos 2\rho = -\frac{\beta w^2}{4\alpha u^2}, \tag{32}$$

in each case. These examples show that the spontaneous $CP$ violation could occur through the scalar $S$ as long as suitable values of the parameters are chosen. In fact, for example, if $\mu^2 = -4\alpha u^2$ is satisfied for $\alpha \ll 1$ and $|\mu^2| \ll u^2$, the maximum $CP$ phase $\rho \simeq \frac{\pi}{2}$ could be realized in the case (i). We should note that these conditions on $\alpha$ and $\mu^2$ are consistent with the requirement for which $u$ and $w$ are determined as Eq. (31). In the case (ii), the maximum $CP$

[7] This assumption is adopted due to the relation (11) to the up-type quarks which is caused by the $SU(4)$ symmetry.

[8] The model might be embedded into an effective model derived by a suitable compactification of the $E_8 \times E_8'$ superstring [21].

[9] We do not consider such terms for $s$ in the present study.

[10] We note that terms proportional to $S^2$ are induced through the $SU(4)$ breaking from an operator $\Phi^\dagger \Psi S^2$, which is invariant under the imposed symmetry.

phase is obtained for $\beta w^2 \simeq 4\alpha u^2$, which is consistent with the determination of $u$ and $w$ as found from Eq. (31). As a result of this symmetry breaking, the mass of $S$ is fixed as $m_{S_R} = \sqrt{4\kappa_S}u$.

On the other hand, in order for this breaking to cause large $CP$ phases in both the CKM and PMNS matrices, the conditions $\mu_D\mu_D^\dagger < \mathcal{F}^d\mathcal{F}^{d\dagger}$ and $\mu_E\mu_E^\dagger < \mathcal{F}^e\mathcal{F}^{e\dagger}$ should be satisfied as discussed before. They are supposed to require

$$u > w, \tag{33}$$

as long as the relevant Yukawa couplings have a similar magnitude. This condition can be easily satisfied for suitable parameters as found from Eq. (31). Although the tuning of parameters is necessary, the present scenario is found to work as long as the scalar potential takes a suitable form. We can expect that the required $CP$ violation is induced in both the quark and lepton sectors based on the same origin.

## III. PHENOMENOLOGY

In the previous part, we addressed that the $CP$ problem in the SM could be solved in this model. In this section, we order several discussions and comments on other phenomenological issues.

### A. Inflation

The model has candidates for the inflaton, such as $\sigma$ and $S$. They can have nonminimal couplings with the Ricci scalar $R$ [23]:

$$\frac{1}{2}\xi_\sigma\sigma^\dagger\sigma R, \qquad \frac{1}{2}\left[\xi_{S_1}S^\dagger S + \frac{\xi_{S_2}}{2}(S^2 + S^{\dagger 2})\right]R. \tag{34}$$

Although the real and imaginary components of $\sigma$ have the same coupling $\xi_\sigma$, only the real part of $S$ could have a nonzero coupling $\frac{1}{2}\xi S_R^2 R$ in the case $\xi_{S_1} = \xi_{S_2}$ where $S \equiv \frac{1}{\sqrt{2}}(S_R + iS_I)$ and $\xi \equiv \xi_{S_1} + \xi_{S_2}$. If we suppose that a coupling $\xi$ takes a sufficiently large value in such a case, inflation via $S_R$ is expected to occur in the same way as the Higgs inflation [24]. A nice feature in this scenario is that the dangerous unitarity violation caused by a higher order mixing between $S_R$ and $S_I$ [25] is not induced at $\frac{M_{\rm pl}}{\xi}$ but could be suppressed at least up to an inflation scale $\frac{M_{\rm pl}}{\sqrt{\xi}}$ [26].

The potential of the inflaton can be expressed in the Einstein frame as

$$V_E = \frac{\kappa_S}{\left(1 + \frac{\xi S_R^2}{M_{\rm pl}^2}\right)^2}\left[\frac{1}{2}(S_R^2 + S_I^2) - u^2\right]^2. \tag{35}$$

Since the canonically normalized inflaton $\chi$ is defined as

$$\frac{d\chi}{dS_R} = \frac{1}{1 + \frac{\xi S_R^2}{M_{\rm pl}^2}}\left(1 + \frac{\xi S_R^2}{M_{\rm pl}^2} + \frac{6\xi^2 S_R^2}{M_{\rm pl}^2}\right)^{1/2}, \tag{36}$$

$\chi$ and $S_R$ are related each other as $S_R \propto \exp\frac{\chi}{\sqrt{6}M_{\rm pl}}$ at a large field region $S_R^2 \gg \frac{M_{\rm pl}^2}{\xi}$. In that region, the potential of $\chi$ becomes constant $V_E = \frac{\kappa_S M_{\rm pl}^4}{4\xi^2}$ as long as $S_R \gg S_I$ is satisfied. The slow roll parameters for $\chi$ can be expressed as

$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V_E'}{V_E}\right)^2 = \frac{3}{4N_e^2}, \qquad \eta \equiv M_{\rm pl}^2\frac{V_E''}{V_E} = -\frac{1}{N_e} \tag{37}$$

by using the e-foldings number $N_e$. If we take $N_e = 60$, we obtain the spectral index $n_s = 0.97$ and the tensor-to-scalar ratio $r = 3.3 \times 10^{-3}$. On the other hand, since the amplitude of scalar perturbation is given as $A_S = \frac{V_E}{24\pi^2 M_{\rm pl}^4\epsilon}$ and the CMB observation constrains it as $A_S = 2.4 \times 10^{-9}$ at $k_* = 0.002$ Mpc$^{-1}$ [27], $\kappa_S$ has to satisfy $\kappa_S = 4.7 \times 10^{-10}\xi^2$ for $N_e = 60$. Using this constraint, the inflaton mass is found to be determined as

$$m_{S_R} = 4.3 \times 10^{10}\left(\frac{\xi}{10^3}\right)\left(\frac{u}{10^{12}\ \text{GeV}}\right)\ \text{GeV}. \tag{38}$$

The inflaton mass should be fixed in a consistent way with Eqs. (16) and (33). We also note that the assumed vacuum with the spontaneous $CP$ violation could be consistently realized for suitable parameters in this inflation framework.

The reheating after the end of inflation is expected to be caused by the inflaton decay to the singlet neutrino pairs $N_iN_i$ through the couplings in Eq. (12). In the case $y_3^N > \tilde{y}_{1,2}^N$ which is assumed in this study, a dominant process is $S_R \to N_3N_3$. Since singlet fermions $N_i$ interact with other fields only through the neutrino Yukawa couplings, except for the couplings with $S$ and $S^*$, instantaneous reheating is expected to occur for the case $M_{S_R} > 2M_{N_3}$ and $H \simeq \Gamma_S \gtrsim \Gamma_{N_3}$, where $\Gamma_S$ and $\Gamma_{N_3}$ are the decay width of $S_R \to N_3N_3$ and $N_3 \to \bar{\ell}_i\phi^\dagger$, respectively.[11] If we take account of these conditions which may be expressed as $\sqrt{\kappa_S} > y_3^N$ and $2\sqrt{\kappa_S}y_3^N \gtrsim y_3^{\nu 2}$, the reheating temperature $T_R$ could be bounded as[12]

[11]Here, we do not consider a possibility for nonthermal leptogenesis which could be expected to occur for the case $\Gamma_{N_i} > \Gamma_S$ [28].

[12]The restoration of the PQ symmetry could occur in the reheating process depending on the parameters. However, since the domain wall number is one in this model, no domain wall problem is induced even if the PQ symmetry is restored.

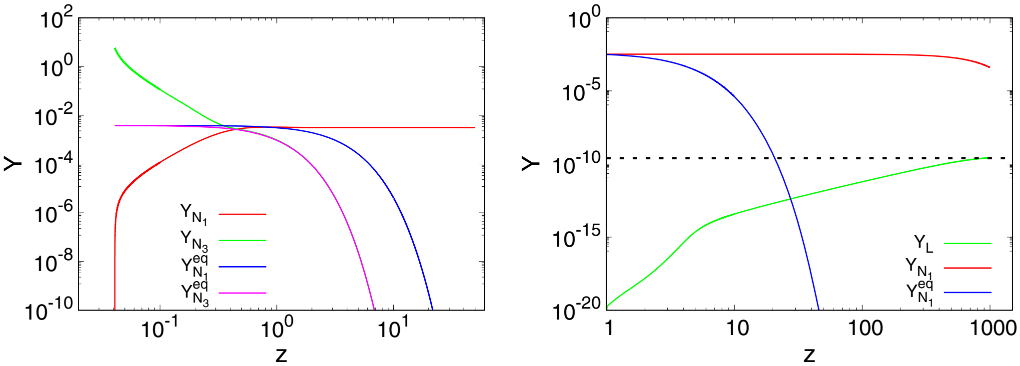


FIG. 2. Left panel: A typical solution of the Boltzmann equations for $Y_{N_1}$ and $Y_{N_3}$. Their equilibrium values $Y^{\rm eq}_{N_1}$ and $Y^{\rm eq}_{N_3}$ are also plotted in the same panel. While $Y_{N_3}$ is found to follow $Y^{\rm eq}_{N_3}$ at $z \gtrsim 0.4$, $Y_{N_1}$ keeps a constant value until $N_1$ starts decaying. Right panel: The evolution of the lepton number asymmetry $Y_L$ generated through the out-of-equilibrium decay of $N_1$. Horizontal dotted lines show the value of $Y_L$ required in this model to realize the baryon number asymmetry in the Universe [20].

$$T_R \simeq 1.6\times 10^8 y_3^\nu (y_3^N u)^{1/2} < 5\times 10^{11}\left(\frac{\xi}{10^3}\right)^{3/2}\left(\frac{u}{10^{12}\ \mathrm{GeV}}\right)^{1/2}\ \mathrm{GeV}. \quad (39)$$

Although this shows that $T_R > M_{N_1}(\equiv \tilde{y}_1^N u)$ could be satisfied for suitable parameters, $N_1$ is not expected to be thermalized as a relativistic particle since the Yukawa coupling $y_1^\nu$ of $N_1$ is supposed to be very small. Fortunately, it could be expected to reach the thermal equilibrium through the scattering process $N_3N_3 \to N_1N_1$ mediated by the scalar $S_R$ before it becomes nonrelativistic ($T < M_{N_1}$). This allows the present model to generate the lepton number asymmetry sufficiently through the out-of-equilibrium decay of $N_1$, although the Yukawa coupling $y_1^\nu$ of $N_1$ is very small. We discuss this possibility in the next part.

### B. Leptogenesis

The model has two possible decay processes $N_1 \to \ell_i \phi^\dagger$ and $\Sigma_\alpha \to \ell_i \eta^\dagger$ which could contribute to the generation of the lepton number asymmetry since these processes violate the lepton number. However, $\Sigma_\alpha$ has the $SU(2)$ gauge interaction so that its out-of-equilibrium decay is impossible, at least before the electroweak symmetry breaking. On the other hand, as addressed above, $N_1$ could reach the equilibrium abundance through the scattering mediated by $S_R$ even if its neutrino Yukawa coupling $y_1^\nu$ is very small. In that case, its decay could generate the lepton number asymmetry through the out-of-equilibrium decay at $T < M_{N_1}$. As long as the couplings $y_i^N$ or $\tilde{y}_i^N$ between the inflaton and $N_i$ have sufficient magnitude, such as $y_3^N > \tilde{y}_2^N \gtrsim \tilde{y}_1^N \gtrsim 10^{-3}$, the equilibrium number density of $N_1$ can be easily realized at $T > M_{N_1}$ as shown below.[13] On the other hand, since the mass of $N_i$ is generated through the Yukawa coupling $(y_i^N S + \tilde{y}_i^N S^*)\bar{N}_i^c N_i$, $N_i$ cannot be light if we take account of the values of $y_i^N$ and $\tilde{y}_i^N$ mentioned above. In fact, under the constraints (16) and (33), the mass of $N_1$ has to be $M_1 > 10^7$ GeV at least.

In order to check whether this scenario works, we present a typical solution of the Boltzmann equations for $Y_{N_1}$ and $Y_{N_3}$ as functions of $z(\equiv \frac{M_{N_1}}{T})$ in the left panel of Fig. 2. Here, $Y_{N_i}$ is defined as $Y_{N_i} = \frac{n_{N_i}}{s}$ with the $N_i$ number density $n_{N_i}$ and the entropy density $s$. In this calculation, as an example, we assume $u = 2\times 10^{12}$ GeV and $\xi = 500$, and then the inflaton mass is fixed as $m_{S_R} = 4.3\times 10^{10}$ GeV. Taking account of the constraints in Eq. (28), we fix other relevant parameters at the following values[14]

$$y_3^N = 10^{-2}, \quad \tilde{y}_2^N = 10^{-0.2} y_3^N, \quad \tilde{y}_1^N = 10^{-0.5} y_3^N$$
$$y_3^\nu = 2.8\times 10^{-3}, \quad y_2^\nu = 10^{-5}, \quad y_1^\nu = 10^{-6}. \quad (40)$$

Since $m_{S_R} > 2M_{N_3}$ is satisfied, $N_3$ is allowed to be produced through the inflaton decay $S_R \to N_3N_3$. The following $N_3$ decay $N_3 \to \ell_i \phi^\dagger$ caused by the coupling $y_3^\nu$ is considered to be a substantial process for the thermalization. Thus, the initial value of $Y_{N_3}$ is fixed as the one produced through this inflaton decay assuming the instantaneous reheating. The figure shows that $Y_{N_1}$ reaches the equilibrium value $Y^{\rm eq}_{N_1}$ around $z \simeq 1$ for the assumed value of $\tilde{y}_1^N$ and leaves its equilibrium value at $z \gtrsim 1$, where the out-of-equilibrium decay could generate the lepton number asymmetry.[15]

[13]We should recall that $y_{1,2}^N = 0$ and $\tilde{y}_3^N = 0$ are assumed.

[14]These values of $y_i^\nu$ require some overall suppression effect compared with the Yukawa couplings of the up-type quarks in Eq. (11). We just assume it in this setting.

[15]For a smaller value of $\tilde{y}_1^N$ or a larger value of $y_3^\nu$, $Y_{N_1}$ cannot reach an equilibrium value $Y^{\rm eq}_{N_1}$ for $z < 1$, although $Y_{N_1}$ keeps a constant value in the same way as the one in the left panel of Fig. 2. A larger value of $\tilde{y}_1^N$ realizes $Y_{N_1} = Y^{\rm eq}_{N_1}$ at an earlier stage.

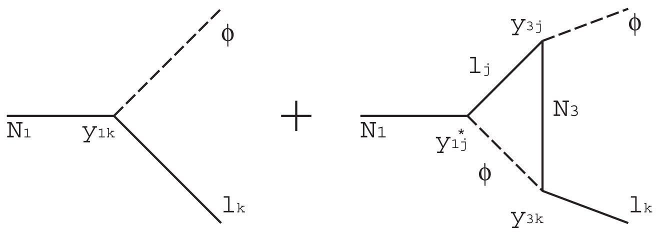


FIG. 3. The $N_1$ decay diagrams which contribute to the generation of the lepton number asymmetry. The interference between them causes the $CP$ asymmetry $\varepsilon$.

The generated lepton number asymmetry through the $N_1$ decay is converted to the baryon number asymmetry through the sphaleron process as in the usual leptogenesis [29,30]. In the present model, the $CP$ asymmetry $\varepsilon$ for the decay $N_1 \to \ell\phi^\dagger$ [31] is dominantly caused by an interference between a tree diagram and a one-loop diagram mediated by $N_3$ which are shown in Fig. 3. Under the assumption given in Eq. (27), it can be estimated as

$$\begin{aligned}\varepsilon &\equiv \frac{\Gamma(N_1\to\ell\phi^\dagger)-\Gamma(N_1^c\to\bar{\ell}\phi)}{\Gamma(N_1\to\ell\phi^\dagger)+\Gamma(N_1^c\to\bar{\ell}\phi)} \\ &= \frac{1}{8\pi}\frac{\mathrm{Im}(\sum_i \tilde{y}^\nu_{1i}e^{-i\frac{\rho}{2}}y^{\nu*}_{3i}e^{-i\frac{\rho}{2}})^2}{(\sum_i y^\nu_{1i}y^{\nu*}_{1i})}F\left(\frac{M^2_{N_3}}{M^2_{N_1}}\right) \\ &= \frac{1}{4\pi}|y^\nu_3|^2 F\left(\left(\frac{y^N_3}{\tilde{y}^N_1}\right)^2\right)\sin(-2\rho), \end{aligned} \tag{41}$$

where $F(x)$ is defined as

$$F(x)=\sqrt{x}\left(1-(1+x)\ln\frac{1+x}{x}\right). \tag{42}$$

In the following analysis, we assume $\sin(-2\rho)=1$ which makes $\varepsilon$ maximal.

If $N_1$ is in the thermal equilibrium at $z<1$, the out-of-equilibrium decay of $N_1$ could start at $z\sim 1$ and the lepton number asymmetry is effectively generated at $z>1$. By introducing an efficiency factor for the washout of the generated lepton number asymmetry as $\kappa$, the lepton number asymmetry $Y_L$, which is defined as $Y_L\equiv\frac{n_L}{s}$ by using a net lepton number density $n_L$, is roughly estimated as $Y_L=\varepsilon\kappa Y^{\rm eq}_{N_1}|_{z=1}$. It suggests that $\varepsilon\gtrsim 8\times10^{-8}\kappa^{-1}$ is necessary to realize a value $Y_L\gtrsim 2.5\times 10^{-10}$ at a sphaleron decoupling temperature in order to produce the sufficient baryon number asymmetry in the Universe for $Y^{\rm eq}_{N_1}|_{z=1}\simeq 3.1\times10^{-3}$. Since $y^\nu_1$ is supposed to be very small in this model, $N_1$ is considered to start its substantial decay at a later stage, such as $z\gg 1$, where the washout caused by $N_3$ and $\Sigma_\alpha$ could be largely Boltzmann suppressed as long as $\frac{M_{N_3}}{M_{N_1}},\frac{M_{\Sigma_\alpha}}{M_{N_1}}>1$ are satisfied. Thus, in such a case, the almost all lepton number asymmetry generated there could be kept and the sufficient lepton number asymmetry is expected to be generated through the out-of-equilibrium decay of $N_1$.

TABLE I. The $CP$ asymmetry $\varepsilon$ and the generated baryon number asymmetry $Y_B$ for the parameters in Eq. (40) with $u=2\times10^{12}$ GeV and $\xi=500$, which realize the spectrum $\frac{m_{S_R}}{2}>M_{N_3}>M_{N_2}>M_{N_1}$. The Yukawa couplings $h_{1,2}$ of $\Sigma_{1,2}$ are determined through the neutrino oscillation conditions (28) by assuming the values of $|\lambda_5|$ and $M_{\Sigma_{1,2}}$.

| $M_{\Sigma_{1,2}}$ | $\lvert\lambda_5\rvert$ | $h_1$ | $h_2$ | $\lvert\varepsilon\rvert$ | $Y_B$ |
|---|---|---|---|---|---|
| $3M_{N_1}$ | $10^{-1}$ | $3.7\times10^{-2}$ | $1.3\times10^{-2}$ | $9.6\times10^{-8}$ | $9.5\times10^{-11}$ |
| $3M_{N_1}$ | $10^{-1.5}$ | $6.6\times10^{-2}$ | $2.4\times10^{-2}$ | $9.6\times10^{-8}$ | $9.4\times10^{-11}$ |
| $3M_{N_1}$ | $10^{-2}$ | $1.2\times10^{-1}$ | $4.2\times10^{-2}$ | $9.6\times10^{-8}$ | $7.2\times10^{-11}$ |
| $5M_{N_1}$ | $10^{-1.5}$ | $8.4\times10^{-2}$ | $3.0\times10^{-2}$ | $9.6\times10^{-8}$ | $9.4\times10^{-11}$ |
| $10M_{N_1}$ | $10^{-1.5}$ | $1.2\times10^{-1}$ | $4.2\times10^{-2}$ | $9.6\times10^{-8}$ | $9.4\times10^{-11}$ |

In the right panel of Fig. 2, we present the evolution of the lepton number asymmetry $Y_L$ generated through the out-of-equilibrium decay of $N_1$ using the same parameters given in Eq. (40), which can prepare an initial value $Y_{N_1}(1)\simeq Y^{\rm eq}_{N_1}(1)$ as shown in the left panel. In this analysis of Boltzmann equations, we fully take account of the washout processes and use the neutrino Yukawa couplings $h_{1,2}$ which are fixed by taking account of the condition (28) with $M_\eta=10^3$ GeV, $M_{\Sigma_{1,2}}=3M_{N_1}$, and $|\lambda_5|=10^{-1.5}$. The small neutrino Yukawa coupling $y^\nu_1$ makes the $N_1$ decay be delayed until the temperature where the washout processes could be frozen out due to the Boltzmann suppression. This feature can be found in the behavior of $Y_N$ and $Y_L$ in the right panel. As its result, almost all the lepton number asymmetry generated through the out-of-equilibrium $N_1$ decay could be converted to the baryon number asymmetry in the Universe as discussed above. The model is found to present a successful leptogenesis framework. Results of the analysis for several parameter settings are also listed in Table I.

Here, we order a few remarks related to these results. First, since a smaller $|\lambda_5|$ makes $h_{1,2}$ larger through the neutrino mass condition (28) for the fixed $M_{\Sigma_{1,2}}$, the washout processes mediated by $\Sigma_{1,2}$ are considered to suppress the generation of the lepton number asymmetry at an early stage where it is not frozen out. Second, the $N_1$ mass seems to be bounded as $M_1>10^9$ GeV in the present model in order to produce the required baryon number asymmetry. This bound is similar to the one given in [32]. Third, for the present parameter settings, $w\gtrsim10^{10}$ GeV seems to be required to avoid the washout of the generated lepton number asymmetry, which is consistent with the requirement from the PQ symmetry breaking scale. Finally, the coexistence of the couplings $y^N_i$ and $\tilde{y}^N_i$, such as $y^N_i\neq\tilde{y}^N_i$ in Eq. (12), is crucial for the leptogenesis. We should recall that the same feature is required in the explanation of the CKM phase through the mass matrix (17).

### C. Dark matter

The model has three dark matter (DM) candidates, that is, the axion, the neutral component of $\Sigma_\alpha$, and the lightest neutral component of $\eta$. The axion could explain the required DM abundance as long as $w \simeq 10^{12}$ GeV is satisfied [16]. The latter two have odd parity of the remnant $Z_2$ of the global $U(1)$ symmetry, which makes them stable and then DM candidates. However, $\Sigma_\alpha^0$ is supposed to have a large mass so that it cannot be DM in the present model.[16] On the other hand, $\eta$ is assumed to have a mass of $O(1)$ TeV as discussed in the neutrino mass generation. In that case, the lightest neutral component of $\eta$ can be DM. Moreover, even if the VEV $w$ is not large enough to guarantee the sufficient axion density for the explanation of the DM energy density, the thermal relics of $\eta^0$ could explain it as long as the quartic couplings $\lambda_{3,4}$ in Eq. (14) take suitable values [33,34]. As a result, the breaking scale $w$ of the PQ symmetry could be free from the explanation of the DM energy density in this model.

### D. Quark and lepton mass hierarchy

Yukawa coupling constants for quarks and leptons are related to each other by Eq. (11) at a $SU(4)$ breaking scale $\Lambda$. On the other hand, their weak scale values, which determine mass eigenvalues of the quarks and the leptons, are fixed through the renormalization group equations taking them as the initial values. It can bring about a difference of a factor three due to the color effect between quarks and leptons. The mass difference between the down-type quarks and the charged leptons seems to be partially explained by this effect, but it is not satisfactory. Even if corrections caused by the mixing with heavy fermions in these sectors are taken into account, this situation is not improved and then some new ingredients are needed to be introduced for it.

On the other hand, in the up-type quarks and the neutrinos, several additional parameters related to the neutrino mass generation could give a different feature in these sectors. Especially since neutrino masses are determined by the type III seesaw contribution, the relation among the Yukawa couplings of quarks and leptons at the high energy scale does not directly affect their mass matrices. These features could make the large difference found in the CKM and PMNS matrices be consistently realized in the present unification scheme. Since details depend on the model parameters, and this issue is beyond the scope of present study, we will not discuss it further here and leave it to future study. Finally, it may be useful to note the fact that the present unification scheme could make the leptogenesis work well. A requirement that the third generation Yukawa coupling of the up-quark sector should be much larger than others brings about the relation $y^\nu_{1,2} \ll y^\nu_3$ in the neutrino sector, which plays a crucial role in the present leptogenesis scenario as shown in the above study.

[16]The DM study in the cases where $\Sigma$ has a mass of $O(1)$ TeV can be found in [18].

## IV. SUMMARY

We proposed a model which gives the origin of the $CP$ violation at an intermediate scale. In this model, the $CP$ symmetry is supposed to be spontaneously broken, but it does not cause the strong $CP$ problem and $\bar{\theta} = 0$ is kept even if the radiative corrections are taken into account. We showed that such a model could be realized in a Pati-Salam-type unification model, in which $CP$ phases in both the CKM and PMNS matrices are derived from the same source. Neutrino masses are generated in a hybrid way by the tree level type I seesaw and the one-loop type III seesaw. The required baryon number asymmetry can be produced through the leptogenesis. The out-of-equilibrium decay of $N_1$ occurs at a later stage where the washout effects are almost frozen out. As a result, the generated lepton number asymmetry could be effectively converted to the baryon number asymmetry. This feature comes from the present unification based on the fact that the top Yukawa coupling is much larger than others. The model has two DM candidates and the dominant DM is fixed depending on the intermediate symmetry breaking scale. Since the axion needs not to be DM, the PQ symmetry breaking scale can be free from the condition for the DM energy density realization. We also note a possibility such that the model might be derived as the low energy effective model of the $E_8 \times E_8'$ superstring. It will be discussed elsewhere.

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## APPENDIX: EXAMPLES OF THE CKM MATRIX AND THE PMNS MATRIX

In this Appendix, we present a simple example which could bring about a phase in the CKM matrix. We assume the relevant couplings $y^d$, $y^D$, and $\tilde{y}^D$ to be written as[17]

$$y^d = c\begin{pmatrix} \epsilon^4 & \epsilon^3 & x\epsilon^3 \\ \epsilon^3 & \epsilon^2 & y\epsilon^2 \\ \epsilon^2 & 1 & -1 \end{pmatrix},$$
$$y^D = (a_1, a_2, a_3), \qquad \tilde{y}^D = (b_1, b_2, b_3), \tag{A1}$$

[17]A similar Yukawa coupling matrix for the down-type quarks has been considered in a different context [35]. There is no background to explain its hierarchical structure in the present model.

by using real constants $a_i$, $b_i$, $c$, and $x$, $y$. As long as $\epsilon$ satisfies $\epsilon \ll 1$, the down-type quark mass matrix $m^d(\equiv y^d\langle\tilde{\phi}\rangle)$ has hierarchical mass eigenvalues. Here, we introduce $X_{ij}$ and $Y_{ij}$ whose definition is given as

$$X_{ij} = 1 + p_i p_j + \frac{(a_2+b_2)^2 + (a_3+b_3)^2 p_i p_j + \{a_2 b_3 + b_2 b_3 + (a_2 b_3 + a_3 b_2)\cos 2\rho\}(p_i + p_j)}{a_2^2 + a_3^2 + b_2^2 + b_3^2 + 2(a_2 b_2 + a_3 b_3)\cos 2\rho},$$
$$Y_{ij} = \frac{(a_2 b_3 - a_3 b_2)(p_i - p_j)\sin 2\rho}{a_2^2 + a_3^2 + b_2^2 + b_3^2 + 2(a_2 b_2 + a_3 b_3)\cos 2\rho}, \tag{A2}$$

where $p_i$ is fixed as $p_1 = x$, $p_2 = y$, and $p_3 = -1$. If we define $R_{ij}$ and $\theta_{ij}$ by using these quantities as

$$R_{ij} = \sqrt{X_{ij}^2 + Y_{ij}^2}, \qquad \tan\theta_{ij} = \frac{Y_{ij}}{X_{ij}}, \tag{A3}$$

the component of Eq. (23) is found to be expressed as

$$(A^{-1} m^2 A)_{ij} = c^2 \langle\tilde{\phi}\rangle^2 \epsilon_{ij} R_{ij} e^{i\theta_{ij}}, \tag{A4}$$

where $\mu_D^2 \ll \mathcal{F}^d \mathcal{F}^{d\dagger}$ is assumed. $\epsilon_{ij}$ is defined as

$$\epsilon_{11} = \epsilon^6, \quad \epsilon_{22} = \epsilon^4, \quad \epsilon_{33} = 1, \quad \epsilon_{12} = \epsilon_{21} = \epsilon^5, \quad \epsilon_{13} = \epsilon_{31} = \epsilon^3, \quad \epsilon_{23} = \epsilon_{32} = \epsilon^2. \tag{A5}$$

By solving Eq. (A4), we find that $A$ is approximately written as

$$A \simeq \begin{pmatrix} 1 & -\lambda & \lambda^3\left(\frac{X_{23}}{|\alpha|^2 X_{33}} e^{i\theta} - \frac{X_{13}}{|\alpha|^3 X_{33}}\right) \\ \lambda & 1 & -\lambda^2 \frac{X_{23}}{|\alpha|^2 X_{33}} e^{i\theta} \\ \lambda^3 \frac{X_{13}}{|\alpha|^3 X_{33}} & \lambda^2 \frac{X_{23}}{|\alpha|^2 X_{33}} e^{-i\theta} & 1 \end{pmatrix}, \tag{A6}$$

where the constants $\lambda$, $\alpha$, and $\theta$ are defined as

$$\alpha = \frac{X_{12} X_{33} - X_{13} X_{23} e^{-i(\theta_{23}+\theta_{12}-\theta_{13})}}{X_{22} X_{33} - X_{23}^2}, \qquad \lambda = |\alpha|\epsilon, \quad \theta = \arg(\alpha) + \theta_{23} + \theta_{12} - \theta_{13}. \tag{A7}$$

This expression shows that $A$ could have a nontrivial phase which gives the origin of the CKM phase as long as $a_2 b_3 - a_3 b_2 \neq 0$ and $x \neq y$ are satisfied. If the diagonalization matrix $O^L$ for the mass matrix of the up-type quarks takes an almost diagonal form, an interesting matrix could be obtained as the CKM matrix, such as $V_{\rm CKM} \simeq A$. In this case, the mass eigenvalues for the down-type quarks are obtained as

$$X_{33}^{1/2} c\langle\tilde{\phi}\rangle, \qquad \left(X_{22} - \frac{X_{23}^2}{X_{33}}\right)^{1/2} \epsilon^2 c\langle\tilde{\phi}\rangle,$$
$$\left\{X_{11} - \frac{X_{13}^2}{X_{33}} + |\alpha|^2\left(X_{22} - \frac{X_{23}^2}{X_{33}} - 2\right)\right\}^{1/2} \epsilon^3 c\langle\tilde{\phi}\rangle. \tag{A8}$$

A diagonalization matrix $\tilde{A}$ for the charged lepton mass matrix takes the same form as $A$ as a result of the Pati-Salam $SU(4)$ symmetry in the model. However, since the Yukawa couplings which induce the neutrino mass matrix could be irrelevant to the ones in the up-type quarks as discussed in the text, the large mixing in the PMNS matrix could be obtained if large flavor mixings are realized in the neutrino mass matrix. If we use the assumption in Eq. (27), the PMNS matrix in this example is found to be written as

$$V_{\mathrm{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{6}}(2-\lambda) & \frac{1}{\sqrt{3}}(1+\lambda) & \frac{1}{\sqrt{2}}\lambda \\ \frac{1}{\sqrt{6}}(-1-2\lambda+\beta\lambda^2) & \frac{1}{\sqrt{3}}(1-\lambda-\beta\lambda^2) & \frac{1}{\sqrt{2}}(1+\beta\lambda^2) \\ \frac{1}{\sqrt{6}}(1+\beta^*\lambda^2) & -\frac{1}{\sqrt{3}}(1+\beta^*\lambda^2) & \frac{1}{\sqrt{2}}(1-\beta^*\lambda^2) \end{pmatrix} + O(\lambda^3), \tag{A9}$$

where $\beta = \frac{X_{23}}{|\alpha|^2 X_{33}} e^{i\theta}$ and the Majorana phases are not taken into account.


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