Dissertation

Inflation and Reheating with The Singlet Scalars Related to CP Violation

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Date of Submission: 24 June 2021

Kanazawa University
Graduate School of Natural Science and Technology
Division of Mathematical and Physical Sciences

Thesis submitted for the degree

Doctor of Philosophy

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by

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Abstract

"Inflation and Reheating with the Singlet Scalars Related to CP Violation"

–Norma Sidik Risdianto–

We proposed a simple extension of the standard model which contains new fermions and scalars. This extension will perhaps solve the problems of the early universe and also explain the origin of the CP violation. In the inflation cosmology, we proposed that inflaton is the linear combination of the introduced singlet scalars. This new-defined inflaton gives the new features in both inflation and reheating scenarios. Also, we expect the new fermions can give the suitable parameters which solves the phase in CKM and PMNS matrices. Finally, we show that both introduced fermions and scalars can have a new feature in low-scale leptogenesis.

keywords singlet scalars, inflation, leptogenesis, CP violation

Acknowledgment

In considering this thesis, I am owing greatly to Suematsu Daijiro-San for everything during the progress and completion of this manuscript. I am truly indebted to Tomita Katsuki, Idham Syah Alam, Hashimoto Tsuyoshi, Yang Jinbo, Kaneko Daiki, Mr. Mirza Satriawan, Mr. Romy Hanang Setya Budhi, Khoirul Faiq Muzakka, (especially to) Ahsani Hafizhu Shali, Qidir Maulana Binu Soesanto, and *Bartholomew Kuma*, for their great help during the research.

I am also grateful for the hospitality of all members of the Institute for Theoretical Physics, Kanazawa University, including Nishikawa-san for her tremendous help, especially about language problems. Not to forget, the kindness of the Ueno Hoikuen staffs for taking cares of my son during my study. In addition for Elizabeth Nakamura for her help when applying to this nursery. Also, thanks to the Indonesian community in Kanazawa for numerous help during the stay, eg: Mohamad Iman Fauzi, Alifian Mahardhika, and especially all members of PCNU Kanazawa. Finally, with great respect, I am truly thankful to the Ministry of Religious Affairs, Republic of Indonesia, for their funding in the form of scholarship.

Least but always be the honorable mention, I am also indebted to Kunti Robi'atul Mahmudah for her support as a wife and the son, Haidar Nurshiddiq Ibrahim, for his smile.

This work is dedicated to my Mom and Dad as the only "two" who trust me when I was still nobody.

Contents

Introduction

It has been believed that the universe experienced inflation prior to radiation domination epoch [1]. The inflationary phase is needed to solve the almost nearly flat spectrum of temperature, flatness, and homogeneity of the universe observed in Cosmic Microwave Background (CMB) [1, 2]. Inflation theory was introduced and developed in the early 1980's by Alexei Starobinsky [3], Alan Guth [4], and Andrei Linde [5].

Many inflationary models have been proposed, the most famous among them are Starobinsky Inflation [3] and Higgs[1] Inflation [6]. The other inflationary models seem to be derived from these two. This last 20 years, we can find many papers about the growth of the research including this inflationary topic. The development on the observational cosmology, including the data from e.g: [2, 7], may encourage physicists around the world to propose their models. However, due to the lack of observational bound, many speculations appear on the models, especially during preheating and reheating. The reason is clear, the reheating temperature is poorly constrained. In that case, many have construct the reheating temperature related to leptogenesis. It is really interesting to take a lower bound such as Davidson-Ibarra [8] bound on the right-handed neutrino mass for successful leptogenesis. However in our model, we propose the successful leptogenesis could be generated for much lower bound. We also discuss the connection on the origin of CP violation in the inflation.

In the construction of this Ph.D thesis, we construct the **first** chapter as the preliminary before going into the proposed model. In this part, the basic discussion of inflation is presented, with the special feature on the preheating stage for the different case of potential model: $\frac{1}{2}m^2\phi^2$ and $\frac{1}{4}\lambda\phi^4$. They showed the number of particle production differently. Both models show distinct features compared by each others but we conclude them into the single model to generalize the case and obtain the generalized calculation. In the **second** chapter we discuss the Higgs boson as inflaton. This way, we derive the condition for inflation with non-minimal coupling ξ, therefore Higgs inflation may be the best example

[1]This model can be generalized to inflation with non-minimal coupling

since it is the simplest model. Hence, we derive the inflaton condition during inflation and (p)reheating. This unique feature of Higgs inflation can be generalized for many models of inflation which contains non-minimal coupling. That's why we introduce its feature in this thesis. For the **third** chapter, we proposed a model with the simple extension of the standard model. We introduce some new fermions and scalars and we build a complete set of features which discuss the origin of the CP violation connected to the condition of early universe. Finally, we put our conclusion in the **Concluding Remarks**.

Notes: in this thesis we use ϕ as the inflaton field and its definition will vary depending on how we define it. For instance, in chapter 2, the ϕ corresponds to the Higgs field in Einstein frame. In chapter 3, ϕ is the inflaton field which corresponds to the linear combination of σ and S. Please note, for every sequence chapters, the other symbols may be reused with the different definitions.

Chapter 1

The General Approach to Inflation

1.1 Preamble

1.1.1 Inflation and The Timeline of Our Universe

It said, the age of the universe is about > 13.7 billion years [2, 9]. As we don't know when the universe will collapse, it is unclear whether it is appropriate to address the universe to be 'old'. As we desperately search for the remnant of the early universe, the theory about the beginning of the universe seems attract for questions. However, it is also challenging for any scientist to solve the problem with such tiny remnants available. As the early universe energy is in order of Planck mass $\sim 10^{18}$GeV, there still a lot of physics which needs to be solved.

If we consider the timeline, we won't talk much about the Planck epoch, which happens in the pre-inflationary stage followed by Grand Unified epoch[1], due to our current physics the mystery is still progressing to be solved. The next of it: inflationary period, the condition which the space-time was inflated by e-folds[2] from its original size. This period happens in about $< 10^{-32}s$. After the inflation ends, our universe is cooled so much. In addition, it will be reheated via reheating mechanism. We will discuss inflation and reheating mechanism in the rest of this chapter.

Interestingly, inflation is solved two bugging problems in cosmology, namely *horizon* and *flatness* problems. Imagine, there is a horizon which is described as the most observable direction from our observation facilities, such as PLANCK [2] and BICEP2 [7], that the horizon's radius is ~ 45 billion light-years. Given the age of universe we have mentioned, it is impossible that the information (which should run at speed of light)

[1] it has been believed that during this stage three fundamental forces of Standard Model are unified
[2] we will discuss it later in 1.2.3

from one edge is transferred in the opposite edge, but we observed that they are similar. Such a problem is called by the *horizon problem.* Also, this problem appeared that the universe is truly homogeneous in its density, given the 'almost' same temperature everywhere ~ 2.73K. The *flatness problem* requires the universe to be extremely flat near critical value in the very beginning, since small kink of un-flatness may affect greatly in our today universe. Both problems are (perhaps) solved beautifully in the cosmic inflation.

1.1.2 The Friedman-(Lemaître)-Robertson-Walker Geometry

In the preliminary of this topic, one may suggest that in order to learn the meaning behind the cosmological theory, the pre-requisite of General Relativity (GR) is necessary. However, a deeper of understanding GR requires a lot of mathematical tools, such as topology and manifolds [10], we may reduce the whole infinitesimal dimensions to just only a four-dimensional space-time manifold. Even though the newly reduced dimensions are introduced, the whole theoretical aspects of cosmology are explained quite perfectly.

For further understanding, we may show the line element or *metric* of the space-time in infinitesimal dimensions is [11]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1.1}$$

where $g_{\mu\nu}$ is the metric tensor of coordinates $x^\mu; x^\nu$ with $\mu; \nu = 0, 1, 2, 3, ...n$ for any n-dimensional manifold. In the special case of 4-dimensions, the line element of manifold, literally the space-time, can be prescribed by

$$ds^2 = -c^2 dt^2 + g_{ij} dx^i dx^j \tag{1.2}$$

as $i, j = 1, 2, 3$ represents the spatial indices. Here we are still using the SI units, later we will use notation $G = c = \hbar = 1$ and metric signature $(-, +, +, +)$ for this chapter only.

As the matter of fact, once we talk about inflation, one needs to consider the homogeneity and isotropy [12] of the universe. So we need to rewrite the metric (1.1) and (1.2) so it can be used to explain the homogeneous and isotropic universe. The metric best-suited to answer these problems is [10–12]

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \mathcal{K} r^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{1.3}$$

where $\mathcal{K}$ takes the values $-1, 0, +1$ as *open*, *flat*, and *closed manifolds* respectively. Equation (1.3) is the famous Friedman-(Lemaître)-Robertson-Walker(FRW) metric. The $a(t)$ shown in equation (1.3) is the *scale factor*, which roughly speaking, expand the whole spatial coordinates $a(t)$ times. If the space-time is sliced by δt, the infinitesimal slices are marked by t_i, which $i = 0, 1, 2, ...n$. It means the $i+1$th slice was magnified by the factor of $a(t_{i+1})$ from the previous ith slice.

For the purpose of our next discussion, it was clear that $a(t) \propto dr$. So it is obvious that the matter density of the universe should be in the form of $\rho_m \propto a(t)^{-3}$. We will see later in the future that the last definition is truly helpful to understand inflation. To explore the property of $a(t)$, let us expand $a(t)$ in Taylor Series around t_0 and obtains [11]

$$\begin{aligned} a(t) &= a(t_0) - (t_0 - t)\dot{a}(t_0) + \frac{1}{2}(t_0 - t)^2 \ddot{a}(t_0) - ... \\ &= a(t_0)\left[1 - (t_0 - t)H(t_0) - \frac{1}{2}(t_0 - t)^2 q(t_0) H^2(t_0) - ...\right]. \end{aligned} \tag{1.4}$$

Thus we have defined

$$\begin{aligned} H(t) &\equiv \frac{\dot{a}(t)}{a(t)} \\ q(t) &\equiv -\frac{\ddot{a}(t)a(t)}{\dot{a}^2(t)} \end{aligned} \tag{1.5}$$

as Hubble parameter and deceleration parameter respectively. The first line of eq. (1.5), which is $H(t)$ will be our core of interest. Later, this definition of Hubble parameter H will be the most important factor that drives the derivation of many aspects in inflation.

1.1.3 The Cosmological Field Equation

In this part, we may explore further the scale factor $a(t)$. But in order to do that, we should investigate the gravitational field in the presence of matter. We start with gravitational field equation with the presence of cosmological constant, which can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = -\frac{T_{\mu\nu}}{M_p^2}, \tag{1.6}$$

where $R_{\mu\nu}$ is Riemann tensor, $R = g_{\mu\nu}R^{\mu\nu}$ is Ricci scalar, and $M_p = m_p/\sqrt{8\pi G} = 1/\sqrt{\kappa} = 2.44 \times 10^{18}$GeV [13] is the reduced Planck Mass[3]. Lastly, $T_{\mu\nu}$ is the Energy-Momentum

[3] $m_p = 1.22 \times 10^{19}$GeV is the 'true' Planck mass,

Tensor of the perfect fluid and can be expressed as [11]

$$T^{\mu\nu} = (\rho + P)\, u^{\mu} u^{\nu} - P g^{\mu\nu}. \tag{1.7}$$

In order to be suitable for the homogeneous and isotropic universe, the density ρ and pressure P must be the function of cosmic time t only [11]. Combining equation (1.3), (1.5), and (1.6) with the quite-long calculation of Christoffel symbols[4], we obtain the *Friedman equation* as

$$\begin{aligned} \ddot{a} &= \frac{1}{6M_p^2}(\rho + 3P)\, a + \frac{1}{3}\Lambda R, \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{\rho}{3M_p^2} + \frac{1}{3}\Lambda - \mathcal{K}. \end{aligned} \tag{1.8}$$

As for $\Lambda = 0$ and regard $\mathcal{K} \approx 0$ for nearly flat space. Finally, the second line of equation (1.8) can be rewritten as

$$H^2 = \frac{\rho}{3M_p^2}, \tag{1.9}$$

where $\dot{a}/a = H$ has been introduced earlier in equation (1.5) as Hubble parameter, which estimated today to be around the value of $H_0 = 67.4 \pm 0.5\text{km} \cdot s^{-1}\ \text{Mp}c^{-1}$ [2]. The last equation will later be used in this entire work.

One can retract from (1.9) and get $H^2 \propto \rho$, so $H^2 \propto a^{-3}$ for $w = 0$ (heavy matter)[5]. With a little calculation we get $a \propto t^{2/3}$ and $\dot{a} \propto (2/3)t^{1/3}$. Putting back to $H = \dot{a}/a$, one obtains

$$H_i = \frac{2}{3(t_f - t_i)}, \tag{1.10}$$

where H_i represents the Hubble parameter in early inflation and $t_f - t_i$ represents the time required from the start of inflation until the end of inflation.

1.2 The Slow-Roll Inflation

1.2.1 The Energy-Momentum Tensor

In order to further get the details of the inflation, this topic has been described by the term of quantum field theory (QFT). To make it straightforwardly clear, we consider the Lagrangian with scalar ϕ-field

[4] The full derivation can be obtained in ref. [11] in chapter 14

[5] The definition of w will be clear in the next section.

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi). \tag{1.11}$$

Hence, from Lagrangian (1.11) we may construct the equation of motion as

$$\Box \phi + \frac{dV}{d\phi} = 0. \tag{1.12}$$

Until this point we already obtain what we need. Let's continue or calculation on (1.12), we obtain[6]

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{1.13}$$

It is no other than the a semi-classical equation of motion with the Hubble friction.

We can construct energy-momentum tensor from (1.11), which can be deduced from the Noether theorem, which says

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}, \tag{1.14}$$

We will continue to calculate $\nabla_\mu T^{\mu 0} = 0$, which showed the energy conservation, it can be derived by

$$\begin{aligned} \nabla_\mu T^{\mu 0} &= \partial_\mu T^{\mu 0} + \Gamma^\mu_{a\mu} T^{a0} + \Gamma^0_{a\mu} T^{a\mu} \\ &= \partial_0 T^{00} + \Gamma^i_{0i} T^{00} + \Gamma^0_{ij} T^{ii} \\ &= \frac{\partial \rho}{\partial t} + 3H(\rho + P) = 0. \end{aligned} \tag{1.15}$$

Until this point, we need to define the three conditions considering the last equation which depend on its property, namely: heavy matter ($P = 0$), radiation ($\rho = P/3$), and dark energy $\rho = -P$. This way, by using (1.15) we get the solution

$$\rho = a^{-3(1-w)}, \tag{1.16}$$

where $w = P/\rho$. Here we get $w = 0$ for heavy matter, $w = 1/3$ for radiation, and $w = -1$ for dark energy.

[6] Here we used $\Box \phi \equiv \frac{1}{\sqrt{-g}} \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right)$

1.2.2 The Slow-Roll Parameters

In order to obtain the complete and acceptable theory, one needs to tune the theory with observation. As it is widely accepted, the inflationary model is accepted if it fits the observational data, for instance, the data from the Planck satellite [2]. As result, we must obtain the favored slow-roll parameters. Finally, we need to investigate equation (1.13) and putting the slow-roll limit $\ddot{\phi} \approx 0$, hence we obtain

$$\dot{\phi} = -\frac{V'(\phi)}{3H}, \tag{1.17}$$

where the prime denotes the derivative with respect to ϕ. Recalling (1.9) and substitute its value into (1.17), resulting

$$\dot{\phi} = -\frac{M_p}{\sqrt{3}}\frac{V'}{\sqrt{V}}. \tag{1.18}$$

Next, we differentiate the last equation with respect to time and obtain

$$\begin{aligned}\ddot{\phi} &= -\frac{M_p}{\sqrt{3}}\frac{d}{dt}\left(\frac{V'}{\sqrt{V}}\right) = \frac{-1}{\sqrt{3}}M_p\left[\frac{(dV'/dt)\sqrt{V} - (1/2)(V')^{1/2}(dV/dt)}{V}\right] \\ &= \frac{-1}{\sqrt{3}}M_p\left[\frac{V''\dot{\phi}}{\sqrt{V}} - \frac{(V')^2\dot{\phi}}{2V^{3/2}}\right] = \frac{-1}{\sqrt{3}}M_p\sqrt{V}\dot{\phi}\left[\frac{V''}{V} - \frac{(V')^2}{2V^2}\right] \\ &= -M_p^2 H\dot{\phi}\left[\frac{V''}{V} - \frac{(V')^2}{2V^2}\right].\end{aligned} \tag{1.19}$$

Or we may rewrite it in our desired form

$$-\frac{\ddot{\phi}}{H\dot{\phi}} = M_p^2\frac{V''}{V} - \frac{1}{2}M_p\left(\frac{V'}{V}\right)^2. \tag{1.20}$$

Here the denominator of the left-hand side represents the *Hubble friction*[7] [14]. It is obvious that in order for the slow-roll inflation to happen, the friction must be far greater than $\ddot{\phi}$. Therefore, both sides approximate to be zero in the slow-roll regime. Furthermore, we have to determine (from equation (1.20)) the right-hand-side of the equation as the functions of the slow-roll parameters ϵ and η which are depicted as follows

$$\begin{aligned}\epsilon &\equiv \frac{1}{2}M_p\left(\frac{V'}{V}\right)^2 \\ \eta &\equiv M_p^2\frac{V''}{V}.\end{aligned} \tag{1.21}$$

[7] See eq. (1.13) for further details

1.2.3 The e-Folds

E-folds are based on the idea that the universe is multiplied by its size in $e^{\mathcal{N}}$ times during inflation, with $\mathcal{N}$ corresponds to the *number of folds*. Before we straightforwardly move into e-folding, let's re-investigate the Hubble parameter as $H = \dot{a}/a$, which has the solution

$$a(t) = e^{-\int H(t)dt} = e^{-\mathcal{N}}. \tag{1.22}$$

Surely, with everything we got so far in the previous section, we may obtain the $\mathcal{N}$-term in inflationary potential as

$$\begin{aligned}\mathcal{N} &= \int_{t_i}^{t_f} H(t)dt = \int_{\phi_i}^{\phi_f} H(t)\frac{dt}{d\phi}d\phi = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}}d\phi = \int_{\phi_i}^{\phi_f} \frac{3H^2}{-V'}d\phi \\ &= \frac{1}{M_p^2}\int_{\phi_f}^{\phi_i} \frac{V}{V'}d\phi,\end{aligned} \tag{1.23}$$

where we have used the help of equation (1.9)[8]. The e-folds could be any number greater than 50, but with $\mathcal{N} = 60$, it may solve the baryon asymmetry problem [15, 16] if the inflationary energy scale is around $O(10^{16})$GeV.

1.3 The Curvature Perturbation

We consider the scalar function of f in space-time. We will see if this field is changing in the change of time coordinate $t \to t' = t + \Delta t$ and define the new perturbed function as [11]

$$f'(t') = f(t). \tag{1.24}$$

In that case, we may write

$$\begin{aligned}f'(t) &= f'(t' - \Delta t) = f'(t') - \dot{f}'\Delta t + ... \\ &= f(t) - \dot{f}\Delta t.\end{aligned} \tag{1.25}$$

Hence we obtain

$$\Delta f = -\dot{f}\Delta t. \tag{1.26}$$

We may apply the same idea of the perturbation on f to the arbitrary perturbed metric as [11]

$$ds^2 = -(1 - 2\Psi)dt^2 + (1 - 2\psi)a^2(t)(dx^2 + dy^2 + dz^2). \tag{1.27}$$

[8] The ρ in (1.9) equals $V(\phi)$

We are working only in the role of spatial curvature and obtain

$$\begin{aligned}(1-2\psi')a^2 &= (1-2\psi)a^2 - \left(\frac{d}{dt}(1-2\psi)a^2(t)\right)\Delta t \\ &= (1-2\psi)a^2 - [2a\dot{a}(1-2\psi) - 2a^2\dot{\psi}]\Delta t\end{aligned} \tag{1.28}$$

, and finally we obtain

$$\psi' = \psi + \left[H(1-2\psi) - \dot{\psi}\right]\Delta t. \tag{1.29}$$

By using approximation $\psi \ll 1$, Finally, we get

$$|\Delta\psi| \approx H\Delta t = \frac{H}{\dot{\phi}}\Delta\phi. \tag{1.30}$$

This last equation will shortly be used in our calculation for the power spectrum in the next section.

1.4 The Power Spectrum

Before we proceed to the main idea about the power spectrum, firstly we will discuss the spectator field. The spectator field is a field that literally contributes almost 'nothing' during inflation [17]. It has a small energy density compared with the inflaton field yet could give major effects in some parts, e.g. reheating[9].

Consider the scalar field in de Sitter space whom the Action is

$$\begin{aligned}S &= \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi_x\partial_\mu\phi_x = \frac{1}{2}\int d^4x\sqrt{-g}(g^{00}\partial_0\phi_x\partial_0\phi_x + g^{ii}\partial_i\phi_x\partial_i\phi_x) \\ &= \frac{1}{2}\int dtd^3xa^3(-\partial_0\phi_x\partial_0\phi_x + a^2\partial_i\phi_x\partial_i\phi_x) \\ &= \frac{1}{2}\int(ad\tau)d^3xa^3(-(\partial_\tau\phi_x/a)(\partial_\tau\phi_x/a) + a^{-2}\partial_i\phi_x\partial_i\phi_x) \\ &= \frac{1}{2}\int d\tau d^3xa^2\left[-\dot{\phi}_x^2 + (\partial_i\phi_x)^2\right] = -\frac{1}{2}\int d\tau d^3xa^2\left[\dot{u}^2 - (\partial_i u)^2 + \frac{\ddot{a}}{a}u^2\right],\end{aligned} \tag{1.31}$$

where we have used $a(\tau) \equiv -1/H\tau$, $u = a\phi_x$, $dt \equiv ad\tau$ and $\partial_0\phi_x \equiv \partial_\tau\phi_x/a \equiv \dot{\phi}_x/a$. We may use *dot* above a variable to represent the derivative with comoving time instead of time *only in this part* for simplicity. We may decompose u [10] in Heisenberg representation

[9]Depends on the models

[10]Actually u_k equals u_{-k} since $\phi = u/a$ doesn't have a directional orientation, after all, it is a scalar field to begin with

as

$$
\begin{aligned}
u(\tau, x) &= \int \frac{d^3k}{(2\pi)^3}(a_k u_k(\tau)e^{ik\cdot x} + a_k^\dagger u_k^*(\tau)e^{-ik\cdot x}) \\
&= \int \frac{d^3k}{(2\pi)^3}(a_k u_k(\tau) + a_{-k}^\dagger u_{-k}^*(\tau))e^{ik\cdot x} \\
&= \int \frac{d^3k}{(2\pi)^3}(a_k u_k(\tau) + a_{-k}^\dagger u_k^*(\tau))e^{ik\cdot x}
\end{aligned} \tag{1.32}
$$

and it should obey the commutation relation

$$
\begin{aligned}
&[u(\tau, x), \Pi(\tau, x')] = i\delta^{(3)}(x - x'), \\
&[u_k(\tau, k), \Pi_k(\tau, k')] = i(2\pi)^3\delta^{(3)}(k - k'), \\
&[\hat{a}_k, \hat{a}_k^\dagger] = i(2\pi)^3\delta^{(3)}(k - k'),
\end{aligned} \tag{1.33}
$$

where we have used $\Pi \equiv \partial\mathcal{L}/\partial u$. Next, we shall write the equation of motion from action (1.31) and obtain

$$
\ddot{u}_k + \left(k^2 - \frac{\ddot{a}}{a}\right) u_k = 0, \tag{1.34}
$$

which no other than *Sasaki-Mukhanov equation.* This equation shows the semi-classical behavior of the scalar field in the inflationary universe. In the horizon exit $k \gg H^2$ the general solution of (1.34) is

$$
u_k(\tau) = c_1 \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} + c_2 \left(1 + \frac{i}{k\tau}\right) e^{ik\tau}, \tag{1.35}
$$

where c_1 and c_2 are all constants. Using constraint on eq. (1.33), there is $u_k\Pi_k^* - \Pi_k u_k^* = i(2\pi)^3$, we arrive at

$$
|c_1|^2 + |c_2|^2 = \frac{1}{2k}. \tag{1.36}
$$

Setting $|c_2| = 0$, we can have

$$
|c_1|^2 = \frac{1}{2k}. \tag{1.37}
$$

Thus, we determined the *Bunch-Davies* mode function

$$
u_k(\tau) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}. \tag{1.38}
$$

After all, pieces have been already there, we may proceed to calculate the expectation value of $u(\tau, x)$ as

$$\langle u(\tau, x)\rangle = \langle 0|u(\tau, x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\,\langle 0|(\hat{a}_k u_k(\tau) + \hat{a}^\dagger_{-k} u^*_{-k}(\tau))|0\rangle\, e^{ik\cdot x} = 0. \tag{1.39}$$

As we find the value of $\langle u(\tau, x)\rangle$ is zero, we don't have to stop here. In contrast, we may find the variance of inflaton fluctuations experienced the non-zero value as

$$\begin{aligned}\langle |u(\tau, x)|^2\rangle &= \int \frac{d^3k}{(2\pi)^3}\int \frac{d^3k'}{(2\pi)^3}\,\langle 0|(\hat{a}^\dagger_k u^*_k(\tau) + \hat{a}_{-k}u_k(\tau))(\hat{a}'_k u'_k(\tau) + \hat{a}^\dagger_{-k'}u^*_{k'}(\tau))|0\rangle \\ &= \int \frac{d^3k}{(2\pi)^3}\int \frac{d^3k'}{(2\pi)^3} u_k(\tau)u^*_{k'}(\tau)\,\langle 0|[\hat{a}'_k, \hat{a}^\dagger_{-k'}]|0\rangle = \int \frac{d^3k}{(2\pi)^3}|u_k(\tau)|^2 \\ &= \int \frac{dk}{(2\pi)^3}k^2(4\pi)|u_k(\tau)|^2 = \int d\ln k \frac{k^3}{2\pi^2}|u_k(\tau)|^2 = \int d\ln k P_u(\tau, k).\end{aligned} \tag{1.40}$$

Thus we defined the *power spectrum* [18]

$$P_u(\tau, k) \equiv \frac{k^3}{2\pi^2}|u_k(\tau)|^2, \tag{1.41}$$

which is dimensionless. Later, by using (1.38) for $\kappa\tau \gg 1$, we can obtain the final form of the power spectrum as

$$P_u(\tau, k) \approx \frac{k^3}{2\pi^2}\frac{1}{2k^3\tau^2} = \frac{k^3}{2\pi^2}\frac{a^2H^2}{2k^3} = \frac{a^2H^2}{4\pi^2}. \tag{1.42}$$

As we used the relation $u = a\phi_x$ or $\phi_x = u/a$, we can also write

$$P_\phi(\tau, k) = \frac{P_u(\tau, k)}{a^2} = \left(\frac{H}{2\pi}\right)^2. \tag{1.43}$$

Alternatingly, using the metric perturbation of (1.30), we can rewrite (1.43) in the form

$$P_\psi(\tau, k) = \left(\frac{H}{\dot{\phi}_x}\right)^2 P_\phi(\tau, k) = \left(\frac{H}{\dot{\phi}_x}\right)^2\left(\frac{H}{2\pi}\right)^2 = \frac{H^4}{4\pi^2\dot{\phi}_x^2}. \tag{1.44}$$

Above equation can be simply solved by substituting (1.9) with small $\dot{\phi}$ and (1.17). With these, equation (1.44) can be written in the term of potential:

$$P_\psi(\tau, k) = \frac{V^3}{12\pi^2 M_p^6 V'^2}, \tag{1.45}$$

since the last equation is applicable for any desired inflationary models (such as R^2 inflation [3], Higgs Inflation [6], and Inert-Doublet Inflation [15]) by calculating their potential, one can constraint the power spectrum based on Planck 2018 result [2]. We will apply this equation in the next chapter.

1.5 End of Inflation and Reheating

The end of inflation is marked when $\ddot{\phi}$ is increasing until the point that slow-roll parameter $\epsilon \sim 1$. Thus, $\ddot{\phi}$ is comparable with the Hubble friction term. During this time, the inflaton field oscillates around a minimum of its potential. The inflaton decays to Standard Model (SM) particles or perhaps dark matter [1]. This process has been originally studied once the theory of inflation was proposed [19] and considered only perturbative decay. This process is later known as reheating. As inflation left us with a cold and empty universe, then the reheating process is proposed to explain the large entropy and energy of our universe at present.

As previously remarked, the inflaton will decay perturbatively to SM particles during the end of inflation. The idea of this process was really simple. During the end of inflation, as the universe dilutes from the inflaton field, its energy density is converted completely to relativistic particles ρ as [12],

$$\rho = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2 - m^2)^{1/2}E^2}{\exp([E-\mu]/T) - 1} dE \tag{1.46}$$

where E is the total energy, m is the mass of combined SM relativistic particles, thus we take the their internal degrees of freedom g by $O(100)$ [20]. Also, μ is chemical potential at temperature T.

As it correspond to the relativistic case, we can neglect the mass m and solve the last equation using the Riemann-Zeta function, but we will skip the details. Finally, we can have

$$\rho = \frac{g\pi^2}{30}T^4. \tag{1.47}$$

The energy density ρ should satisfy eq. (1.9) where $H^2 = \rho/3M_p^2$. One should also remember, at reheating stage, the decay rate of inflaton Γ_{total} should be[11] $\Gamma_{total} \sim H$. By substituting above results one obtains

$$T_R = \left(\frac{90}{g\pi^2}\right)^{1/4}\sqrt{M_p\Gamma_{tot}}. \tag{1.48}$$

[11] The reheating stage ends when $\Gamma > H$ [20]

In calculating the decay channel, we should take the Lagrangian of interaction as

$$\mathcal{L}_{int} = y\phi\psi^2 + \sigma\phi\bar{\zeta}\zeta, \tag{1.49}$$

where ψ and ζ are respectively corresponding to the scalar and spinor fields of decayed particles. Given above Lagrangian, we can simply calculate the decay rate of the inflaton as

$$\Gamma_\psi = \frac{y^2}{8\pi m} \qquad \text{and} \qquad \Gamma_\varphi = \frac{\sigma^2 m}{8\pi}. \tag{1.50}$$

Using these decay rates of the above result, we can construct the total decay rate of all species[12] as

$$\Gamma_{total} = \Gamma_\psi + \Gamma_\zeta. \tag{1.51}$$

In this (old) theory of reheating, the temperature predicted by this theory is extremely large. For instance, if we apply this theory in Higgs inflation [6], the reheating tmeperature is about $\sim 10^{15}$GeV. In the next section, we will describe another mode to drain the inflaton energy via oscillation which is called by preheating.

1.6 Preheating—The Preamble

During the end of inflation, the energy stored by inflaton field tends to be transferred to relativistic particle via decay or annihilation. The decay can be perturbative or non-perturbative. In perturbative decay, the process can be treated by a tree-level decay process. However, if the drain of inflaton field is occurred that way, the reheating temperature tends to be very high[13]. So the idea non-thermal decay is appeared, which inflaton can decay to another particle via non-perturbative effect or parametric resonance. This effect is first depicted in [21], where the preliminary stage should be added before the reheating stage, which later is called *preheating*. In following of this thesis, we will use the description mainly from the paper [22] for $V = \frac{1}{2}m^2\phi^2$ theory and [23] for $V = \frac{1}{4}\lambda\phi^4$ theory. Also, we used the notation ϕ as the inflaton in both potentials to save some introduction to new symbols. However, please be careful in taking ϕ in these separate ways.

The perturbative decay is not so efficient, In that case, the non-perturbative decay becomes the main role. It is the decay that cannot be explained by the perturbative effect. Thus, the decay due to non-perturbative effect is defined differently. It is the

[12]This is just the simplest case, we can add as many particles if necessary, depend on the model used
[13]See discussion in the last section in 2.2 about the reheating temperature without preheating

decay that is caused by the oscillation of the inflaton field. As it is rather unusual, this method has been prescribed in some early papers. Historically, the particle production due to oscillating inflaton field was developed by [24] and [25] which described the *narrow resonance*[14]. The narrow resonance describes the particle production suitable for the late stage of preheating, which is less efficient. In contrary, *broad resonance* is described as more efficient particle production which may appear in the early stage of the preheating.

There is some interesting feature of this non-perturbative mode, mass of the produced particle created by this mode can be much larger than inflaton mass. For further usage in this paper, we will only focus on *stochastic resonance* in which the particle production from oscillation occurs in series of kicks during the zero crossing [22]. In that case, we do not discuss the condition provided by narrow and broad resonance. The reason is: we only consider the most important part for the chapter 2 and 3, since we truly consider only the most efficient particle production during preheating.

1.7 Preheating With The Potential $V = \frac{1}{2}m^2\phi^2$

For the potential $V = \frac{1}{2}m^2\phi^2$, we will rewrite the Friedman equation (1.9), eq.(1.13) and second Friedman equation as

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$$\begin{aligned} H^2 &= \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right), \\ \ddot{\phi} &+ 3H\dot{\phi} + m^2\phi = 0, \\ \dot{H} &= \frac{\dot{\phi}^2}{2M_p^2}. \end{aligned} \tag{1.52}$$

One can define from the first line of Eq. (1.52) [20]

$$\dot{\phi} = \sqrt{6}HM_p\cos\theta \quad \text{and} \quad m\phi = \sqrt{6}HM_p\sin\theta. \tag{1.53}$$

From the last line of eq.(1.52), we have

$$\dot{H} = -\frac{1}{2M_p^2}\dot{\phi}^2 = -\frac{1}{2M_p^2}\left[6H^2M_p^2\cos^2\theta\right] = -3H^2\cos^2\theta. \tag{1.54}$$

Taking the derivation in respect to time from the right equation of (1.53), one obtains

[14]The application for oscillation is described at first by these papers, but the method was developed far earlier (see [26])

$$m\dot{\phi} = \sqrt{6}M_p \left(\dot{H}\sin\theta - H\dot{\theta}\cos\theta\right). \tag{1.55}$$

Inserting the last equation with eq. (1.53) and (1.54) :

$$m\left(\sqrt{6}HM_p\cos\theta\right) = \sqrt{6}M_p\left[\left(-3H^2\cos^2\theta\right)\sin\theta - H\dot{\theta}\cos\theta\right]$$

Finally, we obtain

$$\dot{\theta} = -m - \frac{3}{2}H\cos 2\theta. \tag{1.56}$$

If $m \gg H$, then the second term can be neglected, hence the solution is $\theta = mt$. Using this result, we can rewrite eq. (1.54) as

$$\frac{1}{H^2}\frac{dH}{dt} = -3\cos^2(mt)$$

Solving the above calculation, we simply obtain

$$H(t) = \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right)^{-1} \simeq \frac{2}{3t}\left(1 + \frac{\sin(2mt)}{2mt}\right). \tag{1.57}$$

Substituting this result to the right-hand side of eq. (1.53), we get

$$\phi(t) \sim \tilde{\phi}(t)\sin(mt)\left(1 + \frac{\sin(2mt)}{2mt}\right), \tag{1.58}$$

where $\tilde{\phi}(t)$ is the amplitude of $\phi(t)$ which is defined as

$$\tilde{\phi}(t) \equiv 2\sqrt{\frac{2}{3}}\frac{M_p}{mt}. \tag{1.59}$$

One should note that (1.57), which is $H \simeq \frac{2}{3t}$–correspond to $a \propto t^{2/3}$, show that preheating in $\frac{1}{2}m^2\phi^2$ model is belong to matter dominated region.

1.7.1 The Oscillation Phase For $\frac{1}{2}m^2\phi^2$ Theory

Consider the Lagrangian of the created particle

$$\mathcal{L} \supset \frac{1}{2}m_\psi^2\psi^2 + \frac{1}{2}g\phi^2\psi^2 \tag{1.60}$$

after we take $\delta\mathcal{L} = 0$, it is correspond to the equation of motion:

$$\Box\psi - \left(m_\psi^2 + g\phi^2\right)\psi = 0. \tag{1.61}$$

We can decompose ϕ in Heisenberg representation, namely

$$\psi(x,t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \psi_k(t) e^{-i\bar{k}\cdot\bar{x}} + \hat{a}_k^\dagger \psi_k^*(t) e^{i\bar{k}\cdot\bar{x}}\right), \tag{1.62}$$

and hence we can obtain

$$\ddot{\psi}_k + 3H\dot{\psi}_k + \left(\frac{k^2}{a^2} + m_\psi^2 + g\phi^2\right)\psi_k = 0. \tag{1.63}$$

Neglecting the expansion $H \sim 0$, the bare mass of ψ_k ($m_\psi = 0$), and we defined $\phi = \tilde{\phi}\sin(mt)$, we finally obtain

$$\ddot{\psi}_k + \left(\frac{k^2}{a^2} + g\tilde{\phi}^2 \sin^2(mt)\right)\psi_k = 0. \tag{1.64}$$

This is the obvious result which belongs to the type of *Lame Equation.*

1.8 Preheating with the potential $V = \frac{1}{4}\lambda\phi^4$

In this part, we will derive the amplitude of ϕ in the potential $V = \frac{1}{4}\lambda\phi^4$. One can use the Friedman equation from (the first line of) eq. (1.52) with potential $V \gg \dot{\phi}^2$, here we obtain

$$H^2 \simeq \frac{1}{3M_p^2}\left(\frac{1}{4}\lambda\phi^4\right) \rightarrow H^2 \simeq \frac{1}{3M_p^2}\left(\frac{1}{4}\lambda\tilde{\phi}^4\right) \rightarrow \tilde{\phi}(t) \simeq H^{1/2}\left(\frac{3M_p^2}{\lambda}\right)^{1/4}, \tag{1.65}$$

where $\tilde{\phi}$ is the amplitude of ϕ. Hence, for radiation dominated[15] $a \propto t^{1/2}$ then $H = \frac{1}{2t}$, we can substitute above result[16]:

$$\tilde{\phi}(t) \simeq \frac{1}{\sqrt{t}}\left(\frac{3M_p^2}{\lambda}\right)^{1/4}. \tag{1.66}$$

[15] One can see ref. [27] especially in eq. 5.56 in that book.

[16] this potential, correspond to radiation dominated, one can refer [1] and see [27] page 242.

1.8.1 The Oscillation Phase For $\frac{1}{4}\lambda\phi^4$ Theory

In this part, we will study the oscillation of the inflaton field in the $\frac{1}{4}\lambda\phi^4$ model. It will be used for most of the inflationary model, as we will use it in chapter 3 extensively. Before we proceed, we may write the conformal time τ as

$$\tau \equiv \int \frac{dt}{a(t)}. \tag{1.67}$$

For the reminder, we already used the exact definition in eq. (1.32). In addition, we also define the conformal field

$$\varphi \equiv a\phi. \tag{1.68}$$

The Klein-Gordon equation for the inflaton is depicted in (1.13). For $\frac{1}{4}\lambda\phi^4$ theory, with the addition of changes in conformal time τ and conformal field φ, it turns out to be[17]

$$\varphi'' + \lambda\varphi^3 = 0, \tag{1.69}$$

which superscript–prime denotes the derivative in respect of conformal time. In the same way, the Friedman equation (1.52) with potential $\frac{1}{4}\lambda\phi^4$, can be presented as

$$a'^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\varphi'^2 + \frac{1}{4}\lambda\varphi^4\right). \tag{1.70}$$

Before we proceed, it is more convenient to introduce $\tilde{\varphi}$ as the 'constant' amplitude of φ, by this method, neglecting the kinetic term in (1.70), we find

$$\left(\frac{da}{d\tau}\right)^2 \simeq \frac{1}{3M_p^2}\left(\frac{1}{4}\lambda\tilde{\varphi}^4\right) \quad \rightarrow \quad a(\tau) = \frac{1}{2}\sqrt{\frac{\lambda}{3}}\frac{\tilde{\varphi}^2}{M_p}\tau. \tag{1.71}$$

Again, we can recall the definition of conformal time (1.67) and the last equation to obtain

$$\frac{dt}{d\tau} = a(\tau) = \frac{1}{2}\sqrt{\frac{\lambda}{3}}\frac{\tilde{\varphi}^2}{M_p}\tau \quad \rightarrow \quad t = \frac{1}{4}\sqrt{\frac{\lambda}{3}}\frac{\tilde{\varphi}^2}{M_p}\tau^2. \tag{1.72}$$

Lastly, we can redefine the conformal time to dimensionless variable $\mathbf{x}$ as

$$\mathbf{x} = \sqrt{\lambda}\tilde{\varphi}\tau = 2(3M_p^2\lambda)^{1/4}\sqrt{t}. \tag{1.73}$$

[17] We neglect the a'' terms, for its smallness compared by others.

and we will use it shortly[18]. One should look back at eq. (1.69), we can write φ as

$$\varphi = \tilde{\varphi} f(\mathbf{x}), \tag{1.74}$$

where $f(x)$ is the some function which depend on dimensionless conformal time $\mathbf{x}$. Furthermore, the solution of eq. (1.69) requires φ to be elliptic cosine function:

$$\varphi = \tilde{\varphi}\mathrm{cn}\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right). \tag{1.75}$$

1.9 Particle Fluctuation

During preheating, particle production is presented by means of the oscillation of the inflaton field. For preliminary, we will discuss the production of self-excitation: production of ϕ from inflaton ϕ. In here, we will use the quartic coupling λ from $\frac{1}{4}\lambda\phi^4$. However, we will see late whether this process is dominant or not. To make it clear, we should look back on eq. (1.12) for potential $\frac{1}{4}\lambda\phi^4$ and rewrite it as

$$\Box\phi - 3\lambda\phi^3 = 0. \tag{1.76}$$

We can decompose ϕ in Heisenberg representation, namely

$$\phi(x,t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \phi_k(t) e^{-i\bar{k}\cdot\bar{x}} + \hat{a}_k^\dagger \phi_k^*(t) e^{i\bar{k}\cdot\bar{x}}\right), \tag{1.77}$$

and hence we can obtain

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2} + 3\lambda\phi^2\right)\phi_k = 0 \tag{1.78}$$

from eq. (1.76)[19]. The next step is extremely important. In this part, we will rewrite eq. (1.78) with dimensionless conformal time $\mathbf{x}$ in (1.73) and conformal field $\varphi = a\phi$. With

[18] Actually, its kind of ineffective to redefine the $t \to \tau \to \mathbf{x}$, We can straightforwardly take from $t \to \mathbf{x}$ directly, but here we just follow [23] to follow up the paper.

[19] Here we used $\Box\phi \equiv \frac{1}{\sqrt{-g}}\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)$. Also $g_{00} = -1$, $g_{ii} = a^2$, $g = a^6$, and $\sqrt{-g} = a^3$

these, we can transform eq. (1.78) as

$$\begin{aligned} 0 &= \ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2} + 3\lambda\phi^2\right)\phi_k \\ &= \frac{d}{dt}\left(\frac{d\phi_k}{dt}\right) + 3\left(\frac{da}{adt}\right)\left(\frac{d\phi_k}{dt}\right) + \left(\frac{k^2}{a^2} + 3\lambda\phi^2\right)\phi_k \\ &= \frac{d}{ad\tau}\left(\frac{d(\varphi_k/a)}{ad\tau}\right) + 3\left(\frac{da}{a^2d\tau}\right)\left(\frac{d(\varphi_k/a)}{ad\tau}\right) + \left(\frac{k^2}{a^2} + 3\lambda\phi^2\right)\frac{\varphi_k}{a} \\ &= \lambda\tilde{\varphi}^2\frac{d}{ad\mathbf{x}}\left(\frac{d(\varphi_k/a)}{ad\mathbf{x}}\right) + 3\lambda\tilde{\varphi}^2\left(\frac{da}{a^2d\mathbf{x}}\right)\left(\frac{d(\varphi_k/a)}{ad\mathbf{x}}\right) + \left(\frac{k^2}{a^2} + 3\lambda\phi^2\right)\frac{\varphi_k}{a}. \end{aligned} \tag{1.79}$$

Solving the above result, we can have

$$\varphi_k'' + \left[\frac{k^2}{\lambda\tilde{\varphi}^2} + \frac{3\lambda\phi^2a^2}{\lambda\tilde{\varphi}^2}\right]\varphi_k = \varphi_k'' + \left[\frac{k^2}{\lambda\tilde{\varphi}^2} + \frac{3\varphi^2}{\tilde{\varphi}^2}\right]\varphi_k = \varphi_k'' + \left[\frac{k^2}{\lambda\tilde{\varphi}^2} + 3f(\mathbf{x})^2\right]\varphi_k = 0 \tag{1.80}$$

or simply

$$\varphi_k'' + \left[\kappa^2 + 3\mathrm{cn}^2\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right)\right]\varphi_k = 0. \tag{1.81}$$

The prime in the two last equations corresponds to the derivative of dimensionless conformal time $\mathbf{x}$ and here we also redefined $\kappa^2 = \frac{k^2}{\lambda\tilde{\varphi}}$.

During oscillation, the inflaton ϕ may decay to another particle which couple with it. In order to understand this decay channel, assume that we have Lagrangian

$$\mathcal{L} \supset g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - g\phi^2\psi^2. \tag{1.82}$$

Using the same method to derive (1.81) for ψ_k, we can obtain

$$\psi_k'' + \left[\kappa^2 + \frac{g}{\lambda}\mathrm{cn}^2\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right)\right]\psi_k = 0 \tag{1.83}$$

The above equation will be our master equation. The ratio g/λ will play a special role in preheating and we will see it later shortly. In another case, we can also define

$$\omega_k^2 = \kappa^2 + \frac{g}{\lambda}\mathrm{cn}^2\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right). \tag{1.84}$$

Before we are moving forward, it is important to describe particle production through oscillation. In this case, we will work on the particle production of arbitrary field ψ. It is clear, in order for the inflaton field to perform a decay to another particle, say two ψ particles, it must have mass at least twice larger than ψ. During oscillation period, inflaton mass $3\lambda\phi^2$, while ψ particle has mass $g\phi^2$. The problem lies in how large the couplings? Generally, g is much larger than λ, this way, the inflaton field can only decay to two ψs only during the zero-crossing[20]. Actually, we should emphasize the value of g, hence we will get introduced to the distinction of narrow resonance and broad resonance. But in this chapter, we will only consider the $g \gg \lambda$ case. Which will be best described by *broad resonance*. Also we note the appearance of *Stochastic Resonance* [22], which is really important in this paper, where the explosive particle production mostly happens in zero crossings. In contrary, if $g/\lambda \ll \mathcal{O}(1)$ is satisfied, the loss of energy density from the inflaton field by resonance may not be so important in the reheating process, and it is belongs to the *narrow resonance*[21]. Let's say, we have g_1/λ and g_2/λ, which correspond to the two types of particles, if both satisfy $g_1/\lambda \gg 1$ and $g_2/\lambda \ll 1$, the particle with g_2/λ doesn't have a chance to compete with particle g_1/λ in depleting the inflaton field's energy. Hence, it can be neglected. In this way, it is also clear that the self-producing inflaton field showed by eq. (1.81) will not so important. As a result, we can neglect such a case in discussing the reheating temperature.

1.10 Particle Production

In this part, we can borrow the result from eq. (1.83) with field ψ_k. Here, we describe the particle production in general case to build up necessary theory for preheating. Let us start with the solution for *before particle production*:

$$\begin{aligned} \psi_k &= \psi_k^+ a_{\mathbf{k}}^\dagger + \psi_k^- a_{-\mathbf{k}} \\ \psi^\dagger &= (\psi_k^+)^* a_{\mathbf{k}} + (\psi_k^-)^* a_{-\mathbf{k}}^\dagger, \end{aligned} \tag{1.85}$$

and *after particle production* is

$$\begin{aligned} \psi_k &= \bar{\psi}_k^+ \bar{a}_{\mathbf{k}}^\dagger + \bar{\psi}_k^- \bar{a}_{-\mathbf{k}} \\ \psi^\dagger &= (\bar{\psi}_k^+)^* \bar{a}_{\mathbf{k}} + (\bar{\psi}_k^-)^* \bar{a}_{-\mathbf{k}}^\dagger, \end{aligned} \tag{1.86}$$

where $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the same operators mentioned in (1.32). We can also simply define

$$N_{\mathbf{k}} = a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \qquad \text{and} \qquad \bar{N}_{\mathbf{k}} = \bar{a}_{\mathbf{k}}^\dagger \bar{a}_{\mathbf{k}} \tag{1.87}$$

[20]When inflaton field ϕ crosses zero, the inflaton mass is zero, together with induced ψ mass. The decay 'gate' will open and $\phi \to \psi\psi$ is successful

[21]In narrow resonance, inflaton energy density is depleted much slower than *broad resonance*, wherein the opposite drawing the inflaton energy density is much faster. Here *Stochastic Resonance* is even wilder

as the number of particle operators *before* and *after* particle production. The initial condition of a particle can be described by [28]

$$ {}_{in}\langle 0|\, N_{\mathbf{k}}\, |0\rangle_{in} = {}_{in}\langle 0|\, a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}}\, |0\rangle_{in} = 0. \tag{1.88} $$

Thus, we can relate $a_{\mathbf{k}}$ and $\bar{a}_{\mathbf{k}}$ via

$$ \begin{aligned} \bar{a}_{\mathbf{k}} &= \alpha_k a_{\mathbf{k}} + \beta_k a^{\dagger}_{\text{-}\mathbf{k}} \\ \bar{a}^{\dagger}_{\mathbf{k}} &= \alpha^*_k a^{\dagger}_{\mathbf{k}} + \beta^*_k a_{\text{-}\mathbf{k}}, \end{aligned} \tag{1.89} $$

where both α_k and β_k are *Bogoliobov coefficients.* With these definition, we can write the particle number operator *after particle production* as

$$ \begin{aligned} {}_{in}\langle 0|\, \bar{N}_k\, |0\rangle_{in} = {}_{in}\langle 0|\, \bar{a}^{\dagger}_{\mathbf{k}} \bar{a}_{\mathbf{k}}\, |0\rangle_{in} &= {}_{in}\langle 0| \left(\alpha^*_k a^{\dagger}_{\mathbf{k}} + \beta^*_k a_{\text{-}\mathbf{k}}\right)\left(\alpha_k a_{\mathbf{k}} + \beta_k a^{\dagger}_{\text{-}\mathbf{k}}\right) |0\rangle_{in} \\ \langle \bar{N}_k\rangle &= |\beta_k|^2 (2\pi)^3 \delta(0) = |\beta_k|^2 \times \text{Volume}, \end{aligned} \tag{1.90} $$

where we have used the relation $(2\pi)^3\delta(\mathbf{k}'-\mathbf{k}) = \int d^3x \exp(i[\mathbf{k}'-\mathbf{k}]\cdot\mathbf{x})$. Giving $\mathbf{k}'=\mathbf{k}$, we obtain $(2\pi)^3\delta(0) = \int d^3x = \text{Volume}$. This way we can define n_k as the particle density via

$$ n_k = \frac{\langle \bar{N}_k\rangle}{\text{Volume}} = |\beta_k|^2. \tag{1.91} $$

One should also note that the commutation relation for this creation and annihilation $a_{\mathbf{k}}$ must be valid, hence for this operator *after particle production* we have

$$ \begin{aligned} (2\pi)^3\delta(\mathbf{k}'-\mathbf{k}) &= \left[\bar{a}_{\mathbf{k}'}, \bar{a}^{\dagger}_{\mathbf{k}}\right] \\ &= \left[\left(\alpha_{k'} a_{\mathbf{k}'} + \beta_{k'} a^{\dagger}_{-\mathbf{k}'}\right), \left(\alpha^*_k a^{\dagger}_{\mathbf{k}} + \beta^*_k a_{-\mathbf{k}}\right)\right] \\ &= \alpha_{k'}\alpha_k \left[a_{\mathbf{k}'}, a^{\dagger}_{\mathbf{k}}\right] + \beta_{k'}\beta_k \left[a_{-\mathbf{k}'}, a^{\dagger}_{-\mathbf{k}}\right]. \end{aligned} \tag{1.92} $$

Finally we can get the desired result from the last calculation as

$$ |\alpha_k|^2 - |\beta_k|^2 = 1. \tag{1.93} $$

1.11 The WKB approximation of Particle production

In this part we will combine the description of particle production on 2 models with potential $\frac{1}{2}m^2\phi^2$ and $\frac{1}{4}\lambda\phi^4$. So basically, we can get the simpler version regardless of the models. Firstly, we can recall eq. (1.64) and (1.83) in their separate models

$$\begin{array}{cc} \frac{1}{2}m^2\phi^2 \quad \text{model} & \frac{1}{4}\lambda\phi^4 \quad \text{model} \\ \boxed{\ddot{\psi}_k + \left(\frac{k^2}{a^2} + g\tilde{\phi}^2\sin^2(mt)\right)\psi_k = 0} & \boxed{\psi_k'' + \left[\kappa^2 + \frac{g}{\lambda}\mathrm{cn}^2\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right)\right]\psi_k = 0}. \end{array} \tag{1.94}$$

Both equations are related to the particle production during zero crossing. In that case, some approximation may apply such as $\sin^2(mt) \simeq (mt)^2$ and $\mathrm{cn}^2(\mathbf{x}, 1/\sqrt{2}) \simeq \frac{1}{2}\mathbf{x}^2$. With these, we can rewrite eq. (1.94) as

$$\begin{array}{cc} \frac{1}{2}m^2\phi^2 \quad \text{model} & \frac{1}{4}\lambda\phi^4 \quad \text{model} \\ \boxed{\ddot{\psi}_k + \left(k^2/a^2 + g\tilde{\phi}^2m^2t^2\right)\psi_k = 0} & \boxed{\psi_k'' + \left[\kappa^2 + \frac{g}{2\lambda}\mathbf{x}^2\right]\psi_k = 0}. \end{array} \tag{1.95}$$

Finally, we can serve these equations into a single form which is

$$\boxed{\frac{d^2\psi_k}{dq^2} + \left(\kappa_u^2 + q^2\right)\psi_k = 0}. \tag{1.96}$$

The bracket can be defined by $\kappa_u^2 + q^2 = \omega_u^2$ to simplify the result. Also, if

$$q^2 = \begin{cases} \sqrt{g}\tilde{\phi}mt^2, & \text{for } \frac{1}{2}m^2\phi^2 \quad \text{model} \\ \sqrt{\frac{g}{2\lambda}}\mathbf{x}^2, & \text{for } \frac{1}{4}\lambda\phi^4 \quad \text{model.} \end{cases} \tag{1.97}$$

$$\kappa_u^2 = \begin{cases} \frac{k^2}{a^2\sqrt{g}\tilde{\phi}m}, & \text{for } \frac{1}{2}m^2\phi^2 \quad \text{model} \\ \frac{\kappa^2}{\sqrt{g^2/2\lambda}}, & \text{for } \frac{1}{4}\lambda\phi^4 \quad \text{model,} \end{cases} \tag{1.98}$$

the solution of the last result can be written by[22]

$$\bar{\psi}_k^{\pm} = \frac{1}{\sqrt{\omega_u}}\exp\left(\pm i\int_1^q \left(\kappa_u^2 + q^2\right)^{1/2} dq\right), \tag{1.99}$$

where the lower limit 1 will be described soon. Here, the bar corresponds to the condition of post-scattering. We can also approximate some terms:

$$\pm i\int_1^q \left(\kappa_u^2 + q^2\right)^{1/2} dq \approx \pm i\int_1^q \left(q + \frac{\kappa_u^2}{2q}\right) dq = \pm i\left(\frac{q^2}{2} - \frac{1}{2} + \frac{\kappa_u^2}{2}\ln q\right), \tag{1.100}$$

where we assumed $q \gg \kappa_u$. In this subsection, we will proceed to Wentzel–Kramers–Brillouin

[22] one can find this subsection derived beautifully in [27]

(WKB) which can be a good approximation when the adiabaticity does not violated so much. The adiabaticity condition requires $\mathcal{R} = \frac{1}{\omega_u}\frac{d\omega_u}{dt} > 1$, which corresponds to the constraint $\kappa_u < q^{2/3} - q^2$. The maximum value is obtained when $q = 0.44$, and can be inferred that adiabaticity can be violated when $|q| < 1$. Here is the reason for the lower limit on the integral of eq. (1.100).

It is also allowed to simplify our argument using approximation $\omega \approx q^{1/2}$ and the result given by (1.100). In that way, $\bar{\psi}_k^{\pm}$ can be written by

$$\bar{\psi}_k^{\pm} = q^{\pm i\frac{\kappa_u^2}{2} - \frac{1}{2}} \exp\left(\pm i\frac{q^2}{2}\right) \tag{1.101}$$

For the pre-scattering solution, one can trace back from eq. (1.99) and (1.100). The integration will be different, say

$$\pm i \int_{-q}^{-1} \left(\kappa_u^2 + q^2\right)^{1/2} dq \approx \pm i \int_{-q}^{-1} \left(q + \frac{\kappa_u^2}{2q}\right) dq = \mp i \left(\frac{q^2}{2} - \frac{1}{2} + \frac{\kappa_u^2}{2}\ln q\right), \tag{1.102}$$

hence we can obtain

$$\psi_k^{\pm} = q^{\mp i\frac{\kappa_u^2}{2} - \frac{1}{2}} \exp\left(\mp i\frac{q^2}{2}\right). \tag{1.103}$$

Thus, the relation of pre and post-scattering can be related by

$$\begin{aligned} A_+\psi_k^+ &\to B_+\psi_k^+ + C_+\psi_k^- = \bar{\psi}_k^+ \\ A_-\psi_k^- &\to B_-\psi_k^- + C_-\psi_k^+ = \bar{\psi}_k^-, \end{aligned} \tag{1.104}$$

or more beautifully

$$\begin{pmatrix} \bar{\psi}_k^+ \\ \bar{\psi}_k^- \end{pmatrix} = \begin{pmatrix} B_+ & C_+ \\ C_- & B_- \end{pmatrix} \begin{pmatrix} \psi_k^+ \\ \psi_k^- \end{pmatrix}, \tag{1.105}$$

where $A_\pm$, $B_\pm$, and $C_\pm$ are all constants which are truly our consent right now. The method solving these constants can be seen in [29]. Firstly, we want to connect the pre and post-scattering. However, it can't be done simply by moving q from the right axis to another axis. The problem is: when q is small enough, remember that this approximation is broken once $|q| < 1$. We should push the q into the complex plane while maintaining its quite large value$|q| > 1$. In that case, it is allowed to write $q = \tilde{q}\exp(i\theta)$. Thus we move q into the semicircular path in a complex plane. Finally, we can insert $q = \tilde{q}\exp(i\pi)$

to eq. (1.101), let's say for + mode as

$$\begin{aligned}\bar{\psi}_k^+ &= \tilde{q}^{i\frac{\kappa_u^2}{2}-\frac{1}{2}} \exp\left(\left[i\frac{\kappa_u^2}{2}-\frac{1}{2}\right] i\pi\right) \exp\left(i\frac{\tilde{q}^2}{2}\exp(2i\pi)\right) \\ &= -i\exp\left(-\frac{\pi\kappa_u^2}{2}\right)\tilde{q}^{i\frac{\kappa_u^2}{2}-\frac{1}{2}}\exp\left(i\frac{\tilde{q}^2}{2}\right).\end{aligned} \tag{1.106}$$

In opposite, we can obtain

$$\bar{\psi}_k^- = i\exp\left(-\frac{\pi\kappa_u^2}{2}\right)\tilde{q}^{-i\frac{\kappa_u^2}{2}-\frac{1}{2}}\exp\left(-i\frac{\tilde{q}^2}{2}\right). \tag{1.107}$$

Comparing $\bar{\psi}_k^\pm$ with the pre-scattering case, we will have

$$C_\pm = \mp i\exp\left(-\frac{\pi\kappa_u^2}{2}\right)A_\pm. \tag{1.108}$$

However, the constant $B_\pm$ can't be obtained in the same way. In that case, we need to write the Wronskian

$$W(\psi_k^-,\psi_k^+) = i\left(\psi_k^+\frac{d\psi_k^-}{dq} - \psi_k^-\frac{d\psi_k^+}{dq}\right). \tag{1.109}$$

Taking the derivative dW/dq as

$$\frac{dW}{dq} = i\left(\psi_k^+\frac{d^2\psi_k^-}{dq^2} - \psi_k^-\frac{d^2\psi_k^+}{dq^2}\right) = i\left(\psi_k^+(-\omega_u^2\psi_k^-) - \psi_k^-(-\omega_u^2\psi_k^+)\right) = 0, \tag{1.110}$$

where we used the relation $\frac{d^2\psi_k^\pm}{dq^2} = -\omega_u^2\psi_k^\pm$, here it is clear that Wronskian W is constant with respect to conformal time q. Once again we can calculate

$$W(A_-\psi_k^-, A_+\psi_k^-) = W((B_+\psi_k^+ + C_+\psi_k^-),(B_-\psi_k^- + C_-\psi_k^+)) \tag{1.111}$$

and obtain

$$i\left(\psi_k^+\frac{d^2\psi_k^-}{dq^2} - \psi_k^-\frac{d^2\psi_k^+}{dq^2}\right)(A_+A_-) = i\left(\psi_k^+\frac{d^2\psi_k^-}{dq^2} - \psi_k^-\frac{d^2\psi_k^+}{dq^2}\right)(B_+B_- - C_+C_-).$$

Finally we can obtain the desired result

$$|A|^2 = |B|^2 - |C|^2, \tag{1.112}$$

where we have used the relation that the Wronskian *before* and *after* time q. Thus, we can imply $B_\pm$ as

$$B_\pm = \sqrt{1+\exp(-\pi\kappa_u^2)}\exp(\pm i\delta)A_\pm, \tag{1.113}$$

where δ is an arbitrary phase.

On the other side, we can write the complete set of eq. (1.104) as

$$\begin{aligned}\bar{\psi}_k &= \bar{\psi}_k^+ + \bar{\psi}_k^- \\ &= (B_+\psi_k^+ + C_+\psi_k^-) + (B_-\psi_k^- + C_-\psi_k^+) \\ &= (B_+ + C_-)\psi_k^+ + (B_- + C_+)\psi_k^-\end{aligned} \tag{1.114}$$

and we can write the pre-scattering process by

$$\psi_k = A_+\psi_k^+ + A_-\psi_k^-. \tag{1.115}$$

Before we proceed, it is important to note, the discussion in this entire part is depicted from two separate sources. The first half (before this paragraph), is entirely taken from [27]. In another side, starting from this paragraph, we will follow the most people used in their papers. In many literature (see eg. [22, 23]) the solution of eq. (1.96) in j-th (with j is some integer)

$$\psi_k^j = \frac{\alpha_k^j}{\sqrt{2\omega_u}}e^{-i\int_0^t \omega_u dt} + \frac{\beta_k^j}{\sqrt{2\omega_u}}e^{+i\int_0^t \omega_u dt}, \tag{1.116}$$

and for $j+1$-th we get

$$\psi_k^{j+1} = \frac{\alpha_k^{j+1}}{\sqrt{2\omega_u}}e^{-i\int_0^t \omega_u dt} + \frac{\beta_k^{j+1}}{\sqrt{2\omega_u}}e^{+i\int_0^t \omega_u dt}. \tag{1.117}$$

The α_k and β_k in j-th and $j+1$-th are related by

$$\begin{pmatrix}\alpha_k^{j+1} \\ \beta_k^{j+1}\end{pmatrix} = \begin{pmatrix} B_+ & C+ \\ C_- & B_-\end{pmatrix}\begin{pmatrix}\alpha_k^j \\ \beta_k^j\end{pmatrix}, \tag{1.118}$$

where this result is taken similarly using the definition of eq. (1.105). One should remember, α_k and β_k are *Bogoliubov Coefficients* similar to the coefficients used in the subsection 1.10. The eq. (1.120) are related with transmission coefficient: $\left(\frac{1}{B_+}\right)$, and reflection coefficient: $\left(\frac{C_+}{B_+}\right)$. In addition, they should obey $|B|^2 - |C|^2 = 1$. Hence we also retrieve another relation from that subsection (in 1.10):

$$n_k = |\beta_k|^2 \qquad \text{and} \qquad |\alpha_k|^2 = n_k + 1, \tag{1.119}$$

where the last part is coming from eq. (1.92).

By obtaining all requirements, we can rewrite Eq. (1.120) in the complete form

$$\begin{pmatrix} \alpha_k^{j+1} \\ \beta_k^{j+1} \end{pmatrix} = \begin{pmatrix} \sqrt{1+e^{-\pi\kappa_u^2}}e^{+i\delta} & -ie^{-\frac{\pi\kappa_u^2}{2}} \\ +ie^{-\frac{\pi\kappa_u^2}{2}} & \sqrt{1+e^{-\pi\kappa_u^2}}e^{-i\delta} \end{pmatrix} \begin{pmatrix} \alpha_k^{j} \\ \beta_k^{j} \end{pmatrix} \tag{1.120}$$

From above, we can find $n_k^{j+1} = |\beta_k^{j+1}|^2$, and we have

$$n_k^{j+1} = e^{-\pi\kappa_u^2}|\alpha_k^j| + (1+e^{-\pi\kappa_u^2})|\beta_k^j|^2 + ie^{-\pi\kappa_u^2/2}\sqrt{1+e^{\pi\kappa_u^2}}\left(e^{i\delta}-e^{-i\delta}\right)|\alpha_k^j\beta_k^{j*}|. \tag{1.121}$$

Using eq. (1.119) and $(e^{i\delta}-e^{-i\delta}) = 2i\sin(\delta)$, we finally obtain [23]

$$n_k^{j+1} = e^{-\pi\kappa_u^2} + (1+2e^{-\pi\kappa_u^2})n_k^j - 2\sin(\delta)e^{-\pi\kappa_u^2/2}\sqrt{n_k^j(n_k^j+1)}\sqrt{1+e^{\pi\kappa_u^2}}. \tag{1.122}$$

As for further usage, it is preferred to describe the occupation number as

$$n_k^{j+1} = n_k^j \exp\left(2\pi\mu_k^j\right) \tag{1.123}$$

as we defined μ_k^j as the growth index. Thus we simply related the growth index μ_k^j for large occupation number n_k^j via

$$\mu_k^j \approx \frac{1}{2\pi}\ln\left(1+2e^{-\pi\kappa_u^2} - 2\sin(\delta)e^{-\pi\kappa_u^2/2}\sqrt{1+e^{\pi\kappa_u^2}}\right), \tag{1.124}$$

where we left the first term in eq. (1.122). We skip the reason here as we will describe it later in chapter 2.

Chapter 2

The Higgs Inflation

The discussion about the advanced inflationary models requires the non-minimal coupling ξ with Ricci scalar R, which can be considered as the extension of the preliminary theory stated in the chapter 1. In this chapter, we will discuss the Standard Model Higgs as the inflaton which is proposed by [6].

2.1 The Action

The action in this model is

$$S_J = \int d^4x\sqrt{-g_J}\left[-\frac{1}{2}M_p^2 R - \frac{1}{2}\xi h^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu h \partial_\nu h - \frac{1}{4}\lambda h^4\right], \tag{2.1}$$

where R is the Ricci scalar, $-g = \det g^{\mu\nu}$ is the determinant of the metric tensor, h and λ are radial component and quartic coupling of the Higgs Boson[1]. The subscript "J" in (2.1) corresponds to the Jordan frame. We used the metric signature (+,-,-,-) for this chapter. Later we will transform (2.1) to the Einstein frame removing the non-minimal coupling part using Weyl transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_p^2}. \tag{2.2}$$

Then, we may redefine the field h as

$$\phi = \sqrt{\frac{3}{2}}M_p \ln\Omega^2 \qquad \Omega^2\frac{d\phi}{dh} = \sqrt{\Omega^2 + \frac{6\xi^2 h^2}{M_p^2}}. \tag{2.3}$$

[1]It is natural to use the condition $h \gg \nu$, where ν is the vev oh Higgs.

Thus, the action in the Einstein frame is

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}M_p^2\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4\Omega^4}\lambda h^4\right]. \tag{2.4}$$

In this case, it is better to define the critical value or ϕ_{crit}, which correspond to the value below which the Higgs field in the Jordan frame coincides with the one in the Einstein frame . In order fix it, one can recall eq. (2.3) and apply $h \simeq \phi_{crit}$, also taking $\ln\Omega^2 = \ln(1 + \frac{\xi h^2}{M_p^2}) \approx \frac{\xi h^2}{M_p^2}$, resulting

$$\phi_{crit} = \sqrt{\frac{2}{3}}\frac{M_p}{\xi}. \tag{2.5}$$

In this way, one can conclude

$$\phi = \begin{cases} h, & \text{if } h < \phi_{crit} \\ \sqrt{\frac{3}{2}}M_p\ln\Omega^2 & \text{if } h > \phi_{crit}. \end{cases} \tag{2.6}$$

After all, we may simply obtain the potential corresponding to the parametrization in eq. (2.6) as follows [30]

$$V(\phi) = \begin{cases} \frac{1}{4}\lambda\phi^4, & \text{if } \phi < \phi_{crit} \\ \frac{\lambda M_p^4}{4\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right) & \text{if } \phi > \phi_{crit}. \end{cases} \tag{2.7}$$

The second line of the potential can be easily derived from potential eq. (2.4) by substituting eq. (2.3).

2.2 Confront with CMB

This model should match with the CMB observation, for instance from the Planck [2] and BICEP [7]. Before we proceed, it is necessary to calculate the slow-roll parameters ϵ and

η in this model. Firstly, we calculate the slow-roll parameter ϵ as[2]

$$\begin{aligned}\epsilon = \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2 &= \frac{1}{2}M_p^2\left[\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right]^{-2}\left[\frac{\partial}{\partial\phi}\left\{\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right\}\right]^2 \\ &= \frac{4}{3}\left[-1+e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right]^{-2}.\end{aligned} \tag{2.8}$$

The end of inflation is determined by $\epsilon \simeq 1$, which correspond to $\phi_{end} \simeq 0.94M_p$. We used the index prime here for derivative with respect to ϕ. For slow-roll parameter η we can write

$$\eta = M_p^2\frac{V''}{V} = \frac{4}{3}\frac{\left[2-e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right]}{\left[-1+e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right]^2}. \tag{2.9}$$

The number of e-fold $\mathcal{N}_k$ taken from scale k horizon exit to the end of inflation can be calculated by

$$\begin{aligned}\mathcal{N}_k &= \frac{1}{M_p^2}\int_{\phi_{end}}^{\phi_k}\frac{V}{V'}d\phi \\ &= \frac{1}{M_p^2}\int_{\phi_{end}}^{\phi_k}\left[\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right]\left[\frac{\partial}{\partial\phi}\left\{\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right\}\right]^{-1}d\phi \\ &= \frac{3}{4}\left[e^{\sqrt{\frac{2}{3}}\frac{\phi_k}{M_p}}-\sqrt{\frac{2}{3}}\frac{\phi_k}{M_p}\right]_{\phi_{end}}^{\phi_k} = \frac{3}{4}\left[e^{\sqrt{\frac{2}{3}}\frac{\phi_k}{M_p}}-e^{\sqrt{\frac{2}{3}}\frac{\phi_{end}}{M_p}}-\sqrt{\frac{2}{3}}\frac{\phi_k}{M_p}+\sqrt{\frac{2}{3}}\frac{\phi_{end}}{M_p}\right].\end{aligned} \tag{2.10}$$

In this part, if we use $\mathcal{N}_k = 60$, we can obtain $\phi_k \approx 5.3M_p$. One should remember that the number of e-fold should be more than 50 to solve the flatness and horizon problems and cannot be much larger than 60 [15]. In addition, we can obtain the tensor to scalar ratio r and spectral index n_s via

$$r = 16\epsilon \qquad\qquad n_s = 1-6\epsilon+2\eta. \tag{2.11}$$

In fig. 2.1, the prediction by this model is contained in the allowed region from WMAP [9] and Planck data [2].

To match the CMB observation, we can use the potential in (2.7) to define the ampli-

[2]the procedure in this subsection follows the same way as [15], except for purely Higgs, this paper is talking about the Inert doublet model

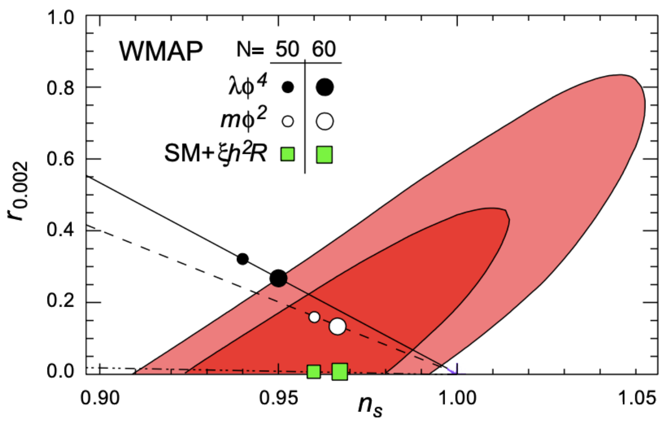

Figure 2.1: The green square-shaped correspond to the prediction from parameters r and n_s of Higgs Inflation, it is under the allowed region from WMAP. The prediction from $\lambda\phi^4$ and $m\phi^2$ models is also shown. The image has been taken from [6] with a little modification

tude of observable power spectrum (1.45).

$$\begin{aligned} P_k &= \left.\frac{V^3}{12\pi^2 M_p^6 V'^2}\right|_{\phi=\phi_k} \\ &= \frac{1}{12\pi^2 M_p^6}\left[\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right]^3\left.\left[\frac{\partial}{\partial\phi}\left\{\frac{\lambda M_p^4}{4\xi^2}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2\right\}\right]^{-2}\right|_{\phi=\phi_k} \\ &= \left.\frac{1}{128\pi^2}\left(\frac{\lambda}{\xi^2}\right)\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)e^{2\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right|_{\phi=\phi_k}. \end{aligned} \tag{2.12}$$

It can be constrained by PLANCK satellites [2], in this thesis, we pick $P_k = 2.101^{+0.031}_{-0.034} \times 10^{-9}$ at $k_* = 0.05 Mpc^{-1}$, where ϕ_k stands for ϕ for which correspond to e-fold in this chosen k_*. Interestingly, with this constraint, we can obtain ξ from (2.12) as

$$\lambda = 4.7 \times 10^{-10}\xi^2. \tag{2.13}$$

This correspond to the value $\xi = 0(10^4)$ for $\lambda = 0(10^{-2})$. The largeness of non-minimal coupling gives the problem in the naturalness of Higgs inflation. In this way, some extension of this model has been proposed to obtain the small ξ which is in order of near unity to make the naturalness is not violated[3].

The reheating temperature which happens during the end of inflation is calculated by

[3]See for example ref. [15].

using the conversion from the energy density during the h_{end} which correspond to

$$\rho_m = \frac{1}{4}\lambda h_{end}^4 = \frac{1}{4}\lambda\left(\sqrt{\frac{2}{3}}\frac{M_p}{\sqrt{\xi}}\right)^4 = \frac{1}{9}\lambda\frac{M_p^4}{\xi^2}. \tag{2.14}$$

Substitute the last result to eq.(1.47), we obtain [6]

$$T_{reh} \simeq \left(\frac{3\lambda}{g^*\pi}\right)^{1/4}\frac{M_p}{\sqrt{\xi}} \sim 10^{15}\text{GeV}. \tag{2.15}$$

Of course, the above temperature should be taken as the (extremely) *upper bound.* This idea only happens if preheating stage is negligible. Please remember, the preheating stage is depleting the inflaton energy density without (in most models) thermalizing the universe. Also, the reheating stage could positively happen in the region much below the h_{end}. Those 'two' are the main reasons why the reheating temperature should be much lower.

2.3 Preheating in Oscillating Quadratic Potential

In this section, we will describe the condition when the Higgs field value in Einstein frame is $\sim 1M_p$, such condition is applied when inflation is ended. However, this field value is still in the region of $\phi > \phi_{crit}$. In this region, the potential (2.7) can be expressed via

$$V(\phi) = \frac{\lambda M_p^4}{4\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}}\right)^2 \simeq \frac{1}{2}\frac{\lambda M_p^2}{3\xi^2}\phi^2. \tag{2.16}$$

Thus, the potential is turned into quadratic form with a mass term

$$m_\phi^2 = \frac{\lambda M_p^2}{3\xi^2}. \tag{2.17}$$

It is also clear, during this time, the universe behaves like a matter-dominated era with $H = 2/3t$. With the adoption of the result from eq. (1.58) and substituting m with m_ϕ we can approximate ϕ as

$$\phi(t) = \tilde{\phi}(t)\sin(m_\phi t) \simeq 2\sqrt{2}\frac{\xi}{t\sqrt{\lambda}}\sin(m_\phi t), \tag{2.18}$$

where the amplitude is defined by $\tilde{\phi}(t) = 2\sqrt{2}\frac{\xi}{t\sqrt{\lambda}}$. This equation shows the amplitude of the inflaton decreases over time. Replacing eq. (2.18) with eq. (2.5), we get the critical amplitude

$$\tilde{\phi}_{crit} = 2\sqrt{2}\frac{\xi}{t_{crit}\sqrt{\lambda}} = \sqrt{\frac{2}{3}}\frac{M_p}{\xi}, \tag{2.19}$$

where we simply define the critical time when inflaton changes from (the beginning of) quadratic potential to quartic potential. It is obtained from this model as

$$t_{crit} = \frac{2\xi}{m_\phi}. \tag{2.20}$$

It is important to note, in this model the largeness of non-minimal coupling ξ discussed in the previous part tends to make the quadratic potential stay longer.

The Lagrangian of the inflaton ϕ in the Einstein frame is written as

$$\mathcal{L} \supset \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{2.21}$$

Taking $\delta\mathcal{L} = 0$, we can simply get

$$\Box\phi - m_\phi^2\phi = 0. \tag{2.22}$$

In addition, we can represent ϕ in the Heisenberg picture as[4]

$$\phi(x,t) = \frac{1}{(2\pi)^{3/2}}\int d^3k \left(\hat{a}_k\phi_k(t)e^{-i\bar{k}\cdot\bar{x}} + \hat{a}_k^\dagger\phi_k^*(t)e^{i\bar{k}\cdot\bar{x}}\right) \tag{2.23}$$

We then finally obtain

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left(\frac{k^2}{a^2} + m_\phi^2\right)\phi_k = 0. \tag{2.24}$$

2.3.1 Induced Masses

During this oscillation period, the produced particles from the inflaton will drain the inflaton energy density. For Higgs as inflaton, its main tree-level decay could be the ones to gauge bosons and top quarks. In case of relevant couplings are coming from the kinetic

[4]Please note $\Box\phi = \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi)$, $\tilde{g}_{00} = -1$, $\tilde{g}_{ii} = a^2$, $\tilde{g} = a^6$, and $\sqrt{-\tilde{g}} = a^3$.

term of the Higgs field, namely

$$\begin{aligned} D_\mu\phi D_\nu\phi &= \left| \left(\partial_\mu\partial_\nu + \frac{i}{2} g\tau^k W_\mu^k + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix} \right|^2 \\ &= \frac{h^2}{8} \left| \left(g\tau^k W_\mu^k + g' B_\mu \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\ &= \frac{h^2}{8} \left| \begin{pmatrix} gW_\mu^1 - igW_\mu^2 \\ -gW_\mu^3 + g' B_\mu \end{pmatrix} \right|^2, \end{aligned} \tag{2.25}$$

where we used the Pauli three matrices τ^k here, and also we define

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2). \tag{2.26}$$

From the first of the row of eq. (2.25), we obtain

$$\frac{1}{2} \left(\frac{gh}{2} \right)^2 W_\mu^\dagger W^\mu. \tag{2.27}$$

Thus, the induced mass of W bosons during this period can be expressed by usinng the second line of eq. (2.6) for $h > \phi_{crit}$ as

$$m_W^2 = \frac{g^2}{4} h^2(\phi) = \frac{g^2}{4} \left(\sqrt{\frac{2}{3}} \frac{M_p}{\xi} \right) = \frac{g^2}{2\sqrt{6}} \frac{M_p}{\xi} |\phi(t)|. \tag{2.28}$$

In addition, we can also write the second line of (2.25) as

$$\frac{h^2}{8} |(-gW_\mu^3 + g' B_\mu)|^2 = \frac{h^2}{8} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix}^\dagger \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix}. \tag{2.29}$$

We can solve the above 2×2 matrix and obtain the corresponding 2 eigenvalues and 2 eigenvectors as

$$\begin{aligned} &\text{Eigenvalue} = (g^2 + g'^2), \qquad &&\text{Eigenvector} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g \\ -g' \end{pmatrix}, \\ &\text{Eigenvalue} = 0, &&\text{Eigenvector} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g' \\ g \end{pmatrix}. \end{aligned} \tag{2.30}$$

Using this result, we can reconstruct eq. (2.29) with diagonal mass matrix

$$\begin{aligned}
&\frac{h^2}{8(g^2+g'^2)}\begin{pmatrix} W^3_\mu & B_\mu \end{pmatrix}^\dagger \begin{pmatrix} g & g' \\ -g' & g \end{pmatrix} \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix} \\
&= \frac{h^2}{8(g^2+g'^2)}\begin{pmatrix} gW^3_\mu - g'B_\mu & g'W^3_\mu + gB_\mu \end{pmatrix}^\dagger \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} gW^{\mu 3} - g'B^\mu \\ g'W^{\mu 3} + gB^\mu \end{pmatrix} \\
&\equiv \frac{h^2}{8}\begin{pmatrix} Z_\mu & A_\mu \end{pmatrix}^\dagger \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \frac{1}{2}\left(h^2\frac{(g^2+g'^2)}{4}\right) Z^\dagger_\mu Z^\mu + \frac{1}{2}\left(h^2\frac{0}{4}\right) A^\dagger_\mu A^\mu,
\end{aligned} \tag{2.31}$$

where in the last line we used the definition of Z bosons and photon A_μ. Photon is still massless, but the induced mass of Z boson is characterized by using (2.6) as

$$m_Z^2 = \frac{(g^2+g'^2)}{4} h^2(\phi) = \frac{(g^2+g'^2)}{2\sqrt{6}}\frac{M_p}{\xi}|\phi(t)|. \tag{2.32}$$

Lastly, for the top quark, the mass is obtained by Yukawa interaction. We skip some details and obtain [30]

$$m_t^2 = y_t^2 \frac{M_p}{\sqrt{6}}\phi(t). \tag{2.33}$$

The W, Z, and top-quark are all have large couplings, which make them are very heavy and non-relativistic during this period.

2.3.2 The W Boson Production

In this subsection, we will describe particle production during preheating in quadratic potential (2.16). In this stage, the number of oscillations of the inflaton is proportional to ξ. To prove it, we need to consider the amplitude during the end of inflation $\tilde{\phi}_{end}$ and critical amplitude $\tilde{\phi}_{crit}$. They satisfy

$$\begin{aligned}
\tilde{\phi}_{end} = 2\sqrt{2}\frac{\xi}{\sqrt{\lambda}}\frac{1}{t_{end}} = M_p \qquad &\rightarrow \qquad t_{end} = 2\sqrt{2}\frac{1}{\sqrt{\lambda}}\frac{\xi}{M_p} \\
\tilde{\phi}_{crit} = 2\sqrt{2}\frac{\xi}{\sqrt{\lambda}}\frac{1}{t_{crit}} = \frac{M_p}{\xi} \qquad &\rightarrow \qquad t_{crit} = 2\sqrt{2}\frac{1}{\sqrt{\lambda}}\frac{\xi^2}{M_p}.
\end{aligned} \tag{2.34}$$

We also need to consider the average time t of one oscillation via

$$m_\phi t = 2\pi \quad \rightarrow \quad \sqrt{\frac{\lambda}{3}}\frac{M_p^2}{\xi}t = 2\pi \quad \rightarrow \quad t = 2\pi\sqrt{\frac{3}{\lambda}}\frac{\xi}{M_p}. \tag{2.35}$$

Finally, the number of oscillations needed from quadratic potential to quartic potential can be defined by

$$n_{osc} = \frac{t_{crit} - t_{end}}{t} = \frac{1}{\pi}\sqrt{\frac{2}{3}}(\xi - 1). \tag{2.36}$$

Thus, the largeness of non-minimal coupling may extend the longevity of preheating stage in quadratic potential mode. This assumption is only valid when $\xi > 1$. Otherwise, the quadratic potential regime won't appear and inflaton energy density drains more effectively, since it is drained much faster. When the quadratic potential mode does not appear, the behavior of preheating will purely follow the quartic potential mode.

At the first glance, based on eq. (2.36), it is clear that the produced particle may not relativistic due to the largeness of the mass. Firstly,for simplicity, we can treat a vector boson as scalar particles [30]. Hence we can write the evolution of field W_k as

$$\ddot{W}_k + 3H\dot{W}_k + \left(\frac{k^2}{a^2} + m_W^2(t)\right) W_k = 0, \tag{2.37}$$

where the dots correspond to the derivative with respect of time. Let us redefine the field W_k as $\mathcal{W}_k = a^{3/2} W_k$, and then we can obtain the simpler form as

$$\begin{aligned} \ddot{\mathcal{W}}_k + \left(\frac{k^2}{a^2} + m_W^2(t)\right)\mathcal{W}_k = \ddot{\mathcal{W}}_k + \left(\frac{k^2}{a^2} + \frac{g^2}{2\sqrt{6}}\frac{M_p}{\xi}|\phi(t)|\right)\mathcal{W}_k = \\ \ddot{\mathcal{W}}_k + \left(\frac{k^2}{a^2} + \frac{g^2}{2\sqrt{6}}\frac{M_p}{\xi}\Phi(t)\sin(m_\phi t)\right)\mathcal{W}_k = 0, \end{aligned} \tag{2.38}$$

and finally we obtain

$$\ddot{\mathcal{W}}_k + \left(\frac{k^2}{a^2} + \frac{g^2 M_p^2}{6\xi^2}\sqrt{\frac{\lambda}{2}}\Phi(t)t\right)\mathcal{W}_k = 0. \tag{2.39}$$

Furthermore, we can redefine (2.38) with

$$\kappa^2 = \frac{k^2}{K^2 a^2} = \frac{k_a^2}{K^2} \qquad \text{and} \qquad \tau = Kt, \tag{2.40}$$

where

$$K \equiv \left[\frac{g^2 M_p^2}{6\xi^2}\sqrt{\frac{\lambda}{2}}\Phi(t)\right]^{1/3}. \tag{2.41}$$

Finally, we obtain

$$\frac{d^2\mathcal{W}_k}{d\tau^2} + \left(\kappa^2 + \tau\right)\mathcal{W}_k = 0 \tag{2.42}$$

In solving this equation analytically, we will use the result from the WKB approximation depicted in eq. (1.122) as

$$n_k^{j+1} = e^{-\pi\kappa^2} + \left(1 + 2e^{-\pi\kappa^2} - 2\sin(\delta)e^{-\pi\kappa^2/2}\sqrt{1+e^{\pi\kappa^2}}\right) n_k^j, \tag{2.43}$$

where κ is defined by eq. (2.40), n_k^{j+1} is the occupation number of the W- boson, and δ is an arbitrary phase.

2.3.3 The Non-Resonance Production of W-boson

In this part, we calculate the produced particle using the first term in eq. (2.43). The occupation number integrated into the momentum space can be calculated by

$$\begin{aligned}
n^W &= \int_0^\infty \frac{d^3k_a}{(2\pi)^3} n_{k_{1st}}^{j+1} = \int_0^\infty \frac{d^3k_a}{(2\pi)^3} e^{-\pi\kappa^2} = \int_0^\infty \frac{d^3k_a}{(2\pi)^3} e^{-\frac{k_a^2}{(K^2/\pi)}} \\
&= \int_0^\infty 4\pi k_a^2 \frac{dk_a}{(2\pi)^3} e^{-\frac{k_a^2}{(K^2/\pi)}} = \frac{1}{2\pi^2}\int_0^\infty k_a^2 dk_a e^{-\left(\frac{\pi}{K^2}\right)k_a^2} \\
&= \frac{1}{2}\left(\frac{K^2}{\pi}\right)^{3/2}\Gamma(3/2) = \frac{1}{4}\left(\frac{K^2}{\pi}\right)^{3/2}\Gamma(1/2) = \frac{1}{4}\left(\frac{K^2}{\pi}\right)^{3/2}\sqrt{\pi} \\
&= \frac{K^3}{4\pi}.
\end{aligned} \tag{2.44}$$

The last integration is evaluated by using Gamma Function[5] [31] :

$$\int_0^\infty x^m e^{-kx^n} = \frac{\Gamma\left(\frac{m+1}{n}\right)}{nk^{(m+1)/n}}. \tag{2.45}$$

The W-Bosons created in this mode are non-relativistic.

2.3.4 The Stochastic Resonance Production of W-boson

The calculation of the stochastic resonance may be derived from the second term of eq. (2.46). This contribution could be effective if the occupation number n_k exceeds 1. In that case, the exponentially rapid creation of the corresponding particles happen. If the first term is neglected in eq. (2.43), we obtain

$$n_k^{j+1} \simeq n_k^j \left(1 + 2e^{-\pi\kappa^2} - 2\sin(\delta)e^{-\pi\kappa^2/2}\sqrt{1+e^{\pi\kappa^2}}\right) = n_k^j e^{2\pi\mu_k^j}. \tag{2.46}$$

[5]Where Gamma function is defined by $\Gamma(n) = (n-1)!$. We used some identities: $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$

Here, the averaged exponential growth μ_k^j can be calculated by (see eq. (1.124))

$$\begin{aligned}\mu^j_{k(average)} &= \frac{1}{2\pi}\int_0^{2\pi}\frac{d\delta}{2\pi}\log\left(1+2e^{-\pi\kappa^2}-2\sin(\delta)e^{-\pi\kappa^2/2}\sqrt{1+e^{\pi\kappa^2}}\right)\\ &\simeq \frac{1}{2\pi^2}\int_0^{2\pi} d\delta\left(e^{-\pi\kappa^2}-\sin(\delta)e^{-\pi\kappa^2/2}\sqrt{1+e^{\pi\kappa^2}}\right)\\ &\simeq \frac{1}{2\pi^2}\int_0^{2\pi} d\delta\,(1-\sin(\delta))\,e^{-\pi\kappa^2} = \frac{1}{\pi}e^{-\pi\kappa^2}\simeq\frac{1}{\pi}-\kappa^2,\end{aligned} \tag{2.47}$$

where we have assumed $\exp(-\pi\kappa) \ll 1$, since the decay channel has large coupling compared to the Higgs boson (see eq. (2.40)). Here we calculate the growth only roughly.

We already discuss shortly non-resonance and stochastic production of W-boson. Two of them have one distinguishing feature. For large K corresponding with the large induced mass of the produced particle (in this case, W-boson), the non-resonance production becomes dominant and the stochastic production is rather suppressed. On contrary, for the smallness of the induced mass, the corresponding particle production is greatly enhanced, while the non-resonance production is rather suppressed. But there is some defect in stochastic production since the induced mass is quite small, smaller than inflaton's induced mass, the particle will be created even without the zero-crossing effect, which causes the production even though large, it is inefficient to produce necessary temperature for the universe.

2.4 Preheating in the Oscillating Quartic Potential

We can say that this stage corresponds to the second part of preheating before the perturbative reheating happens. This stage starts when ϕ is below $\phi_{crit} \sim M_p/\xi$. Also, during this stage, the Einstein frame and Jordan frame coincide and there is nothing to make any distinction between those two ($\phi \simeq h$). We can write the evolution of field W_k not in the form of eq. (2.38) but we will follow eq. (1.83) as

$$\mathcal{W}_k'' + \left[\kappa^2 + \frac{g_W}{\lambda}cn^2\left(\mathbf{x}, \frac{1}{\sqrt{2}}\right)\right]\mathcal{W}_k = 0, \tag{2.48}$$

where we have defined[6]

$$\mathcal{W}_k = aW_k, \quad g_w = \frac{g}{2}, \quad \kappa = \frac{k^2}{\lambda\tilde{\varphi}}, \tag{2.49}$$

[6] this definition of $\mathcal{W}_k$ in this section is different from eq. (2.38), but preventing in introducing a new symbol seems to a lot wiser, so let us just follow the new definition

One should understand that we used the conformal field $\varphi = \tilde{\varphi} f(\mathbf{x}) = a\phi$. The prime corresponds to the derivative with respect to conformal field $\mathbf{x} = 2(3M_p^2\lambda)^{1/4}\sqrt{t}$. The definition has been taken literally from section 1.8. In the case of this region, since g is only by one order larger than λ, the occupation number density n^W created during this process is not so large compared to the production during the previous stage, which is during the quadratic potential regime. Thus we can conclude that in this part, the production of W-boson doesn't have any significant contribution.

2.5 Some Additional Remarks

We only consider the inflaton decay to W-boson as an example in Higgs Inflation. We should actually consider the self-production of Higgs boson, also we should consider the decay to fermions. However we skip these discussions. It is also remarked, if preheating is important to reheat the universe, the temperature should be fixed by the decay of W-boson decay to relativistic particles (for example). Here we are skipping this part.

Chapter 3

The Inflation with Singlet Scalars and CP Violation

3.1 The Lagrangian

In this chapter, we will start our discussion with the introduction of the Lagrangian

$$\begin{aligned}-\mathcal{L}_Y =& y_D\sigma\bar{D}_L\bar{D}_R + y_E\sigma\bar{E}_L\bar{E}_R + \sum_{j=1}^{3}\left[\frac{y_{N_J}}{2}\sigma\bar{N}_J^c N_j + y_{D_j}S\bar{D}_L d_{R_j} + \tilde{y_{d_j}}S^\dagger\bar{D}_L d_{R_j}\right.\\ &\left.+ y_{e_j}S\bar{E}_L e_{R_j} + \tilde{y}_{e_j}S^\dagger\bar{E}_L e_{R_j} + \sum_{\alpha=1}^{3} h^*_{\alpha j}\eta\bar{l}_\alpha N_j\right]\end{aligned} \tag{3.1}$$

and potentials

$$\begin{aligned}V =& \lambda_1(\mathcal{H}^\dagger\mathcal{H})^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\mathcal{H}^\dagger\mathcal{H})(\eta^\dagger\eta) + \lambda_4(\mathcal{H}^\dagger\eta)(\eta^\dagger\mathcal{H}) + \frac{\lambda_5}{2M_*}\left[\sigma(\eta^\dagger\mathcal{H})^2 + \text{h.c}\right]\\ &+ \kappa_\sigma(\sigma^\dagger\sigma)^2 + \kappa_S(S^\dagger S)^2 + (\kappa_{H\sigma}\mathcal{H}^\dagger\mathcal{H} + \kappa_{\eta\sigma}\eta^\dagger\eta)(\sigma^\dagger\sigma) + (\kappa_{HS}\mathcal{H}^\dagger\mathcal{H} + \kappa_{\eta S}\eta)(S^\dagger S)\\ &+ \kappa_{\sigma S}(\sigma^\dagger\sigma)(S^\dagger S) + m_H^2\mathcal{H}^\dagger\mathcal{H} + m_\eta^2\eta^\dagger\eta + m_\sigma^2\sigma^\dagger\sigma + m_S^2 S^\dagger S + V_b.\end{aligned} \tag{3.2}$$

We have introduced the extension of SM with global $U(1) \times Z_4$ symmetry and several additional fields. We introduced vector-like down-type quarks (D_L, D_R), a pair of vector-like charged leptons (E_L, E_R), and three right-handed singlet fermions N_j $(j = 1, 2, 3)$. We introduced the doublet η, which later in this model can be chosen as the dark matter candidate for the lightest neutral component with Z_2 odd parity, and singlet-scalars σ and S which later be used as the linear combination of inflaton in this model. The SM

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)$	Z_4		$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)$	Z_4
D_L	3	1	$-\frac{1}{3}$	0	2	D_R	3	1	$-\frac{1}{3}$	2	0
E_L	1	1	-1	0	2	E_R	1	1	-1	2	0
σ	1	1	0	-2	2	S	1	1	0	0	2
N_k	1	1	0	1	1	η	1	2	$-\frac{1}{2}$	-1	1

Table 3.1: The additional fields in respect to this model which charge under $[SU(3)_C \times SU(2)_L \times U(1)_Y] \times U(1) \times Z_4$

particles do not have a charge in this global symmetry. This global U(1) has a color anomaly similar to the KSVZ model for the strong CP problem [32, 33], then it can play the role of PQ symmetry. Please note that $\mathcal{H}^\dagger = \frac{1}{\sqrt{2}}(0 \;\; h^*)$ is the SM Higgs doublet. Also, we have d_{R_j} and e_{R_j} as the SM down-type quarks and charged leptons. We assume, all parameters in Lagrangian are real-positive and the dominant parts are up to dimension five with the cut-off scale of M_*. Lastly, we identify V_b as another potential, its terms are invariant under global symmetry but violate the S number:

$$V_b = \alpha(S^4 + S^{\dagger 4}) + \beta\sigma^\dagger\sigma(S^2 + S^{\dagger 2}) = \frac{1}{2}\tilde{S}^2(\alpha\tilde{S}^2\cos 4\rho + \beta\tilde{\sigma}^2\cos 2\rho), \tag{3.3}$$

where we define $\sigma = \frac{\tilde{\sigma}}{\sqrt{2}}e^{i\theta}$ and $S = \frac{\tilde{S}}{\sqrt{2}}e^{i\rho}$. For the clearer version, we will put the representation of the introduced fields which charge is under symmetry $[SU(3)_C \times SU(2)_L \times U(1)_Y] \times U(1) \times Z_4$ as seen in the table 3.1

In addition, to build the inflationary model, we assume in our model that the inflaton is the linear combination of σ and S. Thus, we will write the potential necessary for the inflation model, that is

$$\begin{aligned} V =& \frac{1}{4}\kappa_\sigma(\tilde{\sigma}^2 - w^2)^2 + \frac{1}{4}\kappa_S(\tilde{S}^2 - u^2)^2 + \frac{1}{4}\kappa_{\sigma S}(\tilde{\sigma}^2 - w^2)(\tilde{S}^2 - u^2) \\ &+ \frac{1}{2}\alpha(\tilde{S}^2 - u^2)^2\cos 4\rho + \frac{1}{2}\beta(\tilde{\sigma}^2 - w^2)(\tilde{S}^2 - u^2)\cos 2\rho. \end{aligned} \tag{3.4}$$

if we assume that inflaton is following minimum of ρ, which is $\frac{\partial V_b}{\partial \rho} = 0$, we obtain $\cos 2\rho = -\frac{\beta}{4\alpha}\frac{(\tilde{\sigma}^2 - w^2)}{(\tilde{S}^2 - u^2)}$. Substitute this result to (3.4) we obtain

$$V(\tilde{\sigma}, \tilde{S}) = \frac{1}{4}\tilde{\kappa}_\sigma(\tilde{\sigma}^2 - w^2)^2 + \frac{1}{4}\tilde{\kappa}_S(\tilde{S}^2 - u^2)^2 + \frac{1}{4}\kappa_{\sigma S}(\tilde{\sigma}^2 - w^2)(\tilde{S}^2 - u^2), \tag{3.5}$$

where we define

$$\tilde{\kappa}_\sigma = \kappa_\sigma - \frac{\beta^2}{4\alpha} \qquad \tilde{\kappa}_S = \kappa_S - 2\alpha. \tag{3.6}$$

Also, w and u are vacuum expectation values (VEVs) of $\tilde{\sigma}$ and $\tilde{S}$ respectively. These

VEVs are assumed much larger than weak scale and break the global symmetry $U(1) \times Z_4$ to its diagonal sub-group Z_2. Here we have three couplings related to the inflation, and they should relevant to the stability condition which can be found if we redefined $(\tilde{\sigma}^2 - w^2) = \sin\delta$ and $(\tilde{S}^2 - u^2) = \cos\delta$, then substitute to (3.5) as

$$\begin{aligned} V(\tilde{\sigma}, \tilde{S}) &= \frac{1}{4}\tilde{\kappa}_\sigma \cos^2\delta + \frac{1}{4}\tilde{\kappa}_S \sin^2\delta + \frac{1}{4}\kappa_{\sigma S}\cos\delta\sin\delta \\ &= \frac{1}{4}\left(\frac{\tilde{\kappa}_\sigma + \tilde{\kappa}_S}{2}\right) - \frac{1}{4}\left(\frac{\tilde{\kappa}_\sigma - \tilde{\kappa}_S}{2}\right)\cos 2\delta + \frac{1}{8}\kappa_{\sigma S}\sin 2\delta. \end{aligned} \tag{3.7}$$

Taking the condition $\partial V/\partial\delta = 0$ and serve it in the quadratic form, we may get

$$(\tilde{\kappa}_\sigma - \tilde{\kappa}_S)^2 \sin^2 2\delta = \kappa_{\sigma S}^2 \cos^2 2\delta. \tag{3.8}$$

Finally, we can simply obtain

$$\sin\delta = \sqrt{\frac{\kappa_{\sigma S}^2}{(\tilde{\kappa}_\sigma - \tilde{\kappa}_S)^2 + \kappa_{\sigma S}^2}} \qquad \text{and} \qquad \cos\delta = \sqrt{\frac{(\tilde{\kappa}_\sigma - \tilde{\kappa}_S)^2}{(\tilde{\kappa}_\sigma - \tilde{\kappa}_S)^2 + \kappa_{\sigma S}^2}}. \tag{3.9}$$

By substituting these last results in the potential (3.7) and using the fact that stability of potential requires $V > 0$, we then obtain

$$\tilde{\kappa}_\sigma, \tilde{\kappa}_S > 0 \qquad 4\tilde{\kappa}_\sigma\tilde{\kappa}_S > \kappa_{\sigma S}^2. \tag{3.10}$$

If we consider the fluctuation of both fields: $\tilde{\sigma}$ and $\tilde{S}$ around their respective vacua, we can simply write the potential (3.5) as

$$\begin{aligned} V(\tilde{\sigma}, \tilde{S}) =& \frac{1}{4}\tilde{\kappa}_\sigma((\tilde{\sigma} + w)^2 - w^2)^2 + \frac{1}{4}\tilde{\kappa}_S((\tilde{S} + u)^2 - u^2)^2 \\ &+ \frac{1}{4}\kappa_{\sigma S}((\tilde{\sigma} + w)^2 - w^2)((\tilde{S} + u)^2 - u^2). \end{aligned} \tag{3.11}$$

We can get the mass eigenstates:

$$\frac{1}{2}\begin{pmatrix} \tilde{\sigma} & \tilde{S} \end{pmatrix}^\dagger \begin{pmatrix} 2\tilde{\kappa}_\sigma w^2 & \kappa_{\sigma S} uw \\ \kappa_{\sigma S} uw & 2\tilde{\kappa}_S u^2 \end{pmatrix} \begin{pmatrix} \tilde{\sigma} \\ \tilde{S} \end{pmatrix}. \tag{3.12}$$

Thus, the mass eigenvalues can simply be obtained and we also retrieve

$$m_{\tilde{\sigma}}^2 \simeq 2\tilde{\kappa}_\sigma w^2 \qquad m_{\tilde{S}}^2 \simeq \left(\tilde{\kappa}_S - \frac{\kappa_{\sigma S}}{4\tilde{\kappa}_\sigma}\right) u^2 \equiv \hat{\kappa}_S u^2. \tag{3.13}$$

Here we assume $\tilde{\kappa}_S u^2, \kappa_{\sigma S} uw \ll \tilde{\kappa}_\sigma w^2$.

The Axion decay constant is given by $f_a = w$, and we assume the global U(1) symmetry works as the PQ symmetry. In this way, the VEV of w should satisfy

$$10^9\text{GeV} \lesssim w \lesssim 10^{11}\text{GeV}. \tag{3.14}$$

The NG-boson caused by the spontaneous symmetry breaking of this global U(1) becomes Axion [34, 35], if is associated by coupling with photons [36]

$$g_{a\gamma\gamma} = \frac{1.51}{10^{10}\text{GeV}}\left(\frac{m_a}{\text{eV}}\right). \tag{3.15}$$

3.2 The CP Violation Phases in CKM and PMNS Matrices

To determine the Yukawa couplings of down-type quarks and charged leptons in the Lagrangian (3.1), the mass terms can be written by

$$\begin{pmatrix}\bar{f}_{L_i} & \bar{F}_L\end{pmatrix}\mathcal{M}_f\begin{pmatrix}f_{R_j}\\ F_R\end{pmatrix} + \text{h.c.}, \quad \mathcal{M}_f = \begin{pmatrix}m_{f_{ij}} & 0\\ \mathcal{F}_{fj} & \mu_F\end{pmatrix} = \begin{pmatrix}h_{f_{ij}}\langle h\rangle & 0\\ (y_{f_j}ue^{i\rho} + \tilde{y}_{f_j}ue^{-i\rho}) & y_F w\end{pmatrix}. \tag{3.16}$$

The f and F correspond to $f = d, e$ and $F = D, E$ for down-type quarks and charge leptons, also $\mathcal{M}_f$ is 4×4 mass matrix written in eq. (3.16). This mass matrix has a similar form [37]. Here, we suggest that global U(1) symmetry works as the PQ symmetry and all parameters are assumed to be real, the $\arg(\det \mathcal{M}_f) = 0$ is fulfilled, whether the radiative effects are taken into account after the braking of PQ symmetry (see [38, 39]).

By taking the diagonalization of $\mathcal{M}_f\mathcal{M}_f^\dagger$ we obtain

$$\begin{pmatrix}A_f & B_f\\ C_f & D_f\end{pmatrix}\begin{pmatrix}m_f m_f^\dagger & m_f\mathcal{F}_f^\dagger\\ \mathcal{F}_f m_f^\dagger & \mu_F^2 + \mathcal{F}_f\mathcal{F}_f^\dagger\end{pmatrix}\begin{pmatrix}A_f^\dagger & C_f^\dagger\\ B_f^\dagger & D_f^\dagger\end{pmatrix} = \begin{pmatrix}\tilde{m}_f^2 & 0\\ 0 & \tilde{M}_f^2,\end{pmatrix}. \tag{3.17}$$

the $\tilde{m}_f^2$ is 3×3 diagonal matrix[1]. Based on eq. (3.17) we also obtain

$$\begin{aligned}&m_f m_f^\dagger = A_f^\dagger \tilde{m}_f^2 A_f + C_f^\dagger \tilde{M}_f^2 C_f \qquad \mu_F^2 + \mathcal{F}_f\mathcal{F}^\dagger = B_f^\dagger \tilde{m}_f^2 B_f + D_f^\dagger \tilde{M}_F^2 D_f\\ &\mathcal{F}_f m_f^\dagger = B_f^\dagger m^2 A_f + D_f^\dagger M^2 C_f.\end{aligned} \tag{3.18}$$

[1]the matrix will not explicitly written here

In this thesis, we assume that $\mu_F^2 + \mathcal{F}_f\mathcal{F}^\dagger \gg \mathcal{F}_f m_f^\dagger$. It can be obviously satisfied as we assume $u, w \gg \langle h \rangle$. We can also have

$$B_f \simeq -\frac{A_f m_f \mathcal{F}_f^\dagger}{\mu_F^2 + \mathcal{F}_f\mathcal{F}_f^\dagger}, \qquad C_f \simeq \frac{\mathcal{F}_f m_f^\dagger}{\mu_F^2 + \mathcal{F}_f\mathcal{F}_f^\dagger}, \qquad D_f \simeq 1. \tag{3.19}$$

These approximately guarantee the unitarity of the matrix A_f. It is easy to find

$$A_f^{-1}\tilde{m}_f^2 A_f \simeq m_f m_f^\dagger - \frac{1}{\mu_F + \mathcal{F}_f\mathcal{F}_f^\dagger}(m_f\mathcal{F}_f^\dagger)(\mathcal{F}_f m_f^\dagger). \tag{3.20}$$

The first term on the right-hand side corresponds to the effective squared mass matrix of ordinary fermions. The second term found to have complex phases in off-diagonal components if $y_{f_i} \neq \tilde{y_{f_i}}$ is satisfied, thus A_f can be complex. Thus, the second term can be comparable with the first term if $\mu_F^2 \gg \mathcal{F}_f\mathcal{F}_f^\dagger$, so let's work on this one. Indeed, it is sufficient to consider some parameters here for instance

$$\langle h \rangle \ll w < u \qquad\qquad y_{f_i} \sim \tilde{y}_{f_j} < y_F. \tag{3.21}$$

The CKM Matrix is found to be $V_{CKM} = O_u^T A_d$, where O_u is an orthogonal matrix that is used to diagonalize the mass matrix for up-type quarks. Thus it is obvious that the CP phase is due to the components of A_d. In another case, for the lepton sector, the PMNS matrix has the structure $V_{PMNS} = A_e^\dagger U_\nu$, where U_ν is the diagonalization matrix of neutrino mass matrix. The Dirac CP phases in both CKM and PMNS matrix can be induced from the same origin. We provide the example of A_d in the Appendix A.3.

3.3 Effective Model at Lower Energy Region

In this section, we turn back to the Lagrangian eq. (3.1) and (3.2) and integrating out the heavy fields σ and S, as in this part we discuss the effective model in the lower energy region. It is leaving us with the SM with extended lepton sector as it sounds in Scotogenic model referred in [40] which invariant under Z_2 symmetry. The Lagrangian is

$$\begin{aligned} -\mathcal{L}_{scotogenic} = &\sum_{j=1}^{3}\left[\frac{M_{N_J}}{2}\bar{N}_J^c N_j + \sum_{\alpha=1}^{3}\tilde{h}_{\alpha j}^*\eta\bar{l}_\alpha N_j + \text{h.c.}\right] + +\tilde{m}_H^2\mathcal{H}^\dagger\mathcal{H} + \tilde{m}_\eta^2\eta^\dagger\eta \\ &+ \tilde{\lambda}_1(\mathcal{H}^\dagger\mathcal{H})^2 + \tilde{\lambda}_2(\eta^\dagger\eta)^2 + \tilde{\lambda}_3(\mathcal{H}^\dagger\mathcal{H})(\eta^\dagger\eta) + \lambda_4(\mathcal{H}^\dagger\eta)(\eta^\dagger\mathcal{H}) + \frac{\tilde{\lambda}_5}{2}\left[\sigma(\eta^\dagger\mathcal{H})^2 + \text{h.c}\right], \end{aligned} \tag{3.22}$$

which the redefined coupling constant (with tilde) are appearing after the symmetry breaking of $\tilde{\sigma}$ and $\tilde{S}$:

$$\begin{aligned}
\tilde{\lambda}_1 &= \lambda_1 - \frac{\kappa_{H\sigma}^2}{4\tilde{\kappa_\sigma}} - \frac{\kappa_{HS}^2}{4\tilde{\kappa_S}} + \frac{\kappa_{\sigma S}\kappa_{H\sigma}\kappa_{HS}}{4\tilde{\kappa_\sigma}\tilde{\kappa}_S}, \qquad \tilde{\lambda}_2 = \lambda_2 - \frac{\kappa_{\eta\sigma}^2}{4\tilde{\kappa_\sigma}} - \frac{\kappa_{\eta S}^2}{4\tilde{\kappa_S}} + \frac{\kappa_{\sigma S}\kappa_{\eta\sigma}\kappa_{\eta S}}{4\tilde{\kappa_\sigma}\tilde{\kappa}_S} \\
\tilde{\lambda}_3 &= \lambda_3 - \frac{\kappa_{H\sigma}\kappa_{\eta\sigma}}{2\tilde{\kappa}_\sigma} - \frac{\kappa_{HS}\kappa_{\eta S}}{2\tilde{\kappa}_S} + \frac{\kappa_{\sigma S}\kappa_{H\sigma}\kappa_{\eta S} + \kappa_{\sigma S}\kappa_{\eta\sigma}\kappa_{HS}}{4\tilde{\kappa}_\sigma\tilde{\kappa}_S}, \qquad \tilde{\lambda}_5 = \lambda_5\frac{w}{M_*}.
\end{aligned} \tag{3.23}$$

In the present study, we take $\bar{M} = w$. Stability of the potential written in eq. (3.22) can be satisfied in scales $\mu < \bar{M}$. To find the stability condition, we can rewrite the fields as

$$\begin{aligned}
\mathcal{H}^\dagger &= \sqrt{-i\sin\delta} \qquad & \eta^\dagger &= \sqrt{-i\cos\delta} \\
\mathcal{H} &= \sqrt{i\sin\delta} \qquad & \eta &= \sqrt{i\cos\delta}.
\end{aligned} \tag{3.24}$$

Thus, using the same method in obtaining eq. (3.10), we get the requirements of stability as

$$\tilde{\lambda}_1\tilde{\lambda}_2 > 0 \qquad \tilde{\lambda}_3, \tilde{\lambda}_3 + \lambda_4 - |\tilde{\lambda}_5| > -2\sqrt{\tilde{\lambda}_1\tilde{\lambda}_2}. \tag{3.25}$$

The stability of potential (3.22), (3.25) and perturbativity for the weak scale to Planck scale can be examined by using the renormalization group equation (RGEs) of the coupling constants. Also, we can see the relevant RGEs at $\mu > \bar{M}$ depicted in appendix A.4. In addition, the mass parameters in this low scale can be written as

$$\begin{aligned}
M_{N_j} &= y_{N_j} w \\
\tilde{m}_H^2 &= m_H^2 + \left(\kappa_{H\sigma} + \frac{\kappa_{HS}\kappa_{\sigma S}}{2\tilde{\kappa}_S}\right) w^2 + \left(\kappa_{HS} + \frac{\kappa_{HS}\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}\right) u^2, \\
\tilde{m}_\eta^2 &= m_\eta^2 + \left(\kappa_{\eta\sigma} + \frac{\kappa_{\eta S}\kappa_{\sigma S}}{2\tilde{\kappa}_S}\right) w^2 + \left(\kappa_{\eta S} + \frac{\kappa_{\eta S}\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}\right) u^2,
\end{aligned} \tag{3.26}$$

where in this model η has no VEV if $\tilde{m}_\eta^2 > 0$. We assume both $\tilde{m}_H$ and $\tilde{m}_\eta$ are in order of $\sim 0(1)$TeV, parameters tuning are required for this.

In various studies [41–50], the phenomenology of neutrinos and dark matter have been studied, and here we adopt those results. The deviation from those studies may apply if we consider Axion as the dominant part of the dark matter. Still in those results, if we assume η_R is the dominant component of dark matter, both relic abundance and direct search of dark matter may constraint the parameters $\tilde{\lambda}_3$ and λ_4. In fig. 3.1, we show the constraint in $\lambda_+, \tilde{\lambda}_3$ plane, where $\lambda_+ = \tilde{\lambda}_3 + \lambda_4 + \tilde{\lambda}_5$ and we choose $M_{\eta_R} = (0.9, 1, 1.1)$TeV — not to forget we define $M_{\eta_R}^2 = m_\eta^2 + \lambda_+ \langle h\rangle^2$. Perturbativity constraint requirements at $\mu > \bar{M}$ are also affected since dark matter abundance strictly required both $\tilde{\lambda}_3$ and λ_4

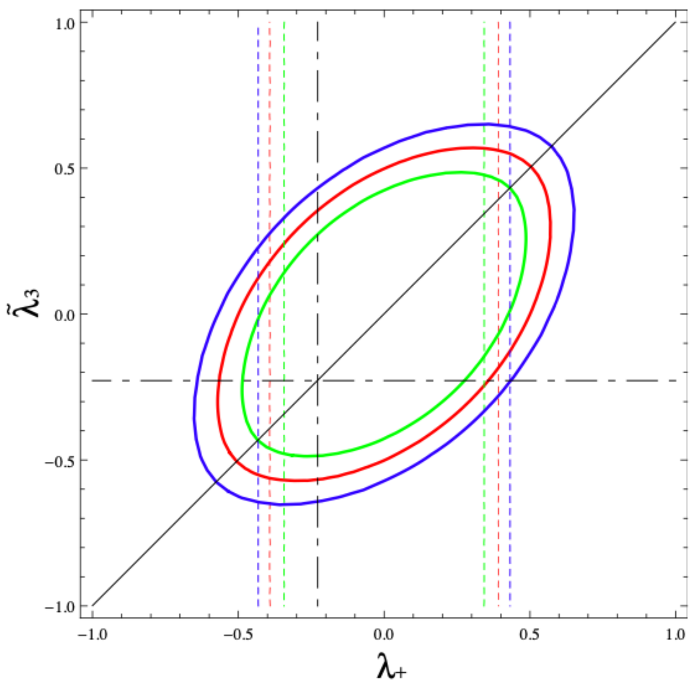

Figure 3.1: This is the contours of $\Omega_{DM}h^2 = 0.12$. They are plotted in the λ_+ and $\tilde{\lambda}_3$ plane. The solid line green, red, and blue correspond to $M_{\eta_R} = (0.9, 1, 1.1)$ in TeV unit respectively. Since η_R is supposed to be lighter than its charged components, we impose $\lambda_4 < 0$ which is depicted in the region above the diagonal **black** solid line. As we assume $\tilde{\lambda}_2 = 0.1$. The allowed region constrained by eq. (3.25) is depicted in square-shaped region in above-right side than two-**black** dash dotted lines.

to rather large values.

As in the original Scotogenic model, the neutrino mass is forbidden at tree level due to Z_2 symmetry but it can be generated through a 1-loop diagram. The result is given by[2]

$$\mathcal{M}^{\nu}_{\alpha\beta} \simeq \sum_{j=1}^{3} \tilde{h}_{\alpha j}\tilde{h}_{\beta j}\tilde{\lambda}_5\Lambda_j \qquad \text{with} \qquad \Lambda = \frac{\langle h\rangle^2}{8\pi}\frac{1}{M_{N_j}}\ln\frac{M^2_{N_j}}{M^2_\eta}, \tag{3.27}$$

where we supposed $M_\eta^2 = \tilde{m}_\eta^2 + (\tilde{\lambda}_3 + \lambda_4)\langle h\rangle^2$ and $M_{N_j} \gg M_\eta$. As an example, we can adjust some parameters of neutrino Yukawa coupling as [51].

$$\tilde{h}_{ei} = 0,\ \ \tilde{h}_{\mu i} = h_{\tau i} = h_i\ \ (i = 1, 2);\ \ \tilde{h}_{e3} = \tilde{h}_{\mu 3} = -\tilde{h}_{\tau 3} = h_3 \tag{3.28}$$

This is the realization of tri-bimaximal mixing which gives the zeroth-order approximation for neutrino oscillation data and leptogenesis [49, 50]. We find the squared mass difference

[2]See A.2 for original Scotogenic model.

from the eigenvalues of (3.27) (we assume $h_1 \ll h_2, h_3$)

$$h_2^2 \Lambda_2 |\tilde{\lambda}_5| = \frac{1}{2}\sqrt{\Delta m_{32}^2}, \qquad h_3^2 \Lambda_3 |\tilde{\lambda}_5| = \frac{1}{3}\sqrt{\Delta m_{21}^2}. \tag{3.29}$$

As we have $\Lambda_{2,3} = O(1)$eV for $M_{2,3} = O(10^7)$GeV, the neutrino oscillation data can be taken with the parameters as follows

$$y_{N_j} = O(10^{-2}), \qquad h_{2,3} = O(10^{-3}), \qquad |\tilde{\lambda}_5| = 10^{-3}. \tag{3.30}$$

On another note, the smallness of h_1 may play a crucial role in low-scale leptogenesis, as we will see later.

3.4 Inflation

In this section, we will discuss the Inflation due to singlet scalars. It can be traced back to the similar problems mentioned in ref. [52–57]. For the remaining of this inflation part, we borrow the relevant method from ref. [23] and [58]. Straightforwardly, in this model, the Action relevant to inflation is given by

$$\begin{aligned} S_J = \int d^4x\sqrt{-g}\Big[&-\frac{1}{2}M_p^2 R - \xi_\sigma \sigma^\dagger \sigma R - \xi_{S1} S^\dagger S R - \frac{\xi_{S_2}}{2}(S^2 + S^{\dagger 2})R \\ &+ \partial^\mu \sigma^\dagger \partial_\mu \sigma + \partial^\mu S^\dagger \partial_\mu S - V(\sigma, S)\Big], \end{aligned} \tag{3.31}$$

where supscript J correspond to the Jordan frame and M_p is the reduced Planck mass, also coupling of S is affected by the Z_4 symmetry. $V(\sigma, S)$ correspond the potential depicted in eq. (3.5). If we assume $\xi_{S1} = -\xi_{S2}$, the coupling with the Ricci scalar is reduced to $\frac{1}{2}\xi_S S_I^2 R$, as $S = \frac{1}{\sqrt{2}}(S_R + iS_I)$ and $\xi_S = \xi_{S1} - \xi_{S2}$. As we expect, S moves along the minimum of ρ which is determined by $\partial V_b/\partial\rho = 0$, thus we get the radial component $\tilde{S}$ couples with Ricci scalar via $\frac{1}{2}\tilde{\xi}_S \tilde{S}^2 R$ as $\tilde{\xi}_S$ is defined as $\tilde{\xi}_S = \xi_S \sin^2\rho$. Here we consider only $\tilde{\xi}_S$ and ξ_σ are all positive. This stability condition is preserved in (3.10). The VEVs w and u are expected to be neglected due to our range of energy level during inflation, the same situation also applied for any other fields except $\tilde{\sigma}$ and $\tilde{S}$.

We impose the conformal transformation depicted as eq. (2.3)

$$\tilde{g_{\mu\nu}} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = 1 + \frac{\xi_\sigma \tilde{\sigma}^2 + \tilde{\xi}_S \tilde{S}^2}{M_p^2}. \tag{3.32}$$

. Using the transformation, we can obtain the Action in the Einstein frame as

$$
\begin{aligned}
S_E = \int d^4x\sqrt{-g}\Big[&-\frac{1}{2}M_p^2 R_E + \frac{1}{2}\partial^\mu\phi_\sigma\partial_\mu\phi_\sigma + \frac{1}{2}\partial^\mu\phi_S\partial_\mu\phi_S \\
&+ \frac{6\xi_\sigma\tilde{\xi}_S\frac{\tilde{\sigma}\tilde{S}}{M_p^2}}{\left[\left(\Omega^2 + \frac{6\xi_\sigma^2}{M_p^2}\tilde{\sigma}\right)\left(\Omega^2 + \frac{6\tilde{\xi}_S^2}{M_p^2}\tilde{S}\right)\right]^{1/2}}\partial^\mu\phi_\sigma\partial_\mu\phi_S - \frac{1}{\Omega^4}V(\tilde{\sigma},\tilde{S})\Big],
\end{aligned}
\tag{3.33}
$$

where the supscript E represents the Einstein frame. The redefined fields: ϕ_σ and ϕ_S are depicted by

$$
\frac{\partial\phi_\sigma}{\partial\tilde{\sigma}} = \frac{1}{\Omega^2}\sqrt{\Omega^2 + 6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}, \qquad \frac{\partial\phi_S}{\partial\tilde{S}} = \frac{1}{\Omega^2}\sqrt{\Omega^2 + 6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}. \tag{3.34}
$$

We will introduce ϕ which can be connected to field $\tilde{\sigma}$ and $\tilde{S}$ via $\sigma = \phi\cos\varphi$ and $\tilde{S} = \phi\sin\varphi$. The potential at the large field regions characterized by $\xi_\sigma\tilde{\sigma}^2 + \tilde{\xi}_S\tilde{S}^2 \gg M_p^2$ can be written by

$$
V(\phi,\varphi) = \frac{M_p^4}{4}\frac{\tilde{\kappa}_S\sin^4\varphi + \tilde{\kappa}_\sigma\cos^4\varphi + \kappa_{\sigma S}\sin^2\varphi\cos^2\varphi}{\xi_\sigma\cos^2\varphi + \tilde{\xi}_S\sin^2\varphi}. \tag{3.35}
$$

There are three valleys that correspond to this potential. Two of them are

$$
\text{(i)}\quad \varphi = 0 \quad \text{for} \quad 2\tilde{\kappa}_\sigma\tilde{\xi}_S < \kappa_{\sigma S}\xi_\sigma \qquad\qquad \text{(ii)}\quad \varphi = \frac{\pi}{2} \quad \text{for} \quad 2\tilde{\kappa}_S\xi_\sigma < \kappa_{\sigma S}\tilde{\xi}_S. \tag{3.36}
$$

Using these valleys, the kinetic mixing between ϕ_σ and ϕ_S disappears and inflaton can be identified only by using ϕ_σ for (i), and ϕ_S for (ii). For another valley, let's say, the (iii), it can be obtained by taking $\partial V(\phi,\varphi)/\partial\varphi = 0$. Thus, the valleys are characterized by

$$
\sin^2\varphi = \frac{2\tilde{\kappa}_\sigma\tilde{\xi}_S - \kappa_{\sigma S}\xi_\sigma}{(2\tilde{\kappa}_S\xi_\sigma - \kappa_{\sigma S}\tilde{\xi}_S) + (2\tilde{\kappa}_\sigma\tilde{\xi}_S - \kappa_{\sigma S}\xi_\sigma)}, \tag{3.37}
$$

under the condition $2\tilde{\kappa}_\sigma\tilde{\xi}_S > \kappa_{\sigma S}\xi_\sigma$ and $2\tilde{\kappa}_S\xi_\sigma > \kappa_{\sigma S}\tilde{\xi}_S$. Also, in this model, we would like to be S-like inflation. Hence, the requirements condition for this type of inflaton imposed

$$
\tilde{\xi}_S \gg \xi_\sigma \qquad \kappa_{\sigma S} < 0 \qquad \tilde{\kappa}_S < |\kappa_{\sigma S}| < \kappa_\sigma. \tag{3.38}
$$

The canonically normalized inflaton ϕ can be described as

$$\Omega^2 \frac{d\phi}{d\tilde{S}} = \sqrt{b\Omega^2 + 6\tilde{\xi}_S^2 \frac{\tilde{S}}{M_p^2}}. \tag{3.39}$$

with $b = 1 - \frac{\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}$, this prescription is identically derived from ref. [58]. Also, the solution of (3.39) can be written as

$$\frac{\phi}{M_p} = -\sqrt{6}\sinh^{-1}\left(\frac{\sqrt{\frac{6}{b}\frac{\tilde{\xi}_S \tilde{S}}{M_p}}}{\sqrt{1 + \frac{\tilde{\xi}_S}{M_p^2}\tilde{S}^2}}\right) + \sqrt{\frac{b + 6\tilde{\xi}_S}{\tilde{\xi}_S}}\sinh^{-1}\left(\frac{\sqrt{\tilde{\xi}_S(1 + 6\tilde{\xi}_S/b)}\tilde{S}}{M_p}\right). \tag{3.40}$$

In addition, three regions are containing the inflaton potential

$$V(\phi) = \begin{cases} \frac{\hat{\kappa}_S}{4\tilde{\xi}_S^2} M_p^4 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_p}\right)\right]^2 & \text{if } \phi > M_p \\ \frac{\hat{\kappa}_S}{6\tilde{\xi}_S^2} M_p^2 \phi^2 & \text{if } \frac{M_p}{\tilde{\xi}_S} < \phi < M_p \\ \frac{1}{4}\hat{\kappa}\phi^4, & \text{if } \phi < \frac{M_p}{\tilde{\xi}_S}, \end{cases} \tag{3.41}$$

where $\hat{\kappa}_S = \tilde{\kappa}_S - \frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}$ is derived in eq. (3.13). The first region ($\phi > M_p$) correspond to the inflationary phase, the inflation ends when $\phi \simeq M_p$. After the end of inflation, if $\tilde{\xi}_S > 1$, the inflation will reach the quadratic potential depicted in the second region ($M_p/\tilde{\xi}_S < \phi < M_p$). Larger $\tilde{\xi}_S$ may prolong the inflaton to stay at this stage. After the inflaton oscillates $\frac{1}{2\pi\sqrt{3\pi}}(\tilde{\xi}_S - 1)$ times[3]. It will fall into quartic potential depicted in the third region ($\phi < M_p/\tilde{\xi}_S$), where the Jordan and Einstein frames are coincided. As in our study, we only consider $\tilde{\xi}_S < 10$, this quartic potential becomes our main attention in preheating stage.

To calculate the slow-roll parameters ϵ and η, we can recall the definitions on eq. (1.21)

$$\epsilon \equiv \frac{1}{2}M_p\left(\frac{V'}{V}\right)^2 = \frac{8M_p^4}{b\tilde{\xi}_S\left(1 + 6\tilde{\xi}_S/b\right)\tilde{S}^4}, \qquad \eta \equiv M_p^2\frac{V''}{V} = -\frac{8M_p^2}{b\left(1 + 6\tilde{\xi}_S/b\right)\tilde{S}^2}. \tag{3.42}$$

The number of e-folds $\mathcal{N}_k$ with scale k exits the Horizon to the end of inflation can be

[3]This result is derived the same way as seen in the beginning of subsection 2.3.2

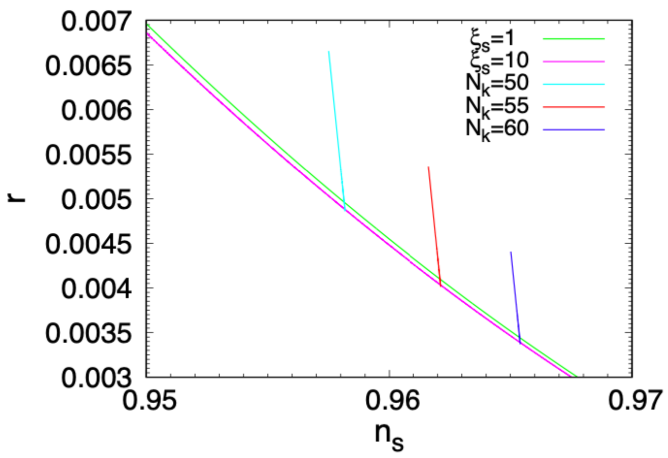

Figure 3.2: We varied the coupling constant $\hat{\kappa} \sim 10^{-7} - 10^{-10}$ with the values of n_s and r can be read-off by the intersection of fixed $\tilde{\xi}_S$ or $\mathcal{N}_k$

calculated as (see. eq. (2.10))

$$\mathcal{N}_k = \frac{1}{M_p}\int_{\phi_{end}}^{\phi_k} \frac{V}{V'} d\phi = \frac{1}{8M_p^2}(b + 6\tilde{\xi}_S)(\tilde{S}_k^2 - \tilde{S}_{end}^2) - \frac{3}{4}\ln\frac{M_p^2 + \tilde{\xi}_S \tilde{S}_k^2}{M_p^2 + \tilde{\xi}_S \tilde{S}_{end}^2}, \tag{3.43}$$

where we have used eq. (3.39). With this, we have another approximate relation, there are: $\epsilon \simeq \frac{3}{4\mathcal{N}_K}$ and $\eta \simeq -\frac{1}{\mathcal{N}k}$. The potential during the end of inflation is approximated to be $V(\phi) \simeq 0.072\frac{\hat{\kappa}_S}{\tilde{\xi}_S^2} M_p^4$.

The scalar power spectrum can be written as (see. (1.45))

$$\mathcal{P}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \qquad A_s = \frac{V^3}{12\pi^2 M_p^6 V'^2} = \frac{V}{24\pi^2 M_p^4 \epsilon}\bigg|_{k_*}. \tag{3.44}$$

If we used the Planck data $A_s = (2.101^{+0.031}_{-0.034}) \times 10^{-9}$ at $k^* = 0.05\text{Mpc}^{-1}$ [2]. We find the constraint

$$\hat{\kappa}_S \simeq 4.13 \times 10^{-10} \tilde{\xi}_S^2 \left(\frac{60}{\mathcal{N}_{k*}}\right)^2. \tag{3.45}$$

As we already have all requirements to calculate the spectral index and the tensor-to-scalar ratio respectively

$$r = 16\epsilon \qquad\qquad n_s = 1 - 6\epsilon + 2\eta, \tag{3.46}$$

we can plot the $n_s - r$ plane in fig. 3.2. Since $\hat{\kappa}_S$ is free parameter in our model, using the the fixed values of $\tilde{\xi}_S$ and $\mathcal{N}_\parallel$, we choose the range $10^{-10} \leq \hat{\kappa}_S \leq 10^{-7}$. The constraint of CMB (3.45) can be obtained in the intersection points of the fixed $\tilde{\xi}_S$ and $\mathcal{N}_k$.

3.5 End of Inflation: Preheating and Reheating

During the end of Inflation, the friction ($3H\dot{\phi}$) is getting smaller, and comparable with other terms. We can rewrite the Klein-Gordon equation of eq. (1.13) as

$$\ddot{\phi} + \frac{dV(\phi)}{d\phi} \simeq 0. \tag{3.47}$$

As we already stated, the quadratic potential is neglected due to the reason we mentioned before[4]. Thus the quartic potential plays a substantial role to drain the inflaton energy. If we introduce the dimensionless conformal time τ as $a\tau = \sqrt{\hat{\kappa}_S}\phi_{end}$ and also we redefine the field $f = \frac{a\phi}{\phi_{end}}$, eq. (3.47). Hence, it can be approximated by

$$\frac{d^2 f}{d\tau^2} + f^3 = 0. \tag{3.48}$$

The solution of this equation is belong to Jacobi Elliptic function $f(\tau) = \mathrm{cn}\left(\tau - \tau_i, \frac{1}{\sqrt{2}}\right)$ [23, 58]. The derivation of this one is similar with our calculation in chapter 1. We also find the conformal time for this oscillation is found to be (see eq. (1.72) and (1.73))

$$a(\tau) = \frac{\phi_{end}}{2\sqrt{3}M_p}\tau \qquad\qquad \tau = 2(3\hat{\kappa}_S M_p^2)^{1/4}\sqrt{t}. \tag{3.49}$$

The last equations are necessary for later use. Before we proceed, it is important to find the minimum of $\tilde{\sigma}$. We can turn back at the potential eq. (3.5). Taking $\partial V/\partial\tilde{\sigma} = 0$ for minimum, we obtain $\tilde{\sigma} = \sqrt{\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_S}}\tilde{S} \to \sqrt{\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_S}}\phi,$. Hence we got $\sigma|_{\theta=0} = \frac{1}{\sqrt{2}}\tilde{\sigma} = \frac{1}{\sqrt{2}}\sqrt{\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_S}}\phi$. In the purpose of our discussion, we will recall some terms in Lagrangian (3.1) and (3.2) which correspond to the the field $\tilde{\sigma}$ and $\tilde{S}$ and substitute both fields with ϕ, we obtain

$$\begin{aligned}
&\Bigg[-\frac{y_D}{\sqrt{2}}\frac{\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}\phi\bar{D}_L\bar{D}_R - \frac{y_E}{\sqrt{2}}\frac{\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}\phi\bar{E}_L\bar{E}_R - \sum_{j=1}^{3}\Bigg\{\frac{1}{\sqrt{2}}(y_{d_j}e^{i\rho} + \tilde{y}_{d_j}e^{-i\rho})\phi\bar{D}_L d_{R_j} \\
&+ \frac{1}{\sqrt{2}}(y_{e_j}e^{i\rho} + \tilde{y}_{e_j}e^{-i\rho})\phi\bar{E}_L e_{R_j} + \frac{y_{N_j}}{2\sqrt{2}}\frac{\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}\phi\bar{N}_j^c N_j\Bigg\} + \mathrm{h.c}\Bigg] \\
&+ \frac{1}{2}(\kappa_{\sigma S}\mathcal{H}^\dagger\mathcal{H} + \kappa_{\eta S}\eta^\dagger\eta)\phi^2 - \frac{1}{2}\frac{\kappa_{\sigma S}}{2\tilde{\kappa}_\sigma}(\kappa_{H\sigma}\mathcal{H}^\dagger\mathcal{H} + \kappa_{H\sigma}\eta^\dagger\eta)\phi^2.
\end{aligned} \tag{3.50}$$

[4]One should note that the potential in eq. (3.47) on this section is quartic as we already stated, we neglect the quadratic potential regime

Thus, the mass term from eq. (3.50) can be extracted explicitly

$$
\begin{aligned}
M_{N_j} &= \frac{y_{N_j}}{\sqrt{2}}\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_\sigma}\phi & \tilde{M}_F &= \frac{\phi}{\sqrt{2}}\left[\sum_{j=1}^{3}(y_{f_j}^2+\tilde{y}_{f_j}^2)+y_F^2\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma^2}\right]^{1/2} \\
m_H^2 &= \frac{1}{2}\left(\kappa_{HS}+\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_\sigma}\kappa_{H\sigma}\right)\phi^2 & m_\eta^2 &= \frac{1}{2}\left(\kappa_{\eta S}+\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_\sigma}\kappa_{\eta\sigma}\right)\phi^2.
\end{aligned}
\tag{3.51}
$$

Where $F=D$ or E should be regarded similar with $f=d$ or e. In addition, we will recall back on eq. (3.5) and split $\tilde{S}^2$ to be $\tilde{S}^2=S_\parallel^2+S_\perp^2$, also the same way with $\tilde{\sigma}$, we finally have

$$
\frac{\tilde{\kappa}_S}{4}(S_\parallel^2+S_\perp^2-u^2)^2+\frac{\kappa_{\sigma S}}{4}(S_\parallel^2+S_\perp^2-u^2)(\sigma_\parallel^2+\sigma_\perp^2-w^2)+\frac{\tilde{\kappa}_\sigma}{4}(\sigma_\parallel^2+\sigma_\perp^2-w^2)^2. \tag{3.52}
$$

Using the last potential, we can obtain the masses of each component:

$$
\begin{aligned}
m_{S_\parallel}^2 = \frac{\partial^2 V}{\partial S_\parallel^2} &= \left(3\tilde{\kappa}_S S_\parallel^2+\frac{\kappa_{\sigma S}}{2}(\sigma_\parallel^2+\sigma_\perp^2-w^2)\right) = \left(3\tilde{\kappa}_S S_\parallel^2-\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}(S_\parallel^2+S_\perp^2-u^2)\right) \\
&\simeq \left(3\tilde{\kappa}_S S_\parallel^2-\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}S_\parallel^2\right) = \left(3\tilde{\kappa}_S S_\parallel^2-\frac{\kappa_{\sigma S}^2}{2\tilde{\kappa}_\sigma}S_\parallel^2+\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}S_\parallel^2\right) \\
&= \left(3\hat{\kappa}_S S_\parallel^2-\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}S_\parallel\right) = \left(3\hat{\kappa}_S^2+\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}\right)\phi^2.
\end{aligned}
\tag{3.53}
$$

where we have used $\tilde{\sigma}=\sqrt{\frac{|\kappa_{\sigma S}|}{2\tilde{\kappa}_S}}\tilde{S}$ and $S_\parallel \gg S_\perp, u$, since in this part we regard $S_\parallel$ as inflaton[5]. Using the same method, as we skipped their calculation for better reason, the other masses can be obtained:

$$
m_{S_\perp}^2=\hat{\kappa}_S\phi^2, \qquad m_{\sigma_\parallel}^2=\left(|\kappa_{\sigma S}|+\frac{\kappa_{\sigma S}^2}{4\tilde{\kappa}_\sigma}\right)\phi^2, \qquad \text{and} \qquad m_{\sigma_\perp}^2=\frac{\kappa_{\sigma S}}{4\tilde{\kappa}_\sigma}\phi^2. \tag{3.54}
$$

The largeness of these coupling constants is constrained by our assumption on inflation and the realization of the CP phase in previous sections. The constraints are

$$
\hat{\kappa}_S < |\kappa_{\sigma S}| < \tilde{\kappa}_\sigma \qquad \text{and} \qquad \hat{\kappa}_S < y_{N_j}, y_{f_i}, \tilde{y}_{f_i} \tag{3.55}
$$

Furthermore, it is more convenient to write the mass terms in more general form

$$
m_\psi^2 = g_\psi \phi^2. \tag{3.56}
$$

[5] we understand this thing is quite confusing, hence we recommend the reader to read [58] as we using the same method here

where g_ψ is the coupling for each species $\psi = \sigma, S_\perp, h, \eta$.

additionally, we also have a constraint

$$\hat{\kappa}_S < \begin{cases} g_H = \kappa_{HS} + \frac{|\kappa_{\sigma S}|}{\tilde{\kappa}_\sigma}\kappa_{H\sigma} \\ g_\eta = \kappa_{\eta S} + \frac{|\kappa_{\sigma S}|}{\tilde{\kappa}_\sigma}\kappa_{\eta\sigma} \end{cases} . \tag{3.57}$$

Since the mass or frequency of the inflaton is $\sqrt{\tilde{\kappa}_S}\phi$, decays or annihilation of inflaton are kinematically forbidden , but only for $\sigma_\perp$ is allowed (see (3.54)). We will see later, the smallness mass of produced particles will be neglected due to the smallness of energy transfer from inflaton to produced particles[6].

It is clear, the masses of particles are ϕ dependent, so the decay channel is opened once $\phi \simeq 0$. Hence, the particles are produced during zero crossings. Here we write the equation of oscillation for the self-production of the inflaton

$$\frac{d^2\Phi_k}{d\tau^2} + \omega_k^2\Phi_k = 0, \qquad \omega^2 = \bar{k}^2 + 3f(\tau)^2, \tag{3.58}$$

and the equation of the created particles

$$\frac{d^2F_k}{d\tau^2} + \tilde{\omega}_k^2F_k = 0, \qquad \tilde{\omega}^2 = \bar{k}^2 + \frac{g_\psi}{\hat{\kappa}_S}f(\tau)^2, \tag{3.59}$$

where we have used the rescaled variables

$$\Phi_k = \frac{a\phi_k}{\phi_{end}} \qquad F_k = \frac{a\psi_k}{\phi_{end}} \qquad \bar{k} = \frac{ak}{\phi_{end}\sqrt{\hat{\kappa}_S}}. \tag{3.60}$$

Here $f(\tau)$ is the solution of Eq. (3.48). Both Φ_k and F_k shown the exponential behavior $\propto e^{\mu_k\tau}$, where μ_k represents the characteristic exponent [22, 23]. This μ_k is determined by the ratio of $g_\psi/\hat{\kappa}_S$. The number density of produced particle of each species ψ can be written as

$$n_k^\psi = \frac{\tilde{\omega}_k}{2\hat{\kappa}_S}\left(\frac{|F_k|^2}{\tilde{\omega}_k} + |F_k|^2\right) - \frac{1}{2}. \tag{3.61}$$

We then parametrize the coupling g_ψ into 5 groups

$$\text{(A)}\ \frac{g_{\sigma_\parallel}}{\hat{\kappa}_S} \gg 1, \quad \text{(B)}\ \frac{g_{S_\parallel}}{\hat{\kappa}_S} = 3, \quad \text{(C)}\ \frac{g_{S_\perp}}{\hat{\kappa}_S} = 1, \quad \text{(D)}\ \frac{g_{\sigma_\perp}}{\hat{\kappa}_S} \ll 1, \quad \text{(E)}\ \frac{g_H}{\hat{\kappa}_S}, \frac{g_\eta}{\hat{\kappa}_S} > 1. \tag{3.62}$$

We conclude here, the (A)-(D) cases are well constrained by the composition of the inflationary model, meanwhile, (E) are not. Case (D) produces a tiny characteristic exponent

[6] the reaction rate is much smaller than Hubble parameter, thus, the energy drain from $\sigma_\perp$ is ineffective

μ_k, hence it is obviously neglected. For (B) and (C) cases, the production of $S_\parallel$ and $S_\perp$ are produced really fast, but stops when the certain values of $\langle|S_\parallel|^2\rangle$ and $\langle|S_\perp|^2\rangle$ are $\sim 0.5\phi_{end}^2/a^2$ and will decrease slowly without zero crossing [23, 58]. Even though the maximum value of the characteristic exponent of the case (B) smaller than (C) [23], the interaction of $S_\parallel S_\perp$ accelerates the $S_\parallel$ production due to rescattering and finally reaches a similar value [58]. The backreaction of this fluctuation to the oscillation affected the resonance band, the resonant particle production is stopped before the conversion of inflaton energy get further[7]. As stated before, the inflaton seems not to undergo the zero-crossing mode, the decay is kinematically blocked, hence it cannot contribute to reheating. For (A), as $\sigma_\parallel$ couples directly to the inflaton, the resonant production is stopped for the same reason as cases (B) and (C). Even though it can decay to F and N_j, the decay widths are much smaller than Hubble parameters. On another side, if the process (E) is ineffective for the reheating temperature, the whole process of preheating cannot play any role in reheating. In addition, the reheating process only happens by perturbative process, when inflaton's amplitude is smaller than u.

There is a chance for (E) to make it possible for preheating process affects reheating. The particles mentioned in (E): $\mathcal{H}$ and η, have a decay channel to relativistic particles, so these properties are somehow different with (B) and (C). The $\mathcal{H}$ and η are produced via zero crossing when adiabaticity condition is violated (when $\frac{d\tilde{\omega}_k}{d\tau} > \tilde{\omega}_k^2$). By taking the eq. (1.122) to derive analytically. For simplicity, we only consider the non-resonance effect, thus only the first term is used here. The momentum distribution of produced particle ψ-species through the one-zero crossing of inflaton ϕ is

$$n_{\bar{k}}^{\psi} = e^{2\mu_k \frac{\tau_0}{2}} = e^{-\left(\frac{\bar{k}}{\bar{k}_c}\right)^2}, \qquad \bar{k}_c^2 = \sqrt{\frac{g_\psi}{2\pi^2, \hat{\kappa}_S}} \tag{3.63}$$

where τ_0 is the inflaton period. The resonance is efficient for $\bar{k} < \bar{k}_c$, then the particle number density of species ψ can be calculated via

$$n^{\psi} = \int \frac{d^3\bar{k}}{(2\pi)^3} n_{\bar{k}}^{\psi} = \int \frac{d^3\bar{k}}{(2\pi)^3} e^{-\left(\frac{\bar{k}}{\bar{k}_c}\right)^2} = \frac{\bar{k}_c^3}{8\pi^{3/2}}. \tag{3.64}$$

The energy transfer from inflaton to relativistic particles happens from the decay of $\mathcal{H}$ and η. Thus, the relativistic particles are created by indirect ones. The $\mathcal{H} \to \bar{q}t$ decay process with top Yukawa coupling h_t while $\eta \to \bar{l}N$ with neutrino Yukawa coupling h_j. During the oscillation period, the induced mass η can be larger than N_j. The decay width

[7]this discussion can be seen following ref. [58], instead, in this paper, they used σ as the inflaton.

of $\psi = \mathcal{H}, \eta$ can be written by

$$\bar{\Gamma}_\psi = \frac{c_\psi y_\psi^2}{8\pi}\bar{m}_\psi \qquad\qquad \bar{m}_\psi = \frac{a m_\psi}{\phi_{end}\sqrt{\hat{\kappa}_S}} = \sqrt{\frac{g_\psi}{\hat{\kappa}_S}} f(\tau), \tag{3.65}$$

where c_ψ is internal degrees of freedom, $c_H = 3$ and $c_\eta = 1$. The Yukawa coupling y_ψ represents $y_H = h_t$ and $y_\eta = h_j$. For $\bar{\Gamma}_\psi^{-1} < \tau_0/2$ is satisfied with $g_\psi > 4 \times 10^{-7}\left(\frac{\hat{\kappa}_S}{10^{-8}}\right)$, the produced ψ-species decays to the relativistic fermions are finished before next zero-crossing [59]. If we fix $\tau = 0$ at the first zero-crossing, we can approximate $f(\tau) = \sin(cf_0\tau)$. The energy transferred by ψ decay can be written by

$$\delta\bar{\rho}_r = \int_0^{\tau_0/2} d\tau \bar{\Gamma}_\psi \bar{m}_\psi \bar{n}_\psi e^{-\int_0^\tau \bar{\Gamma}_\psi \tau'} = \frac{1}{8\pi^{3/2}(2\pi^2)^{3/4}}\left(\frac{g_\psi}{\hat{\kappa}_S}\right)^{5/4} Y(f_0, \gamma_\psi). \tag{3.66}$$

where γ_ψ and $Y(f_0, \gamma_\psi)$ are defined as

$$\gamma_\psi = \frac{c_\psi y_\psi^2}{8\pi c}\sqrt{\frac{g_\psi}{\hat{\kappa}_S}}, \qquad Y(f_0, \gamma_\psi) = c\gamma_\psi \int_0^{\tau_0/2} d\tau f_0^2 \sin^2(cf_0\tau) e^{-2\gamma_\psi \sin^2(\frac{cf_0\tau}{2})}, \tag{3.67}$$

here we used[8] $c = 2\pi/\tau_0$. The energy density which is transferred to the light particles is accumulated for each zero-crossing, it can be approximated using average value of τ as

$$\bar{\rho}_r = \frac{2\tau}{\tau_0}\delta\bar{\rho}_r = 6.5 \times 10^{-4}\left(\frac{g_\psi}{\hat{\kappa}_S}\right)^{5/4} Y(f_0, \gamma_\psi)\tau, \tag{3.68}$$

where we assumed f_0 to be constant. The total energy density of the inflation energy $\bar{\rho}_\phi$ and transfer energy $\bar{\rho}_r$ are conserved, the reheating temperature can be found by using relation $\bar{\rho}_\phi = \bar{\rho}_r$. We found

$$\frac{1}{4\hat{\kappa}_S}\left(\frac{\sqrt{\kappa}_S \phi_{end}}{a}\right)^4 = \frac{\pi^2}{30} g_* T_R^4, \tag{3.69}$$

we used $\bar{\rho}_\phi = \frac{1}{4\hat{\kappa}_S}$ and $g_* = 130$. Using relation (3.49) and (3.68) to (3.69) we obtain

$$T_R = 5.9 \times 10^{15} g_\psi^{5/4} Y(f_0, \gamma_\psi)\text{GeV}. \tag{3.70}$$

Since $h_t \gg h_j$, reheating can be determined by the $\mathcal{H}$. If preheating cannot effectively produce relativistic particles, the dominant energy is kept in the oscillation which is depleted slowly. When the inflaton ϕ amplitude reaches $\sim O(u)$, it starts to decay perturbatively.

[8] following ref. [23], we used $\tau = 7.416$ for one oscillation

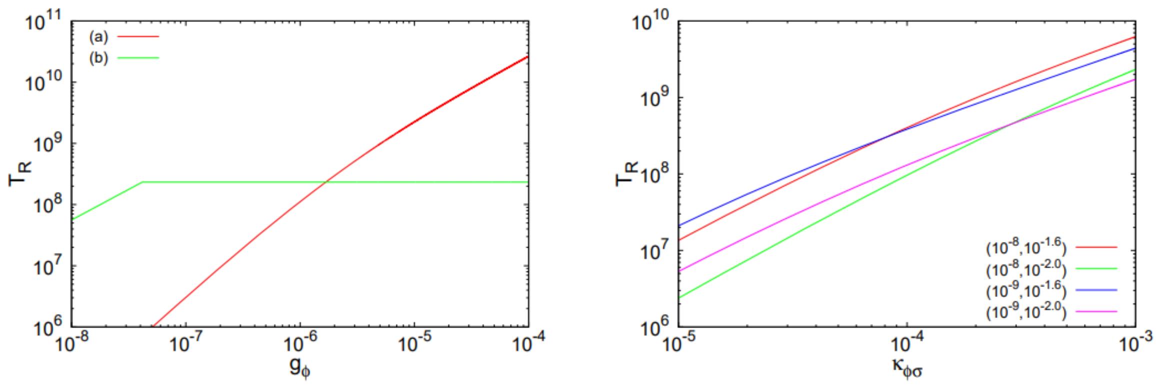

Figure 3.3: left: the comparison of the reheating temperature T_R in (a) preheating and (b) perturbative process, where the assumed perturbative process is $h \to \bar{q}t$. **right**: the reheating temperature due to σ. We fix $\tilde{\kappa}_\sigma = (10^{4.5}, 10^{5.3})$ and vary the parameters $\tilde{\kappa}_S, \kappa_{\sigma S}/\tilde{\kappa}_\sigma$.

however, since we assume

$$2\tilde{m}_\eta < m_\phi < \tilde{M}_D, \tilde{M}_F, \tag{3.71}$$

the decay of inflaton will mainly occur by tree-levels $\phi \to \eta^\dagger \eta$ and $\phi \to \mathcal{H}^\dagger \mathcal{H}$. The decay width can be approximated by[9]

$$\Gamma_\psi = \frac{g_\psi^2}{16\pi \hat{\kappa}_S} m_\phi. \tag{3.72}$$

With $\Gamma_\psi > H$ is satisfied if[10] $g_\psi > 10^{7.1} \left(\frac{\hat{\kappa}_S}{10^{-8}}\right)^{1/2} \left(\frac{u}{10^{11}\mathrm{GeV}}\right)^{1/2}$, when $\phi \simeq u$. The reheating temperature can be approximated via $\frac{1}{4}\hat{\kappa}_S u^4 = \frac{\pi^2}{30} g_* T_R^4$ as

$$T_R \simeq 2.8 \times 10^8 \left(\frac{\hat{\kappa}_S}{10^{-8}}\right)^{1/4} \left(\frac{u}{10^{11}\mathrm{GeV}}\right) \mathrm{GeV}, \tag{3.73}$$

interestingly independent of g_ψ. The small g_ψ can make $\Gamma_\psi \geq H$ not fulfilled.

Based on the left panel of fig. 3.3, the reheating temperature in both processes is plotted as a function of g_H, please note, that we only consider the dominant process caused by Higgs. In this figure, we used $\tilde{\kappa}_S = 10^{-8}$, $\tilde{\kappa}_\sigma = 10^{-4.5}$ and $|\kappa_{\sigma S}|/\tilde{\kappa}_\sigma = 10^{-1.2}$ and $u = 10^{11}$GeV. For $g_H < 10^{-6}$, the reheating temperature is determined by the perturbative process, on contrary for $g_H > 10^{-6}$, the reheating process is determined by preheating. In the right panel of Fig. 3.3, if we assume $\kappa_{HS} = 0$, hence it means there is no decay channel of S to $\mathcal{H}$, the reheating process can still be obtained via σ by varying $\kappa_{\sigma S}/\tilde{\kappa}_\sigma$.

[9] by using $m_\phi = \sqrt{\tilde{\kappa}_S}\phi \to \sqrt{\tilde{\kappa}_S}u$

[10] with $H = \sqrt{\frac{\rho}{3M_p^2}}$, and $\rho \simeq \frac{1}{4}\hat{\kappa}_S u^4$. Also use relation $\Gamma_\psi > H$, with Γ_ψ taken from eq. (3.72)

Based on these figures, the reheating temperature $T_R > 2.3 \times 10^8$GeV can be reached if $g_H > 4 \times 10^{-8}$.Anyway, since the perturbative decay is violated at $g_H > 10^{-4.4}$, resulting the values $\kappa_{H\sigma} < 10^{-4.4}$ and $\kappa_{H\sigma} > 10^{-2.6}$ makes the reheating temperature cannot exceed 10^{10}GeV. Since the $\mathcal{H}$ is truly effective, hence it decays too soon even before the amplitude oscillation of inflaton becomes large, thus lead to inefficient preheating.

In the leptogenesis of the seesaw model, the right-handed neutrino is thermalized [60, 61] (see also [39]). Hence, it depends solely on its Yukawa coupling h_j. In this model, we constraint the coupling with $h_{2,3}$ to be $\sim O(10^{-3})$ from neutrino mass eigenvalues. In addition, the reheating temperature found in the above discussion to be $T_R \geq 10^8$GeV [39]. The decay width of $\Gamma_{N_{2,3}}$ satisfy $\Gamma_{N_{2,3}} > H$ and $T_R > M_{N_{2,3}}$. Also, $N_{2,3}$ are expected to be in thermal equilibrium. This one happens due to the inverse decay at reheating period.

3.6 Leptogenesis

In the ordinary seesaw model, the neutrino mass is generated by Yukawa interaction $h_{\alpha j}\bar{l}_\alpha \eta N_j$. Baryon number asymmetry is assumed to be generated by the same interaction through thermal leptogenesis [60, 61]. If we assumed, the sufficient number of lepton asymmetry is generated in out-of-equilibrium decay of the lightest right-handed-neutrino, which is of course in thermal equilibrium at that time, the reheating temperature T_R must be larger than the mass M_{N_1}. Furthermore, $h_{\alpha 1}$ should not so small to be sufficiently produced. On another side, the neutrino mass formula gives the strict upper bound on $h_{\alpha 1}$ with smaller M_{N_1} under the constraint of neutrino oscillation data. The lower bound for M_{N_1} given by the data realized at 10^9GeV [8]. This mass value does not change even if $T_R \gg 10^9$GeV. The problem arises in the model that both production and the out-of-equilibrium decay of right-handed-neutrino caused by the same Yukawa coupling, as it doesn't change in the original scotogenic model [49, 50]. In that model, by taking the smaller value of $|\lambda_5|$, the smaller right-handed-neutrino can be achieved less than 10^9GeV while keeping the Yukawa coupling larger, this still consistent with neutrino oscillation data. However, one should realize, the washout effect of the generated lepton number due to inverse decay becomes truly effective. As a result, the successful leptogenesis cannot be realized for $M_{N_1} < 10^8$GeV. Thus, it is a notable aspect of this model that it can be changed by the particles which are introduced, this introduction may explain the CP issues.

The production of N_1, the lightest right-handed-neutrino, in the thermal bath, is due

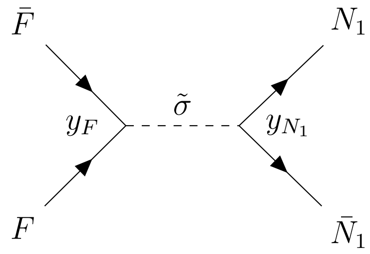

Figure 3.4: The scattering process of $\tilde{D}_L D_R, \tilde{E}_L E_R \to N_1 N_1$ mediated by $\tilde{\sigma}$

to the scattering process of $\tilde{D}_L D_R, \tilde{E}_L E_R \to N_1 N_1$ mediated by $\tilde{\sigma}$ if both fermions are in thermal equilibrium (see Fig. 3.4). The conditions $T > \tilde{M}_F, M_{N_1}$ and $\Gamma_{FF} \simeq H$ must be satisfied, where Γ_{FF} is the reaction rate of the scattering. The estimation of the temperature using relation of $\Gamma_{FF} \simeq H(T)$ gives

$$T \simeq 5.8 \times 10^8 \left(\frac{y_F}{10^{-1.2}}\right)^2 \left(\frac{y_{N_1}}{10^{-2}}\right)^2 \text{GeV}. \tag{3.74}$$

Thus, $T > \tilde{M}_F, M_{N_1}$ if y_F and y_{N_1} satisfied. This feature is not dependent on Yukawa coupling h_1 of N_1. Thus, the smallness of h_1 is allowed and lead to successful leptogenesis and consistent with neutrino oscillation data even with $M_{N_1} < 10^9$GeV.

After N_1 is created by scattering of extra fermions, the decay product $l_\alpha \eta^\dagger$ is expected by suppressed Yukawa coupling $h_{\alpha 1}$. After the washout process which is labeled as frozen out, the decay occurs and so the lepton number asymmetry which is generated can be efficiently converted to baryon number asymmetry via sphaleron processes. We can investigate this case by solving Boltzmann equation for Y_{N_1} and $Y_L (\equiv Y_l - Y_{\bar{l}})$. $Y_\psi = \frac{n_\psi}{s}$, with number density n_ψ and entropy density s. The Boltzmann equation for Y_{N_a} and Y_L are

$$\begin{aligned} \frac{dY_{N_1}}{dz} &= -\frac{z}{sH(M_{N_1})}\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\left[\gamma^{N_1}_D + \left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\sum_{F=D,E}\gamma_F\right] \\ \frac{dY_L}{dz} &= -\frac{z}{sH(M_{N_1})}\left[\epsilon\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\gamma^{N_1}_D - \frac{2Y_L}{Y^{eq}_l}\sum_{j=1,2,3}\left(\frac{\gamma^{N_j}_D}{4} + \gamma_{N_j}\right)\right], \end{aligned} \tag{3.75}$$

where we used $z = \frac{M_{N_1}}{T}$, Y^{eq}_ψ is the equilibrium value of Y with species ψ. The ε represent the CP asymmetry for the decay $N_1 \to l_\alpha \eta^\dagger$ and it is expressed as

$$\begin{aligned}
\varepsilon &= \frac{\Gamma(N_1 \to l\eta^\dagger) - \Gamma(N_1^c \to \bar{l}\eta)}{\Gamma(N_1 \to l\eta^\dagger) + \Gamma(N_1^c \to \bar{l}\eta)} \\
&= \frac{1}{8\pi} \sum_{j=2,3} \frac{\mathrm{Im}\left(\sum_\alpha (\tilde{h}_{\alpha 1}\tilde{h}_{\alpha j})\right)^2}{\sum_\alpha \tilde{h}_{\alpha 1}\tilde{h}_{\alpha 1}} F\left(\frac{M_{N_j}^2}{M_{N_1}^2}\right) \\
&= \frac{1}{8\pi}\left[4|h_2|^2 F\left(\frac{y_{N_2}^2}{y_{N_1}^2}\right)\sin 2(\theta_1 - \theta_2) + |h_2|^2 F\left(\frac{y_{N_3}^2}{y_{N_1}^2}\right)\sin 2(\theta_1 - \theta_3)\right],
\end{aligned} \tag{3.76}$$

where $h_j = |h_j|e^{i\theta_j}$ and $F(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$. We have here $\gamma_D^{N_j}$ which represent the reaction density of $N_j \to l_\alpha \eta^\dagger$, γ_{N_j} represents the reaction density of lepton number violation scattering mediated by N_j, and γ_F represents the reaction density of $\bar{D}_L D_R, \bar{E}_L E_R \to N_1 N_1$. In our case, we assume both extra fermions species in thermal equilibrium and $Y_{N_1} = Y_L = 0$ at $z = z_R(\equiv \frac{M_{N_1}}{T_R})$. From here, we will numerically fix the parameter by solving eq. (3.75) numerically. We will adjust the model with two cases for the VEVs of the Singlet scalars:

$$\text{(I) } w = 10^9\text{GeV},\ u = 10^{11}\text{GeV} \qquad \text{(II) } w = 10^{11}\text{GeV},\ u = 10^{12.5}\text{GeV}, \tag{3.77}$$

where in case (II), Axion can be the dominant component of dark matter (see eq. (3.14)).

In adjusting the parameters, for $\tilde{\kappa}_S$, $\tilde{\kappa}_\sigma$, and $\kappa_{\sigma S}$ which correspond to the inflation are fixed (see Fig. 3.3). One can recall $\mathcal{F}_f\mathcal{F}_f^\dagger > \mu_F^2$ can be formulated as $\delta \equiv \tilde{M}_F/\mu_F > \sqrt{2}$. As for simplicity, if we adjust $y_f = (0, 0, y)$ and $\tilde{y}_f = (0, \tilde{y}, 0)$, we obtain $y^2 + \tilde{y}^2 = (\delta^2 - 1)\frac{w^2}{u^2}y_f^2$. Thus for our purpose, let's fix them by adjust $\delta = \sqrt{3}$, $\tilde{y}/y = 0.5$ and $y_D = y_E = 10^{-1.2}$ at the scale $\bar{M}$. For neutrino mass generation, we can fix

$$\begin{aligned}
&y_{N_2} = 2\times 10^{-2}, \quad y_{N_3} = 4\times 10^{-2}, \quad \text{for (I) and (II) cases} \\
&y_{N_1} = 7\times 10^{-3}, \quad |h_1| = 6\times 10^{-7}, \quad |\tilde{\lambda}_5| = 10^{-3}, \quad M_\eta = 1\text{TeV} \quad \text{for (I)} \\
&y_{N_1} = 10^{-3}, \quad |h_1| = 6\times 10^{-5}, \quad |\tilde{\lambda}_5| = 5\times 10^{-2}, \quad M_\eta = 0.9\text{TeV} \quad \text{for (II)}.
\end{aligned} \tag{3.78}$$

Using these parameters the ϵ for CP asymmetry for N_1 takes the value $O(10^{-6})$ if we assume the CP phase is maximum. The parameters $\tilde{\lambda}_3$ and λ_4 can be constrained for dark matter abundance if we use the neutral component of η and requirements depicted in case (I), thus depicted in Fig. 3.1. Those parameters are not constrained well if Axion is the dominant of dark matter, followed by requirements of (II).

The solution of the Boltzmann equation is depicted in Fig.3.5 with case (I) on the left

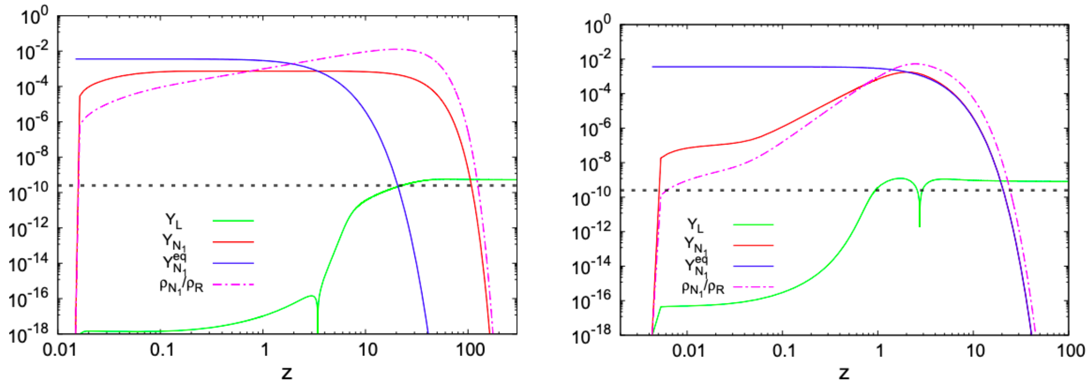

Figure 3.5: Both Figures correspond to the evolution of Y_L and Y_{N_1}. Case (I) is depicted in left-panel with $\kappa_{H\sigma} = 10^{-4}$ and $\kappa_{HS} = \kappa_{\eta S} = 0$ Case (II) is depicted in right-panel for $\kappa_{HS} = \kappa_{\eta S} = 10^{-6}$ while $\kappa_{H\sigma} = \kappa_{\eta\sigma} = 0$. Please note, the other parameters depicted in the the text. The initial condition is simply depicted as $Y_L = Y_{N_1} = 0$ at $z = z_R$ while ρ_{N_1}/ρ_R depicted as ratio of energy density of N_1 and radiation.

side figure and case (II) on the right side. We assume the right-handed neutrino in case (I) to be $M_{N_1} = 7 \times 10^6$GeV and case (II) with $M_{N_1} = 7 \times 10^8$GeV. We can see on the figure, a sufficient amount of baryon number asymmetry is produced, with some remarks on both cases.

For the case (I): we clearly show, Y_{N_1} reaches $Y_{N_1}^{eq}$ by the scattering of the fermions. The lepton number asymmetry found to be $z > 10$ due to the out-of-equilibrium decay[11]. For case (II), the corresponding mass is near the Davidson-Ibarra bound for right-handed neutrino [8] which is substantial for baryon asymmetry in the (original) scotogenic model [49, 50]. In the right figure, the contribution of the inverse decay of N_1 is starting at $z \approx 0.1$ and expected to become effective at[12] $z \sim 10$ until $100 \approx \left(\frac{6.3\times10^{-5}}{h_1}\right)\left(\frac{M_{N_1}}{10^8\text{GeV}}\right)$, as h_1 is mentioned in order of $\sim 10^{-7}$ (see eq. (3.78)).

The case (1) show the suitable parameters in low reheating temperature $T_R < 10^9$GeV to obtain the amount of sufficient baryon number asymmetry by leptogenesis. This is eligible as long as condition $T_R > M_{N_1}$ is satisfied. In our model, both $M_{N_j} = y_{N_J} w$ and $\tilde{M}_F = \delta y_F w$. By adjusting the value of w same as PQ breaking scale and $\delta > 1$, their masses, by the condition of the scattering of the Fermions and right-handed neutrinos, should not under 10^9GeV. This is the remark of this model compared from the original

[11] the N_1 decay's delay is caused by the smallness of h_1, resulting in the ineffectiveness of the lepton number asymmetry

[12] with $z = M_{N_1}/T$, and $T \propto h_1$ for Temperature due to the decay of N_1. The effective z is around $10 - 100$, that's why we obtain that number.

one, as we put into the low scale leptogenesis. The successful leptogenesis realized when $M_{N_1} > 4 \times 10^6 \left(\frac{y_F}{10^{-1.2}}\right)^{-1/2}$GeV for $T_R > 10^8$GeV. As for sufficient N_1 production the requirement of T in (3.74) must satisfy $T > \tilde{M}_F, M_{N_1}$.

3.7 Dark Matter and Isocurvature Fluctuations

In this model, we propose 2 dark matters candidate. The first is the lightest component of η with Z_2 odd parity [49, 50, 62–64]. As we are thrown back at Fig 3.1, the dark matter abundance and its direct search can be preserved if $\tilde{\lambda}_3$ and $|\lambda_4|$ take the suitable values. However, these parameters may affect the perturbativity of the quartic coupling due to radiative corrections, but we can safely stay away from this problem in certain regions. This part is obtained by considering case (I) where η_R is the main component of the dark matter.

The other candidate for dark matter is Axion. This happens if $f_a \sim 10^{11}$GeV[13]. This case is can be inferred to the case (II). The PQ symmetry is spontaneously broken during inflation[14]. The Axion is depicted as the phase θ in $\sigma = \frac{1}{\sqrt{2}}\tilde{\sigma}e^{i\theta}$ and its potential is flat during inflation. The Axion gets the quantum fluctuation $\delta A = \left(\frac{H}{2\pi}\right)^2$ and causes the isocurvature fluctuation and affects the CMB amplitude [66–68].

We consider the canonically normalized Axion, say A, from Einstein frame of eq. (3.33) and (3.34) as

$$\frac{\partial A}{\partial \theta} = \frac{\tilde{\sigma}}{\Omega^2}\sqrt{\Omega^2 + 6\xi_\sigma \frac{\tilde{\sigma}^2}{M_p^2}} \simeq \frac{\sqrt{M_p}}{\tilde{\xi}_S^{1/4}}\frac{|\kappa_{\sigma S}|}{\tilde{\kappa}_\sigma}\sqrt{\phi} = \phi_{iso} \tag{3.79}$$

The Axion causes isocurvature fluctuation due to its weakly interaction with other fields and has its number density–say n_A. The power spectrum can be written by

$$\mathcal{P}_i(k) = \left\langle \left|\frac{\delta n_A}{n_A}\right|^2 \right\rangle = \frac{H_k^2}{\pi^2 \phi_{iso}^2 \langle\theta^2\rangle} \tag{3.80}$$

As we assumed the Axion is the only component of isocurvature fluctuation in the present model. The fraction of the power spectrum is calculated as

$$\alpha_A = \frac{R_a^2 \mathcal{P}_i(k)}{R_a^2\mathcal{P}_i(k) + \mathcal{P}_s(k)} \simeq 8\epsilon\tilde{\xi}_S^{1/2}\frac{M_p}{\phi_k}\frac{R_a^2}{\langle\theta^2\rangle}\left(\frac{\tilde{\kappa}_\sigma}{\kappa_{\sigma S}}\right)^2 \tag{3.81}$$

$P_s(k) = A_s$ is given by eq. (3.44) and R_a is the energy density fraction of Axion in the

[13]even though it depends on the contribution on the string decay [65]
[14]note that inflaton has the component of radial σ

cold dark matter (CDM) ($R_a = \Omega_a/\Omega_{CDM}$). As we taken from [69][15], that the fraction is related by

$$R_a = \frac{\langle\theta\rangle}{6\times 10^{-6}}\left(\frac{f_a}{10^{16}\mathrm{GeV}}\right)^{7/6}, \tag{3.82}$$

thus we simply obtain

$$\alpha = 3.25\times 10^{-5}\tilde{\xi}_S^{1/2} R_a \left(\frac{M_p}{\phi_k}\right)\left(\frac{55}{\mathcal{N}_k}\right)^2\left(\frac{f_a}{10^{10}\mathrm{GeV}}\right)^{7/6}\left(\frac{\tilde{\kappa}_\sigma}{\kappa_{\sigma S}}\right)^2 \tag{3.83}$$

The constraint from Planck data $\alpha \lesssim 0.037$ at $k + 0.05Mpc^{-1}$[2], we can roll back at R_a and get the value

$$R_a < \frac{67}{\tilde{\xi}_S^{1/2}}\left(\frac{\phi_k}{M_p}\right)\left(\frac{\mathcal{N}_k}{55}\right)^2\left(\frac{10^{11}}{w}\right)^{7/6}\left(\frac{\kappa_{\sigma S}}{\tilde{\kappa}_\sigma}\right)^2 \tag{3.84}$$

as we used $f_a = w$. In case (I), the last result doesn't seem to make any constraint correspond to the reheating temperature and baryon number asymmetry is still consistent. The dark matter dominant component is η_R in this case. for case (II), if $|\kappa_{\sigma S}/\tilde{\kappa}_\sigma| = 10^{-1.6}$ we obtain $R_a < 0.21$ and for $|\kappa_{\sigma S}/\tilde{\kappa}_\sigma| = 10^{-2}$ we have $R_a < 0.034$. This constraint resulting from the idea forbids Axion to be the dominant component of dark matter. Corresponding both cases as the conclusion, both scenarios provide the neutral component η as the dominant part of dark matter.

[15] see eq. (67) within mentioned reference

Concluding Remarks

In this proposed model, we introduce the new fermions and some scalars. These introduced fields, perhaps, can solve some problems which we have discussed earlier. In this model, we discussed the origin of CP violation on CKM an PMNS matrices by the mixing of extra fermions and SM quarks or charged leptons. After the symmetry breaking due to the singlet scalars, the scotogenic model is appeared from the leptonic sector. In this way we can explain the smallness of neutrino masses also the abundance of dark matter. We regard the neutral component of η as the dominant part of dark matter. In the scenario we proposed, axion cannot be the dominant component of dark matter. The singlet scalars we proposed to explain the CP issues play the role as inflaton if they couple with gravity non-minimally. We supposed this non-minimal coupling is in order of 1, hence it solves the unitarity problem which seems to be the problem of the conventional Higgs inflation. Also, the chosen non-minimal coupling to be small may evade the quadratic potential and shortened the oscillation era.

The notable feature of this model can be explained. The extra fermions can generate the sufficient baryon number asymmetry via thermal leptogenesis. This scenario is applied well even if the right-handed neutrino mass is below 10^9GeV as this value is considered the lower bound for successful leptogenesis in conventional seesaw model. By using the constraint from isocurvature fluctuation, we argue in both cases (I) and (II), refer to section 3.7, the dominant component of dark matter is mainly composed by neutral component of η. Giving this taken into account, we can predict the relic abundance of dark matter. Luckily it is not contradicting with other predictions that we made for other problems.

Appendix A

Calculation of the selected Equations

A.1 The Conformal Transformation

This calculation is necessary to transform the Action in Jordan Frame to Einstein Frame via Weyl Transformation. So I provide the complete set of calculations[1]. Let us rewrite the Action in Jordan frame depicted in eq. (3.31) as

$$\begin{aligned} S_J &= \int d^4x\sqrt{-g}\bigg[-\frac{1}{2}M_p^2R - \xi_\sigma\sigma^\dagger\sigma R - \tilde{\xi}_S S^\dagger S R + \partial^\mu\sigma^\dagger\partial_\mu\sigma + \partial^\mu S^\dagger\partial_\mu S - V(\sigma,S)\bigg] \\ &= \int d^4x\sqrt{-g}\bigg[-\frac{1}{2}M_p^2\left(1+\xi_\sigma\frac{\tilde{\sigma}^2}{M_p^2}+\tilde{\xi}_S\frac{\tilde{S}^2}{M_p^2}\right)R + \partial^\mu\sigma^\dagger\partial_\mu\sigma + \partial^\mu S^\dagger\partial_\mu S - V(\sigma,S)\bigg] \\ &= \int d^4x\sqrt{-g}\bigg[-\frac{1}{2}M_p^2\Omega^2R + \partial^\mu\sigma^\dagger\partial_\mu\sigma + \partial^\mu S^\dagger\partial_\mu S - V(\sigma,S)\bigg]. \end{aligned} \tag{A.1}$$

One should remember that $S = \frac{1}{\sqrt{2}}\tilde{S}e^{i\rho}$ and $\sigma = \frac{1}{\sqrt{2}}\tilde{\sigma}e^{i\theta}$, the definition of these terms have been discussed in the main contents. $V(\sigma,S)$ is depicted in eq. (3.5). For the Weyl transformation some information are needed, they are

$$\tilde{g}_{\mu\nu E} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1+\xi_\sigma\frac{\tilde{\sigma}^2}{M_p^2}+\tilde{\xi}_S\frac{\tilde{S}^2}{M_p^2}, \tag{A.2}$$

here we have

$$\sqrt{-\tilde{g}} = \sqrt{-\det(\tilde{g}_{\mu\nu})} = \sqrt{-\det(\Omega^2 g_{\mu\nu})} = \sqrt{-\Omega^8\det(g_{\mu\nu})} = \Omega^4\sqrt{-g}. \tag{A.3}$$

[1] I admit, at some parts I just too lazy to write

The Christoffel symbol is written as

$$\begin{aligned}
\Gamma^{\rho}_{\mu\nu} &= \frac{1}{2}g^{\rho\sigma}\{\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}\} \\
&= \frac{1}{2}\Omega^2\tilde{g}^{\rho\sigma}\{\partial_\nu(\Omega^{-2}\tilde{g}_{\sigma\mu}) + \partial_\mu(\Omega^{-2}\tilde{g}_{\sigma\nu}) - \partial_\sigma(\Omega^{-2}\tilde{g}_{\mu\nu})\} \\
&= \frac{1}{2}\Omega^2\tilde{g}^{\rho\sigma}\{-2\Omega^{-3}\tilde{g}_{\sigma\mu}\partial_\nu\Omega - 2\Omega^{-3}\tilde{g}_{\sigma\nu}\partial_\mu\Omega + 2\Omega^{-3}\tilde{g}_{\mu\nu}\partial_\sigma\Omega\} + \tilde{\Gamma}^{\rho}_{\mu\nu} \\
&= \frac{1}{\Omega}\tilde{g}_{\mu\nu}\partial^\rho\Omega - \frac{1}{\Omega}\delta^\rho_\mu\partial_\nu\Omega - \frac{1}{\Omega}\delta^\rho_\nu\partial_\mu\Omega + \tilde{\Gamma}^{\rho}_{\mu\nu},
\end{aligned} \tag{A.4}$$

and the Ricci tensor can be derived from

$$R_{\mu\nu} = \partial_\rho\Gamma^\rho_{\nu\mu} - \partial_\nu\Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\nu\mu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\rho\mu}, \tag{A.5}$$

inserting $\Gamma^\rho_{\mu\nu}$ and obtains (we skipped this step) the Ricci Scalar in 4-dimensions as

$$R = \Omega^2\tilde{R} + \frac{6}{\Omega}\Box\Omega. \tag{A.6}$$

Working out for R we can write

$$\begin{aligned}
R &= \Omega^2\tilde{R} + \frac{6}{\Omega}\Box\Omega = \Omega^2\tilde{R} + \frac{6}{\Omega}g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega = \Omega^2\tilde{R} + \frac{6}{\Omega}\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\Omega] \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}\frac{\Omega^4}{\sqrt{-\tilde{g}}}\partial_\mu\left[\frac{\sqrt{-\tilde{g}}}{\Omega^4}\Omega^2\tilde{g}^{\mu\nu}\partial_\nu\Omega\right] \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}\frac{\Omega^4}{\sqrt{-\tilde{g}}}\partial_\mu\left[\frac{1}{\Omega^2}\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\Omega\right] \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}\frac{\Omega^4}{\sqrt{-\tilde{g}}}\left(\frac{1}{\Omega^2}\right)\partial_\mu\left[\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\Omega\right] + \frac{\Omega^4}{\sqrt{-\tilde{g}}}\left[\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\Omega\right]\partial_\mu\left(\frac{1}{\Omega^2}\right) \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}\frac{\Omega^2}{\sqrt{-\tilde{g}}}\partial_\mu\left[\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\Omega\right] - 2\Omega\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}(\Omega^2\tilde{\Box}\Omega - 2\Omega\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega) \\
&= \Omega^2\tilde{R} + \frac{6}{\Omega}(\Omega^2\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu\Omega - 2\Omega\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega) \\
&= \Omega^2\tilde{R} + 6\Omega\tilde{g}^{\mu\nu}\partial_\mu\partial_\nu\Omega - 12\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega.
\end{aligned} \tag{A.7}$$

We can use integral $\int udv = uv - \int vdu$ and write

$$\int \partial_\mu\partial_\nu\Omega dx^\mu = \partial_\nu\Omega - \int \partial_\mu\Omega\frac{-1}{\Omega}\partial_\nu\Omega dx^\mu, \tag{A.8}$$

the first term can be vanished by adjusting the boundary conditions. In that case we can rewrite R as

$$\begin{aligned} R &= \Omega^2\tilde{R} + 6\Omega\tilde{g}^{\mu\nu}\partial_\mu\partial_\nu\Omega - 12\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega = \Omega^2\tilde{R} + 6\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega - 12\tilde{g}^{\mu\nu}\partial_\nu\Omega\partial_\mu\Omega \\ R &= \Omega^2\tilde{R} - 6\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega. \end{aligned} \tag{A.9}$$

Finally, we should rewrite eq. (A.1), with the supscript of E refer to Einstein frame, using eq. (A.2) and (A.3) also (A.9) for R as

$$\begin{aligned} S_E &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2}M_p^2\Omega^2 R + \partial^\mu\sigma^\dagger\partial_\mu\sigma + \partial^\mu S^\dagger\partial_\mu S - V(\sigma,S)\Big] \\ &= \int d^4x\frac{\sqrt{-\tilde{g}}}{\Omega^4}\Big[-\frac{1}{2}M_p^2\Omega^2\left(\Omega^2\tilde{R} - 6\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega\right) \\ &\qquad + \Omega^2\tilde{g}^{\mu\nu}\partial_\mu\sigma^\dagger\partial_\nu\sigma + \Omega^2\tilde{g}^{\mu\nu}\partial_\mu S^\dagger\partial_\nu S - V(\sigma,S)\Big] \\ &= \int d^4x\sqrt{-\tilde{g}}\Big[-\frac{1}{2}M_p^2\tilde{R} + \frac{3M_p^2}{\Omega^2}\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega \\ &\qquad + \frac{\tilde{g}^{\mu\nu}}{\Omega^2}\partial_\mu\sigma^\dagger\partial_\nu\sigma + \frac{\tilde{g}^{\mu\nu}}{\Omega^2}\partial_\mu S^\dagger\partial_\nu S - \frac{1}{\Omega^4}V(\sigma,S)\Big]. \end{aligned} \tag{A.10}$$

For compactification, this is also useful for n-fields (not only for 2 fields), we can generalize some terms inside eq. (A.10) using $\phi_i = \phi(\tilde{S},\tilde{\sigma},\rho,\theta)$ as

$$\partial_\mu\Omega\partial_\nu\Omega = \frac{1}{4\Omega^4}\partial_\mu\Omega^2\partial_\nu\Omega^2 \to \frac{1}{4\Omega^4}\frac{\partial\Omega^2}{\partial\phi_i}\frac{\partial\Omega^2}{\partial\phi_j}\partial_\mu\phi_i\partial_\nu\phi_j. \tag{A.11}$$

Above components represent the mixing of kinetic term. Here we also write the non-mixed kinetic term by

$$\begin{aligned} \frac{\tilde{g}^{\mu\nu}}{\Omega^2}\partial_\mu\sigma^\dagger\partial_\nu\sigma + \frac{\tilde{g}^{\mu\nu}}{\Omega^2}\partial_\mu S^\dagger\partial_\nu S &= \frac{\tilde{g}^{\mu\nu}}{2\Omega^2}\left(\partial_\mu\tilde{\sigma}\partial_\nu\tilde{\sigma} + \partial_\mu\tilde{S}\partial_\nu\tilde{S} + \tilde{S}^2\partial_\mu\rho\partial_\nu\rho + \tilde{\sigma}^2\partial_\mu\theta\partial_\nu\theta\right) \\ &\equiv \frac{\delta_{ij}}{2\Omega^2}\tilde{g}^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_j. \end{aligned} \tag{A.12}$$

Please note δ_{ij} here is not the conventional delta Kronecker, instead the condition $\delta_{ij} =$ 0still valid for $i \neq j$, there are $\delta_{\tilde{\sigma}\tilde{\sigma}} = 1$, $\delta_{\tilde{S}\tilde{S}} = 1$, , and $\delta_{\theta\theta} = \tilde{\sigma}^2$ and $\delta_{\rho\rho} = \tilde{S}^2$. Substituting eq. (A.11) and (A.12) to (A.10) and finally we obtain

$$\begin{aligned} S_E &= \int d^4x\sqrt{-\tilde{g}}\Big[-\frac{1}{2}M_p^2\tilde{R} + \frac{1}{2}\left(\frac{\delta_{ij}}{\Omega^2} + \frac{3M_p^2}{4\Omega^4}\frac{\partial\Omega^2}{\partial\phi_i}\frac{\partial\Omega^2}{\partial\phi_j}\right)\partial_\mu\phi_i\partial_\nu\phi_j - \frac{1}{\Omega^4}V(\sigma,S)\Big] \\ &= \int d^4x\sqrt{-\tilde{g}}\Big[-\frac{1}{2}M_p^2\tilde{R} + \frac{1}{2}G_{ij}\partial_\mu\phi_i\partial_\nu\phi_j - \frac{1}{\Omega^4}V(\sigma,S)\Big]. \end{aligned} \tag{A.13}$$

The pre-factor G_{ij} can be written in the matrix form via

$$G = \begin{pmatrix} \frac{\Omega^2+6\tilde{\xi}_S^2\tilde{S}^2/M_p^2}{\Omega^4} & 6\frac{\tilde{\xi}_S\xi_\sigma}{M_p^2\Omega^2}\tilde{S}\tilde{\sigma} & 0 & 0 \\ 6\frac{\tilde{\xi}_S\xi_\sigma}{M_p^2\Omega^2}\tilde{S}\tilde{\sigma} & \frac{\Omega^2+6\xi_\sigma^2\tilde{\sigma}^2/M_p^2}{\Omega^4} & 0 & 0 \\ 0 & 0 & \frac{\Omega^2+6\tilde{\xi}_S^2\tilde{S}^2/M_p^2}{\Omega^4}\tilde{S}^2 & 0 \\ 0 & 0 & 0 & \frac{\Omega^2+6\xi_\sigma^2\tilde{\sigma}^2/M_p^2}{\Omega^4}\tilde{\sigma}^2 \end{pmatrix}. \tag{A.14}$$

Focusing on the kinetic term we can write

$$\begin{aligned} \frac{1}{2}G_{ij}\partial_\mu\phi_i\partial_\nu\phi_j =& \frac{1}{2}\tilde{g}^{\mu\nu}\left(\frac{1}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}\partial_\mu\tilde{S}\right)\left(\frac{1}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}\partial_\nu\tilde{S}\right) \\ &+\frac{1}{2}\tilde{g}^{\mu\nu}\left(\frac{1}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}\partial_\mu\tilde{\sigma}\right)\left(\frac{1}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}\partial_\nu\tilde{\sigma}\right) \\ &+\frac{1}{2}\tilde{g}^{\mu\nu}\left(\frac{\tilde{S}}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}\partial_\mu\tilde{\rho}\right)\left(\frac{\tilde{S}}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}\partial_\nu\tilde{\rho}\right) \\ &+\frac{1}{2}\tilde{g}^{\mu\nu}\left(\frac{\tilde{\sigma}}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}\partial_\mu\tilde{\theta}\right)\left(\frac{\tilde{\sigma}}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}\partial_\nu\tilde{\theta}\right) \\ &+6\frac{\tilde{\xi}_S\xi_\sigma}{M_p^2\Omega^2}\tilde{S}\tilde{\sigma}\partial_\mu\tilde{S}\partial_\nu\tilde{\sigma} \\ =&\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi_S\partial_\nu\phi_S+\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi_\sigma\partial_\nu\phi_\sigma+\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi_\rho\partial_\nu\phi_\rho+\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu A\partial_\nu A \\ &+6\frac{\tilde{\xi}_S\xi_\sigma}{M_p^2\Omega^2}\tilde{S}\tilde{\sigma}\partial_\mu\tilde{S}\partial_\nu\tilde{\sigma}. \end{aligned} \tag{A.15}$$

Where we defined

$$\frac{\partial\phi_S}{\partial\tilde{S}} = \frac{1}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}, \qquad \frac{\partial\phi_\sigma}{\partial\tilde{\sigma}} = \frac{1}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}, \tag{A.16}$$

which explains eq. (3.34)[2]. Here we also have redefinition for canonically normalized ϕ_ρ and Axion A as

$$\frac{\partial\phi_\rho}{\partial\rho} = \frac{\tilde{S}}{\Omega^2}\sqrt{\Omega^2+6\tilde{\xi}_S^2\frac{\tilde{S}^2}{M_p^2}}, \qquad \frac{\partial A}{\partial\tilde{\theta}} = \frac{\tilde{\sigma}}{\Omega^2}\sqrt{\Omega^2+6\xi_\sigma^2\frac{\tilde{\sigma}^2}{M_p^2}}, \tag{A.17}$$

where the latter explains eq. (3.79).

[2] also eq. (3.33)

A.2 The Original Scotogenic Model

This part will derive the result of neutrino mass in eq. (3.27) but in its original Scotogenic model of Ma-Model [40]. The Lagrangian of Yukawa terms corresponding this model can be written as

$$-\mathcal{L} = \frac{1}{2}M_i\bar{N}_iN_i^c + \frac{1}{2}M_i^*\bar{N}_i^cN_i - h_{\alpha i}\bar{N}_i\eta^\dagger l_\alpha - h_{\alpha i}^*\tilde{l}_\alpha\eta N_i \tag{A.18}$$

The fields charge assignment under $SU(2)_L \times U(1)_Y \times Z_2$ are

$$\begin{gathered}\begin{pmatrix}\nu_i\\ l_i\end{pmatrix} \sim (2,-1/2,+), \qquad l_i{}^c \sim (1,1,+), \qquad \mathcal{H} = \begin{pmatrix}h^+\\ h\end{pmatrix} \sim (2,1/2,+),\\ N_i \sim (1,0,-), \qquad \eta = \begin{pmatrix}\eta^+\\ \eta_R^0 + i\eta_I^0\end{pmatrix} \sim (2,1/2,-).\end{gathered} \tag{A.19}$$

The potential in the Scotogenic model is[3]

$$\begin{aligned} V =& m_1^2\mathcal{H}^\dagger\mathcal{H} + m_2^2\eta^\dagger\eta + \lambda_1(\mathcal{H}^\dagger\mathcal{H})^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\mathcal{H}^\dagger\mathcal{H})(\eta^\dagger\eta)\\ &+ \lambda_4(\mathcal{H}^\dagger\eta)(\eta^\dagger\mathcal{H}) + \frac{1}{2}\left[\lambda_5(\mathcal{H}^\dagger\eta)^2 + \lambda_5^*(\eta^\dagger\mathcal{H})^2\right].\end{aligned} \tag{A.20}$$

expanding all terms using eq.(A.19) we get

$$\begin{aligned} V =& m_1^2h^+h^- + m_1^2h^2 + m_2^2\eta^-\eta^+ + m_2(\eta_R^0)^2 + m_2(\eta_I^0)^2\\ &+ \lambda_1(h^-h^+)^2 + \lambda_1h^4 + 2\lambda_1(h^-h^+)h^2\\ &+ \lambda_2(\eta^-\eta^+)^2 + \lambda_2(\eta_I^0)^4 + 2\lambda_2(\eta^-\eta^+)\left[h^2 + (\eta_R^0)^2 + (\eta_I^0)^2\right]\\ &+ \lambda_3(h^-h^+)[(\eta^-\eta^+) + (\eta_R^0)^2 + (\eta_I^0)^2] + \lambda_3(h^2)[(\eta^-\eta^+)^2 + (\eta_R^0)^2 + (\eta_I^0)^2]\\ &+ \lambda_4h^-\eta^+\eta^-h^+ + \lambda_4h^2\left[(\eta_R^0)^2 + (\eta_I^0)^2\right] + \lambda_4h^-\eta^+\lambda_4h\left[\eta_R^0 - i\eta_I^0\right]\\ &+ \lambda_4h\eta^-h^+\left[\eta_R^0 + i\eta_I^0\right] + \lambda_5(h^-\eta^+)^2 + \lambda_5h^2\left[(\eta_R^0)^2 - (\eta_I^0)^2\right]\\ &+ \frac{1}{2}\lambda_5h^-\eta^+h\left[\eta_R^0 + i\eta_I^0\right] + \frac{1}{2}\lambda_5h^+\eta^-h\left[\eta_R^0 - i\eta_I^0\right].\end{aligned} \tag{A.21}$$

Finding the masses of m_H, $m_{\eta^\pm}$, m_{η_R}, and m_{η_I}

- (m_H^2); Taking the minimum $\partial V/\partial h = 0$, we obtain $m_1^2 = -2\lambda_1\langle h^2\rangle = -2\lambda_1\nu^2$. The mass of m_H^2 can be obtained by the term with only have h^2 by expanding around

[3] in this part we used m_1 and m_2 to represent m_H and m_η, since in the context we used the latter. However only in this appendix we stick into m_1 and m_2.

minimum, there is

$$\begin{aligned} V_{min} &= m_1^2(h+\nu)^2 + \lambda_1(h+\nu)^2 \\ &= (-\lambda_1\nu^2)(h+\nu)^2 + \lambda_1(h+\nu)^2 \\ &= 4\lambda_1\nu^2h^2 + ...(\text{non } h^2 \text{ terms}) = m_H^2h^2 + ...(\text{non } h^2 \text{ terms}). \end{aligned} \tag{A.22}$$

Thus, we obtain $m_H^2 = 4\lambda_1\nu^2$.

- ($m_{\eta^\pm}$); taking the minimum $\partial V/\partial\eta^+$ we obtain $\lambda_2(\eta^+\eta^-) = -\frac{1}{2}(m_2^2 + \lambda_3\nu^2)$. Thus we can write

$$\begin{aligned} \lambda_2(\eta^+\eta^-)^2 = \lambda_2(\eta^+\eta^-)(\eta^+\eta^-) &= -\frac{1}{2}(m_2^2 + \lambda_3\nu^2)(\eta^+\eta^-) \\ &= -\frac{1}{2}m_{\eta^\pm}^2(\eta^+\eta^-). \end{aligned} \tag{A.23}$$

- (m_{η_R}); taking the minimum $\partial V/\partial\eta_R$ we obtain $\lambda_2(\eta_R^0)^2 = -\frac{1}{2}(m_2^2+(\lambda_3+\lambda_4+\lambda_5)\nu^2)$. Thus we can write

$$\begin{aligned} \lambda_2(\eta_R^0)^4 = \lambda_2(\eta_R^0)^2(\eta_R^0)^2 &= -\frac{1}{2}(m_2^2 + (\lambda_3+\lambda_4+\lambda_5)\nu^2)(\eta_R^0)^2 \\ &= -\frac{1}{2}m_{\eta_R}^2(\eta_R^0)^2. \end{aligned} \tag{A.24}$$

- (m_{η_I}); taking the minimum $\partial V/\partial\eta_I$ we obtain $\lambda_2(\eta_I^0)^2 = -\frac{1}{2}(m_2^2+(\lambda_3+\lambda_4-\lambda_5)\nu^2)$. Thus we can write

$$\begin{aligned} \lambda_2(\eta_I^0)^4 = \lambda_2(\eta_I^0)^2(\eta_I^0)^2 &= -\frac{1}{2}(m_2^2 + (\lambda_3+\lambda_4+\lambda_5)\nu^2)(\eta_I^0)^2 \\ &= -\frac{1}{2}m_{\eta_I}^2(\eta_I^0)^2. \end{aligned} \tag{A.25}$$

We can simplify the results as

$$\begin{aligned} m_H^2 &= 4\lambda_1\nu^2, & m_{\eta^\pm}^2 &= m_2^2 + \lambda_3\nu^2, \\ m_{\eta_R}^2 &= m_2^2 + (\lambda_3+\lambda_4+\lambda_5)\nu^2, & m_{\eta_I}^2 &= m_2^2 + (\lambda_3+\lambda_4-\lambda_5)\nu^2, \end{aligned} \tag{A.26}$$

where ν is the VEV of Higgs boson. The next step is applying Feynman rule on the diagram in Fig. A.1 and calculate the amplitude

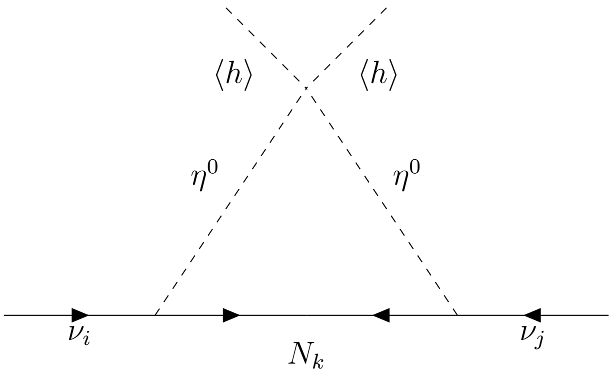

Figure A.1: The neutrino mass generation via one-loop in Scotogenic model

$$\begin{aligned} -i\mathcal{M}_R(p) &= \int \frac{d^4q}{(2\pi)^4}(-ih_{iR})\frac{i(\not{q}+M_R)}{q^2-M_R^2}(-ih_{Rj})\frac{-i}{(p-q)^2-m_{\eta_R}^2} \\ &= \int \frac{d^4q}{(2\pi)^4}h_{iR}h_{Rj}\frac{(\not{q}+M_R)}{(q^2-M_R^2)[(p-q)^2-m_{\eta_R}^2]}, \end{aligned} \tag{A.27}$$

then, using the Feynman parameter (see [70] page 189)[4] Finally, we can write it in the form

$$-\,i\mathcal{M}_R(p) = \int \frac{d^4q}{(2\pi)^4}h_{iR}h_{Rj}\int_0^1 \frac{(\not{q}+M_R)}{\{(q^2-M_R^2)x+(1-x)[(p-q)^2-m_{\eta_R}^2]\}^2}dx \tag{A.28}$$

the denominator reads

$$\begin{aligned} &\int_0^1 \left[(q^2-M_R^2)x+(1-x)((p-q)^2-m_{\eta_R}^2)\right]^{-2}dx = \\ &\int_0^1 \left[q^2-M_R^2x+p-2pq+q^2-p^2x+2pqx-q^2x-m_{\eta_R}^2-m_{\eta_R}^2x\right]^{-2}dx = \\ &\int_0^1 \left[-(M_R^2-m_{\eta_R}^2)x-m_{\eta_R}^2+p^2-2pq+q^2-p^2x+2pqx\right]^{-2}dx = \\ &\int_0^1 \left[-(M_R^2-m_{\eta_R}^2)x-m_{\eta_R}^2+q^2-2pqx+p^2x^2-p^2x^2+4pqx-2pq+p^2-p^2x\right]^{-2}dx = 0, \end{aligned} \tag{A.29}$$

[4] there is $\frac{1}{AB}=\int_0^1 dx\frac{1}{[xA+(1-x)B]^2}$.

we define $\bar{q} = q - px$ and $\Lambda_R^2 = (M_R^2 - m_{\eta R}^2)x + m_{\eta R}^2$, and finally get

$$\int_0^1 \big[- \Lambda_R^2 + \bar{q}^2 - p^2x^2 + 4pqx - 2pq + p^2 - p^2x\big]^{-2} dx. \tag{A.30}$$

The amplitude can be re-written as

$$-i\mathcal{M}_R(p) = \int \frac{d^4\bar{q}}{(2\pi)^4} h_{iR}h_{Rj} \int_0^1 \frac{(\not{q} + M_R)}{\big[- \Lambda_R^2 + \bar{q}^2 - p^2x^2 + 4pqx - 2pq + p^2 - p^2x\big]^2} dx. \tag{A.31}$$

for $p \to 0$ and large M_R we obtain

$$-i\mathcal{M}_R(0) = \int \frac{d^4\bar{q}}{(2\pi)^4} h_{iR}h_{Rj} \int_0^1 \frac{M_R}{\big[\bar{q}^2 - \Lambda_R^2\big]^2} dx. \tag{A.32}$$

We can use the mathematical tools from [70] on page 249:

$$\int \frac{d^d l_E}{(2\pi)^4} \frac{1}{(l_E^2 + \Delta^2)} = \int \frac{d\Omega_d}{(2\pi)^4} \int_0^\infty dl_E \frac{l_E^{d-1}}{(l_E^2 + \Delta^2)}, \qquad \text{where } d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}, \tag{A.33}$$

or by checking [70] in page 193, which $\bar{q} = i\bar{q}_E$, then we found

$$\begin{aligned} \int \frac{d^4\bar{q}}{(2\pi)^4} \frac{1}{(\bar{q}^2 - \Lambda^2)^2} &= i(-1)^{4/2} \int \frac{d\Omega_4}{(2\pi)^4} \int_0^\infty d\bar{q}_E \frac{\bar{q}_E^{4-1}}{(\bar{q}_E^2 + \Lambda_R^2)^2} \\ &= \frac{i}{8\pi^2} \int_0^\infty d\bar{q}_E \frac{\bar{q}_E^3}{(\bar{q}_E^2 + \Lambda_R^2)^2}. \end{aligned} \tag{A.34}$$

Adding the result back to $\mathcal{M}(0)$, or later we rename it by $\mathcal{M}(0)^{(1)}$ (the reason will be clear later) and we have

$$
\begin{aligned}
\mathcal{M}(0)^{(1)} &= \int_0^1 dx \int_0^\infty h_{iR}h_{Rj}\frac{M_R}{8\pi^2}\frac{\bar{q}_E^3}{(\bar{q}_E^2+\Lambda_R^2)^2}d\bar{q}_E \\
&\quad \text{let } u=\bar{q}_E^2+\Lambda_R^2;\ du = 2\bar{q}_E d\bar{q}_E, \text{ hence we can write} \\
&= \int_0^1 dx \int_0^\infty h_{iR}h_{Rj}\frac{M_R}{8\pi^2}\int_0^u \frac{1}{2}\frac{(u-\Lambda_R^2)}{u^2}du \\
&= \int_0^1 dx\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\left(\ln u+\frac{\Lambda_R^2}{u}\right)\Bigg|_0^u \\
&= \int_0^1 dx\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\left(\ln(\bar{q}_E^2+\Lambda_R^2)+\frac{\Lambda_R^2}{(\bar{q}_E^2+\Lambda_R^2)}\right)\Bigg|_0^u \\
&= \int_0^1 dx\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\left(\ln(\infty+\Lambda_R^2)-\ln(0+\Lambda_R^2)+\frac{\Lambda_R^2}{(\infty+\Lambda_R^2)}-\frac{\Lambda_R^2}{(0+\Lambda_R^2)}\right)\Bigg|_0^u \\
&= -\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\int_0^1 dx\left[\ln(\Lambda_R^2)-1\right].
\end{aligned}
\tag{A.35}
$$

Since $\mathcal{M}_R(0)^{(1)}$ is divergence, to make it finite, we introduce $\mathcal{M}_R(0) = \mathcal{M}_R(0)^{(1)} - \mathcal{M}_R(0)^{(2)}$. $\mathcal{M}_R(0)^{(2)}$ is defined same as $\mathcal{M}_R(0)^{(1)}$ but Λ_R is replaced by cut-off Λ, thus

$$
\begin{aligned}
&\mathcal{M}_R(0) = \mathcal{M}_R(0)^{(1)} - \mathcal{M}_R(0)^{(2)} = -\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\int_0^1 dx \ln\left(\frac{\Lambda_R^2}{\Lambda^2}\right) \\
&= -\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\int_0^1 dx\ln\left(\frac{(M_R^2-m_{\eta_R}^2)x+m_{\eta_R}^2}{\Lambda^2}\right) \\
&= -\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\left[\left(x+\frac{m_{\eta_R}^2}{M_R^2-m_{\eta_R}^2}\right)\ln\left[(M_R^2-m_{\eta_R}^2)x+m_{\eta_R}^2\right]-x-\ln(\Lambda^2)\right]\Bigg|_0^1 \\
&= -\frac{M_R}{16\pi^2}h_{iR}h_{Rj}\times \\
&\qquad \left[\left(1+\frac{m_{\eta_R}^2}{M_R^2-m_{\eta_R}^2}\right)\ln(M_R^2)-\left(\frac{m_{\eta_R}^2}{M_R^2-m_{\eta_R}^2}\right)\ln(m_{\eta_R}^2)-1-\ln(\Lambda^2)\right] \\
&= \frac{M_R}{16\pi^2}h_{iR}h_{Rj}\left[1+\left(\frac{m_{\eta_R}^2}{M_R^2-m_{\eta_R}^2}\right)\ln\left(\frac{m_{\eta_R}^2}{M_R^2}\right)+\ln\left(\frac{\Lambda^2}{M_R^2}\right)\right],
\end{aligned}
\tag{A.36}
$$

where an integral identity[5] is used.As $\mathcal{M}_R(0)$ is coming with η_R^0, we also obtain $\mathcal{M}_I(0)$ corresponds field η_I^0. The neutrino mass can be obtained by

$$
\mathcal{M}_{ij}^\nu = \mathcal{M}_R - \mathcal{M}_I, \tag{A.37}
$$

[5]we used $\int \ln(ax+b)dx = \left(x+\frac{b}{a}\right)\ln(ax+b)-x$

then one obtains

$$\mathcal{M}^{\nu}_{ij} = \frac{M_R}{16\pi^2} h_{iR} h_{Rj} \left[\left(\frac{m^2_{\eta_R}}{M^2_R - m^2_{\eta_R}} \right) \ln \left(\frac{m^2_{\eta_R}}{M^2_R} \right) - \left(\frac{m^2_{\eta_I}}{M^2_R - m^2_{\eta_I}} \right) \ln \left(\frac{m^2_{\eta_I}}{M^2_R} \right) \right]. \tag{A.38}$$

In addition, we define $\Delta m^2 = (m^2_{\eta_R} - m^2_{\eta_I})/2$ and $m^2_\eta = (m^2_{\eta_R} + m^2_{\eta_I})/2$ and write

$$\begin{aligned}
\ln \left(\frac{m^2_{\eta_R}}{M^2_R} \right) &= \ln \left(\frac{m^2_\eta + \Delta m^2}{M^2_R} \right) = \ln \left(\frac{m^2_\eta}{M^2_R} \right) + \ln \left(1 + \frac{(\Delta m^2/M^2_R)}{(m^2_\eta/M^2_R)} \right) \\
&= \ln \left(\frac{m^2_\eta}{M^2_R} \right) + \ln \left(1 + \frac{\Delta m^2}{m^2_\eta} \right) \\
&\simeq \ln \left(\frac{m^2_\eta}{M^2_R} \right) + \frac{\Delta m^2}{m^2_\eta} \\
\ln \left(\frac{m^2_{\eta_I}}{M^2_R} \right) &= \ln \left(\frac{m^2_\eta + \Delta m^2}{M^2_R} \right) \simeq \ln \left(\frac{m^2_\eta}{M^2_R} \right) - \frac{\Delta m^2}{m^2_\eta}.
\end{aligned} \tag{A.39}$$

We put back in the eq. (A.38) and obtains

$$\begin{aligned}
\mathcal{M}^{\nu}_{ij} = \sum_{R=1}^{3} \frac{M_R}{16\pi^2} h_{iR} h_{Rj} \Bigg[& \left(\frac{m^2_{\eta_R}}{M^2_R - m^2_{\eta_R}} \right) \left(\ln \left(\frac{m^2_\eta}{M^2_R} \right) + \frac{\Delta m^2}{m^2_\eta} \right) \\
& - \left(\frac{m^2_{\eta_I}}{M^2_R - m^2_{\eta_I}} \right) \left(\ln \left(\frac{m^2_\eta}{M^2_R} \right) - \frac{\Delta m^2}{m^2_\eta} \right) \Bigg].
\end{aligned} \tag{A.40}$$

We then skip some calculation, using approximation $m_{\eta_R} \approx m_{\eta_I} \approx m_\eta$, we obtain

$$\mathcal{M}^{\nu}_{ij} = \frac{M_R}{8\pi^2} h_{iR} h_{Rj} \frac{\Delta m}{m^2_\eta - M^2_R} \left[1 - \frac{M^2_R}{(m^2_\eta - M^2_R)} \ln \left(\frac{m^2_\eta}{M^2_R} \right) \right], \tag{A.41}$$

we substitute $\Delta m = \lambda_5 \nu^2$ and get

$$\mathcal{M}^{\nu}_{ij} = \frac{M_R}{8\pi^2} h_{iR} h_{Rj} \frac{\lambda_5 \nu^2}{m^2_\eta - M^2_R} \left[1 - \frac{M^2_R}{(m^2_\eta - M^2_R)} \ln \left(\frac{m^2_\eta}{M^2_R} \right) \right]. \tag{A.42}$$

If $M_R \gg m_\eta$, we can write

$$\mathcal{M}^{\nu}_{ij} = \sum_{R=1}^{3} h_{iR} h_{Rj} \frac{\lambda_5 \nu^2}{8\pi^2 M_R} \ln \left(\frac{M^2_R}{m^2_\eta} \right). \tag{A.43}$$

A.3 The Simple example of A_f

This part is considered for the matrix example depicted in the CKM matrix. We assume $w = 10^9$GeV and $u = 10^{11}$GeV and also we write

$$h_d = c \begin{pmatrix} \epsilon^4 & \epsilon^3 & p_1\epsilon^3 \\ \epsilon^3 & \epsilon^2 & p_2\epsilon^2 \\ \epsilon^2 & p_3 & -p_3 \end{pmatrix}, \qquad \tilde{y}_d = (0, 0, \tilde{y}) \qquad y_d = (0, y, 0), \tag{A.44}$$

We introduce X_{ij} and Y_{ij} as

$$\begin{aligned} X_{ij} &= 1 + p_i p_j + \left(1 - \frac{1}{\delta^2}\right) \frac{y^2 + \tilde{y}^2 p_i p_j + y\tilde{y}(p_i + p_j)\cos 2\rho}{y^2 + \tilde{y}^2}, \\ Y_{ij} &= \left(1 - \frac{1}{\delta^2}\right) \frac{y\tilde{y}(p_i - p_j)\sin 2\rho}{y^2 + \tilde{y}^2}. \end{aligned} \tag{A.45}$$

where we have defined δ as $\delta = \tilde{M}_F/\mu_F$. We also define

$$R_{ij} = \sqrt{X_{ij}^2 + Y_{ij}^2}, \qquad \tan\theta_{ij} = \frac{Y_{ij}}{X_{ij}}. \tag{A.46}$$

One can recall from eq. (3.20), using assumption $\mu_D < \mathcal{F}_d\mathcal{F}_d^\dagger$, we obtain

$$(A_d^{-1} m^2 A_d)_{ij} = c^2 \langle h \rangle^2 \epsilon_{ij} R_{ij} e^{i\theta_{ij}}, \tag{A.47}$$

where ϵ_{ij} is defined by

$$\epsilon_{13} = \epsilon_{31} = \epsilon^3, \;\; \epsilon_{23} = \epsilon_{32} = \epsilon^2, \;\; \epsilon_{11} = \epsilon^6, \;\; \epsilon_{22} = \epsilon^4, \;\; \epsilon_{33} = 1, \;\; \epsilon_{12} = \epsilon_{21} = \epsilon^5. \tag{A.48}$$

With these, we can solve (A.47) in approximate form as

$$A_d \simeq \begin{pmatrix} 1 & -\lambda & \lambda^3\left(\frac{X_{23}}{|\alpha|^2 X_{33}} e^{i\vartheta} - \frac{X_{13}}{|\alpha|^3 X_{33}}\right) \\ \lambda & 1 & -\lambda^2 \frac{X_{23}}{|\alpha|^2 X_{33}} e^{i\vartheta} \\ \lambda^3 \frac{X_{13}}{|\alpha|^3 X_{33}} & \lambda^2 \frac{X_{23}}{|\alpha|^2 X_{33}} e^{-i\vartheta} & 1 \end{pmatrix}. \tag{A.49}$$

where there are constants mentioned in above relation and defined by

$$\alpha = \frac{X_{13}X_{23}e^{-(\theta_{23}+\theta_{12}-\theta_{13})} - X_{12}X_{33}}{X_{23}^2 - X_{22}X_{33}}, \quad \lambda = |\alpha|\epsilon, \quad \vartheta = \arg(\theta) + \theta_{12} + \theta_{13} + \theta_{23}. \tag{A.50}$$

If diagonalization of O^L turns out to be almost diagonal form for mass matrix of the up-type quarks, The condition $V_{CKM} \simeq A_d$ is fulfilled. For simplify, we assume $\cos\rho = \pi//4$, thus we have the parameters

$$\begin{aligned} &y_F = 10^{-1.5} \quad y = 4\times 10^{-4} \quad \delta = \sqrt{3} \quad \tilde{y} = 2\times 10^{-4} \\ &p_1 = 1.1 \quad \epsilon = 0.2 \quad c = 0.014 \quad p_2 = -0.9 \quad p_3 = 1, \end{aligned} \tag{A.51}$$

hence we obtain $\lambda = 0.22$. The Jarlskog Invariant [71] obtained as $J \equiv \mathrm{Im}[A_{12}A_{13}^{*}A_{23}A_{22}^{*}] = -1.6\times 10^{-6}$. Thus, the mass eigenvalues of the down quarks are

$$\begin{aligned} m_d &= \left[|\alpha|^2\left(X_{22} - \frac{X_{23}^2}{X_{33}} - 2\right) + X_{11} - \frac{X_{13}^2}{X_{33}}\right]^{1/2} \epsilon^3 \langle h\rangle \simeq 3.3\text{MeV}, \\ m_s &= \left[X_{22} - \frac{X_{23}^2}{X_3}\right]^{1/2} \epsilon^2 c \langle h\rangle \simeq 138\text{MeV}, \\ m_b &= X_{33}^{1/2} c \langle h\rangle \simeq 4.2\text{GeV}. \end{aligned} \tag{A.52}$$

A.4 The Renormalization Group Equations (RGEs)

The running coupling constants depicted in this appendix is correspond to the present model in eq. (3.1) and (3.2). For one-loop RGEs of the relevant coupling constants are

$$16\pi^2\mu\frac{\partial\lambda_1}{\partial\mu} = -3\lambda_1(3g^2+g'^2-4h_t^2)+24\lambda_1^2+\lambda_3^2+(\lambda_3+\lambda_4)^4-6h_t^2+\frac{3}{8}(3g^4+g'^4+2g^2g'^2) + \kappa_{H\sigma}^2+\kappa_{HS}^2$$

$$16\pi^2\mu\frac{\partial\lambda_2}{\partial\mu} = -3\lambda_2(3g^2+g'^2)+24\lambda_2^2+4\lambda_2(2h_2^2+3h_3^2)+\lambda_3^2+(\lambda_3+\lambda_4)^2-8h_2^4-18h_3^4 + \kappa_{\eta\sigma}^2+\kappa_{\eta S}^2+\frac{3}{8}(3g^4+g'^4+2g^2g'^2)$$

$$16\pi^2\mu\frac{\partial\lambda_3}{\partial\mu} = \frac{3}{4}(3g^4+g'^4-2g^2g'^2)+2(\lambda_1+\lambda_2)(6\lambda_3+2\lambda_4)+4\lambda_3^2-3\lambda_3(3g^2+g'^2)+2\lambda_4^2$$
$$2\lambda_3(3h_t^2+2h_2^2+3h_3^2)+2\kappa_{H\sigma}\kappa_{\eta\sigma}+2\kappa_{HS}\kappa_{\eta S}$$

$$16\pi^2\mu\frac{\partial\lambda_4}{\partial\mu} = 3g^2g'^2+2\lambda_4(2\lambda_1+2\lambda_2+3h_t^2+2h_2^2+3h_3^2-\frac{9}{2}g^2-\frac{3}{2}g'^2)+8\lambda_3\lambda_4+4\lambda_4^2$$

$$16\pi^2\mu\frac{\partial\kappa_S}{\partial\mu} = 6(2\kappa_S-1)(y_d^2+\tilde{y}_d^2)^2+4\kappa_S(y_e^2+\tilde{y}_e^2)-2(y_e^2+\tilde{y}_e^2)^2+20\kappa_S^2+\kappa_{\sigma S}^2+2(\kappa_{HS}^2+\kappa_{\eta S}^2)$$

$$16\pi^2\mu\frac{\partial\kappa_\sigma}{\partial\mu} = 2(\kappa_{H\sigma}^2+\kappa_{\eta\sigma}^2)+20\kappa_S^2+\kappa_{\sigma S}^2+4\kappa_\sigma\left(3y_D^2+y_E^2+0.5y_{N_3}^2\right)-2\left(3y_D^4+y_E^4+0.5y_{N_3}^4\right)$$

$$16\pi^2\mu\frac{\partial\kappa_{\sigma S}}{\partial\mu} = 2\kappa_{\sigma S}\left(3(y_d^2+\tilde{y}_d^2)+y_e^2+\tilde{y}_e^2+3y_D^2+y_E^2+0.5y_{N_3}^2\right)+4\kappa_{\sigma S}^2 - 4\left(3y_D^2(y_d^2+\tilde{y}_d^2)+y_E^2(y_e^2+\tilde{y}_e^2)\right)+8(\kappa_S+\kappa_\sigma)\kappa_{\sigma S}+2(\kappa_{HS}\kappa_{H\sigma}+\kappa_{\eta S}\kappa_{\eta\sigma})$$

$$16\pi^2\mu\frac{\partial\kappa_{H\sigma}}{\partial\mu} = 2\kappa_{H\sigma}\left(3y_D^2+y_E^2+0.5y_{N_3}^2+2h_2^2+3h_3^2-\frac{9}{4}g^2-\frac{3}{4}g'2\right)+4\kappa_{H\sigma}^2 + 2\kappa_{\sigma S}\kappa_{HS}+2\kappa_{\eta\sigma}(2\lambda_3+\lambda_4)+4\kappa_{H\sigma}(3\lambda_2+2\kappa_\sigma)$$

$$16\pi^2\mu\frac{\partial\kappa_{\eta\sigma}}{\partial\mu} = 2\kappa_{\eta\sigma}\left(3y_D^2+y_E^2+0.5y_{N_3}^2+2h_2^2+3h_3^2-\frac{9}{4}g^2-\frac{3}{4}g'2\right)+\kappa_{\eta\sigma}^2-4(h_2^2+h_3^2)y_{N_3}^2 + 2\kappa_{\sigma S}\kappa_{\eta S}+2\kappa_{H\sigma}(2\lambda_3+\lambda_4)+4\kappa_{\eta\sigma}(3\lambda_2+2\kappa_\sigma)$$

$$16\pi^2\mu\frac{\partial\kappa_{HS}}{\partial\mu} = 2\kappa_{HS}\left(6\lambda_1+4\kappa_S+3(y_d^2+\tilde{y}_d^2)+y_e^2+\tilde{y}_e^2+3h_t^2-\frac{9}{4}g^2-\frac{3}{4}g'^2\right) + 4\kappa_{HS}^2+2\kappa_{\eta S}(2\lambda_3+\lambda_4)+2\kappa_{\sigma S}\kappa_{H\sigma}$$

$$16\pi^2\mu\frac{\partial\kappa_{\eta S}}{\partial\mu} = 2\kappa_{\eta S}\left(6\lambda_2+4\kappa_S+3(y_d^2+\tilde{y}_d^2)+y_e^2+\tilde{y}_e^2+2h_2^2+3h_3^2-\frac{9}{4}g^2-\frac{3}{4}g'^2\right) + 4\kappa_{\eta S}^2+2\kappa_{HS}(2\lambda_3+\lambda_4)+2\kappa_{\sigma S}\kappa_{\eta\sigma}$$

$$
\begin{aligned}
16\pi^2\mu\frac{\partial y_d}{\partial\mu} &= y_d\left(\frac{1}{2}y_D^2 + y_E^2 + \tilde{y}_E^2 - 8g_s^2 - \frac{2}{3}g'^2 + 4y_d^2 + 3\tilde{y}_d^2\right)\\
16\pi^2\mu\frac{\partial \tilde{y}_d}{\partial\mu} &= \tilde{y}_d\left(\frac{1}{2}y_D^2 + y_E^2 + \tilde{y}_E^2 - 8g_s^2 - \frac{2}{3}g'^2 + 3y_d^2 + 4\tilde{y}_d^2\right)\\
16\pi^2\mu\frac{\partial y_e}{\partial\mu} &= y_e\left(3(y_d^2+\tilde{y}_d^2) - 6g'^2 + \frac{1}{2}y_E^2 + 2y_e^2 + \tilde{y}_e^2\right)\\
16\pi^2\mu\frac{\partial \tilde{y}_e}{\partial\mu} &= \tilde{y}_e\left(3(y_d^2+\tilde{y}_d^2) - 6g'^2 + \frac{1}{2}y_E^2 + y_e^2 + 2\tilde{y}_e^2\right)\\
16\pi^2\mu\frac{\partial y_D}{\partial\mu} &= y_D\left(4y_D^2 + y_E^2 - 8g_s^2 - \frac{2}{3}g'^2 + \frac{1}{2}y_{N_3} + \frac{1}{2}y_d^2 + \frac{1}{2}\tilde{y}_d^2\right)\\
16\pi^2\mu\frac{\partial y_E}{\partial\mu} &= y_E\left(2y_E^2 + \frac{1}{2}y_e^2 + \frac{1}{2}\tilde{y}_e - 6g'^2 + 3y_D^2 + \frac{1}{2}y_{N_3}^2\right)\\
16\pi^2\mu\frac{\partial y_{N_3}}{\partial\mu} &= y_{N_3}\left(\frac{3}{2}y_{N_3}^2 + 3y_D^2 + y_E^2 + 2(h_2^2+h_3^2)\right)\\
16\pi^2\mu\frac{\partial h_2}{\partial\mu} &= h_2\left(5h_2^2 + 3h_3^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2\right)\\
16\pi^2\mu\frac{\partial h_3}{\partial\mu} &= h_3\left(2h_2^2 + \frac{15}{2}h_3^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + \frac{1}{2}y_{N_3}^2\right)\\
16\pi^2\mu\frac{\partial h_t}{\partial\mu} &= h_t\left(\frac{9}{2}h_t^2 - 8g_s^2 - \frac{17}{12}g'^2 - \frac{9}{4}g^2\right)\\
16\pi^2\mu\frac{\partial g_s}{\partial\mu} &= -\frac{19}{3}g_s^3 \qquad 16\pi^2\mu\frac{\partial g}{\partial\mu} = -3g^3 \qquad 16\pi^2\mu\frac{\partial g'}{\partial\mu} = \frac{79}{9}g'^3
\end{aligned}
\tag{A.53}
$$

A.5 The Boltzmann Equation

In this Appendix, we will discuss the Boltzmann equation and use the straightforward result. This part only serve for additional improvement to understand the basic theory in the Leptogenesis part in the last chapter. Before we proceed, we should emphasize the particle number conservation is showed by[6]

$$\frac{d}{dt}[a(t)^3 n(t)] = \dot{n} + 3Hn = 0, \tag{A.54}$$

where $n(t)$ is the number density. However, it is obvious, if there is a *leakage*, the right-hand-side can't be zero. Such situation is the true purposed of Boltzmann equation in general. Let us assume that there is (at least) one pair annihilation (or creation) $XX \leftrightarrow f\tilde{f}$ which breaks the equilibrium. Thus, the eq. (A.54) will have additional term

[6]The discussion of this section will mainly come from [12] and [72].

which is cross section multiplied by velocity relative to inertial frame $\sigma_{XX\leftrightarrow f\tilde{f}}\nu$ (or simply σ_{XX}) in the form [72]

$$\dot{n} + 3Hn = -\left\langle \sigma_{XX}\nu \right\rangle (n^2 - n_{eq}^2), \tag{A.55}$$

where n now is correspond to the number density of particle X, of course n_{eq} represents the number density of X during equilibrium. We can rewrite the above relation much further using redefinition $Y = n/T^3$ with T is the temperature as

$$\frac{1}{a^3}\frac{d}{dt}[na^3] = -\left\langle \sigma_{XX}\nu \right\rangle (n^2 - n_{eq}^2) \to T^3\frac{d}{dt}\left[\frac{n}{T^3}\right] = -\left\langle \sigma_{XX}\nu \right\rangle (Y^2 - Y_{eq}^2)T^3$$

and obtain

$$\frac{dY}{dt} = -\left\langle \sigma_{XX}\nu \right\rangle (Y^2 - Y_{eq}^2), \tag{A.56}$$

We can continue by taking redefinition of $x \equiv m/T$, where m is any mass scale, as it is usually taken with the mass of the corresponding particle, say particle–X, thus we take $m = m_X$. During radiation-dominated epoch, x and t are related by [12]:

$$t = 0.301 g_*^{-1/2}\,\frac{\sqrt{8\pi}M_p}{T^2} = 0.301 g_*^{-1/2}\,\frac{\sqrt{8\pi}M_p}{m_X^2}x^2. \tag{A.57}$$

Thus we can simply obtain

$$\frac{dx}{dt} = \frac{g_*^{1/2}}{0.602\sqrt{8\pi}M_p x} = \frac{1.67\, g_*^{1/2}}{x}\frac{m_X^2}{\sqrt{8\pi}M_p} \equiv \frac{H(m_X)}{x}, \tag{A.58}$$

With this equipment, we can substitute the last line of eq. (A.56) for t, we have

$$\begin{aligned}
\frac{H(m_X)}{x}\frac{dY}{dx} &= -T^3\left\langle \sigma_{XX}\nu \right\rangle (Y^2 - Y_{eq}^2) \\
\frac{dY}{dx} &= -\frac{x}{H(m_X)}T^3\left\langle \sigma_{XX}\nu \right\rangle (Y^2 - Y_{eq}^2) \\
\frac{dY}{dx} &= -\frac{x}{H(m_X)}\left(\frac{m_X^3}{x^3}\right)\left\langle \sigma_{XX}\nu \right\rangle (Y^2 - Y_{eq}^2) \\
\frac{dY}{dx} &= -\frac{\lambda_x}{x^2}(Y^2 - Y_{eq}^2) \qquad \text{as} \qquad \lambda_x \equiv \frac{m_X^3}{H^2(m_X)}\left\langle \sigma_{XX}\nu \right\rangle .
\end{aligned} \tag{A.59}$$

In order to solve analytically, we can impose the approximation as: the equilibrium condition drops as exponential rate[7] e^{-x}, when $n(t) \propto Y(x)$ drops much slower ($Y \gg Y_{eq}$),

[7] See ref. [72] eq. (3.24)

in this case, we can approximate the last result as

$$\frac{dY}{dx} = -\frac{\lambda_x}{x^2} Y^2. \tag{A.60}$$

Furthermore, we assume that λ_x is independent of x, which means g_* doesn't change so much. In addition, eq. (A.60) becomes

$$\frac{dY}{Y^2} = -\frac{\lambda_x}{x^2} dx, \tag{A.61}$$

the solution of the above equation is simply written as

$$\frac{1}{Y_\infty} - \frac{1}{Y_{dec}} = \lambda_x \left(\frac{1}{x_{dec}} - \frac{1}{x_\infty} \right). \tag{A.62}$$

Before we proceed, we should understand that Y_{dec} correspond to the condition during freeze out boundary condition[8] x_{dec}, which is around $x_{dec} \approx 10$, or to be more precise $x_{dec} \approx 20$ or 35 [73], hence the value is $Y_\infty \ll Y_{dec}$. In another side, $x_\infty = m_X/T_\infty \approx \infty$ due to the smallness of T_∞. Hence, we can get approximate solution as

$$Y_\infty \simeq \frac{x_{dec}}{\lambda_x}, \tag{A.63}$$

which we need to be skipped for later use.

The energy density of the universe is treated by

$$\begin{aligned} \rho_u = \bar{m} n = m_X \left(\frac{a_{dec}}{a_0} \right) n = m_X \left(\frac{a_{dec}}{a_0} \right) Y_\infty T^3 = m_X \left(\frac{a_{dec}}{a_0} \right) Y_\infty T_0^3 \left(\frac{T_{dec}^3}{T_0^3} \right) \\ = m_X Y_\infty T_0^3 \left(\frac{a_{dec} T_{dec}^3}{a_0 T_0^3} \right). \end{aligned} \tag{A.64}$$

One can relate[9]

$$\left(\frac{a_{dec} T_{dec}^3}{a_0 T_0^3} \right) \propto \frac{\sum g_{*(\text{photons+neutrinos})}}{\sum g_{*(\text{quarks+leptons+photons+gluons and all their antimatters})}}. \tag{A.65}$$

[8]Actually this is model dependent
[9]this relation, came from the entropy relation $s = \frac{2\pi}{45} g_* T^3$ and $sa^3 = \text{const}$

Solving the degrees of freedom g_* in both numerator and denominator requires [74]

$$\begin{aligned} &g_*(\text{photons}) = 2 && g_*(\text{gluons}) = 8 \times 2 = 16 \\ &g_*(\text{quarks}) = 5 \times 3 \times 2 = 30 && g_*(\text{antiquarks}) = 30 \\ &g_*(\text{leptons}) = 6 \times 2 = 12 && g_*(\text{antileptons}) = 12 \\ &g_*(\text{neutrinos}) = 3 \times 2 \times \left(\frac{4}{11}\right)^{\frac{4}{3}} \times \frac{7}{8} = 1.36. \end{aligned} \tag{A.66}$$

With the last result, we can simply obtain

$$\left(\frac{a_{dec} T_{dec}^3}{a_0 T_0^3}\right) \approx \frac{3.36}{91.5} \approx \frac{1}{27} \approx \frac{1}{30}. \tag{A.67}$$

The last approximation is widely used so we will stick in that value[10]. By this, we can proceed with the eq. (A.64) to be

$$\rho_u = \frac{m_X Y_\infty T_0^3}{30}. \tag{A.68}$$

With this, we can calculate the *density parameter* Ω_X as

$$\begin{aligned} \Omega_X h^2 &= \frac{\rho_u}{\rho_{crit}} h^2 = \frac{m_X Y_\infty T_0^3}{30} \frac{h^2}{\rho_{crit}} = \frac{m_X x_{dec} T_0^3}{\lambda_x 30} \frac{h^2}{\rho_{crit}} = \frac{H(m_X) x_{dec} T_0^3}{30 m_X^2 \langle \sigma_{XX} \nu \rangle} \frac{h^2}{\rho_{crit}} \\ &= \frac{1}{30 m_X^2} \left[\frac{1.67\, g_*^{1/2} m_X^2}{\sqrt{8\pi} M_p}\right] \frac{x_{dec}}{\langle \sigma_{XX} \nu \rangle} \frac{T_0^3 h^2}{\rho_{crit}} \\ &= \frac{1}{30} \left[\frac{1.67\, g_*^{1/2}}{\sqrt{8\pi} M_p}\right] \frac{x_{dec}}{\langle \sigma_{XX} \nu \rangle} \frac{T_0^3}{3 M_p^2} \frac{h^2}{H_0^2}. \end{aligned} \tag{A.69}$$

Solving above result requires some numbers. Let's take $g_* = 100$, T_0 as today temperature to be 2.4×10^{-4}eV, $x_{dec} = 20$, $H_0/h = 2.131 \times 10^{-42}$ [73], whereas the reduced Planck Mass $M_p = 2.435 \times 10^{18}$GeV. with these, we can simplify the result as

$$\Omega_X h^2 = \frac{1.554 \times 10^{-10}}{\langle \sigma_{XX} \nu \rangle} \text{GeV}^{-2}. \tag{A.70}$$

We should necessarily take the $\langle \sigma_{XX} \nu \rangle$ in the form

$$\langle \sigma_{XX} \nu \rangle = \sigma_{XX} \nu + \mathcal{O}(\nu^2) + ... \tag{A.71}$$

[10]actually, we already use it in eq. (1.47) and (1.48)

Here, we can approximate our result in the s-wave form which corresponds to the first order in eq. (A.71) [73].

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