Cross Hedging Using Prediction Error Weather Derivatives for Loss of Solar Output Prediction Errors in Electricity Market

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Abstract Predicting future solar conditions is important for electricity industries with solar power generators to quote a day-ahead sales contract in the electricity market. If a prediction error exists, the market-monitoring agent has to prepare another power generation resource to immediately compensate for the shortage, resulting in an additional cost. In this context, a penalty may be required depending on the size of the prediction error, which may lead to a significant loss for solar power producers. Because the main source of such losses is from prediction errors of solar conditions, they can instead effectively utilize a derivative contract based on solar prediction errors. The objective of this work is to provide such a derivative contract, namely, a prediction error weather derivative.

First, defining a certain loss function, we measure the hedge effect of the derivative on solar radiation prediction error, thereby verifying that the existing hedging method for wind power can also be applied to solar power generation with periodic trends. By introducing the temperature derivative on the absolute prediction error, we also propose a cross-hedging method, where we demonstrate not only a further variance reduction effect when used with solar radiation derivatives, but also a certain hedge effect obtained even when only the temperature derivative is used. For temperature derivative pricing and optimal contract volume estimation, we propose a method using a tensor-product spline function that simultaneously incorporates the smoothing conditions of both the direction of intraday time trend and seasonal trend, and consequently verify its effectiveness.

Keywords: Cross hedge・Non-parametric regression・Minimum variance hedge・Prediction errors・Solar power energy・Weather derivatives

1 Introduction

Predicting future solar conditions is important for electricity industries with solar power generators to quote a next-day sales contract (i.e., day-ahead sales contract) in the electricity market. If a prediction error exists, the market-monitoring agent has to prepare another power generation resource to immediately compensate for the shortage, resulting in an additional cost. In this context, a penalty may be required depending on the size of the prediction error, which may lead to a significant loss for solar power generation industries. Because the main source of such losses is from prediction errors of solar conditions (Ministry of Economy, Trade and Industry 2017), they can instead effectively utilize an insurance contract (or a derivative contract) based on solar prediction errors to hedge against loss caused by prediction errors of solar power output. The objective of this work is to provide such a derivative contract, namely, a prediction error weather derivative.

In this study, we propose multiple hedging methods for prediction error losses in solar power generation using derivatives of weather prediction errors, and then measure the hedge effect under a certain loss function. Weather derivatives are widely used mainly for methods using the temperature index (PricewaterhouseCoopers 2011), and previous studies applied weather derivatives to the electricity industry as follows. Yamada et al. (2006) proposes a pricing method using trend prediction for futures and option contracts based on the monthly average temperature. Yamada (2008a and 2008b) propose weather derivatives for effectively hedging the loss risk of power prediction errors for wind power. Yamada (2018) propose a hedging method using temperature prediction error derivatives against the loss associated with the electricity retailers' imbalance. Another related study, Bhattacharya et al. (2015), verified the cross-hedging effect for a solar power producer in the U.S. when using standard temperature derivatives. This study proposes a derivative to hedge the loss due to solar power prediction errors and relies on the design method proposed in Yamada (2008a, 2008b, and 2018). Specifically, we not only design the solar radiation derivatives on prediction error and measure its hedge effect, but also introduce the temperature deliveries on absolute prediction error and validate the cross-hedging effect when using them together. To price temperature derivatives and construct the hedging model, we propose a method using a tensor product spline function that simultaneously incorporates smoothing conditions for both, the direction of time and seasonal trend, to ensure the robustness of estimation while using small sample size data.

This study is structured as follows. Sect. 2 introduces the minimum variance hedging method using prediction errors, especially focusing on the tensor product spline function. Sect. 3 outlines the data we work with in this study and proposes the prediction method for hourly temperature (pricing method for the temperature derivative). In Sect. 4, we construct several hedging models using derivatives of solar radiation and temperature, measure the hedge effects, and add a consideration. Sect. 5 summarizes this study.

2 Minimum Variance Hedging Problem Using Prediction Errors

As previously mentioned, the prediction errors of solar power outputs lead to losses in the form of a penalty for electricity industries with solar power generators. In this section, we introduce hedging methods for such losses using derivatives on weather prediction errors. Here, using a simple market model shown in Fig.1, we explain the related transactions, intended for a solar power producer who sells all the produced power on the spot market (i.e., day-ahead market where next day's power output is exchanged).

First, the power producer prepares weather derivatives contracts with an insurance company in advance (and pays a premium on the derivatives if needed). Then, as part of daily operations, the power producer predicts next-day solar power outputs (the output prediction at time n is denoted as $\widehat{P_n}$) and sells them at spot price S_n; hence, it gets profit $S_n\widehat{P_n}$ on the spot market. Subsequently, on the delivery day, if the realized output P_n deviates from the prediction $\widehat{P_n}$ (i.e., sales contract volume on the spot market), the deviated output, which is corresponding to the output prediction error $\varepsilon_{P,n} := P_n - \widehat{P_n}$, is procured (sold) by the system operator under the supervision of the monitoring agent and settled by the imbalance price I_n that is determined by adding the penalty unit price (corresponding to the prediction error loss) to the spot-market-based price. Concurrently, the payoffs of the contracted derivatives are also determined as the function of weather prediction errors. The aim of hedging method proposed in this study is to minimize the variance of a portfolio comprising the solar output prediction error loss and the derivatives payoffs[1].

2.1 Definition of Loss and Payoff Functions of Derivatives

Loss function

Here, we will outline the prediction error loss dealt with in this study. For simplicity, we define the imbalance price I_n at time n as the price obtained by adding the penalty term δ (k in the case of shortage, or $-l$ in the case of surplus, both of which are time constant values) to the spot price S_n as follows[2]:

$$I_n = S_n + \delta \quad \text{s.t.} \quad \delta = \begin{cases} k & \text{if } \varepsilon_{P,n} < 0 \\ -l & \text{if } \varepsilon_{P,n} > 0 \end{cases}, \ k \text{ and } l > 0. \tag{1}$$

[1] Note that under Japan's current institution, most of the renewable power producers are not subject to the application of imbalance price, since all the output is purchased at a fixed price determined in advance under the Feed-In Tariff Law (Instead, imbalance risk is borne by retailers or system operators). In the future, however, it is assumed that the scheme will be shifted to a system where renewable power producer themselves make predictions and bear the imbalance risk.

[2] In Japan, this penalty term is supposed to be included in the imbalance price applied from April 2019.

Here, the power producer's total profit π_{total}, which is comprising the profit π_S due to the spot market transaction and the profit (loss) π_I by the imbalance price settlement, is obtained from the following equation:

$$\pi_{total} = \pi_S + \pi_I = S_n\widehat{P_n} + I_n\left(P_n - \widehat{P_n}\right) = S_nP_n + \delta\varepsilon_{P,n}. \tag{2}$$

Since the first term S_nP_n of the right side in equation (2) means the profit when the prediction was accurate and the total amounts of realized output were sold out on the spot market (at the spot price), the flipped value of the second term $-\delta\varepsilon_{P,n}$ ($-k\varepsilon_{P,n}$ in the case of shortage, or $l\varepsilon_{P,n}$ in the case of surplus, both of which are positive values) corresponds to the opportunity loss compared to the case where no prediction error occurs. In this study, we refer to such opportunity loss as prediction error loss. This loss function ($-\delta\varepsilon_{P,n}$) is described in Fig.2.

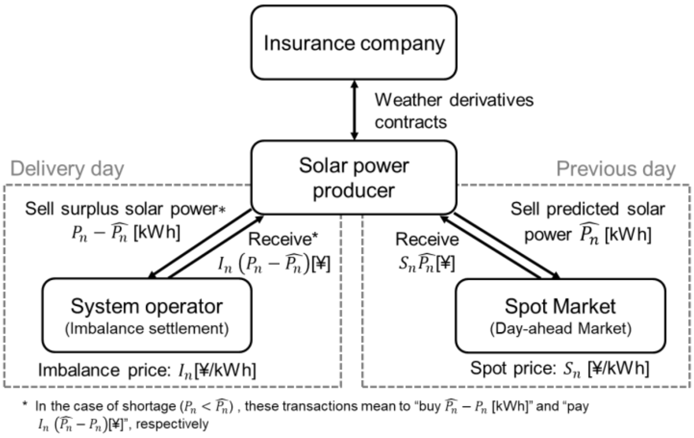

Fig.1 Simple market model for a solar power producer

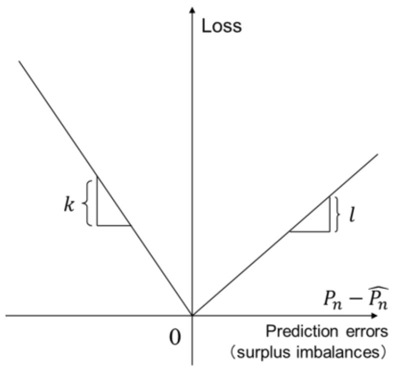

Fig.2 An example of loss function

Here, we define a loss function assuming that the penalties have the same unit price for both the shortage and surplus; that is, we define the loss function proportional to the absolute value of the output prediction error $\varepsilon_{P,n}$ at the time $n(=1,\dots,N)$ as follows:

$$L_n := c\left|\varepsilon_{P,n}\right|. \tag{3}$$

Radiation derivatives on prediction error

Next, we consider the radiation derivatives based on prediction error. This payoff function is denoted by $\psi(\varepsilon_{R,n})$, as a function of solar radiation prediction error $\varepsilon_{R,n} := R_n - \widehat{R_n}$ (the actual solar radiation minus its prediction)[3]. In this study, this derivative is used for solving the optimal payoff function.

Temperature derivatives on absolute prediction error

Then, we introduce the temperature derivatives on absolute prediction error. The payoff of the derivative is defined as $\left|\varepsilon_{T,n}\right|$ (i.e., the absolute value of the temperature prediction error $\varepsilon_{T,n} := T_n - \widehat{T_n}$, the actual temperature minus its prediction). In this study, the temperature derivative is used for solving the optimal contract volume.

2.2 Minimum Variance Hedging Using Smoothing Spline

Estimation of the smoothing spline function

In this study, we use the generalized additive model (GAM; see e.g., Hastie and Tibshirani 1990) for estimating the

[3] The solar radiation derivative is assumed to be contracted without transaction costs, that is, it satisfies the equation $\mathrm{Mean}\left[\psi(\varepsilon_{R,n})\right] = 0$. Note that we assume the same condition for the solar radiation derivative on absolute prediction errors $\psi(|\varepsilon_{R,n}|)$, which will be introduced later.

payoff function or the optimal contract volume of prediction error derivatives, in line with the ideas proposed in Yamada (2008b and 2018). A univariate smoothing spline function is estimated as a function $h(\cdot)$ that minimizes a penalized residual sum of squares (PRSS) as follows (y_n and x_n are the dependent and independent variables):

$$\mathrm{PRSS} = \sum_{n=1}^{N} \{y_n - h(x_n)\}^2 + \lambda \int \{h''(x)\}^2 \, dx. \tag{4}$$

In equation (4), the first term measures closeness to the data while the second term penalizes curvature in the function (penalty term). Estimation of the spline function using GAM corresponds to solving the minimum variance problem under smoothing constraints (Yamada 2008b)[4].

Minimum variance hedging using solar radiation derivatives on prediction error

The method of estimating the univariate smoothing spline described above is applied to the calculation of how to solve the optimal payoff function of solar radiation derivatives. That is, the minimum variance hedging problem to solve the optimal payoff function of solar radiation derivatives is given as follows:

$$\min_{\psi(\cdot)} \mathrm{Var}\left[L_n - \psi(\varepsilon_{R,n})\right] \text{ s.t. } \mathrm{Mean}\left[\psi(\varepsilon_{R,n})\right] = 0. \tag{5}$$

This corresponds to solving the problem of minimizing $\mathrm{Var}\left[L_n - \psi(\varepsilon_{R,n})\right]$, the variance of the portfolio constituted by loss L_n and optimal derivatives payoff $\psi^*(\varepsilon_{R,n})$ under the smoothing conditions.

Note that since this derivative is designed to make its average payoff equal to 0, the buyer does not have to pay premium at the contracting time, as long as it is a fair derivative.

2.3 Minimum Variance Hedging Using a Bivariate Spline Function (Tensor-Product Spline)

Estimation of the tensor product spline function

In this study, we estimate the tensor-product spline function for the pricing of temperature derivatives and calculating its optimal contract volume. Tensor product spline is a type of multivariate spline function whose basis functions are given as tensor products, and in the PRSS to be minimized, penalty terms are added for each explanatory variable. For example, the penalty term $J_{te}(h)$ of a bivariate cubic tensor product spline is given by the following equation (Wood 2017):

$$J_{te}(h) = \int_{x,z} \lambda_x \left(\frac{\partial^2 h}{\partial x^2}\right)^2 + \lambda_z \left(\frac{\partial^2 h}{\partial z^2}\right)^2 dxdz. \tag{6}$$

A typical multivariate spline function other than tensor product spline is a thin plate spline function. An example of a penalty term $J_{tp}(h)$ of a bivariate thin plate spline function is shown in the following equation (Wood 2017):

$$J_{tp}(h) = \lambda \int_{x,z} \left(\frac{\partial^2 h}{\partial x^2}\right)^2 + \left(\frac{\partial^2 h}{\partial x \partial z}\right)^2 + \left(\frac{\partial^2 h}{\partial z^2}\right)^2 dxdz. \tag{7}$$

Comparing equations (6) and (7), it is found that the term related to $\partial^2 h / \partial x \partial z$ is added to the penalty term of the thin plate spline. As can be observed, "thin plate spline" is called so because it is similar to the bent shape of a thin elastic plate, having a feature that the smoothing penalties are isotopically incorporated.[5]

We adopt the tensor- product spline considering that it is appropriate for the time series data handled in this study to be given both the smoothing conditions of time and seasonal trend independently.

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[4] In this study, we construct GAM using the function gam () in the R 3.5.1 package "mgcv" (https://cran.r-project.org), where gam () adopts general cross-validation criterion to calculate the smoothing parameter λ (Wood 2017).

[5] For example, Wood (2013) modeled the density of mackerel eggs in the ocean using a thin plate spline with latitude and longitude as explanatory variables, which can be said to be an intuitive case using the isotropic nature of the function.

Minimum variance cross-hedging using temperature derivatives on absolute prediction error

The method of estimating the tensor-product spline function is applied to the problem of calculating the optimal contract volume of temperature derivatives on prediction error. We work with a different model compared to solar radiation for the following reason. Since the solar power output prediction error is assumed to have an almost constant correlation (for any time) with the solar radiation prediction error, hedging by univariate derivatives such as in equation (5) is effective; on the other hand, as the temperature prediction error, has different sensitivities depending on the time (e.g., at the time in which solar radiation is originally small, such as early mornings or late evenings, the output error's sensitivity to the temperature error is significantly small), the same hedging method as solar radiation is not effective. Therefore, for hedging using temperature derivatives, we apply a method to change the contract volume depending on time. In other words, this hedging model is a problem of estimating the time dependent sensitivity of the absolute temperature prediction error with respect to the loss function (absolute output prediction error). To estimate this sensitivity trend, we use the tensor product spline function. Specifically, we solve the following minimum variance hedging problem:

$$\min_{\Delta(\cdot)} \mathrm{Var}\left[L_n - \Delta(Seasonal_t, m)\left|\varepsilon_{T,n}\right|\right] \tag{8}$$

where t and m represent the date and hour corresponding to the time n, $\Delta(\cdot)$ represents the contract volume of the derivative, and $Seasonal_t$ is the periodicity dummy variable $(= 1, \dots, 365 (or\ 366))$[6]. Equation (8) minimizes the variance of the portfolio consisting of losses L_n and $\Delta(\cdot)$ units of temperature derivative whose payoff is $\left|\varepsilon_{T,n}\right|$.

The average payoff of this derivative, $\mathrm{Mean}\left(\left|\varepsilon_{T,n}\right|\right)$, clearly does not equal to 0; hence, unlike solar radiation derivatives introduced in the previous section, the temperature derivative requires buyers to pay a premium at the contracting time (time 0). Note that the premium can be calculated as the present discounted value of unconditional expected value of the payoff $\mathrm{E}\left(\left|\varepsilon_{T,n}\right|\right)$.

3 Preliminary

3.1 Data Description

To design the prediction error derivatives proposed in this study, actual measured values of solar power output, solar radiation, and temperature, and their respective predicted values on the previous day are required. Since each predicted value is hard to acquire, we obtain it to separately construct the prediction model by using the next-day weather forecast of the past, which is announced by the Japan Meteorological Agency (JMA) every day. Specific data to be used and its sources are shown below (all of the data pertains to Hiroshima city):

(a) Solar power output P_n: Measured value of household's solar power system[7]

(b) Solar radiation R_n, temperature T_n, and weather condition W_n: Measured value by JMA[8]

(c) Prediction of weather F_n, daily maximum temperature $Tmax_t$, and minimum temperature $Tmin_t$: Weather forecast announced by the JMA on the previous morning[9]

(d) Prediction of solar power output $\widehat{P_n}$: Calculated value by prediction model using P_n, F_n, and W_n

(e) Prediction of solar radiation $\widehat{R_n}$: Calculated value by prediction model using R_n, F_n, and W_n

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6 For allocating periodic dummy variables, we use the method proposed in Yamada (2015) (the same applies hereafter).

7 With the permission of the owner, we use the data of the private roof-mounted power system in Hiroshima city.

8 Downloaded from https://www.data.jma.go.jp/gmd/risk/obsdl.

9 Downloaded from http://weather-transition.gger.jp.

(f) Prediction of temperature $\widehat{T_n}$: Calculated value by prediction model using T_n, $Tmax_t$, and $Tmin_t$

Among the above, (a)–(c) are historical data, and (d)–(f) are values calculated by separately constructed prediction models. Of these, (d) and (e) are obtained using the prediction method (calculated value) proposed in Matsumoto and Yamada (2018), and the prediction model of (f) is explained in the next section.

We now pay attention to the relationship between R_n and P_n. Fig.3 shows this in a scatter diagram, and Fig.4 shows a spline regression fitted to it[10]. As shown in Fig.4, the solar radiation and solar power output have the following tendencies: the power output increases proportionally until the solar radiation reaches around 2.0 $[\mathrm{m}^2/\mathrm{J}]$, but beyond that, the slope of the power output decreases with respect to the solar radiation. Originally, the solar power system has technical characteristics such that the power-generating efficiency decreases due to the rise in temperature of the power generation module (Yukawa et al. 1996); hence, this characteristic seems to be reflected.

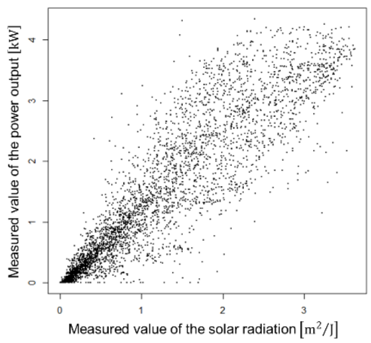

Fig.3 Solar radiation R_n vs. power output P_n

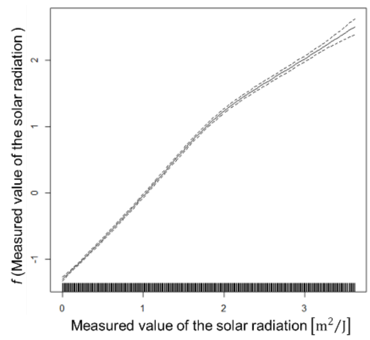

Fig.4 Spline regression curve for Fig.3

3.2 Prediction Model of Hourly Temperature (Pricing Method for Temperature Derivatives)

In this section, we build a model that obtains hourly predicted values of temperature using the daily maximum or minimum temperature as announced by the JMA on the previous morning and the hourly actual temperature. Note that this is also interpreted as a pricing method for temperature derivatives on the absolute prediction error defined in Sec. 2.1, as it determines the hourly payoffs for the derivative. [11]

Here, we build the prediction model by applying GAM using the tensor product spline regression to the hourly measured temperature T_n (which is expressed as $T_t{}^{(m)}$ at hour m on date t) as follows:

$$T_t{}^{(m)} = \gamma(Seasonal_t, m) + s_1(Temp\_max_t) + s_2(Temp\_min_t) + \varepsilon_{T,t}{}^{(m)} \qquad (9)$$

where $\gamma(\cdot)$ is a tensor product spline function, and $s_1(\cdot)$ or $s_2(\cdot)$ is a univariate spline function.

Each spline function estimated by the above equation is shown in Fig.5[12]. The estimated value of the term $\gamma(Seasonal_t, m)$, which refers to the time series trend of the temperature, has two directions of smooth trends such as the intraday time trend (e.g., low in the morning and high in the early afternoon) and the seasonal trend of the same hour (e.g., low in the winter and high in the summer), and it is confirmed that these trends are effectively extracted. As in this case, even when the sample size of available data is small, the tensor-product spline allows for a relatively robust trend estimation by incorporating the smoothing conditions in two different directions.

The spline functions related to the predicted values of the maximum and minimum temperatures are confirmed to be monotonically increasing functions.

[10] We use the data from June 1, 2016 to May 31, 2017.

[11] In the weather forecast publicly announced by the JMA, we cannot obtain the hourly predicted temperature values for the next day, so we newly introduce this pricing method.

[12] We use the data from June 1, 2016 to May 31, 2017, during which it was possible to obtain past weather forecasts.

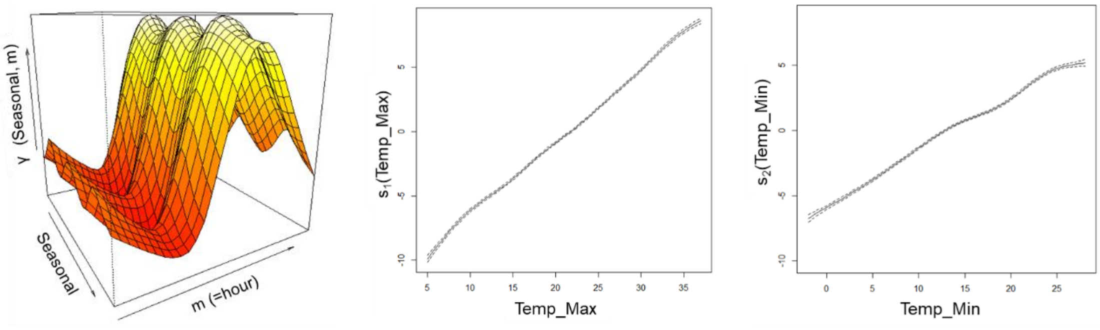

Fig.5 Estimation results for each term in the temperature prediction model

4 Construction of Hedging Model of Prediction Error Loss and Empirical Analysis

In this section, we construct various hedging models using derivatives on the prediction error of solar radiation and temperature against the prediction error loss of solar power output, and verify their hedge effects. We construct hedging models using the prediction errors of solar power output, solar radiation, and temperature ($\varepsilon_{P,n}$, $\varepsilon_{R,n}$ and $\varepsilon_{T,n}$) during the period from June 1, 2016 to May 31, 2017. We also measure the hedge effect of models and add a consideration. Upon measurement, the variance reduction rate (VRR) is determined by the following equation as in Yamada (2008a, 2008b, and 2018), and 1-VRR is hereinafter referred to as the hedge effect.

$$\mathrm{VRR} := \frac{\mathrm{Var[Portfolio\ after\ hedging]}}{\mathrm{Var[Prediction\ error\ loss]}} = \frac{\mathrm{Var}(\varepsilon_n)}{\mathrm{Var}(L_n)} \tag{10}$$

where ε_n indicates the residual term of each hedging model defined in the following sections.

4.1 Hedging Using Solar Radiation Derivatives on Absolute Prediction Error

First, we consider the problem of hedging the loss function L_n with a solar radiation derivative on absolute prediction error. Solving the payoff corresponds to estimating the spline function $\psi(\cdot)$ in the following way:

$$L_n - Mean(L_n) = \psi(|\varepsilon_{R,n}|) + \varepsilon_n. \tag{11}$$

Fig.6 shows the scatter diagram of $|\varepsilon_{R,n}|$ and $|\varepsilon_{P,n}|$, and Fig.7 shows the optimal payoff function of the derivative obtained by the hedging model (11).

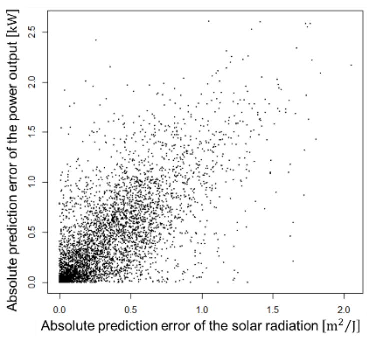

Fig.6 Absolute prediction error of the solar radiation $|\varepsilon_{R,n}|$ vs. absolute prediction error of the solar power output $|\varepsilon_{P,n}|$

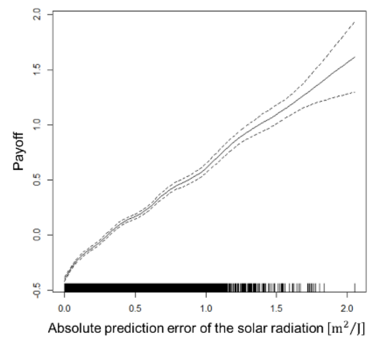

Fig.7 Payoff of solar radiation derivative in model (9): $\varphi^*(|\varepsilon_{R,n}|)$

The horizontal axis is the solar radiation prediction error, and the vertical axis is the payoff[13]. In this case, the 1-

[13] Since the coefficient of the loss on the absolute output prediction error is set to 1, the value obtained by multiplying the vertical axis by c is equivalent to the actual payoff (same applies to the subsequent sections).

VRR is calculated as 0.523, which shows that the variance of the prediction error loss risk is reduced by about 52% of the original value.

4.2 Hedging Using Solar Radiation Derivatives on Prediction Error

Next, we consider the following hedging model. It differs from the previous one as the underlying asset of the derivative is not the absolute value but the pure value of the solar radiation prediction errors:

$$L_n - Mean(L_n) = \psi(\varepsilon_{R,n}) + \varepsilon_n. \tag{12}$$

Fig.8 shows the scatter diagram of $\varepsilon_{R,n}$ and $|\varepsilon_{P,n}|$, and Fig.9 shows the payoff function of the derivative on model (12). The 1-VRR of this model is calculated as 0.536, and the hedge effect is slightly improved over model (11). The estimated payoff function shows that the absolute slope when $\varepsilon_{R,n}$ is positive is smaller than when it is in the negative range. As mentioned above, this is because when the $\varepsilon_{R,n}$ is positive, the influence of output reduction due to temperature tends to increase, which causes the output error to decrease. It is considered that since model (12) can accommodate such asymmetry, the hedge effect is higher than model (11) in the previous section.

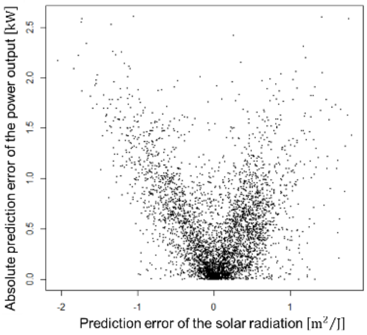

Fig.8 Prediction error of the solar radiation $\varepsilon_{R,n}$ vs. absolute prediction error of the solar power output $|\varepsilon_{P,n}|$

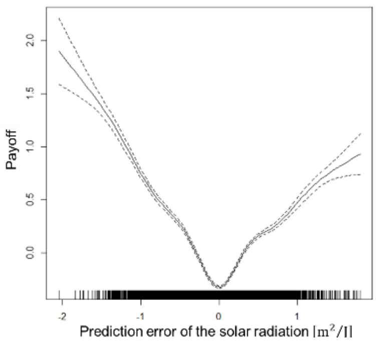

Fig.9 Payoff of solar radiation derivative in model (12): $\psi^*(\varepsilon_{R,n})$

4.3 Cross Hedging Using Solar Radiation Derivative and Temperature Derivatives Together

Next, we consider the following model using the solar radiation derivative on prediction error and the temperature derivative on absolute prediction error in combination[14]:

$$L_n - Mean(L_n) = \Delta(Seasonal_t, m)|\varepsilon_{T,n}| + \psi(\varepsilon_{R,n}) + \varepsilon_n. \tag{13}$$

The estimated value of the tensor product spline $\Delta^*(Seasonal_t, m)$, which is the coefficient of the first term on the right side, is the optimal contract volume of the temperature derivative determined by the delivery time. Fig.10 shows $\Delta^*(Seasonal_t, m)$ and Fig.11 shows $\psi^*(\varepsilon_{R,n})$. The 1-VRR of this model was calculated to be 0.622 and it was confirmed that the variance was further reduced compared to hedging models (11) and (12). In Fig.10, it is considered that the sensitivity of the temperature prediction error which changes depending on the time can be effectively estimated by using the spline model with the intersection variable.

[14] Equation (13) has a term with crossing variables attached to the tensor product spline, but since the intersection term only replaces the new function multiplying the basis function of the tensor product spline by the crossing variable, it can be estimated by the same procedure using normal GAM, in the same idea as described in Yamada (2018).

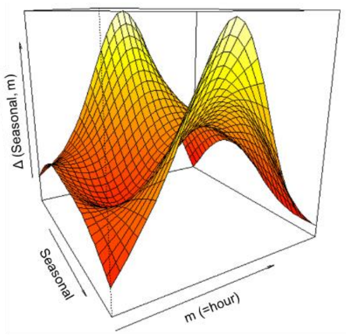

Fig.10 Optimal contract volume of temperature derivative in model (13): $\Delta^*(Seasonal_t, m)$

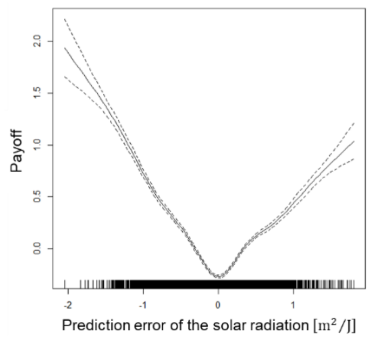

Fig.11 Payoff of solar radiation derivative in model (13): $\psi^*(\varepsilon_{R,n})$

4.4 Cross Hedging Using Only Temperature Derivatives

In this section, we measure the hedge effect when using only temperature derivatives on the absolute prediction error without using solar radiation derivatives. Here, we consider the following hedging model:

$$L_n - Mean(L_n) = \Delta(Seasonal_t, m)\left|\varepsilon_{T,n}\right| + \varepsilon_n. \tag{14}$$

Model (14) differs from model (13) in that there is no term for solar radiation. The optimal contract volume $\Delta^*(Seasonal_t, m)$ of the temperature derivative in this model is shown in Fig.12:

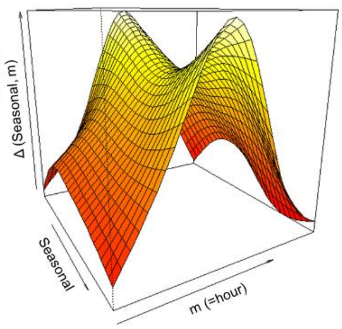

Fig.12 Optimal contract volume of temperature derivative in model (14): $\Delta^*(Seasonal_t, m)$

The 1-VRR of this model is 0.310. This result is interesting because even with the hedging model using only the temperature derivative on the absolute prediction error, the variance of loss decreases by about 31%. This hedge effect can be explained as follows: since there is a positive correlation between the temperature and solar radiation (e.g., high temperature is usually due to large solar radiation) and a strong positive correlation between solar radiation and power output, there is a similar positive correlation between temperature and solar power output even when viewed with the prediction error.

The correlation between solar power output and temperature is a bit complicated: the characteristics of solar power output are such that power-generating efficiency decreases as temperature rises, but this can also be interpreted from the shape of Fig.12, which stands for the sensitivity of the output prediction error to the temperature prediction error. For example, if we look at the value around noon, it can be confirmed that the value in summer (near the saddle point) is lower than that in winter. A possible interpretation of this is, during daytime in summer, the output reduction due to the rise in temperature is remarkable enough for the sensitivity of the output prediction error to become smaller than during winter with respect to the temperature prediction error.

5 Concluding Remarks

In this study, we proposed hedging methods based on derivatives on prediction errors of solar radiation and temperature for the prediction error loss in solar power generation, and measured the resultant hedge effects.

Our proposed method is based on the previously examined research concept of prediction error derivatives in wind power, but it is new in the following points:

- It showed that the previous method can be applied to solar power generation with periodic trends
- It demonstrated the hedge effect using plural weather derivatives such as solar radiation and temperature
- It introduced hedging modeling with a tensor-product spline function and showed its effectiveness

It was found that the variance of prediction error loss in solar power generation can be reduced by about 54% by using solar radiation derivatives on prediction errors, and by about 31% even when only using the temperature derivative on the absolute prediction error. Furthermore, it was demonstrated that the hedge effect was improved by up to 62% by using the solar radiation derivative and temperature derivative together.

The temperature derivative on the absolute prediction error designed from publicly available forecasts and measured values are thought to be effective for hedging the prediction error loss of power demand, which has a strong correlation with temperature; therefore, further application and versatility may be expected. In addition, it is considered that the demonstrated cross-hedging effect obtained when using only the temperature derivative suggests practical recommendations in the case of product development and application.

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