Symmetry restoration due to preheating and lepton number asymmetry

KANAZAWA-17-01

June, 2017

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Daijiro Suematsu*

Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Abstract

We study a possible symmetry restoration due to the radiative effect of particles which are explosively produced in preheating after inflation. As its application, we consider a scenario for leptogenesis based on the lepton number asymmetry generated in the right-handed neutrino sector through the inflaton decay. The scenario is examined in a one-loop radiative neutrino mass model extended with singlet scalars.

*e-mail: suematsu@hep.s.kanazawa-u.ac.jp

1 Introduction

The existence of an inflationary expansion era in the early Universe seems to be justified from cosmic microwave background (CMB) observations [1, 2]. Inflation should be followed by some thermalization processes to realize an initial stage of the standard big-bang Universe. Since inflation is usually considered to be caused by potential energy of inflaton which is a slow-rolling scalar field, this energy should be converted to radiation through certain reheating processes. Reheating is expected to be brought about by interactions of inflaton with contents of the standard model (SM) or others. If we note that inflaton is usually identified with a singlet scalar, we find that couplings with singlet fermions such as right-handed neutrinos could be one of their promising possibilities. In that case, inflaton can also have quartic couplings with other scalar fields in general and the explosive particle production is expected to be induced through resonant instability called preheating [3]. Since preheating cannot convert the inflaton energy to radiation completely, inflaton should have a certain decay process to accomplish the reheating. From this point of view, the existence of the above mentioned coupling with the singlet fermions seems to be favored. Although the final reheating temperature is expected to be fixed through this decay process, it has been suggested that preheating could play an important role in various phenomena which occurred at the early stage of evolution of the Universe, for example, symmetry restoration [4, 5], baryogenesis [6], phase transition [7], secondary inflation [8] and tachyonic preheating [9]. We would like to propose a leptogenesis scenario under the coexistence of such couplings.

In this paper, we discuss the symmetry restoration caused by the explosively produced particles through preheating [4, 5] as a basis of the supposing scenario. In particular, we focus on a possibility of the restoration of lepton number which is usually expected to be broken for the neutrino mass generation at low energy regions. If the lepton number is restored due to such an effect at an early stage of the Universe, a new scenario of non-thermal leptogenesis might be considered as the origin of baryon number asymmetry in the Universe. As the concrete application of this scenario, we adopt an extended one-loop radiative neutrino mass model and examine such a possibility from a viewpoint of the connection with other phenomenology.

The remaining part of this paper is organized as follows. In the next section, we briefly review the symmetry restoration caused by the radiative effect due to the explosively

produced particles through preheating and then we apply it to two inflation scenarios. In section 3, we discuss its application to the lepton number in a one-loop radiative neutrino mass model extended by singlet scalars. After the study of the neutrino mass generation and dark matter abundance in this model, we propose a scenario for non-thermal leptogenesis. We estimate an amount of lepton number asymmetry generated non-thermally through the inflaton decay by using the parameters which are consistent with neutrino oscillation data and dark matter abundance. We summarize the paper in the final section.

2 Symmetry restoration via preheating

2.1 Preheating

We briefly review the basics of the symmetry restoration due to preheating at first. The explosive particle production in the background of the inflaton oscillation is known as parametric resonance or preheating [4, 5]. Inflation is induced by a certain slow-roll potential $V_{\rm inf}(\sigma)$ of a real scalar σ called inflaton, which is assumed to have a minimum at $\langle\sigma\rangle = 0$. If the inflaton σ couples with a complex scalar S, the model could have a $U(1)$ symmetry. We suppose that this $U(1)$ symmetry is spontaneously broken and then the potential is represented as[a]

$$V(\sigma, S) = V_{\rm inf}(\sigma) + \lambda_S \left(|S|^2 - \frac{u^2}{2} \right)^2 + g_S \sigma^2 |S|^2. \tag{1}$$

The mass of S could be expressed as $m_S^2 = g_S\sigma^2 - \lambda_S u^2$ during the slow-roll inflation. After the end of the inflation, the inflaton σ oscillates around the potential minimum $\langle\sigma\rangle = 0$. The oscillation is described by the equation

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV(\sigma)}{d\sigma} = 0, \tag{2}$$

[a]Since this potential looks like one of the hybrid inflation [10] with a waterfall field S, one might wonder if the scenario is based on the hybrid inflation. However, it should be noted that we assume that the potential during the inflation is dominated by the inflaton potential energy $V_{\rm inf}(\sigma)$ but not by the vacuum energy $\frac{\lambda_S}{4}u^4$. The field value of the inflaton σ is assumed to be of $O(M_{\rm pl})$ as the usual chaotic inflation. The amplitude of σ is much larger than that of S.

where a dot stands for a time derivative and S is assumed to stay initially at its local minimum $\langle S\rangle = \frac{u}{\sqrt{2}}$. H is the Hubble parameter given by

$$H^2 = \frac{\frac{1}{2}\dot{\sigma}^2 + V_{\rm inf}(\sigma)}{3M_{\rm pl}^2}, \tag{3}$$

where we use the reduced Planck mass $M_{\rm pl} = \frac{m_{\rm pl}}{\sqrt{8\pi}}$. The solution of eq. (2) might be represented by using the inflaton mass $\tilde{m}_\sigma$ as $\sigma(t) = \Sigma(t)\sin(\tilde{m}_\sigma t)$. Its amplitude $\Sigma(t)$ decreases due to the expansion of the Universe and rapidly approaches to its asymptotic value $\Sigma(t) = 2\sqrt{\frac{2}{3}}\frac{M_{\rm pl}}{\tilde{m}_\sigma t}$. At the first stage of this oscillation, the U(1) symmetry could be restored for a certain period since the amplitude $\Sigma(t)$ is large enough to be $\Sigma^2(t) > \frac{\lambda_S}{g_S}u^2$. Because of both the expansion of the Universe and the production of σ and S, which could happen depending on a self-coupling in $V_{\rm inf}(\sigma)$ and g_S, the oscillation amplitude $\Sigma(t)$ decreases to result in $\Sigma^2(t) < \frac{\lambda_S}{g_S}u^2$. The $U(1)$ symmetry seems to be broken at this period. However, the explosively produced S could restore this symmetry.

In order to study the S production under the background oscillation of σ, we introduce the shifted field $\tilde{S}$ around the symmetry broken vacuum $\langle S\rangle = \frac{u}{\sqrt{2}}$. It is expressed as $S = \langle S\rangle + \tilde{S}$ and $\tilde{S} = \frac{1}{\sqrt{2}}(S_+ + iS_-)$. The equation of motion for a quantum mode $S_{\pm p}$ with momentum $p(\equiv |\mathbf{p}|)$ is given as [3]

$$\ddot{S}_{\pm p} + 3H\dot{S}_{\pm p} + \omega_{\pm p}^2 S_{\pm p} = 0. \tag{4}$$

The frequency $\omega_{\pm p}$ is defined as

$$\omega_{\pm p}^2 = \frac{p^2}{a^2} + g_S\sigma^2 + m_{S_\pm}^2, \tag{5}$$

where $m_{S_+}^2 = 2\lambda_S u^2$ and $m_{S_-}^2 = 0$.[b] We note that $\omega_{\pm p}^2$ depends on σ due to the last term of eq. (1). The scale parameter $a(t)$ satisfies the equation $\frac{\dot{a}}{a} = H$. The dynamics of σ and the quantum scalar $S_\pm$ in the entire regime of interest can be treated by solving the coupled equations (2) and (4). If the expansion of the Universe is neglected, eq. (4) is reduced to

$$S''_{\pm p} + (A_p - 2q\cos 2z)S_{\pm p} = 0, \tag{6}$$

where $A_p = \frac{p^2}{\tilde{m}_\sigma^2} + 2q$, $q = \frac{g_S\Sigma^2}{4\tilde{m}_\sigma^2}$ and $z = \tilde{m}_\sigma t$ is used. A prime represents the differentiation with respect to z. This equation is known as the Mathieu equation whose solution

[b] In the later study we introduce an additional mass term $m_S^2 S^2$ for leptogenesis. In that case, $m_{S_\pm}^2$ is replaced by the ones in eq. (34).

is characterized by the stability/instability chart in the (q, A_p) plane. The solution is expressed as $S_{\pm p} \propto \exp(\mu_{\pm p}^{(n)} z)$ by using a certain constant $\mu_{\pm p}^{(n)}$ which is fixed within the resonance bands of momenta $\Delta p^{(n)}$ labeled by an integer n. This could be interpreted to show the exponential growth of the number density of the produced particles such as $n_p(t) \propto \exp(2\mu_{\pm p}^{(n)} z)$ [3].

However, if the effect of the expansion of the Universe is taken into account, the simple application of the stability/instability chart for the Mathieu equation is not allowed. In that situation, the amplitude of the background field oscillation decreases and the momentum in the resonance bands cannot keep its position due to the red-shift effect. Thus, the existence of the parametric resonance in the expanding Universe requires that the momenta in the resonance bands should not be red-shifted away from them before the sufficient particle production [3]. The condition for its realization is known to be summarized as $q^2 \tilde{m}_\sigma > H$. If we note that $\left(\frac{H}{\tilde{m}_\sigma}\right)^{1/4} \simeq \left(\frac{\Sigma}{M_{\rm pl}}\right)^{1/4}$ takes an almost stable value of $O(1)$ for the first stage of the inflaton oscillation, this condition is found to be written as

$$\sqrt{g_S}\, \Sigma(t) > \sqrt{2}\tilde{m}_\sigma. \tag{7}$$

The occupation number $n_{\pm p}$ of this produced particle mode $S_{\pm p}$ can be estimated by using the solution $S_{\pm p}$ of eq. (4) as

$$n_{\pm p} = \frac{1}{2\omega_{\pm p}} \left(|\dot{S}_{\pm p}|^2 + \omega_{\pm p}^2 |S_{\pm p}|^2 \right) - \frac{1}{2}. \tag{8}$$

For the estimation of this occupation number, it is useful to use a typical momentum $p_* \simeq \sqrt{g_S^{\frac{1}{2}} \tilde{m}_\sigma \Sigma(t_0)}$ to find the resonance band. $\frac{p_*}{2}$ is expected to be contained in the resonance bands of $S_{\pm p}$ at time $t_0 = \frac{\pi}{2\tilde{m}_\sigma}$ when $\sigma(t_0) = 0$ is realized first after the inflaton starts the oscillation [3].

The above argument shows that the inflaton mass $\tilde{m}_\sigma$ is a crucial parameter in the preheating. The potential $V_{\rm inf}(\sigma)$ is known to be constrained by the data from the CMB observations. If we express the power spectrum of scalar perturbation as $\mathcal{P}_\mathcal{R} = A_S \left(\frac{k}{k_*}\right)^{n_s - 1}$, it suggests $A_S \simeq 2.43 \times 10^{-9}$ at the time t_* when the scale characterized by the wave number $k_* = 0.002$ Mpc^{-1} exits the horizon [1, 2]. Since this condition can be rewritten by using a slow-roll parameter ϵ which is defined by $\epsilon = \frac{M_{\rm pl}^2}{2} \left(\frac{1}{V_{\rm inf}} \frac{dV_{\rm inf}}{d\sigma}\right)^2$ as

$$\frac{V_{\rm inf}}{\epsilon} = (0.0275 M_{\rm pl})^4, \tag{9}$$

it gives a constraint on $V_{\rm inf}(\sigma)$. For example, we may take the chaotic inflation $V_{\rm inf}(\sigma) = \frac{1}{2}\tilde{m}_\sigma^2\sigma^2$ although it is now ruled out from the tensor-to-scalar ratio of the amplitude of the CMB power spectrum. Since $\epsilon \simeq \frac{1}{2N}$ is satisfied for the e-foldings N in this example, it imposes $\tilde{m}_\sigma \simeq 1.5 \times 10^{13}$ GeV for $N_* = 60$ which stands for the e-foldings from t_* to the end of inflation. However, if $V_{\rm inf}(\sigma)$ is described by different functions at the inflation era, the inflaton mass $\tilde{m}_\sigma$ might not be constrained in the same way. We will focus our study on such examples.

These particles $S_\pm$ produced through the preheating are known to induce the symmetry restoration [4, 5]. In order to describe it, we consider quantum corrections brought about by the produced $S_\pm$ to the potential of S. During the inflaton oscillation, the effective potential for S may be represented as

$$V_{\rm eff}(S) = \lambda_S\left(|S|^2 - \frac{u^2}{2}\right)^2 + g_S\sigma^2|S|^2 + V_1^0(S) + V_1^f(S). \tag{10}$$

$V_1^0(S)$ is the ordinary zero-temperature one-loop potential and $V_1^f(S)$ comes from the one-loop contribution caused by the particles $S_\pm$ produced explosively through the preheating. Their momentum distribution is assumed to be described by a function $f(p)$. Here we use the formalism given in [11] to estimate $V_1^f(S)$. This is because the distribution function $f(p)$ is not the one in the thermal equilibrium and then the usual imaginary time formalism cannot be used. The free propagator of $S_\pm$ in this formalism is written as a 2×2 matrix and the one-loop effective potential V_1 can be given by using its (11)-component D_{11}. Following the procedure given in Appendix B of [12], V_1 satisfies

$$\frac{dV_1}{d\bar{m}_{S_\pm}^2} = \frac{1}{2}\int\frac{d^4\bar{p}}{(2\pi)^4}D_{11}(\bar{p}), \quad D_{11}(\bar{p}) = \frac{i}{\bar{p}^2 - \bar{m}_{S_\pm}^2 + i\varepsilon} + 2\pi f(p)\delta(\bar{p}^2 - \bar{m}_{S_\pm}^2), \tag{11}$$

where $\bar{m}_{S_\pm}$ is the field dependent mass of $S_\pm$. It is expressed as $\bar{m}_{S_\pm}^2 = 2\lambda_S\left(|S|^2 \pm \frac{u^2}{2}\right)$. $V_1^0(S)$ and $V_1^f(S)$ in eq. (10) come from the first and the second term in $D_{11}(\bar{p})$, respectively. For simplicity, we assume that the momentum distribution of the produced $S_\pm$ is written as $f(p,t) = A(t)p\delta(p-p_m)$ taking account of the red-shift effect. In the expanding Universe, we find $V_1^f(S)$ by solving eq. (11) as [5]

$$V_1^f(S) = \int\frac{d^3p}{(2\pi a)^3}f(p)\left(\sqrt{p^2+\bar{m}_{S_+}^2} + \sqrt{p^2+\bar{m}_{S_-}^2}\right) \simeq \frac{A(t)\lambda_S p_m^2}{\pi^2 a^3}|S|^2, \tag{12}$$

where we use $p_m \gg \bar{m}_{S_\pm}$ and omit both $|S|$ independent terms and higher order terms than $|S|^2$ in the last equality. The physical number density $n_S(t)$ obtained from the present

distribution function $f(p,t)$ is expressed by taking account of the Universe expansion as[c]

$$n_S(t) = \frac{p_m^3 A(t)}{2\pi^2 a(t)^3}. \tag{13}$$

The preheating is expected to end at $t \simeq t_f$ when condition (7) is violated. It might be roughly estimated as $t_f \simeq 1.6\frac{\sqrt{g_S}M_{\rm pl}}{\tilde{m}_\sigma^2}$. Since no explosive production of $S_\pm$ is expected after t_f, the maximum number density is determined as $n_S(t_f)$ by using eq. (13). The produced quanta are monotonically diluted by the expansion of the Universe after that. If we take account of it and use the above effective potential whose dominant one-loop contribution comes from $V_1^f(S)$, we find that the effective mass $\tilde{m}_S^2$ of S at $t > t_f$ could be expressed as

$$\tilde{m}_S^2(t) = g_S\langle\sigma\rangle^2 + \lambda_S\left(-u^2 + \frac{2n_S(t_f)}{p_m a(t)^3}\right). \tag{14}$$

Even when the amplitude of the inflaton σ becomes small, the last term induced by the quantum effect of $\tilde{S}$ could make $\tilde{m}_S^2$ positive and then the $U(1)$ symmetry is restored in such a case. This symmetry restoration could be kept until the time t as long as the condition

$$\frac{n_S(t_f)}{a(t)^3} > p_m\frac{u^2}{2} \tag{15}$$

is satisfied. If we impose it until the time when the reheating completes, the condition (15) could be rewritten as

$$n_S(t_f) > p_m\frac{u^2}{2}\left(\frac{t_R}{t_f}\right)^2 = \frac{2}{9}p_m u^2\left(\frac{1}{\Gamma_\sigma t_f}\right)^2, \tag{16}$$

where the matter dominated expansion $H = \frac{2}{3}t^{-1}$ is assumed from t_f to the end of reheating. The reheating completion time t_R could be fixed from $H \simeq \Gamma_\sigma$ by using the inflaton decay width Γ_σ.

In the next part, we numerically estimate the occupation number of the produced particles in two inflation models. It can be proceeded by solving numerically the above coupled equations (2) and (4) for σ, $S_{\pm p}$, and the Hubble equation $\frac{\dot{a}}{a} = H$ for suitable initial values. In the equation of motion of σ, they could be fixed at $\sigma = \sigma_c$ and also $\dot{\sigma}|_{\sigma=\sigma_c} \simeq 0.8\sqrt{V_{\rm inf}}$, where σ_c is taken as an inflaton value at the end of inflation. The latter could be derived by using the slow-roll equation $3H\dot{\sigma} \simeq -\frac{dV_{\rm inf}}{d\sigma}$. On the other hand,

[c]Following the detailed analysis of the preheating in [3], $A(t)$ in the distribution function $f(p)$ might be approximated by using μ which characterizes the particle production rate fixed by the model parameters. In that case, the number density of S at time t could be estimated as $n_S(t) \sim \frac{p_m^3}{4\pi^2 a(t)^3}e^{2\mu\tilde{m}_\sigma t}$.

if we use eq. (8), we could find the appropriate initial condition for eq. (4). At the initial stage of σ oscillation, $n_{\pm p} = 0$ should be satisfied. From this, we adopt $S_{\pm p} = \frac{1}{\sqrt{\omega_{\pm p}}}$ for $\dot{S}_{\pm p} = 0$ as the initial condition.

2.2 Two inflation scenarios

We study the symmetry restoration due to this particle production in two concrete inflation models here. We consider that the slow-roll inflation is caused by the inflaton potential $V_{\rm inf}(\sigma)$ which is expressed as

$$V_{\rm inf}(\sigma) = \begin{cases} V_{\rm I}(\sigma) & \sigma > \sigma_c, \\ \frac{1}{2}\tilde{m}_\sigma^2\sigma^2 & \sigma < \sigma_c. \end{cases} \tag{17}$$

If we tune the model parameters appropriately, the inflaton potential is expected to transit in this way between inflation time and post inflation time. The slow-roll inflation is considered to be induced by V_I and end at $\sigma \simeq \sigma_c$ where the slow-roll condition is violated to be $\epsilon \simeq 1$. At the post inflation era, the potential is supposed to be approximated as $\frac{1}{2}\tilde{m}_\sigma^2\sigma^2$ before the reheating. For example, if the κ term dominates the potential for large σ in $V_I = \frac{\kappa}{4}\sigma^4 + \frac{1}{2}\tilde{m}_\sigma^2\sigma^2$, the CMB condition (9) constrains κ but not the value of $\tilde{m}_\sigma$ directly. Since σ reduces its value as a result of the expansion, both terms can become equal soon at a certain time t_e much before the reheating time t_R. If we take into account that both t_e and t_R are roughly estimated as $t_e \sim \sqrt{\frac{2\kappa}{3}}\frac{M_{\rm pl}}{\tilde{m}_\sigma}$ and $t_R \sim 2g_*^{-1/2}\frac{M_{\rm pl}}{T_R^2}$, we find that $t_e \ll t_R$ could be possible for $\frac{T_R}{\tilde{m}_\sigma} \ll 0.5\kappa^{-1/4}$. In the following part, we consider two examples for this kind of inflaton potential, which can satisfy the present data of the CMB tensor-to-scalar ratio [2].

Model (a)

We consider a real scalar σ whose Jordan frame potential is written as $V(\sigma) = \frac{\kappa}{4}\sigma^4 + \frac{1}{2}\tilde{m}_\sigma^2\sigma^2$ and the κ term is assumed to be dominant for large values of σ. It is also assumed to have a non-minimal coupling $\frac{\xi}{2}\sigma^2 R$ with Ricci scalar [13–16]. In the Einstein frame, a canonically normalized field χ can be defined by

$$\frac{d\chi}{d\sigma} = \frac{\left[1 + (\xi + 6\xi^2)\frac{\sigma^2}{M_{\rm pl}^2}\right]^{1/2}}{1 + \frac{\xi\sigma^2}{M_{\rm pl}^2}}, \tag{18}$$

	g_S	κ	$\tilde{m}_\sigma$ (GeV)	σ_c (GeV)	p_m (GeV)	$n^c_{\pm p}(t_f)$
Model (a)	1.8×10^{-8}	4.7×10^{-8}	3×10^{12}	8.3×10^{17}	9.89×10^{13}	1.4×10^{8}
Model (b)	4.3×10^{-8}	1.2×10^{-7}	4×10^{12}	7.2×10^{17}	1.26×10^{14}	4.2×10^{8}

Table 1 Parameters used in the numerical study. These correspond to $q = 350$ and $\Sigma \simeq \sigma_c$ at the end of inflation. p_m is the momentum at which the number density $n_{\pm p}$ takes a maximum. $N_* = 60$ is assumed here. If we use $t_f \simeq 1.6\frac{\sqrt{g_S}M_{\rm pl}}{\tilde{m}_\sigma^2}$, t_f is estimated as 28 in Model (a) and 32 in Model (b) in a $\frac{2\pi}{\tilde{m}_\sigma}$ unit.

and the scalar potential can be written as

$$V_{\rm inf} = \frac{1}{\Omega^4}\left(\frac{\kappa}{4}\sigma^4 + \frac{1}{2}\tilde{m}_\sigma^2\sigma^2\right), \qquad \Omega^2 = 1 + \frac{\xi\sigma^2}{M_{\rm pl}^2}. \tag{19}$$

By using eq. (18), σ is found to be related to χ as $\sigma \propto \exp\left(\frac{\chi}{\sqrt{6+\frac{1}{\xi}}M_{\rm pl}}\right)$ at $\sigma \gg \frac{M_{\rm pl}}{\sqrt{\xi}}$ and χ reduces to σ at $\sigma \ll \frac{M_{\rm pl}}{\sqrt{\xi}}$. Thus, if we assume that $\frac{\kappa}{4}\sigma^4 > \frac{1}{2}\tilde{m}_\sigma^2\sigma^2$ is satisfied at $\sigma_c = \frac{M_{\rm pl}}{\sqrt{\xi}}$, $V_{\rm I}$ is found to be represented as $V_{\rm I}(\sigma) \simeq \frac{\kappa M_{\rm pl}^4}{4\xi^2}$ at $\sigma > \sigma_c$ and eq. (17) is also realized at $\sigma < \sigma_c$. In this model, the slow-roll parameters ϵ and the e-foldings N are expressed approximately as $\epsilon = \frac{4M_{\rm pl}^4}{3\xi^2\sigma^4}$ and $N = \frac{3\xi\sigma^2}{4M_{\rm pl}^2}$. Thus, the initial conditions for the σ oscillation are found to be summarized as

$$\sigma_c \simeq \left(\frac{4}{3}\right)^{1/4}\frac{M_{\rm pl}}{\sqrt{\xi}}, \qquad \dot{\sigma}|_{\sigma=\sigma_c} \simeq 0.4\frac{\sqrt{\kappa}}{\xi}M_{\rm pl}^2, \tag{20}$$

if we use $\epsilon \simeq 1$ at $\sigma = \sigma_c$ and $\dot{\sigma}|_{\sigma=\sigma_c} \simeq 0.8\sqrt{V_{\rm I}}$.

From the CMB constraint (9), we find

$$\kappa \simeq 1.7 \times 10^{-6}\frac{\xi^2}{N_*^2}. \tag{21}$$

Since these parameters have no phenomenological constraint differently from the Higgs inflation, we can take a value of ξ freely in this study. We note that there appears no unitarity problem related to the inflation even in that case. Here we use $\xi = 10$ as a moderate value. The parameters relevant to the estimation of the particle production are fixed for this value of ξ.

Model (b)

We consider a complex scalar σ whose potential is expressed as [17]

$$\begin{aligned} V_{\rm inf} &= V_I + \tilde{m}_\sigma^2 \sigma^\dagger \sigma + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\sigma^2 \sigma^{\dagger 2}, \\ V_{\rm I} &= \kappa (\sigma^\dagger \sigma)^2 \left[1 + \alpha \left\{ \left(\frac{\sigma}{M_{\rm pl}} \right)^2 \exp\left(i \frac{\sigma^\dagger \sigma}{\Lambda^2} \right) + \left(\frac{\sigma^\dagger}{M_{\rm pl}} \right)^2 \exp\left(-i \frac{\sigma^\dagger \sigma}{\Lambda^2} \right) \right\} \right] \\ &= \frac{\kappa}{4} \varphi^4 \left[1 + 2\alpha \left(\frac{\varphi}{\sqrt{2} M_{\rm pl}} \right)^2 \cos\left(\frac{\varphi^2}{2\Lambda^2} + 2\theta \right) \right] \\ &\equiv \frac{\kappa}{4} \varphi^4 + V_b \cos\left(\frac{\varphi^2}{2\Lambda^2} + 2\theta \right), \end{aligned} \tag{22}$$

where $\sigma = \frac{1}{\sqrt{2}} \varphi e^{i\theta}$. If $V_b \cos\left(\frac{\varphi^2}{2\Lambda^2} \right) \lesssim \frac{\kappa}{4} \varphi^4$ is satisfied at $\varphi \simeq \varphi_c$, $V_{\rm inf}$ could be approximated by V_I at $\varphi > \varphi_c$. In that case, the inflation is induced through the flat direction of $V_{\rm I}$ which is represented by the inflaton χ constrained along the polar angle direction as long as σ stays at the local minimum in the radial direction [18]. The inflaton χ is defined as

$$d\chi = \left[1 + \frac{1}{\varphi^2} \left(\frac{d\varphi}{d\theta} \right)^2 \right]^{1/2} \varphi d\theta = \left[1 + 4 \left(\frac{\Lambda}{\varphi} \right)^4 \right]^{1/2} \varphi d\theta, \tag{23}$$

where in the second equality we use a fact that this constrained path satisfies $\frac{\varphi^2}{2\Lambda^2} + 2\theta = (2m+1)\pi$ for an integer m. Since the e-foldings N and the slow-roll parameter ϵ can be approximately estimated as[d]

$$N \simeq \frac{1}{12} \left(\frac{\varphi}{\sqrt{2} M_{\rm pl}} \right)^6 \left(\frac{M_{\rm pl}}{\Lambda} \right)^4, \qquad \epsilon \simeq 4 \left(\frac{\sqrt{2} M_{\rm pl}}{\varphi} \right)^6 \left(\frac{\Lambda}{M_{\rm pl}} \right)^4, \tag{24}$$

the e-foldings N and the slow-roll parameter ϵ are related to each other as $\epsilon \simeq \frac{1}{3N}$.

In this model, the single field slow-roll inflation picture cannot be applied at the final stage of inflation since the inflaton χ defined by eq. (23) could not describe well the motion of σ which deviates from the local minimum in the radial direction. However, both $3H\dot{\chi} \simeq -\frac{dV_{\rm I}}{d\chi}$ and $\frac{1}{2}\dot{\chi}^2 \simeq V_b$ are considered to be satisfied simultaneously at the end of inflation. If we use these conditions approximately, we can estimate the value of φ at the end of inflation as

$$\frac{\varphi_c}{\sqrt{2} M_{\rm pl}} \simeq \left(\frac{2}{3\alpha} \right)^{1/8} \left(\frac{\Lambda}{M_{\rm pl}} \right)^{1/2}. \tag{25}$$

[d]The contribution from V_b to these values is omitted in these approximated expressions since it is sub-dominant. Even if we use these formulas, the results are not affected in the present study.

Since the inflaton could go over the potential barrier V_b at $\varphi \simeq \varphi_c$, the complex scalar σ cannot be kept in the constrained path and the components $\sigma_{1,2}$ of $\sigma(\equiv \frac{1}{\sqrt{2}}(\sigma_1 + i\sigma_2))$ could be considered to oscillate in the approximated potential,

$$V_{\rm inf} \simeq \frac{1}{2}m_{+\sigma}^2\sigma_1^2 + \frac{1}{2}m_{-\sigma}^2\sigma_2^2, \qquad m_{\pm\sigma}^2 = \tilde{m}_\sigma^2 \pm m_\sigma^2 \tag{26}$$

at the region $\varphi < \varphi_c$. Since the potential for σ_1 and σ_2 is not the same due to the existence of m_σ^2, the supposed $U(1)$ symmetry could be violated in this part.[e] We note that the coupling $g_S\sigma^\dagger\sigma S^\dagger S$ relevant to the particle production is written as $\frac{g_S}{2}(\sigma_1^2 + \sigma_2^2)S^\dagger S$. The initial conditions for the oscillation of $\sigma_{1,2}$ at $\varphi = \varphi_c$ are found to be expressed as

$$\sigma_1 = \varphi_c, \qquad \sigma_2 = 0, \qquad \dot{\sigma}_1 = 0, \qquad \dot{\sigma}_2 = 2\sqrt{\alpha}\,\frac{\tilde{m}_\sigma\Lambda^2}{M_{\rm pl}}. \tag{27}$$

If we impose the CMB constraint (9) on this model, we find that κ should satisfy the condition

$$\kappa \simeq 3.6 \times 10^{-8}\frac{1}{N_*^{5/3}}\left(\frac{M_{\rm pl}}{\Lambda}\right)^{8/3}. \tag{28}$$

Parameters in the potential (22) are adopted as $\alpha = 1.1$ and $\frac{\Lambda}{M_{\rm pl}} = 0.05$ for $N_* = 60$, which can explain the tensor-to-scalar ratio of the CMB perturbation presented by Planck [17].

Now we present results of the numerical study for the resonant $S_\pm$ production in the framework defined by eq. (1), in which $V_{\rm inf}$ is taken as the above ones. Parameters used in this study are listed in Table 1 for each model. In the left panel of Fig. 1, the number density $n_{\pm p_*}$ of the momentum mode $S_{\pm p_*}$ generated in the preheating is shown for each model. This figure shows that the exponential particle production continues from the end of inflation to the time t_f. As discussed in the previous part, the particle production stops there since the condition (7) is violated due to the red shift of the momentum and the decrease of the inflaton amplitude. After the end of preheating t_f, the number density $n_{\pm p_*}$ decreases monotonically due to the expansion of the Universe. In the right panel of Fig. 1, the distribution of the produced momentum mode is plotted. It is obtained by applying the Gaussian fit to the numerical data points for several values of momentum p. The number density of $S_\pm$ obtained from the integration of this fitting function is found to be nicely approximated by $n_{\pm S} = \frac{p_m^3}{64\pi^2}n_{\pm p_m}$, where $n_{\pm p_m}$ stands for a peak value of $n_{\pm p}$ realized at $p = p_m$. This suggests that we can put $A(t_f) = \frac{1}{32}n_{\pm p_m}(t_f)$

[e] We come back to this point later to study the generation of the lepton number asymmetry through the inflaton decay. We assume $m_\sigma = 0.3\tilde{m}_\sigma$ in the numerical study.

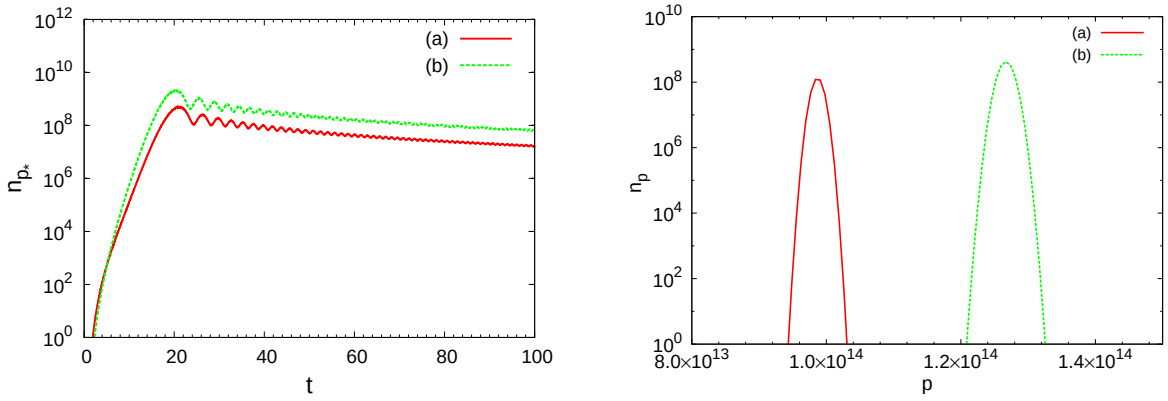

Fig. 1 Left: The evolution of the physical number density of a momentum mode $S_{\pm p_*}$ with a typical momentum p_* which characterizes the position of a resonance band. It can be fixed at $p_* = \sqrt{g_S^{1/2}\tilde{m}_\sigma \Sigma(t_0)}$. The time t is taken as a $\frac{2\pi}{\tilde{m}_\sigma}$ unit. Right: The momentum distribution n_p of the produced particle $S_\pm$. These curves are fixed by the Gaussian fitting to the numerical data points. A unit of the momentum p is taken to be GeV. In both panels, labels (a) and (b) stand for the models discussed in the text and shown in Table 1.

in the previously assumed distribution function. We use these results in the analysis of the symmetry restoration in the model defined by eq. (1). In the following study, the parameters contained in the potential are fixed as

$$u = 1.4 \times 10^{15}\ \text{GeV}, \qquad \lambda_S = 2.5 \times 10^{-11}. \tag{29}$$

The condition (15) for the symmetry restoration is found to be easily satisfied at t_f when the preheating ends. It is crucial for the study of the related physics to know how long this symmetry restoration is kept.

We discuss this problem from a viewpoint to make this symmetry restoration applicable for a new type of non-thermal leptogenesis scenario. For that purpose, we introduce right-handed neutrinos to fix the reheating process and impose them to couple with both σ and S through

$$-\mathcal{L} = \zeta_i \sigma \bar{N}_i^c N_i + \zeta_i^* \sigma \bar{N}_i N_i^c + y_i S \bar{N}_i^c N_i + y_i^* S^\dagger \bar{N}_i N_i^c. \tag{30}$$

If we assign the lepton number N_i and S such as $L(N_i) = 1$ and $L(S) = -2$, the U(1) symmetry discussed above could be identified with this lepton number. The reheating is finally processed through the inflaton decay to $N_i N_i$ which violates the lepton number.

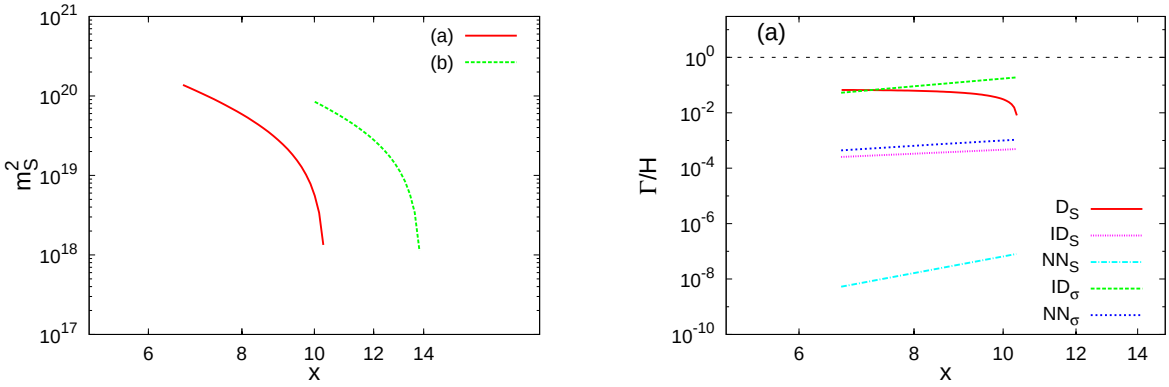

Fig. 2 Left: The effective squared mass $\tilde{m}_S^2$ in a GeV unit as a function of $x\left(\equiv \frac{M_1}{T}\right)$. The right-handed neutrino mass is fixed at $M_1 = 10^{14}$ GeV. The labels (a) and (b) stand for the model shown in Table 1. Right: The reaction rates $\frac{\Gamma}{H}$ for the lepton number violating processes in Model (a). The S decay, the inverse decay of S and σ, and the NN scattering mediated by S and σ are labeled by D_S, $\mathrm{ID}_{S,\sigma}$, and $\mathrm{NN}_{S,\sigma}$, respectively. The left end point of each line corresponds to the reheating temperature T_R.

If we remind that this σ decay completes at $H \simeq \Gamma_\sigma$ for $\Gamma_\sigma = \sum_i \frac{\zeta_i^2}{8\pi}\tilde{m}_\sigma$, the reheating temperature could be estimated as[f]

$$T_R \simeq 1.74 g_*^{-1/4}\sqrt{M_{\rm pl}\Gamma_\sigma}, \tag{31}$$

where $g_* = 116$ is the relativistic degrees of freedom in the model. Using this T_R, eq. (16) which is the condition for the symmetry restoration to be kept until the reheating time can be rewritten as

$$A(t_f) > 0.5\frac{1}{g_S}\left(\frac{u}{p_*}\right)^2\left(\frac{\tilde{m}_\sigma}{T_R}\right)^4. \tag{32}$$

Only the dilution of the number density due to the expansion of the Universe is taken into account in this condition. However, we should note that the breakdown of this symmetry restoration could be caused also by the decrease of the number density $n_{\pm S}$ due to the decay of $S_\pm$ to N_iN_i which is caused by the interactions in eq. (30). This effect can be neglected as long as such a process decouples and $\Gamma_{S_\pm} \ll H(T)$ is satisfied, where $\Gamma_{S_\pm} = \frac{\sum_i y_i^2}{8\pi}\tilde{m}_{S_\pm}$ and $H(T)^2 = \frac{\pi^2}{90}g_*\frac{T^4}{M_{\rm pl}^2}$.

In the left and right panels of Fig. 2, $\tilde{m}_S^2$ and $\frac{\Gamma_{S_\pm}}{H}$ (which is labeled by D_S) are plotted as the function of $x\left(\equiv \frac{M_1}{T}\right)$ at $T < T_R$, respectively. These figures show that the restoration of the lepton number can be kept until a certain temperature T', which is lower than

[f]In the following part, the reheating temperature is fixed at (a) $T_R = 5\tilde{m}_\sigma$ and (b) $T_R = 2.5\tilde{m}_\sigma$.

T_R. The sudden decrease of $\frac{\Gamma_{S_\pm}}{H}$ is caused by the threshold effects due to the generation of the Majorana mass of N_i at T'. In the right panel, we also plot the reaction rates of the N_i scatterings mediated by the exchange of $S_\pm$ and σ and also the inverse decay of $S_\pm$ and σ, which could wash out the lepton number asymmetry since they violate the lepton number explicitly. These results show that any possible lepton number violating processes decouple at $T' \leq T \leq T_R$. If the lepton number asymmetry exists in the N_i sector, it could be conserved at this stage since these processes are freezed out. This means that if the inflaton decay through the coupling $\sigma \bar{N}_i^c N_i$ could generate the lepton number asymmetry in the N_i sector, it could be accumulated in the lepton sector where the lepton number is well defined and it is kept there until T'.

A crucial problem is how the CP symmetry could be violated in the inflaton decay. If its violation is realized at a substantial level, the lepton asymmetry generated through this decay could be distributed in the ordinary lepton sector through the lepton number conserving processes before reaching the symmetry breaking temperature T'. The sphaleron interaction can generate the baryon number asymmetry using this lepton number asymmetry. In that case, the lepton number violating processes caused by the neutrino Yukawa coupling have to be sufficiently suppressed at $T < T'$. It is crucial to avoid the washout of the non-thermally generated lepton number asymmetry in this scenario. If the initial lepton number asymmetry could take a sufficient value and satisfy these conditions, the scenario could be an alternative one to the thermal leptogenesis. It is worth studying whether this could give a new possible scenario for non-thermal leptogenesis in viable neutrino mass models. In the next section, we take a radiative neutrino mass model as an example and propose a realistic framework for this leptogenesis scenario.

3 Application to Leptogenesis

3.1 A particle physics model

We consider an application of the symmetry restoration discussed in the previous section to non-thermal leptogenesis in a one-loop radiative neutrino mass model, which is obtained by extending the Ma model [19] with singlet scalars.[g] It inherits favorable nature of the

[g]Similar extension is studied in [20] in another context.

original Ma model, that is, it can closely relate the neutrino mass generation to the dark matter (DM) existence [21]. The model is composed of an extra doublet scalar η, singlet fermions N_i, a real singlet scalar σ, and a complex singlet scalar S in addition to the SM contents. We impose a Z_2 symmetry on the model and assign its odd parity both η and N_i. All other fields are assigned even parity including the inflaton σ and S. The Lagrangian relevant to these new contents contains the following terms,

$$\begin{aligned}
-\mathcal{L} &= V_{\rm inf}(\sigma) + g_\phi\sigma^2(\phi^\dagger\phi) + g_\eta\sigma^2(\eta^\dagger\eta) + g_S\sigma^2(S^\dagger S) + \zeta_i\sigma\bar{N}N^c + \zeta_i^*\sigma\bar{N}^cN \\
&+ \lambda_S\left(S^\dagger S - \frac{u^2}{2}\right)^2 + \frac{1}{2}m_S^2S^2 + \frac{1}{2}m_S^2S^{\dagger 2} + \kappa_\phi(S^\dagger S)(\phi^\dagger\phi) + \kappa_\eta(S^\dagger S)(\eta^\dagger\eta) \\
&+ y_iS\bar{N}_i^cN_i + y_i^*S^\dagger\bar{N}_iN_i^c + h_{\alpha i}\bar{\ell}_\alpha N_i\eta + h_{\alpha i}^*\bar{N}_i\ell_\alpha\eta^\dagger \\
&+ m_\phi^2\phi^\dagger\phi + m_\eta^2\eta^\dagger\eta + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\phi^\dagger\phi)(\eta^\dagger\eta) + \lambda_4(\eta^\dagger\phi)(\phi^\dagger\eta) \\
&+ \frac{\lambda_5}{2}\left[(\eta^\dagger\phi)^2 + (\phi^\dagger\eta)^2\right], \qquad (33)
\end{aligned}$$

where ℓ_α is the doublet lepton and ϕ is the ordinary doublet Higgs scalar. A concrete form of $V_{\rm inf}(\sigma)$ is presented in the previous section.[h]

This Lagrangian includes the potential (1) as a part of it. However, the minimum of the potential for S is shifted from u^2 to $\bar{u}^2 \equiv u^2 - \frac{m_S^2}{\lambda_S}$ because of the introduction of a new mass term $m_S^2S^2$. The masses of each component of S and η can be expressed as

$$m_{S_+}^2 \simeq 2\lambda_S\bar{u}^2, \quad m_{S_-}^2 \simeq -2m_S^2, \quad M_{\eta^\pm}^2 = \bar{m}_\eta^2 + \lambda_3\langle\phi\rangle^2, \quad M_{\eta_{R,I}}^2 = \bar{m}_\eta^2 + \lambda_\pm\langle\phi\rangle^2, \qquad (34)$$

where $\bar{m}_\eta^2 = m_\eta^2 + \frac{\kappa_\eta}{2}\bar{u}^2$ and $\lambda_\pm = \lambda_3 + \lambda_4 \pm \lambda_5$. The vacuum stability requires $m_S^2 < 0$ and

$$\lambda_1,\ \lambda_2,\ \lambda_S > 0, \quad \lambda_3, \lambda_\pm > -\sqrt{\lambda_1\lambda_2}, \quad \kappa_\phi > -\sqrt{\lambda_1\lambda_S}, \quad \kappa_\eta > -\sqrt{\lambda_2\lambda_S}. \qquad (35)$$

The weak scale is derived as $\langle\phi\rangle^2 = -\frac{1}{2\lambda_1}\left(m_\phi^2 + \frac{\kappa_\phi}{2}\bar{u}^2\right)$. Since the Higgs mass is given as $m_h^2 = 4\lambda_1\langle\phi\rangle^2$, it imposes $\lambda_1 \simeq 0.13$. On the other hand, η is assumed to have no vacuum expectation value (VEV) and then the Z_2 symmetry remains exact. As its result, neutrinos cannot get masses at tree level and the lightest Z_2 odd particle is stable. This stable particle should be neutral to be a good DM candidate. We take it as a neutral component (η_R) of η here. This imposes $\lambda_5 < 0$ and $\lambda_4 + \lambda_5 < 0$.

[h] If we apply Model (b) to this Lagrangian, σ is just replaced by the complex σ and $\sigma_{1,2}$ should be used in the study of the oscillation phenomena.

Before proceeding with further discussion, we order several comments relevant to the lepton number and its assignment to the new ingredients.[i] Since the λ_5 term is indispensable for the small neutrino mass generation at the one-loop level as seen later [20, 21], η should not have the lepton number as long as the lepton number conservation is imposed on this term. As a result, N_i should be assigned the lepton number 1 and then the coupling $S\bar{N}_i^c N_i$ requires that S should have the lepton number -2 as discussed already. Unless the Majorana mass of N_i is caused through the coupling $S\bar{N}_i^c N_i$ as a result of $\langle S\rangle \neq 0$, the neutrino mass cannot be generated at the low energy regions even at the loop level. Thus, the realization of $\langle S\rangle \neq 0$ at low energy regions is required for the neutrino mass generation. It should be also noted that the Z_2 symmetry is kept exact even after S gets a VEV and then the existence of DM is guaranteed. In the next part, we discuss neutrino masses and DM in this model.

3.2 Neutrino mass and dark matter

Here we discuss the constraints derived from the low energy feature of the model after the breakdown of the symmetry restoration for S. Neutrino masses are generated radiatively through one-loop diagrams with N_i in the internal fermion line in the same way as the original Ma model. We apply the value of $\bar{u}$ in eq. (29) to the right-handed neutrino masses $M_i = y_i\bar{u}$. Since $M^2_{\eta_{R,I}} \gg |\lambda_5|\langle\phi\rangle^2$ is satisfied, the neutrino mass formula can be approximately written as

$$\mathcal{M}_{\alpha\beta} = \sum_i h_{\alpha i} h_{\beta i} \lambda_5 \Lambda_i, \qquad \Lambda_i \simeq \frac{\langle\phi\rangle^2}{8\pi^2 M_i} \ln\frac{M_i^2}{\bar{M}_\eta^2}, \tag{36}$$

where $\bar{M}_\eta^2 = \bar{m}_\eta^2 + (\lambda_3+\lambda_4)\langle\phi\rangle^2$. This suggests that the neutrino masses are obtained in almost the same way as the ordinary seesaw model for $|\lambda_5| = O(1)$ in the present case.

In order to take account of the constraints from the neutrino oscillation data, we fix the flavor structure of neutrino Yukawa couplings $h_{\alpha i}$ at the one which induces the tri-bimaximal mixing [23],

$$h_{ej} = 0, \quad h_{\mu j} = h_{\tau j} \equiv h_j \quad (j = 1, 2); \qquad h_{e3} = h_{\mu 3} = -h_{\tau 3} \equiv h_3. \tag{37}$$

[i] If we take another lepton number assignment, a different type of non-thermal leptogenesis could be considered [22].

In that case, the mass eigenvalues are estimated as

$$m_1 = 0, \qquad m_2 = 3|h_3|^2\Lambda_3,$$
$$m_3 = 2\left[|h_1|^4\Lambda_1^2 + |h_2|^4\Lambda_2^2 + 2|h_1|^2|h_2|^2\Lambda_1\Lambda_2\cos 2(\theta_1-\theta_2)\right]^{1/2}, \tag{38}$$

where $\theta_j = \arg(h_j)$. If we use $\bar{u}$ given in eq. (29) and fix the parameters relevant to the neutrino masses as

$$|h_1| = 0.1|\lambda_5|^{-1/2}, \qquad |h_2| = 0.38|\lambda_5|^{-1/2}, \qquad |h_3| = 0.15|\lambda_5|^{-1/2},$$
$$|y_1| = 0.1, \qquad |y_2| = 0.12, \qquad |y_3| = 0.15, \tag{39}$$

the neutrino oscillation data could be explained. Although a certain modification is required to reproduce the favorable mixing structure, it is sufficient for the study in the next section. We note that the smaller $|\lambda_5|$ requires the larger values of neutrino Yukawa couplings.

The value of $|\lambda_5|$ is also constrained by the DM abundance. In the present study, DM is assumed to be the real part η_R of the neutral component of η. Its abundance could be tuned to the observed value as long as the couplings $\lambda_{3,4}$ take suitable values [24]. Here, we should note that the allowed regions of λ_3 and λ_4 are constrained by eq. (35) and the discussion below it. Since $\bar{m}_\eta$ is assumed to be of $O(1)$ TeV, the mass of each component of η is found to be degenerate enough for the allowed values of $\lambda_{3,4}$ and λ_5. This makes the co-annihilation among them effective enough to reduce the DM abundance sufficiently. As an example, the expected relic abundance of η_R for several values of $\lambda_{3,4}$ and $\bar{m}_\eta = 1.75$ TeV is plotted in Fig. 3 for the cases $\lambda_5 = -1$ and -0.5. The larger value of $\bar{m}_\eta$ is required for $|\lambda_5| \gtrsim 1$. In that case, the dependence of the relic abundance on $\lambda_{3,4}$ becomes much weaker compared to the case fixed by the smaller value of $|\lambda_5|$. The possible DM mass is strongly constrained to a narrow region depending on the value of $|\lambda_5|$. Anyway, the simultaneous explanation of the neutrino masses and the DM abundance could be preserved in this extended model. We should stress that no additional constraint from the neutrino physics and the DM physics is brought about by taking the present scenario.

It may be useful to give a remark for another aspect of the model. The VEV of S could give the dominant origin for both the electroweak symmetry breaking and the DM mass through the interaction terms $\kappa_\phi S^\dagger S\phi^\dagger\phi$ and $\kappa_\eta S^\dagger S\eta^\dagger\eta$ unless they are forbidden by

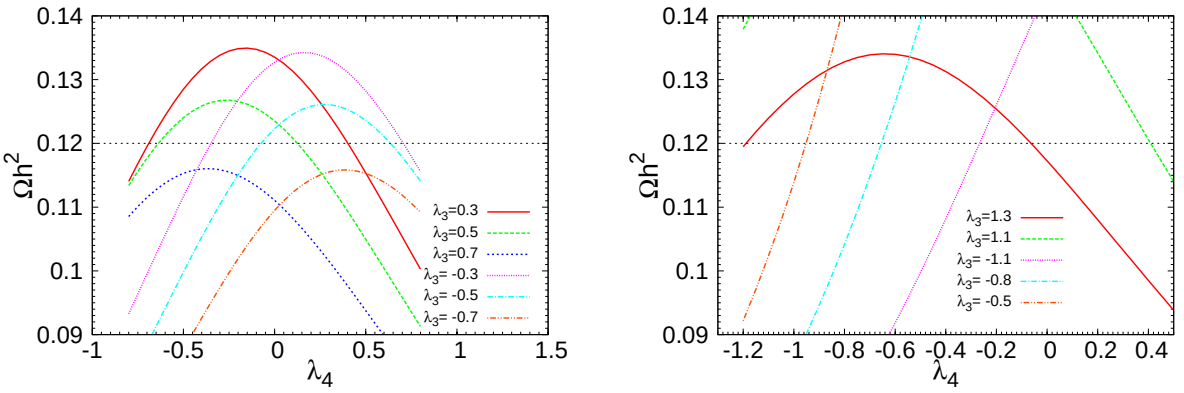

Fig. 3 Relic abundance of η_R in the case $\lambda_5 = -1$ (left panel) and -0.5 (right panel). A horizontal dotted line $\Omega h^2 = 0.12$ is the required value from the observations [2].

a certain reason.[j] Since both scales of the electroweak symmetry breaking and the DM mass could be induced as $\kappa_\phi \bar{u}^2$ and $\kappa_\eta \bar{u}^2$ from the VEV $\bar{u}$, the couplings κ_ϕ and κ_η should take negative and positive tiny values, respectively. Such κ_ϕ and κ_η satisfy the constraints given in eq. (35). Although these couplings should take extremely small values for such a large value of $\bar{u}$ assumed in eq. (29), it might present a possibility to unify the origin of the mass scales at TeV regions. These tiny couplings might be realized as non-renormalizable terms which are suppressed by the Planck mass, for example.

3.3 Lepton asymmetry induced through the inflaton decay

We consider the generation of the lepton number asymmetry through the decay of the inflaton σ to a N_i pair, where the lepton number is supposed to be well defined. This situation is also assumed to be kept until the generated asymmetry has been transferred from them to other particles. These assumptions require that $\langle S \rangle = 0$ is satisfied throughout the period before the completion of the reheating at least. In this conservative situation, the following study is done and then we need not take into account the washout of the generated asymmetry there.[k] Such a situation cannot be realized in the case where the

[j] If we assume the symmetry restoration due to the explosively produced η or ϕ, their couplings κ_η or κ_ϕ with S should take a substantial value as found from (14). In that case, since they could induce large mass terms for η and ϕ at the low energy region via the VEV $\bar{u}$, we could not adopt such a possibility in this model. Only the explosive production of S could not cause such a problem.

[k] We note that the washout processes caused through the coupling $\sigma N_i N_i$ and $m_S^2 S^2$ which break the lepton number explicitly are ineffective as shown in the right panel of Fig. 2.

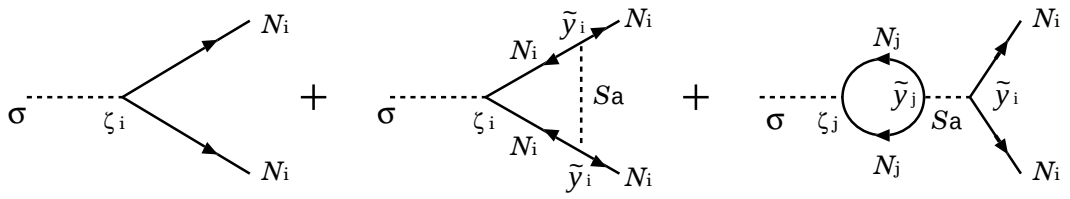

Fig. 4 Feynman diagrams contributing to the generation of lepton number asymmetry. S_a stands for the mass eigenstates $S_\pm$ and the couplings $\tilde{y}_i$ are fixed at the ones shown in eq.(40).

restoration of the lepton number is caused through the finite temperature effect. On the other hand, the symmetry restoration due to preheating discussed in the previous part can realize it as seen before.

Here, it may be useful to compare the present scenario to the one discussed in [25] previously in order to clarify the feature of the scenario. In the latter model, the inflaton decays to the right-handed neutrinos nonthermally in which $U(1)_{B-L}$ is violated. The decay of these right-handed neutrinos generates the lepton number asymmetry. On the other hand, in the present model, the lepton number is considered to be generated through the inflaton decay to the right-handed neutrinos where the lepton number $U(1)_L$ is assumed to be conserved and it is assumed to be kept until they decay to the doublet leptons.

In order to generate the lepton number asymmetry through the lepton number violating decay of the inflaton σ to $N_i N_i$, the CP violation is required there. The mass term $m_S^2 S^2$ in the second line of eq. (33), which breaks the lepton number explicitly,[1] can play a crucial role for this CP violation. On the other hand, since the lepton number violation in the S sector also causes the washout of the generated lepton number asymmetry through the scattering, it has to be taken into account in the estimation of the final lepton number asymmetry. Related to this point, we should remember that the symmetry restoration due to the preheating could be much more effective compared to the one due to the finite temperature effect of the reheating [4]. As its result, these violating effects could be freezed out throughout the symmetry restored period as seen in the right panel of Fig. 2.

Since the lepton number is violated in the interaction which causes the inflaton decay,

[1]This explicit breaking of the lepton number makes a Nambu-Goldstone boson caused by its spontaneous symmetry breaking (SSB) heavy enough as shown in eq. (34). The topological defect which could appear through this SSB is not stable due to the same explicit breaking.

the lepton number asymmetry could be generated if CP is violated in this process. In order to see how the CP could be violated there, we note that S is decomposed into two mass eigenstates $S_\pm$ by the explicit lepton number violation due to the mass term $m_S^2 S^2$ even at the symmetry restored period. Their mass eigenvalues are $m_{S_\pm}^2 = \tilde{m}_S^2 \pm m_S^2$, where $\tilde{m}_S^2$ is the mass brought about through the symmetry restoration due to the preheating. It is given by $\tilde{m}_S^2 \simeq \lambda_S \left(\frac{2n_S(t)}{p_m} - \bar{u}^2 \right)$ as found from eq. (14). If we use these mass eigenstates, the couplings of N_i and S in eq. (33) can be rewritten as

$$y_i S \bar{N}_i^c N_i + y_i^* S^\dagger \bar{N}_i N_i^c = \frac{1}{\sqrt{2}} y_i S_+ \bar{N}_i^c N_i + \frac{i}{\sqrt{2}} y_i S_- \bar{N}_i^c N_i + \frac{1}{\sqrt{2}} y_i^* S_+ \bar{N}_i N_i^c - \frac{i}{\sqrt{2}} y_i^* S_- \bar{N}_i N_i^c. \tag{40}$$

The CP violation in the inflaton decay could be caused from the interference between the tree diagram and the one-loop diagram which is induced by these couplings as shown in Fig. 4.

The CP asymmetry ε in this inflaton decay is defined as

$$\varepsilon = \frac{\Gamma(\sigma \to \sum_i N_i N_i) - \Gamma(\sigma \to \sum_i N_i^c N_i^c)}{\Gamma(\sigma \to \sum_i N_i N_i) + \Gamma(\sigma \to \sum_i N_i^c N_i^c)}. \tag{41}$$

Since the contribution from the self-energy diagram in Fig. 4 is negligible for non-degenerate values of $\tilde{m}_\sigma^2$ and $\tilde{m}_S^2$, we find that ε could be expressed as

$$\begin{aligned} (a) \qquad & \varepsilon = \frac{1}{4\pi} \frac{m_S^2}{\tilde{m}_\sigma^2} \frac{\sum_i \mathrm{Im}(\zeta_i^2 y_i^{*2})}{\sum_i |\zeta_i|^2} \sim \frac{\sum_i |y_i|^2}{12\pi} \frac{m_S^2}{\tilde{m}_\sigma^2}, \\ (b) \qquad & \varepsilon = \frac{1}{2\pi} \frac{m_\sigma^2 m_S^2}{m_{+\sigma}^2 m_{-\sigma}^2} \frac{\sum_i \mathrm{Im}(\zeta_i^2 y_i^{*2})}{\sum_i |\zeta_i|^2} \sim \frac{\sum_i |y_i|^2}{6\pi} \frac{m_\sigma^2 m_S^2}{\tilde{m}_\sigma^4}, \end{aligned} \tag{42}$$

where the maximal CP phase and the universality of ζ_i are assumed in the last expressions for each model. These formulas show that ε is proportional to the mass difference between S_+ and S_- in both models. It is also proportional to the mass difference between σ_1 and σ_2 in Model (b). Thus, these mass differences m_S^2 and m_σ^2 should not be so small compared to $\tilde{m}_\sigma^2$ in order to guarantee a sufficient value for the CP asymmetry ε.

Taking account of the arguments presented by now, we can summarize the necessary conditions for the lepton number asymmetry generated in this scenario to be the origin of the baryon number asymmetry in the Universe as follows:

(i) The symmetry restoration should break down after the completion of reheating. This requires that S gets the VEV at T' which is smaller than T_R. If it is not satisfied, the asymmetry generated before the symmetry breaking is erased by the thermalization at

	x_R $\left(\equiv \frac{M_1}{T_R}\right)$	x' $\left(\equiv \frac{M_1}{T'}\right)$	$\tilde{m}_S(x_R)$	ε	$Y_L(x_R)$
Model (a)	6.7	10.3	3.4×10^{10}	1.4×10^{-8}	6.5×10^{-8}
Model (b)	10	13.8	4.0×10^{10}	1.4×10^{-9}	3.3×10^{-9}

Table 2 Results obtained through the numerical study in each model defined by the parameters in Table 1. The value of x' can be read from Fig. 2. In both models, M_1 is fixed at 10^{14} GeV.

the reheating.

(ii) The Majorana mass $M_i = y_i \bar{u}$ generated through the symmetry breaking should satisfy $M_i > T'$ or the neutrino Yukawa couplings have to be small enough.[m] Otherwise, since the lepton number violating processes containing N_i could be in thermal equilibrium, the existing lepton number asymmetry is washed out immediately through these processes [23]. In that case, the initial lepton number asymmetry plays no role and the scenario is reduced to the usual thermal leptogenesis.

(iii) The inflaton mass and the effective mass of S caused by the symmetry restoration due to the preheating should satisfy $\tilde{m}_\sigma,\ \tilde{m}_S \gg T'$. Since the N_i scatterings mediated by the exchange of σ and $S_\pm$ violate the lepton number, they have to be freezed out to keep the asymmetry generated in the N_i sector.

If these conditions are satisfied, the lepton number asymmetry generated in the N_i sector is expected to be immediately distributed to the SM contents by the interactions which could be in the thermal equilibrium at T_R.

We introduce the lepton asymmetry in the comoving volume as $Y_L \equiv \frac{n_L}{s_R}$ by using the entropy density $s_R = \frac{2\pi^2}{45} g_* T_R^3$, where n_L is defined as the difference between the lepton number density and the antilepton number density. It could be estimated at T_R as

$$Y_L(T_R) = \frac{2\varepsilon n_\sigma(T_R)}{s_R} \simeq 1.5\varepsilon \frac{T_R}{\tilde{m}_\sigma}, \tag{43}$$

where n_σ is defined as $n_\sigma = \frac{\rho_\sigma}{\tilde{m}_\sigma}$ by using ρ_σ which is the energy density of σ and determined by $H \simeq \Gamma_\sigma$. The baryon number asymmetry is generated through the conversion of this lepton number asymmetry Y_L by the $B-L$ conserving sphaleron interaction. If we solve the equilibrium conditions for the chemical potential, the baryon number asymmetry is found to be obtained as $Y_B = -\frac{8}{15} Y_L$ in this model. Thus, the present Y_B is calculated

[m] Since these conditions should be satisfied consistently with the explanation of neutrino oscillation data, the study in the previous part shows the latter one is not allowed in the present model.

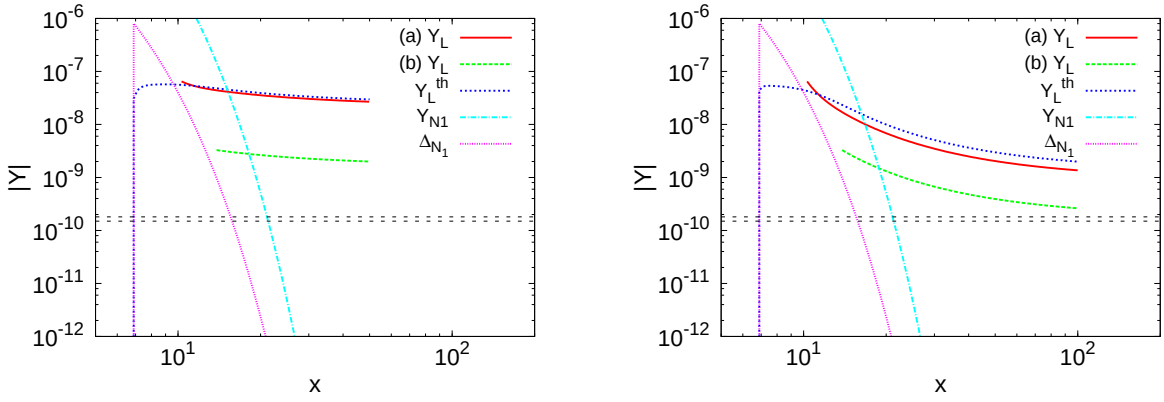

Fig. 5 The evolution of the lepton number asymmetry Y_L at $x > x'$, which is obtained as the solution of the Boltzmann equations for Models (a) and (b). As a reference, $Y_L^{\rm th}$, Y_{N_1} and $\Delta_{N_1}(\equiv |Y_{N_1} - Y_{N_1}^{\rm eq}|)$ for the thermal leptogenesis in Model (a) are plotted at $x > x_R$. The value of Y_L required to explain the observed baryon number asymmetry is shown as the range sandwiched by the horizontal dotted lines. The left and right panels correspond to $\lambda_5 = -1$ and -0.5, respectively.

from $Y_L(T_{\rm EW})$ where $T_{\rm EW}$ is the sphaleron decoupling temperature $T_{\rm EW} \simeq 100$ GeV. The evolution of the lepton number asymmetry after the breaking of the symmetry restoration at T' follows the Boltzmann equations. In that study, we can use the lepton number asymmetry $\frac{5}{8}Y_L(T_R)$ in the ordinary doublet leptons as an initial value for Y_L at T'. It could take a sufficient value only if the scalar mass differences are not strongly suppressed. For example, they should satisfy $m_S^2 > 10^{-7}\tilde{m}_\sigma^2$ in Model (a) and $m_S^2 m_\sigma^2 > 10^{-7}\tilde{m}_\sigma^4$ in Model (b) for $T_R \sim \tilde{m}_\sigma$ and $|y_i| \simeq 0.1$.

In order to estimate $Y_L(T_{\rm EW})$ correctly, it is necessary to take into account the washout effect of the lepton number asymmetry at $T < T'$. It is induced through the inverse decay and the scattering processes which include N_i in them. One may consider a situation such as $M_i \gg T_R$ as a specific situation. Since the washout effects could be almost freezed out at T_R in this case, we can expect $Y_L(T_{\rm EW}) \simeq Y_L(T_R)$. Thus, the baryon number asymmetry is determined as $|Y_B| \simeq 0.5\varepsilon\frac{T_R}{\tilde{m}_\sigma}$. On the other hand, in the marginal case $M_i \gtrsim T'$, the washout effects are crucial and we need to solve the Boltzmann equations which include their effects appropriately. The relevant Boltzmann equations at $T \leq T'$

are given as [26]

$$\frac{dY_{N_1}}{dx} = -\frac{x}{sH(M_1)}\left(\frac{Y_{N_1}}{Y_{N_1}^{\rm eq}} - 1\right)\left\{\gamma_D^{N_1} + \sum_{j=1,2}\left(\gamma_{N_1N_j}^{(2)} + \gamma_{N_1N_i}^{(3)}\right)\right\},$$
$$\frac{dY_L}{dx} = \frac{x}{sH(M_1)}\left\{\varepsilon_{N_1}\left(\frac{Y_{N_1}}{Y_{N_1}^{\rm eq}} - 1\right)\gamma_D^{N_1} - \frac{2Y_L}{Y_\ell^{\rm eq}}\left(\frac{\gamma_{ID}^{N_{2,3}}}{4} + \gamma_N^{(2)} + \gamma_N^{(13)}\right)\right\}, \quad (44)$$

where a hierarchical right-handed neutrino mass spectrum is assumed. x is a dimensionless variable defined as $x = \frac{M_1}{T}$ and N_1 stands for the lightest one. In these equations, we include the decay of N_1 ($\gamma_D^{N_1}$), the inverse decay of $N_{2,3}$ ($\gamma_{ID}^{N_{2,3}}$), and the lepton number violating scatterings mediated by η ($\gamma_N^{(2)}$) and by ℓ_α ($\gamma_N^{(13)}$). The expression of each reaction density γ can be found in [24].

The initial values of Y_L for these Boltzmann equations are given in Table 2, which are obtained for the parameters used in the symmetry restoration study in the previous part. The results of the numerical calculation are shown in Fig. 5 in the cases $\lambda_5 = -1$ (left panel) and $\lambda_5 = -0.5$ (right panel). In this study, the CP asymmetry in the N_1 decay is assumed to be $\varepsilon_{N_1} = -4.0 \times 10^{-8}$, although it can take $|\varepsilon_{N_1}| = O(10^{-3})$ for the maximal CP phase. This allows us to neglect the lepton number asymmetry generated by the thermal origin in the final result. Since $x' > 10$ is satisfied in both cases, the Boltzmann suppression is effective for the lepton number violating processes. On the other hand, the neutrino oscillation data require that the neutrino Yukawa couplings should not be so small and of $O(10^{-1})$ as found from eq. (39) since the right-handed neutrinos are heavy. As a result, the decoupling of the lepton number violating processes could be marginal. The figure shows that $Y_L(x_{\rm EW}) \simeq Y_L(x')$ is satisfied for $\lambda_5 = -1$ in both models. In the case $\lambda_5 = -0.5$, however, their decoupling is not sufficient and then Y_L decreases gradually to a fixed value. Since the lepton number violating processes sufficiently decouple at $x \gg x'$, the lepton number asymmetry Y_L could keep a substantial value until $x_{\rm EW}$. In the same panels, as a reference, we also plot the results of the thermal leptogenesis for the same parameter sets but the initial values such as $Y_L(x') = 0$ and $Y_{N_1}(x') = Y_{N_1}^{\rm eq}(x')$. The lepton number asymmetry produced through it is found to take the same order values as the non-thermal case. This is because $|\varepsilon_{N_1}|\Delta_{N_1} > 10^{-10}$ is satisfied at $x \gtrsim x'$ for $|\varepsilon_{N_1}| = O(10^{-3})$, which is realized for the maximal CP phase. If the CP phase in the neutrino Yukawa couplings does not take such a large value, ε_{N_1} could not be large enough and the thermal leptogenesis could not produce the required baryon number asymmetry.

This condition is not required for the present non-thermal scenario. It is irrelevant to the CP phase in the neutrino Yukawa couplings. Thus, the present non-thermal leptogenesis scenario could be an alternative origin for the baryon number asymmetry in the Universe under such a situation. We should note that the scenario is closely related to the neutrino mass generation and the DM candidate in a somewhat different way from the thermal leptogenesis.

4 Summary

We have proposed a scenario for the non-thermal leptogenesis associated to the reheating due to the inflaton decay. If inflaton is assumed to couple with the right-handed neutrinos, its out-of-equilibrium decay might generate the lepton number asymmetry in the right-handed neutrinos as long as the lepton number is conserved in this sector at such a period. The lepton number asymmetry generated in the right-handed neutrino sector could be transferred to the doublet lepton sector through the lepton number conserving decay. If the transferred asymmetry could take a substantial value, the sufficient baryon number asymmetry is expected to be generated from it. On the other hand, at low energy regions the lepton number violation in the right-handed neutrino sector is necessary for the neutrino mass generation. Thus, the lepton number should be restored at the era of the inflaton decay for this scenario to work well. Preheating associated to the inflation might realize such symmetry restoration.

In this paper, we have studied such a possibility and its application to a one-loop radiative neutrino mass model extended by the singlet scalars. If the inflaton is a singlet scalar, it could have the couplings necessary for this scenario in general. The present study shows that the model can explain the neutrino masses, the DM abundance and the baryon number asymmetry in the Universe simultaneously. The scenario might be applicable for other various particle models. Especially, the ordinary seesaw model could be such a candidate since the right-handed neutrino masses are in a favorable range in the inflation models studied here. However, the DM cannot be included in that case. In this direction, it may be an interesting subject to combine it with an axion DM model.

Acknowledgements

This work is partially supported by MEXT Grant-in-Aid for Scientific Research on Innovative Areas (Grant No. 26104009).

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