DOCTORAL DISSERTATION

A Study on Structural Changes in Cities: Spatial and Temporal Agglomeration Mechanisms of Economic Activities

Author:

Lu MIAO

Student ID 1924052011

Chief advisor:

Assoc. Prof. Yuki TAKAYAMA

A dissertation submitted in fulfillment of the requirements for the degree of Doctor of Engineering

in the

Division of Environmental Design

Graduate School of Natural Science & Technology

Kanazawa University

September, 2022

Thesis Committee

Thesis Supervisor	Assoc. Prof. Yuki TAKAYAMA Institute of Science and Engineering, Kanazawa University
Thesis Committee	Prof. Shoichiro NAKAYAMA Institute of Philosophy in Interdisciplinary Sciences, Kanazawa University
Thesis Committee	Assoc. Prof. Shota FUJISHIMA Graduate School of Economics, Hitotsubashi University
Thesis Committee	Assoc. Prof. Wataru NAKANISHI Institute of Science and Engineering, Kanazawa University
Thesis Committee	Assoc. Prof. Kentaro WADA Faculty of Engineering, Information, and Systems, University of Tsukuba

Abstract

A Study on Structural Changes in Cities: Spatial and Temporal Agglomeration Mechanisms of Economic Activities

by

Lu Miao

Traffic congestion is a major issue in most cities worldwide. The main reason for traffic congestion is that traffic capacity is less than traffic demand. For recent years, economists have been advocating transportation demand management (TDM) measures to deal with urban traffic congestion.

However, as noted by Takayama (2015), TDM measures are not necessarily socially desirable in the long-run. To accurately evaluate the long-run effects of TDM measures on structural changes in cities, we investigate the mechanism of TDM measures. Specifically, we broadly divide TDM measures into the following 2 types, and investigate each type of TDM measures' long-run effects on (spatial and temporal) structural changes in cities in Part I (Chapter 2 and 3) and Part II (Chapter 4 and 5), respectively.

1) Reduction in road traffic demand (by using public transit to reduce road traffic demand).

2) Reduction in spatial and temporal agglomeration of traffic demand (e.g., staggered work hours, flextime, and road pricing)

In the following, each part and chapter of this dissertation are summarized.

Chapter 1 is the introduction that summarizes the theoretical background and the purpose of the present dissertation.

Part I of the dissertation develops the first departure time choice and mode choice model considering scale economies in public transport and investigating the properties of equilibria under different public transport fare regulations (marginal cost regulation/average cost regulation/without regulation).

We analyze urban public transport into two types: rail transit (Chapter 2), reduces traffic demand by separating a portion of commuters from road traffic; and carpooling (Chapter 3), which which shares the road with automobiles and reduces traffic demand by increasing the number of commuters in an automobile.

Chapter 2 develops a model of multi-modal commute with bottleneck congestion and scale economies in rail transit. To this end, we incorporate the models of de Palma et al. (2017) and Tabuchi (1993) into the standard bottleneck model (Vickrey, 1969). We then show the properties of equilibria when the regulator sets rail fares equal to the marginal, average cost and with no regulation on rail fares. By comparing these equilibria, we clarify the impacts of the regulations on the number of rail commuters and commuting costs.

Chapter 3 develops a model of multi-modal commute with bottleneck congestion and scale economies in carpooling services. Similar to Chapter 2, we show the properties of equilibria when the regulator sets carpooling fares equal to the marginal cost, average cost and with no regulation on carpooling fares. By comparing these equilibria, we clarify the impacts of the regulations on the number of carpooling commuters and commuting costs.

Part II of the dissertation is the first to investigate the spatial and temporal agglomeration mechanisms of economic of activities while considering urban spatial structure as an open city (Chapter 4) and multi-city (Chapter 5).

Chapter 4 investigates the mechanisms of spatial and temporal agglomeration of economic activities by introducing spatial structure (open city and multiple residential locations) into a model of WST choice (Henderson, 1981). By using the properties of the potential game, we characterize equilibrium and optimal distributions of population and WSTs.

Chapter 5 investigates the mechanisms of spatial and temporal agglomeration of economic activities in the context of a different urban structure (multi-city and multiple residential locations in each city) from that of Chapter 4. Then, by using the properties of the potential game, we characterize equilibrium and optimal distributions of intercity and intracity populations and WSTs.

Finally, **Chapter 6** concludes the dissertation by summarizing the main findings, contributions and some directions for future work.

Overall, this dissertation contributes to proceed systematic understanding of mechanisms behind TDM measures, and more accurate evaluation of the long-run effects of various TDM measures on structural changes in cities, so that traffic congestion can be alleviated more effectively.

Key Words: *bottleneck congestion, modal split, scale economies, spatial and temporal agglomeration economies, open city model, system-of-cities model, potential game*

Acknowledgements

My experience in Kanazawa has exceeded my greatest expectations, and I treasure the time I have spent working, learning, and growing on this city and this country. I would not have been able to complete this Ph.D. without the nurturing support of many people. It has been an honor for me to work and develop friendships with all of them.

First and foremost, I am extremely indebted to my dear mentor and advisor Dr. Yuki Takayama. He has always been patient and helpful to me since I was not his student. He supported me unconditionally when I wanted to transfer to his university and get a Ph.D. He has been an inspiring guide for me as I have developed my ideas, my research goals, and this dissertation. It has been a joy to work with him and learn from his example. His ability to see through to the essence of a problem, communicate his understanding with clarity and concision, and coordinate teams flawlessly is a skill that I admire and aspire to develop in myself. He is a great scientist and a good friend, and always the first person to help me when I was confused and distressed. I am immensely grateful to know him, be his first Ph.D. student and work with him. I would have been nothing without him. Thank you so much, Yuki Sensei.

I am also deeply grateful to my dissertation committee: Prof. Shoichiro Nakayama, Assoc. Prof. Shota Fujishima, Assoc. Prof. Kentaro Wada and Assoc. Prof. Wataru Nakanishi for taking the time to review this dissertation and for providing valuable comments and discussions. In particular, I greatly appreciate Assoc. Prof. Fujishima, Assoc. Prof. Wada and Assoc. Prof. Nakanishi not only for their valuable comments from perspectives different from those of my field of expertise, but also for their advice on the direction of future research.

Through seminars and conferences, I have received a lot of comments and advises that make my dissertation improved. Special thank goes to Prof. Se-il Mun (Kyoto University) for being my discussant at Applied Regional Science Conference (ARSC) and giving me a lot of helpful comments and advice. I also would like to appreciate Prof. Tomoya Mori (Kyoto University), Prof. Makoto Okumura (Tohoku University), Prof. Tatsuhito Kono (Tohoku University) and Prof. Daisuke Fukuda (University of Tokyo) for their questions and comments.

There are many great young researchers at Kanazawa University who have helped me a lot and set an example for me. I am very grateful for Asst. Prof. Hiromichi Yamaguchi's selfless sacrifice of his precious time to teach me a lot and give me countless help. I would like thank Asst. Prof. Takao Dantsuji for letting me know what a new generation of young researchers should be like. I am honored to have such an outstanding young researcher by my side. I also want to express my deep gratitude to Asst. Prof. Ayako Baba. Although we only spent a short time together, the last few months I spent as a student with her help are precious memories of my life. Her gentle encouragement would inspire me for a long time to come. I am so lucky to have her in my life.

I am very fortunate to work in Yuki Lab. (Urban System Science Laboratory), my beloved lab, with excellent and smart colleagues. I would like to thank Ms. Miho Yagi and Ms. Machiko Onoe, secretaries of the lab, and the current and former lab members for their great help in carrying out this dissertation. My special thanks go to following three juniors: Naoshi Kito, Mizuki Takada and Hikari Horikoshi. I cannot imagine my life in Kanazawa without the friendship and assistance from them. It has been a pleasure to know you, work with you and share memories with you.

Last but the most important, my deepest gratitude goes to Yin Lu and Huaxun Miao, my parents, and Zhenhua Lu, my maternal grandfather for their unconditional love and endless support through all these 27 years. In addition, I would like to give special thanks to Yumin Fu, my maternal grandmother for raising me with all her love. I hope you will continue to watch over me, love me and bless me from heaven. I also hope that I have become the person you wanted me to be. And I am terribly sorry for not being able to do my filial duty to you, but I will always remember your kindness and redouble my efforts to repay our family. Thank you for everything. This work is dedicated to you and our family.

I thank all the sages, families and friends, who have come along with me during my Ph.D. journey, for smoothing the path and pointing the direction to success. My endless wishes to you all.

DEDICATED

To

My Beloved Parents and Maternal Grandparents

For their endless love, support and encouragement

To

My Dear Friend and Mentor, Dr. Yuki Takayama

For his advice, patience and continuous inspiration

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Introduction

1.1 Background

Traffic congestion is a major issue in most cities worldwide. For example, in 2021, the average American driver lost 36 hours and spent $564 due to congestion. Nationally, traffic congestion in the U.S. cost drivers about 3.4 billion hours and $53 billion (Pishue, 2021).

The main reason for traffic congestion is that traffic capacity is less than traffic demand. For recent years, economists have been advocating transportation demand management (TDM) measures to deal with urban traffic congestion. As shown in Figure 1.1, there are two broad types of TDM measures: 1) reduction in road traffic demand (by using public transit to reduce road traffic demand), 2) reduction in spatial and temporal agglomeration of traffic demand (e.g., staggered work hours, flextime, and road pricing).

However, as noted by Takayama (2015), TDM measures that alleviate traffic congestion

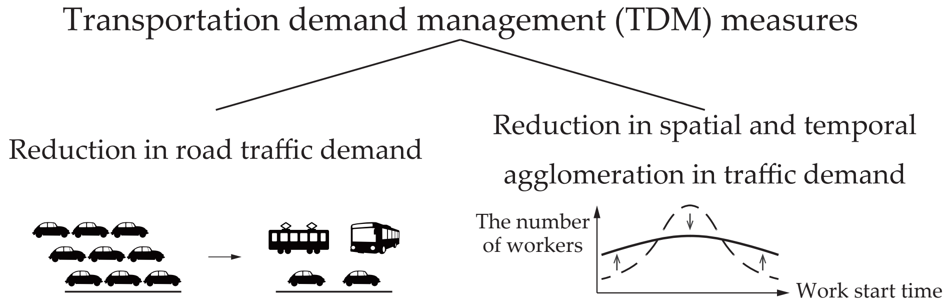


FIGURE 1.1: Types of TDM measures.

during peak hours by adjusting traffic demand (i.e., reducing temporal clustering) are not necessarily socially desirable in the long-run. To accurately evaluate the long-run effects of TDM measures on structural changes in cities, we investigate the mechanism of each type of TDM measures.

First, we focus on the measures of reduction in road traffic demand. Since road traffic demand changes over time, here we discuss effects of measures for road traffic demand reduction on temporal distribution of economic activities. So far, a number of studies (e.g., Takayama, 2015; Li et al., 2020) on departure time choice model have succeeded in developing dynamic frameworks that can adequately describe peak-period congestion. However, frameworks used in most studies cannot properly address urban transit systems because most of them only consider solo-driving commuting and ignore public transport which can significantly reduce road traffic demand and alleviate peak congestion.

Although studies on departure time choice and mode choice that considering public transport have been developed in recent years, most of these studies failed considering the effects of scale economies which is an important characteristic of public transport sector, or assumed that fares for public transport are only set at optimal levels (cf. Introduction of Part I).

Second, we focus on the measures of reduction in spatial and temporal agglomeration of economic activities. It is readily that huge traffic demand that causes traffic congestion (negative congestion externalities) is mainly due to the agglomeration of firms in central business districts (CBDs) and the same work start time (WST) workers have. These phenomena of the spatial and temporal concentration of firms and workers are caused by positive production externalities. Hence, economists also advocate another TDM measures — reducing spatial and temporal agglomeration of economic activities. This type of TDM measures aims to deal with urban traffic congestion by changing the balance of positive and negative temporal externalities. That is, these TDM measures reduce positive temporal externalities (temporal agglomeration economies) alongside negative temporal externalities (temporal agglomeration diseconomies).

In fact, the implementation of TDM measures affects not only temporal distribution of traffic demand but also spatial distribution of economic activities. For example, changes in the distribution of WSTs affect urban land use patterns because traffic demand of peak hours and congestion situations will be changed. Moreover, changes in urban land use patterns affect WSTs distribution because origins and destinations of traffic demand and congestion situations will also change. Thus, it is important to consider the interaction between spatial and temporal distributions of economic activities. However, no study has yet investigated desirable spatial and temporal distributions by considering both spatial and temporal externalities (cf. Introduction of Part II).

1.2 Dissertation Overview

1.2.1 Objectives

The goal of the dissertation is to clarify whether TDM measures are socially desirable in the long-run. To this end, we address the two main issues mentioned in Section 1.1 and investigate the long-run effects of TDM measures on (spatial and temporal) structural changes in cities. More specifically, our objectives are:

1. Verification of long-term effects of reduction in road traffic demand:
   (a) To build dynamic frameworks that can adequately describe peak-period congestion.
   (b) To consider mode choice between solo driving and public transport.
   (c) To consider the characteristics of different public transport and scale economies.
   (d) To investigate the effects of the existence and types of public transport fare regulations on transportation demand and commuting costs by different modes.

   (e) To systematically analyze and compare the properties of all equilibria that arise in situations where public transport fares are set under different regulations.

2. Verification of long-term effects of reduction in spatial and temporal agglomeration of economic activities:

   (a) To establish basic frameworks for analyzing the endogenous distributions of urban population and WSTs.
   (b) To consider different spatial structures of cities.
   (c) To consider spatial and temporal agglomeration economies and diseconomies.
   (d) To investigate the interaction between spatial and temporal externalities.
   (e) To characterize equilibrium and optimal distributions of populations and WSTs by using the properties of potential game.

1.2.2 Main Contributions

The main scientific contributions of this dissertation are summarized hereafter.

1. *Verification of long-term effects of reduction in road traffic demand*

   Part I of the dissertation develops the first departure time choice and mode choice model considering scale economies in public transport and investigating the properties of equilibria under different public transport fare regulations (marginal cost regulation/average cost regulation/without regulation).

   Note that we analyze urban public transport into two types: rail transit, reduces traffic demand by separating a portion of commuters from road traffic; and carpooling, which which shares the road with automobiles and reduces traffic demand by increasing the number of commuters in an automobile. Rail transit and carpooling are analyzed in Chapter 2 and 3, respectively.

   We show that if average cost regulation is implemented simultaneously with the development of public transport, the number of public transport commuters will not increase and equilibrium commuting costs will not change; if average cost regulation is implemented when public transport operators are monopolistically competitive, public transport commuters will increase and equilibrium commuting costs will decrease. We demonstrate that this result holds regardless of whether public transport is assumed to be rail transit or carpooling.

2. *Verification of long-term effects of reduction in spatial and temporal agglomeration of economic activities*

   Part II of the dissertation is the first to investigate the spatial and temporal agglomeration mechanisms of economic of activities while considering urban spatial structure as an open city (Chapter 4) and multi-city (Chapter 5).

   - Chapter 4 shows that greater interaction among different WSTs rural-to-urban migration (spatial agglomeration), and the increase in urban population does not necessarily improve social welfare.
   - Chapter 5 shows that the greater the interaction among different cities, the more clustered the WSTs distribution, and greater interaction among different WSTs leads to intercity population concentration (spatial agglomeration). The equilibrium spatial (temporal) distribution may be less (more) agglomerated than at the optimum.

Note that, it is extremely difficult to extend bottleneck model to urban land use theory with multiple residence locations to consider spatial and temporal agglomeration economies and diseconomies. Hence, in the dissertation, for the first step to investigate the interaction of spatial and temporal agglomeration economies and diseconomies, by using Henderson (1981) model to describe temporal agglomeration economies and diseconomies. For the future works, we will incorporate Part I into Part II to analyze the mechanisms of spatial and temporal agglomeration of economic activities accurately.

1.2.3 Dissertation Outline

This dissertation consists of 6 chapters that are briefly described in the following paragraphs (see also Figure 1.2). The main 4 chapters (excluding Chapter 2, 3, 4, and 5) are organized into 2 parts.

Part I includes Chapters 2 and 3 that investigate long-term effects of reduction in road traffic demand. In this part, we consider multi-modal commute with bottleneck congestion in rail transit and carpooling, respectively, by considering scale economies in these public transports.

Part II investigate long-term effects of reduction in spatial and temporal agglomeration of economic activities In this part, we verify the long-term effects in different urban structures (open city and multiple cities) by considering spatial and temporal agglomeration economies and diseconomies.

Note that each chapter is a complete stand-alone research article including an abstract, introduction, methodology, results, and conclusions with its own (mathematical) notations.

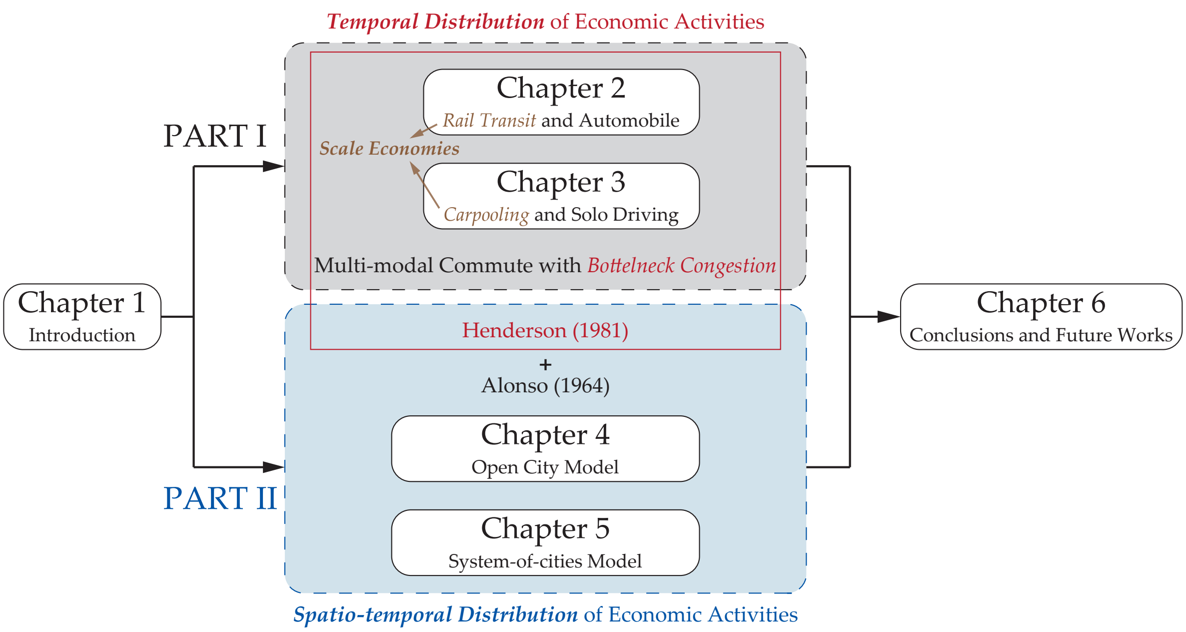


FIGURE 1.2: Organization of the dissertation.

This dissertation is organized in more detail as follows.

Chapter 2 develops a model of multi-modal commute with bottleneck congestion and scale economies in rail transit. To this end, we incorporate the models of de Palma et al. (2017) and Tabuchi (1993) into the standard bottleneck model (Vickrey, 1969). We then show the properties of equilibria when the regulator sets rail fares equal to the marginal cost or average cost and when there is no regulation on rail fares. By comparing these equilibria, we clarify the impacts of the regulations on the number of rail commuters and commuting costs.

Chapter 3 develops a model of multi-modal commute with bottleneck congestion and scale economies considering both carpooling and solo driving. Similar to Chapter 2, we show the properties of equilibria when the regulator sets carpooling fares equal to the marginal cost or average cost and when there is no regulation on carpooling fares. Then by comparing these equilibria, we clarify the impacts of the regulations on the number of carpooling commuters and commuting costs.

Chapter 4 investigates the mechanisms of spatial and temporal agglomeration of economic activities by introducing spatial structure (open city and multiple residential locations) into a model of WST choice (Henderson, 1981). By using the properties of the potential game, we characterize equilibrium and optimal distributions of population and WSTs.

Chapter 5 investigates the mechanisms of spatial and temporal agglomeration of economic activities in the context of a different urban structure (multi-city and multiple residential locations in each city) from that of Chapter 4. Then, by using the properties of the potential game, we characterize equilibrium and optimal distributions of intercity and intracity populations and WSTs.

Finally, **Chapter 6** concludes the dissertation by summarizing the main findings, contributions and some directions for future work.

Part I

Temporal Distribution of Economic Activities

Introduction

Since the seminal work of Henderson (1981), a number of studies (e.g., Wilson, 1992; Arnott et al., 2005; Arnott, 2007) have developed models of WST choice that consider traffic congestion and productivity effects. These studies provide insights into TDM measures by examining the equilibrium and optimal distributions of WSTs and optimal congestion tolls. However, analytical difficulties inevitably that arise in models limit these studies because of their nonconvexities due to considering temporal agglomeration economies and diseconomies. Foremost among their limitations is that they use flow congestion models to describe traffic congestion, which are inappropriate for dealing with peak congestion.

So far, a number of studies (e.g., Takayama, 2015; Li et al., 2020) on departure time choice model have succeeded in clarifying the effects of various TDM measures (e.g., dynamic congestion pricing) based on dynamic frameworks that can adequately describe peak-period congestion. However, frameworks used in most studies cannot adequately address urban transit systems because they only consider solo-driving commuting and ignore public transport.

As mentioned in Section 1.2.2, we analyze urban public transport into two types: **rail transit**, reduces traffic demand by separating a portion of commuters from road traffic; and **carpooling**, which which shares the road with automobiles and reduces traffic demand by increasing the number of commuters in an automobile. We next introduce the previous studies of departure time and mode choice models considering rail transit and carpooling, respectively.

1.2.4 Rail Transit and Automobile

Studies introducing rail transit into standard departure time choice model have made steady progress since Tabuchi (1993) (e.g., Huang, 2000; Kraus, 2003, 2012; Huang et al., 2007; Sean Qian and Michael Zhang, 2011; Gonzales and Daganzo, 2012; Wu and Huang, 2014; Li and Zhang, 2020). However, most of the studies assumed that "convenience of public transport does not depend on the number of its users." In fact, it is widely known that the convenience (e.g., fare, service frequency) of public transport such as rail transit and bus strongly depends on the number of its users. That is, the greater the number of users, the more convenient public transport becomes.

One of the essential factors that make the convenience of public transport highly dependent on the number of users is "scale economies." This is because most of the operating costs required to provide and improve the convenience of public transport services are fixed costs that are independent of the number of users (rather than variable costs that vary depending on the number of users). Hence, in public transport, scale economies that "the more passengers there are, the lower the operating costs per person" work strongly.

To develop and implement appropriate urban transportation policies that take advantage of public transport commuting, it is necessary to analyze the characteristics of the industry, which requires huge fixed costs. It is particularly important to note that these industries are characterized not only by scale economies but also by natural monopolies if there is no regulation. For this reason, policies are generally adopted to regulate fares to average costs (i.e., marginal cost + $\frac{\text{fixed cost}}{\text{number of commuters}}$) rather than marginal costs. Nevertheless, most of the studies assumed that public transport fares are fixed (independent of the number of commuters). Although this assumption can be interpreted as assuming a socially desirable situation in which fares are set equal to marginal cost, it is inconsistent with either no regulation or average cost regulation (where fares change depending on the number of commuters).

Unlike many other studies, Tabuchi (1993) has developed a departure time and mode choice model considering scale economies for rail transit. He then succeeded in comparing the properties of equilibrium, first-best optimum (social optimum), and second-best optimum under marginal and average cost regulations. However, the model for rail commuting is assumed as a simple framework that is static and ignores crowding on trains (i.e., assuming infinite-capacity trains), which is an issue during rush hour. Furthermore, although it has been confirmed that multiple equilibria exist under average cost regulation, the analysis only focused on the equilibrium that minimizes total commuting costs (i.e., large number of rail commuters). [1]

Compared with these studies, this chapter is characterized by a systematic analysis and comparison of all equilibrium characteristics that arise in situations where rail fares are set under marginal cost regulation, average cost regulation, and without regulation. The results of our analysis imply that the equilibrium commuting cost under average cost regulation can be higher than without regulation (i.e., monopoly).

Another feature of this chapter is that rail commuting is modeled in a dynamic framework including crowding on trains. This framework is not only an extension of Tabuchi (1993), but also enables the evaluation of the effects of various TDM measures (including "dynamic" ones such as time-of-day fares), fare regulations, and their combinations on urban transportation system. Therefore, the framework developed in this chapter can provide a theoretical foundation for appropriate urban transportation policies.

1.2.5 Carpooling and Solo Driving

In recent years, "carpooling" has been attracting attention as a new commuting mode of commuting due to the advancement of information technology and the widespread use of smartphones. This is because it is expected that the reduction of traffic demand through carpooling (i.e., sharing an automobile among multiple people) will significantly alleviate traffic congestion during peak hours. Hence, theoretical studies (e.g., Huang, 2000; Sean Qian and Michael Zhang, 2011; Xiao et al., 2016; Ma and Zhang, 2017; Yu et al., 2019) analyzing the effects of carpooling commuting behavior have been accumulating by extending bottleneck model (e.g., Vickrey, 1969; Henderson, 1981; Arnott et al., 1990, 1993) that can represent

[1]We also analyze other equilibria under average cost regulation and natural monopoly, which are not considered in Tabuchi (1993). We then investigate the differences in properties among equilibria, and the effects of rail fare regulations are clarified from the results.

traffic congestion during peak hours.

Since these previous studies have focused on the effect of carpooling on reducing transportation demand (i.e., reducing the number of automobiles by increasing the number of carpool users), the other settings for carpooling commuting have been simplified. That is, all of these studies assumed "the convenience of carpooling itself does not depend on the number of users," and ignored "economies of scale." As a result, the findings of previous studies are likely to be strongly dependent on these settings. Therefore, it is essential to take these settings into account in order to develop and implement appropriate urban policies utilizing carpooling commuting. [2]

[2]The analysis procedure is the same as in Chapter 2.

2

Multi-modal Commute with Bottleneck Congestion: Rail Transit and Automobile

2.1 Departure Time and Mode Choice Model

We develop a commuters' departure time and travel mode choice model considering scale economies in rail transit. To this end, we formulate a model considering commuters' departure time and travel mode choice based on Vickrey (1969) and de Palma et al. (2017), and the behavior of rail operator by extending Tabuchi (1993).

2.1.1 Basic Assumptions

We consider a city consisted of a CBD and a residential area connected by a road and a railroad (Figure 2.1). The N commuters are *ex ante* identical. Each chooses his or her CBD arrival time $t \in \mathbb{R}$ and travel mode from automobile and rail transit (hereafter, subscripts c and p denote automobile and rail transit, respectively). The numbers of respective users are denoted by N_c and N_p. We then assume that commuters have the same work start time t^* and who arrive at CBD at t^* do not have to pay schedule delay costs.

The road has a single bottleneck with capacity μ just before the CBD (commuters arrive at the CBD just after spilling out of the bottleneck). To model queuing congestion, we employ first-in-first-out (FIFO) and a point queue in which vehicles have no physical length as in standard bottleneck models (e.g., Vickrey, 1969; Arnott et al., 1993). Thus, the total travel time for automobile commuters arriving at work at time t is the sum of queuing time $q(t)$ in bottleneck and free-flow travel time T_{0c} from residential area to bottleneck.

There are $2m + 1$ trains, indexed in order of departure, which run at a fixed time interval. We assume that only one train arrives at the CBD exactly at the work start time t^*, and there are m trains before/after t^*. For convenience, we denote the train arriving at the CBD at t^* as train 0, and the trains arriving at the CBD k trains before and after train 0 as train $-k$ and k, respectively. Then, the arrival time of train $k \in \mathcal{K} \equiv \{-m, -m+1, ..., 0, ..., m-1, m\}$ at the CBD is denoted by t_k. Moreover, we assume that the capacity of each train is $s > 0$ and the total travel time from residential area to CBD is T_{0p}, independent of k.

2.1.2 Behavior of Workers

Commuters choose travel mode and CBD arrival time to minimize their commuting costs. In this section, assumptions for automobile commuting and rail commuting are described in order.

Commuting cost $c_c(t)$ of automobile commuters who arrive at work at time t is expressed

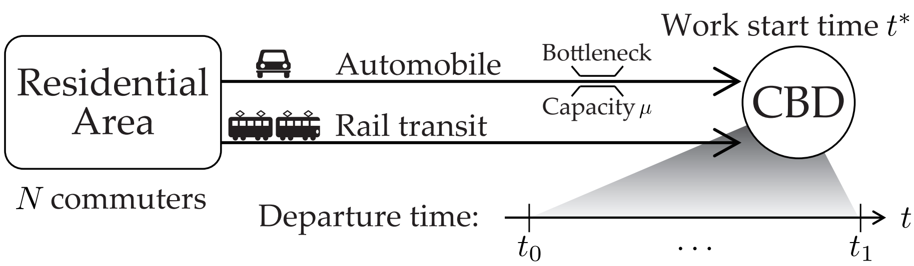


FIGURE 2.1: The Monocentric City.

as the sum of travel time cost $\alpha\{q(t)+T_{0c}\}$, schedule delay cost $\beta|t^{*}-t|$ and fixed cost C_{car}(e.g., fuel, parking): [1]

$$c_c(t) = \alpha q(t) + \beta|t^* - t| + \alpha T_{0c} + C_{\text{car}}. \tag{2.1}$$

where $\alpha > 0$ is the value of per unit travel time of automobile commuters, $\beta > 0$ is early/late delay cost per unit time. We assume $\alpha > \beta$ so that an equilibrium in our model satisfies the FIFO property (i.e., automobiles must leave the bottleneck in the same order as their arrival at the bottleneck) as in the case of the standard departure time choice model.

Rail commuters incur monetary disutility depending on passenger number and train capacity. As in de Palma et al. (2017) and consistent with empirical findings (e.g., Wardman and Whelan, 2011; Haywood and Koning, 2015), we give the crowding cost $g(n_k)$ of train k as a linear function of the passenger number n_k:

$$g(n_k) = \frac{\lambda n_k}{s}. \tag{2.2}$$

where $\lambda > 0$ denotes the magnitude of crowding cost.

Commuting cost $c_p(t_k)$ for rail commuters is given as the sum of crowding cost $g(n_k)$ determined by CBD arrival time t_k, schedule cost $\beta|t^* - t_k|$, travel cost αT_{0p} independent of t_k, and fare p:

$$c_p(t_k) = g(n_k) + \beta|t^* - t_k| + \alpha T_{0p} + p. \tag{2.3}$$

[1]The fixed cost C_{car} can be interpreted as automobile ownership costs minus the benefits of non-commuting purpose.

2.1.3 Behavior of Rail Operator

Rail operator operates trains during the time when passengers exist and sets the fare p and the number of trains $2m + 1$ according to profit π. The profit π is defined as follows:

$$\pi = (p - c)N_p - am - F. \tag{2.4}$$

where c is marginal cost, am is the cost determined by the number of trains, and F is fixed cost.

As is clear from the definition of profit π, scale economies operate in the supply of rail transit services in this model. Therefore, as discussed in the introduction of Part I, the rail transit sector becomes a monopoly if it is left to market competition. In this case, it is socially desirable to regulate rail fares equal to the marginal cost, however, there are drawbacks such as causing deficits for operators. Hence, policies that regulate fares to the level of average cost are generally adopted.

We then consider the following 3 types of situations to investigate the effects of the existence and types of rail fare regulations on rail operator on equilibrium: [2]

Table 2.1: 3 Types of Rail Fare Regulations.

Marginal cost regulation	Rail fares are regulated equal to the marginal cost. The number of trains is set to minimize the social cost in equilibrium.
Average cost regulation	Rail fares are regulated equal to the average cost (i.e., zero profit for rail operators). The number of trains is set to minimize the social cost in equilibrium.
Monopoly	Rail fares and the number of trains are set with no regulation.

[2]We assume that when regulation is implemented, the number of trains is simultaneously specified by the government. This is because under marginal cost regulation, operators' optimal behavior is to set $m = 0$ (minimizing the deficit from fixed costs), and under average price regulation, m has no impact on profit π (i.e., profit is always zero, independent of the number of trains). More specific methods of setting m (i.e., the level at which social costs are adopted in equilibrium) are explained in Section 2.3.2.

Marginal Cost Regulation

We consider a case that the regulator sets marginal cost regulation on rail fares. In this case, the number of trains is set to minimize the social cost (total commuting cost − operator profit) in equilibrium, and rail fare p is set equal to the marginal cost:

$$p = c. \tag{2.5}$$

Average Cost Regulation

We then consider a case that the regulator sets average cost regulation on rail fares. In this case, the number of trains is set to minimize the social cost (total commuting cost − operator profit) in equilibrium as in the case of marginal cost regulation, and rail fare p is set equal to the average cost:

$$p = c + \frac{am + F}{N_p}. \tag{2.6}$$

Equation (2.6) shows that the more the rail commuters, the lower the carpooling fare P, reflecting the positive externalities (scale economies) that characterize rail commuting.

Monopoly

We finally consider a case where a rail operator sets the fare p and the number of $2m + 1$ trains with no regulation. In this case, the operator anticipates the rail transit demand $N_p(p, m)$ corresponding to p, m. That is, p, m are set to satisfy the first-order condition of the profit maximization problem (i.e., to maximize the operators' profit):

$$N_p(p, m) + (p - c)\frac{\partial}{\partial p}N_p(p, m) = 0, \tag{2.7a}$$

$$(p - c)\frac{\partial}{\partial m}N_p(p, m) - a = 0. \tag{2.7b}$$

2.1.4 Assumptions of Rail Transit

We next introduce assumptions regarding rail transit. We assume that the costs independent of CBD arrival time satisfy the following condition:

$$\alpha T_{0p} + c > \alpha T_{0c} + C_{\text{car}}. \tag{2.8}$$

This means that if there is no traffic congestion (i.e., the road capacity μ is infinite), everyone will use an automobile. [3] That is, we assume that the city is favorable for automobile use.

Furthermore, we assume that the time zone $[t_{-m}, t_m]$ during which trains operate is set such that the following conditions are satisfied in equilibrium: commuting costs remain the same for any train $k \in \mathcal{K}$ (i.e, consistent with the equilibrium commuting cost), and commuters have no incentive to use the train at any time outside the time zone because the cost is above the equilibrium commuting cost. [4]

These assumptions imply that rail operators behave to avoid situations such as "trains operate with no passenger (due to high commuting cost)", "no train service during the time zone that commuters want to use." [5] Under these assumptions, the greater the number of trains $2m + 1$, the shorter the time interval between trains.

2.2 Equilibrium Conditions

As in Tabuchi (1993), Huang et al. (2007) and Wu and Huang (2014), we consider commuters' choices regarding CBD arrival time and travel mode in the following 2 stages: commuters first decide their travel modes (first stage), and then choose their CBD arrival times (second stage).

In this case, equilibrium can be obtained by solving backward from the second stage. That is, firstly, given commuters their travel modes (numbers N_s, N_p of commuters by travel modes), we then solve the equilibrium conditions for the choice of CBD arrival time for

[3]This assumption implies that we assume a city (e.g., a regional city) with relatively low fixed costs C_{car} incurred by automobile commuting.

[4]Specifically, we assume that the following conditions are satisfied: $c_p(t_m + \delta) > c_p(t_m) \quad \forall \delta > 0$.

[5]Different operation time zone from the above assumptions can also be considered. However, those situations are not addressed in this paper because they lead to (unnecessary) complications in model analysis.

automobile and rail transit commuters. Next, given the commuting costs c_c^*, c_p^* by travel modes in equilibrium in the second stage, we then solve the equilibrium conditions for the choice of travel mode, and obtain the equilibrium in the first stage.

Hereafter, we call the equilibria in first and second stage described above "equilibrium of mode choice" and "equilibrium of departure time", respectively. In preparation for investigating the characteristics of these equilibria in the following sections, this section formulates the equilibrium conditions for each stage.

2.2.1 Equilibrium Conditions of Departure Time Choice

As mentioned above, CBD arrival time choice for commuters is based on the premise that travel mode is fixed. Therefore, the equilibrium conditions of departure time choice for automobile commuters are consistent with the equilibrium conditions of standard departure time choice model:

$$\begin{cases} c_c^* = c_c(t) & \text{if} \quad n(t) > 0, \\ c_c^* \leq c_c(t) & \text{if} \quad n(t) = 0, \end{cases} \tag{2.9a}$$

$$\begin{cases} n(t) = \mu & \text{if} \quad q(t) > 0, \\ n(t) \leq \mu & \text{if} \quad q(t) = 0, \end{cases} \tag{2.9b}$$

$$\int_{-\infty}^{\infty} n(t)\mathrm{d}t = N_c. \tag{2.9c}$$

where $n(t)$ is the number of commuters arriving at CBD at time t (i.e., CBD arrival rate).

Condition (2.9a) is the no-arbitrage condition for CBD arrival time choices. This condition means that at equilibrium, each commuter has no incentive to change his/her CBD arrival time unilaterally. Condition (2.9b) is the capacity constraint of the bottleneck. It implies that the departure rate $n(t)$ at the bottleneck equals capacity μ if queue occurs at the bottleneck (i.e., $q(t) > 0$) at CBD arrival time (i.e., bottleneck outflow time) t; otherwise, the departure rate is lower than μ. Condition (2.9c) is flow conservation for automobile commuting demand. These conditions give $n(t)$, $q(t)$, and c_c^* at equilibrium as functions of the number of automobile commuters N_c.

Equilibrium conditions for departure time choice of rail commuters are given by the followings as in de Palma et al. (2017):

$$\begin{cases} c_p^* = c_p(t_k) & \text{if} \quad n_k > 0, \\ c_p^* \leq c_p(t_k) & \text{if} \quad n_k = 0, \end{cases} \tag{2.10a}$$

$$\sum_{k \in \mathcal{K}} n_k = N_p. \tag{2.10b}$$

where condition (2.10a) is the no-arbitrage condition for departure time choices, and condition (2.10b) is the conservation law of the population of rail commuters. These conditions give n_k, c_p^* at equilibrium as functions of the number of rail commuters N_p.

2.2.2 Equilibrium Conditions of Mode Choice

Commuters' mode choices are based on the commuting cost c_c^*, c_p^* in the departure time choice equilibrium, hence, equilibrium conditions of mode choice are given as follows:

$$\begin{cases} c_c^*(N_c) = c_p^*(N_p) & \text{if} \quad N_c > 0,\ N_p > 0, \\ c_c^*(N_c) \leq c_p^*(N_p) & \text{if} \quad N_p = 0, \\ c_c^*(N_c) \geq c_p^*(N_p) & \text{if} \quad N_c = 0, \end{cases} \tag{2.11a}$$

$$N_c + N_p = N. \tag{2.11b}$$

Condition (2.11a) is the equilibrium condition for commuters' mode choice, which expresses the following: if both automobile and rail transit are used, the commuting cost of both modes is the same; if only one of the modes has commuters, the commuting cost of the used mode is lower than the other. Condition (2.11b) is the conservation law of the number of commuters.

Equilibrium condition (2.11) implies that the mode choice equilibrium N_c^*, N_p^* is a Nash equilibrium of a population game with the payoff function $(-c_c^*(N_c), -c_p^*(N_p))$. It means that the findings of Sandholm (2010) in the field of population game can be applied to characterize the equilibrium.

We use the findings on potential game in the field of population game. First, we show the properties of potential game: as shown in Sandholm (2001), a game is a potential game if there exists a potential function $P(N_p)$ satisfying the following condition:

$$\frac{\partial}{\partial N_p} P(N_p) = -c_p^*(N_p) + c_c^*(N_c). \tag{2.12}$$

The equilibrium of this potential game coincides with the set of N_p^*, $N_c^*(= N - N_p^*)$ satisfying the Karush-Kuhn-Tucker condition of the following optimization problem (Sandholm, 2001):

$$\max_{N_p} \; P(N_p) \qquad \text{s.t.} \quad 0 \leq N_c \leq N. \tag{2.13}$$

In addition, the potential game has the following properties regarding the local stability of the equilibrium:

> Equilibria that locally maximize the potential function are (locally) stable under a wide class of adjustment dynamics, [6] whereas other equilibria are unstable.

The above properties of potential game imply that the uniqueness and stability of equilibria can be investigated by checking the shape of $P(N_p)$.

The mode choice model constructed in this chapter has the following potential function:

$$P(N_p) = \int_0^{N_p} \left\{ -c_p^*(x) + c_c^*(N - x) \right\} \mathrm{d}x. \tag{2.14}$$

This is because $c_c^*(N_c)$ and $c_p^*(N_p)$ are continuous functions (the proof is in the next section). In the next section, we characterize the equilibrium by using the properties shown in this section.

2.3 Equilibrium of Departure Time and Mode Choices

This section clarifies the equilibrium characteristics of the model shown in the previous sections. Specifically, we show that the commuting cost is uniquely determined in departure

[6] This adjustment dynamics includes the *best response dynamic* (Gilboa and Matsui, 1991), the *Brown-von Neumann-Nash* dynamic (Brown and von Neumann, 1950), and the *projection dynamic* (Dupuis and Nagurney, 1993), which have been adopted in many studies. If the equilibrium is an interior solution, replicator dynamic is also included. See Sandholm (2005) for other dynamics.

time choice equilibrium, and then indicate the uniqueness and stability of mode choice equilibrium.

2.3.1 Departure Time Choice Equilibrium

Equilibrium conditions(2.9), (2.10) of departure time choice of automobile and rail commuters, respectively, are consistent with standard departure time choice models and de Palma et al. (2017) model, as discussed in the previous section. Therefore, equilibrium commuting costs c_c^*, c_p^* are uniquely determined, as proved by Lindsey (2004) and de Palma et al. (2017).

Equilibrium commuting costs c_c^*, c_p^* are given by the following continuous function of N_c, N_p by the same analytical procedure as in previous studies (see **5.A** for their derivations):

$$c_c^*(N_c) = \frac{\beta}{2\mu} N_c + \alpha T_{0c} + C_{\text{car}}, \tag{2.15a}$$

$$c_p^*(N_p) = \frac{\lambda}{sm} N_p + \alpha T_{0p} + p. \tag{2.15b}$$

2.3.2 Mode Choice Equilibrium

In this section, we use the equilibrium commuting costs (2.15) obtained in the previous section to clarify the properties of the mode choice equilibrium for each of the cases where rail fares and number of trains are set under marginal cost and average cost regulations, and with no regulation.

In addition, the main objective of this chapter is to investigate the effects of the existence and types of rail fare regulations on transportation demand and commuting costs by different modes. Hence, we do not consider extreme cases in which everyone uses automobile or rail transit, regardless of the existence and types of rail fare regulations. In order to eliminate these extreme cases, we assume the existence of an equilibrium in which both modes are used in each case where rail fares are set equal to the marginal cost or average cost or without regulation. [7] Furthermore, the conditions for parameter values that satisfy this assumption

[7]Specifically, we assume that in each situation of marginal cost regulation/average cost regulation/monopoly, there exist at least one case of $N_c^*,\ N_p^* > 0$ satisfying $c_c^*(N_c^*) = c_p^*(N_p^*)$.

are shown at the end of this section.

Marginal Cost Regulation

Under marginal cost regulation, fare p is set equal to marginal cost c. From this relationship and the equilibrium condition (2.11a), the numbers $N_p^{\mathrm{MC}*}$, $N_c^{\mathrm{MC}*}$ of commuters under equilibrium is uniquely determined as follows:

$$N_p^{\mathrm{MC}*} = \left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)^{-1} B, \tag{2.16a}$$

$$N_c^{\mathrm{MC}*} = N - N_p^{\mathrm{MC}*}, \tag{2.16b}$$

$$B \equiv \frac{\beta}{2\mu}N - \left(\alpha T_{0p} + c\right) + \left(\alpha T_{0c} + C_{\mathrm{car}}\right). \tag{2.16c}$$

Moreover, the equilibrium commuting cost $c^{\mathrm{MC}*}$ in this case is expressed as follows:

$$c^{\mathrm{MC}*} = \frac{\beta\lambda N + \beta s(\alpha T_{0p} + c)m + 2\mu\lambda(\alpha T_{0c} + C_{\mathrm{car}})}{2\mu\lambda + \beta ms}. \tag{2.17}$$

These results confirm that $N_p^{\mathrm{MC}*}$ and $c^{\mathrm{MC}*}$ are a monotonically increasing function and a monotonically decreasing function of m, respectively. This is because under marginal cost regulation, an increase in the number of trains raises the fixed costs of rail operators while having no effect on rail fares.

Under marginal cost regulation, the number of trains $2m + 1$ is set to minimize the social cost. That is, m is given by the solution of the following social cost minimization problem:

$$\min_m SC = c^{\mathrm{MC}*}N + am + F. \tag{2.18}$$

After solving for this minimization problem, the $m^{\mathrm{MC}*}$ that minimizes the social cost, and the number of rail commuters $N_p^{\mathrm{MC}*}$ under this condition are as follows: [8]

$$m^{\mathrm{MC}*} = N_p^{\mathrm{MC}*}\sqrt{\frac{\lambda}{as}\frac{\beta}{2\mu}\frac{N}{B}}, \tag{2.19a}$$

$$N_p^{\mathrm{MC}*} = \frac{2\mu}{\beta}\left(B - \sqrt{\frac{a\lambda}{s}\frac{2\mu}{\beta}\frac{B}{N}}\right). \tag{2.19b}$$

These results confirm that rail commuters will increase under the following conditions: the road is more likely to be congested, trains are less crowded, and schedule costs are higher (i.e., greater β, s and lower μ, λ), additionally, the number of train will be set more frequently when the road is more likely to be congested and schedule costs are higher.

Average Cost Regulation

We next characterize the mode choice equilibrium in average cost regulation. The rail fare p set by rail operators equals to average cost (2.6). In this case, according to equilibrium condition (2.11a), multiple equilibria exists if there exists $N_p \in (0, N)$ satisfying the following conditions:

$$\left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)N_p^2 - BN_p + (F + am) = 0. \tag{2.20}$$

[8] m, which minimizes social costs, is not necessarily a natural number. Hereafter, for convenience, as in de Palma et al. (2017), we use the m obtained by the analysis that examines the effect of the existence and types of regulations in equilibrium. Since the number of trains that minimize social costs is at $2m + 1$ with the decimals rounded up or down, it does not significantly change the qualitative results.

Then, the number of rail commuters $N_p^{\mathrm{AC}*}$ when multiple equilibria exist is given by

$$N_{p0}^{\mathrm{AC}*} = 0, \tag{2.21a}$$

$$N_{p1}^{\mathrm{AC}*} = \frac{B - \sqrt{B^2 - 4(am + F)\left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)}}{2\left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)}, \tag{2.21b}$$

$$N_{p2}^{\mathrm{AC}*} = \frac{B + \sqrt{B^2 - 4(am + F)\left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)}}{2\left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)}. \tag{2.21c}$$

Note that if $N_p \in (0, N)$ satisfying (2.20) does not exist, $N_{p0}^{\mathrm{AC}*} = 0$ is the only equilibrium. However, this is contrary to the assumption at the beginning of Section 2.3.2 that “there exists an equilibrium in which both modes are used." Therefore, in this chapter, we only consider a situation in which multiple equilibria exist. The condition under which the equilibrium is non-unique is given as follows, will be clarified in the analysis that follows:

$$B > 2\left(\sqrt{\frac{a\lambda}{s}} + \sqrt{\frac{\beta}{2\mu}F}\right). \tag{2.22}$$

Since the multiple equilibria exist, we characterize their stability. From (2.6) and (2.15), potential function of mode choice model under average cost regulation is expressed as follows:

$$\begin{aligned} P(N_p) = &- \left\{\frac{1}{2}\frac{\beta}{2\mu}(N - N_p) + \alpha T_{0c} + C_{\mathrm{car}}\right\}(N - N_p) \\ &- \left\{\int_0^N \frac{\lambda}{s}\frac{N_p}{m} dN_p + (\alpha T_{0p} + c)N_p + a\int_0^N \frac{m}{N_p} dN_p + F\ln(N_p)\right\}. \end{aligned} \tag{2.23}$$

By using the fact that $N_{p1}^{\mathrm{AC}*}, N_{p2}^{\mathrm{AC}*}$ satisfy (2.20), the followings are valid.

$$\frac{\partial}{\partial N_p} P(N_{p0}^{\mathrm{AC}*}) < 0, \tag{2.24a}$$

$$\frac{\partial}{\partial N_p} P(N_{p1}^{\mathrm{AC}*}) = 0, \qquad \frac{\partial^2}{(\partial N_p)^2} P(N_{p1}^{\mathrm{AC}*}) > 0, \tag{2.24b}$$

$$\frac{\partial}{\partial N_p} P(N_{p2}^{\mathrm{AC}*}) = 0, \qquad \frac{\partial^2}{(\partial N_p)^2} P(N_{p2}^{\mathrm{AC}*}) < 0. \tag{2.24c}$$

Thus, it is readily confirm that $N_{p0}^{\mathrm{AC}*}, N_{p2}^{\mathrm{AC}*}$ are stable equilibria (i.e., locally maximize $P(N_p)$), $N_{p1}^{\mathrm{AC}*}$ is unstable equilibrium.

We can also confirm these results through the numerical examples shown in Figure 2.2. In these numerical examples, we use the following parameter values:

$$N = 5000,\ \alpha = 1,\ \beta = 0.8,\ \lambda = 1,\ \mu = 50,\ s = 100,$$
$$a = 1,\ F = 40000,\ C_{\mathrm{car}} = 5,\ c = 1,\ T_{0c} = 5,\ T_{0p} = 10.$$

Since the profit of rail operators is zero, social cost SC is expressed as follows:

$$SC = c^{\mathrm{AC}*} N. \tag{2.25}$$

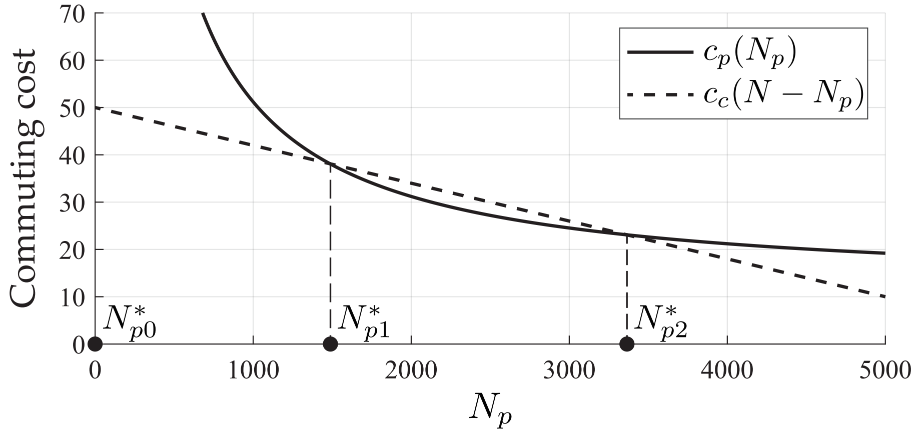
where $c^{\mathrm{AC}*}$ is the average commuting cost, and it can be expressed as follows by denoting the number of rail commuters as $N_p^{\mathrm{AC}*}$:

$$c^{\mathrm{AC}*} = \frac{\beta}{2\mu}\left(N - N_p^{\mathrm{AC}*}\right) + \alpha T_{0c} + C_{\mathrm{car}}. \tag{2.26}$$

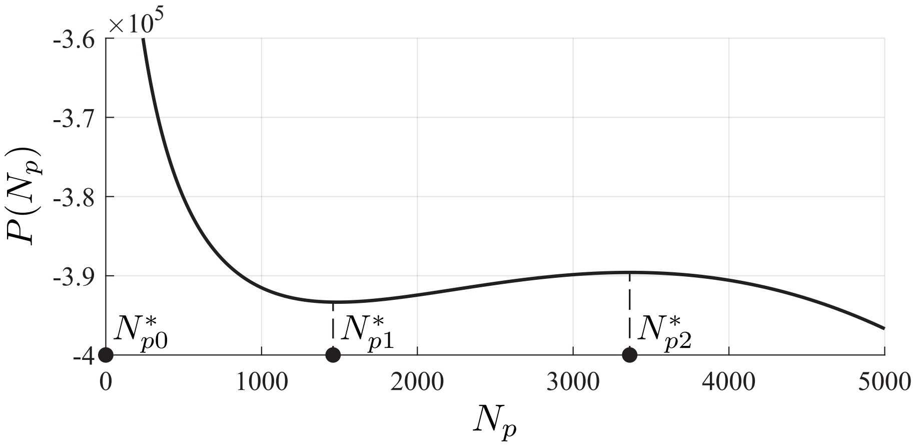
Therefore, social cost SC depends on which equilibrium emerges.

We assume that the number of trains $2m + 1$ is set under the emergence of an equilibrium that has the lowest social cost (i.e., more commuters take rail transit). [9] In this case, $m^{\mathrm{AC}*}$ minimizing the social cost SC satisfies $\frac{\partial}{\partial m} N_p^{\mathrm{AC}*} = 0$ in terms of (2.25), (2.26). Then from (2.20),

[9] This assumption means that "the number $2m + 1$ of trains is set to satisfy $\frac{\partial}{\partial m} N_p^{\mathrm{AC}*} = 0$." As discussed in Footnote 2, the value of m will be undefined without this assumption.



(A) Commuting cost



(B) Potential function

FIGURE 2.2: Comparison between Commuting Cost and Potential Function.

m^{AC*} is given as follows:

$$m^{AC*} = \sqrt{\frac{\lambda}{as}} N_p^{AC*}. \tag{2.27}$$

In this case, the number of rail commuters is as follows:

$$N_{p0}^{\mathrm{AC}*} = 0, \tag{2.28a}$$

$$N_{p1}^{\mathrm{AC}*} = \frac{\mu}{\beta}\left\{D - \sqrt{D^2 - 4\frac{\beta}{2\mu}F}\right\}, \tag{2.28b}$$

$$N_{p2}^{\mathrm{AC}*} = \frac{\mu}{\beta}\left\{D + \sqrt{D^2 - 4\frac{\beta}{2\mu}F}\right\}, \tag{2.28c}$$

$$D \equiv B - 2\sqrt{\frac{\lambda a}{s}}. \tag{2.28d}$$

As is clear from these results, the condition for the existence of multiple equilibria is given by (2.22).

From the above results of average cost regulation show that the equilibrium has the following properties: If the initial state is that the number of rail commuters is less than $N_{p1}^{\mathrm{AC}*}$, the stable equilibrium that eventually emerges is $N_{p0}^{\mathrm{AC}*}$; and if more rail commuters than $N_{p1}^{\mathrm{AC}*}$ in the initial state, stable equilibrium $N_{p2}^{\mathrm{AC}*}$ emerges. That is, the implementation of average cost regulation will reduce the number of rail commuters, unless many commuters use rail transit prior to the implementation. Therefore, if average cost regulation is implemented simultaneously with the development of rail transit, the number of rail commuters will not increase (since the initial number of rail commuters is zero). This is because the fewer the number of rail commuters, the more expensive their fares will be.

Monopoly

We then investigate the equilibrium properties of the case where rail operators determine the fare and the number of trains with no regulation on rail fares. When both modes have commuters, from equilibrium condition (2.10a), the number of rail commuters $N_p(p, m)$ under fare p and the number of trains $2m + 1$ satisfy the following condition:

$$N_p^{\mathrm{m}*} = \left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)^{-1}\{B - (p - c)\}. \tag{2.29}$$

Substituting (2.29) into (2.7), the number of rail commuters N_p^{m*}, m^{m*} and p^{m*} in equilibrium are given as follows:

$$N_p^{m*} = \left(\frac{\lambda}{sm} + \frac{\beta}{2\mu}\right)^{-1} \frac{B}{2}, \tag{2.30a}$$

$$m^{m*} = \sqrt{\frac{\lambda}{sa}} N_p^{m*}, \tag{2.30b}$$

$$p^{m*} = \frac{B}{2} + c. \tag{2.30c}$$

From this relationship, the number of rail commuters N_p^{m*} is as follows:

$$N_p^{m*} = \frac{\mu}{\beta} D. \tag{2.31}$$

The above results show that the equilibrium when rail operators behave monopolistically has similar properties to the equilibrium under marginal cost regulation. In fact, there will be more rail commuters when road is more congested, trains are less crowded, and schedule costs are higher (i.e., greater β and s, lower μ, λ). Moreover, when the road is easily congested and the schedule costs are high, the number of trains is set to be higher.

Additionally, the profit π of rail operator in equilibrium can be expressed as follows:

$$\pi = \left(\frac{B}{2} + \sqrt{\frac{a\lambda}{s}}\right) \frac{\mu}{\beta} D - F. \tag{2.32}$$

Thus, $\pi > 0$ under the condition (2.22).

The Existence Conditions of Equilibrium where Both Modes Are Used

Using the results obtained from the above analysis, we show the conditions for the existence of equilibrium where both modes are used, as assumed at the beginning of Section 2.3.2. In order for both modes to be used, the number of rail commuters, N_p^*, must satisfy the following conditions in equilibrium where fares and the number of trains are set under marginal cost regulation / average cost regulation / no regulation.

$$0 < N_p^* < N. \tag{2.33}$$

Rearranging this conditions, the following conditions can be derived:

$$N > \frac{2\mu}{\beta}\left\{2\left(\sqrt{\frac{a\lambda}{s}} + \sqrt{\frac{\beta}{2\mu}F}\right) - G\right\}, \tag{2.34a}$$

$$N < \frac{2\mu}{\beta}\frac{a\lambda G}{sG^2 - a\lambda}, \tag{2.34b}$$

$$G = (\alpha T_{0c} + C_{\text{car}}) - (\alpha T_{0p} + c). \tag{2.34c}$$

Here, (2.34a), which is derived from the conditions under which rail transit can be used under average cost regulation (i.e., N_{p1}^{AC*}, N_{p2}^{AC*} exist), requires that the population size of the city analyzed must exceed a certain level. This is because cities with small populations have large burdens under fixed costs, thus no one uses rail transit. Meanwhile, (2.34b), which is derived from the conditions under which automobile commuters exist under marginal cost regulation (i.e., $N_p^{MC*} < N$), requires that the population size not be too large. This is because if the population is too large, the timetable will be set very closely, with the effect that everyone will use rail transit.

2.4 Comparison of Equilibria

This chapter compares the characteristics of equilibria under marginal cost regulation/ average cost regulation/no regulation obtained so far. We then use these results to clarify the impacts of the existence and types of regulations on rail operators on urban transportation system.

2.4.1 Comparison of the Number of Rail Commuters

Firstly, let us compare the number of rail commuters. From the results of the previous section, we obtain the following relation (see **Appendix 2.B** for the proof):

$$N_p^{\mathrm{MC}*} > N_{p2}^{\mathrm{AC}*} > N_p^{\mathrm{m}*} > N_{p1}^{\mathrm{AC}*} > N_{p0}^{\mathrm{AC}*} = 0. \tag{2.35}$$

This result is consistent with the intuition that the implementation of marginal cost regulation leads to the maximum use of rail transit. Meanwhile, we can also confirm the following properties of equilibria when the regulator sets rail fares equal to the average cost or without regulation on rail fares: The number of rail commuters $N_p^{\mathrm{m}*}$ in equilibrium with no regulation is more than the unstable equilibrium $N_{p1}^{\mathrm{AC}*}$ and less than the stable equilibrium $N_{p2}^{\mathrm{AC}*}$ under average cost regulation.

These results imply the following equilibrium properties:

1) If the average cost regulation is implemented simultaneously with the development of rail transit (the number of rail commuters is zero when the regulation is implemented), the number of rail commuters will not increase;

2) If rail operators implement average cost regulation after monopoly (the number of rail commuters is $N_p^{\mathrm{m}*}$ when the regulation is implemented), the number of rail commuters will increase.

That is, the results in this chapter show that a hasty implementation of average cost regulation will hinder the use of rail transit.

In addition, the above results do not change whether m is given as a parameter (i.e., assuming the same level regardless of the existence or type of regulation). In fact, using (2.16a), (2.21) and (2.30a) gives exactly the same relationship as (2.35).

2.4.2 Comparison of the Number of Trains

We then consider m, which represents the number of trains. Using the relation (2.35) and (2.8) of the number of rail commuters, the relation of m is given as follows (see **Appendix 2.C** for the proof):

$$m^{\mathrm{MC}*} > m_2^{\mathrm{AC}*} > m^{\mathrm{m}*} > m_1^{\mathrm{AC}*} > m_0^{\mathrm{AC}*} = 0. \tag{2.36}$$

where $m_i^{\mathrm{AC}*}$ denotes the level of m when the number of rail commuters is $N_{pi}^{\mathrm{AC}*}$ under average cost regulation. This relationship shows that the higher the number of rail commuters, the higher the number of trains.

2.4.3 Comparison of Equilibrium Commuting Costs

Finally, we compare equilibrium commuting costs c^*.

Since automobile commuters exist in all types of equilibrium, equilibrium commuting cost satisfies the following relationship:

$$c^* = \frac{\beta}{2\mu} N_c^* + \alpha T_{0c} + C_{\mathrm{car}}. \tag{2.37}$$

This means that the more rail commuters, the lower the equilibrium commuting cost.

Based on the above, we obtain the following relationship for equilibrium commuting costs:

$$c^{\mathrm{MC}*} < c_2^{\mathrm{AC}*} < c^{\mathrm{m}*} < c_1^{\mathrm{AC}*} < c_0^{\mathrm{AC}*}. \tag{2.38}$$

where c_i^{AC*} denotes the equilibrium commuting cost when the number of public transport commuters is $N_{pi}^{\mathrm{AC}*}$ under the average cost regulation. From this result, we can confirm that the marginal cost regulation leads to the lowest commuting cost, whereas the commuting cost under the average cost regulation can be higher than no regulation on rail fares.

2.5 Summary and Discussions

In this chapter, we developed a framework considering commuters' departure time and mode choice behavior, and scale economies in rail transit. We then show the properties of equilibria when the regulator sets rail fares equal to the marginal cost or average cost and when there is no regulation on rail fares.

By comparing these equilibria, we obtained the following findings:

1) The implementation of marginal cost regulation leads to the highest number of rail commuters and the lowest equilibrium commuting costs;

2) If average cost regulation is implemented simultaneously with the development of rail transit, the number of rail commuters will not increase and equilibrium commuting costs will not change;

3) If average cost regulation is implemented when rail operators are monopolistically competitive, rail commuters will increase and equilibrium commuting costs will decrease.

These results imply that when it is difficult to implement marginal cost regulation, a hasty implementation of average cost regulation can lead to a socially undesirable situation (i.e., high rail fares and non-increasing in the number of rail commuters). The above results also indicate that one way to alleviate the problem is to "allow rail operators to be a monopoly to increase the number of rail commuters, and then implement average cost regulation." These results are unique to this chapter and have not been presented in previous studies.

Since this chapter focused on the impacts of the marginal cost/average cost regulations on rail fares, we only consider the equilibria under those regulations/without regulation. Thus, it is important to clarify the impacts of other regulations such as price-cap regulation (Kidokoro, 2006), compare social (i.e., first-best) optimum and second-best optimum as in Tabuchi (1993) and obtain insights on policies to achieve them (e.g., subsidies, time-of-day-varying fare, congestion pricing). Additionally, the departure time and mode choice model used in this analysis is base on the assumption that only one type of public transportation, rail transit, exists. Therefore, it is also important to develop a framework for public transportation such as buses (which share road space with automobiles) based on the results of this chapter.

Appendix

2.A Derivation of Equilibrium Commuting Costs

2.A.1 Automobile Commuting Costs

As proven in the studies using standard departure time choice model, $n(t)$ satisfying equilibrium conditions satisfies the following condition:

$$n(t) = \begin{cases} \mu & \text{if} \quad t \in [t^{\mathrm{E}}, t^{\mathrm{L}}] \\ 0 & \text{otherwise} \end{cases} \tag{2.39}$$

where, t^{E} [t^{L}] is the earliest [latest] CBD arrival time of commuters and is given as follows:

$$t^{\mathrm{E}} = t^* - \frac{N_c}{2\mu}, \qquad t^{\mathrm{L}} = t^* + \frac{N_c}{2\mu}. \tag{2.40}$$

Therefore, equilibrium commuting cost c_c^* is expressed as follows:

$$c_c^*(N_c) = \beta \frac{N_c}{2\mu} + \alpha T_{0c} + C_{\mathrm{car}}. \tag{2.41}$$

2.A.2 Rail Commuting Costs

Based on equilibrium condition (2.10a) and the assumption in Section 2.3, if there exists an equilibrium, the following conditions are satisfied: [10]

$$c_p^* = \frac{\lambda n_k}{s} + \beta|t^* - t_k| + p + \alpha T_{0p} \qquad \forall k \in \mathcal{K}. \tag{2.42}$$

Note that this can be rewritten as

$$n_k = \frac{s}{\lambda}\left\{c_p^* - \beta|t^* - t_k| - p - \alpha T_{0p}\right\} \quad \forall k \in \mathcal{K}. \tag{2.43}$$

Substituting (2.43) into the conservation law of the population of rail commuters (2.10b) yields the following relationship:

$$N_p = \frac{s}{\lambda}\left\{(2m+1)\left(c_p^* - p - \alpha T_{0p}\right) - TSC\right\}. \tag{2.44}$$

where TSC is the total scheduling cost. Assuming that the time interval of trains is τ, then TSC is given as follows:

$$TSC = \sum_{k \in \mathcal{K}} \beta|t^* - t_k| = \beta(m+1)m\tau \tag{2.45}$$

Substituting this into (2.44), c_p^* is expressed as follows:

$$c_p^* = \frac{\lambda N_p}{(2m+1)s} + \frac{\beta(m+1)m\tau}{2m+1} + p + \alpha T_{0p}. \tag{2.46}$$

Then using $t_m = t^* + \tau m$, $n_m = 0$ ($\because$ the assumption introduced in Section 2.1.3), (2.42) can be rewritten as follows:

$$c_p^* = \beta m\tau + p + \alpha T_{0p}. \tag{2.47}$$

[10]Specifically, we use the following assumptions: commuting cost remain the same no matter which train a commuter chooses ($c_p^* = c_p(t_k) \quad \forall k \in \mathcal{K}$); commuters will not use the train ($n_m = n_{-m} = 0$) if it runs at any time other than time zone $[t_{-m}, t_m]$.

Substituting (2.46) into (2.47), τ satisfies the following condition:

$$\tau = \frac{\lambda N_p}{\beta m^2 s}. \tag{2.48}$$

Finally, substituting (2.48) into (2.47), equilibrium commuting costs for rail commuters is given by a function of N_p:

$$c_p^*(N_p) = \frac{\lambda N_p}{ms} + p + \alpha T_{0p}. \tag{2.49}$$

2.B Comparison of the Number of Rail Commuters

Based on (2.28) and (2.31), the numbers of rail commuters under average cost regulation and monopoly have following relationship:

$$N_{p2}^{\mathrm{AC}*} > N_p^{\mathrm{m}*} > N_{p1}^{\mathrm{AC}*} > N_{p0}^{\mathrm{AC}*} = 0. \tag{2.50}$$

Moreover, since $\frac{\beta}{2\mu}N > B$ under (2.8), the number of rail commuters under marginal cost regulation (2.19b) satisfies following relationship:

$$N_p^{\mathrm{MC}*} > \frac{2\mu}{\beta}D. \tag{2.51}$$

Therefore, $N_p^{\mathrm{MC}*} > N_{p2}^{\mathrm{AC}*}$ holds and (2.35) is obtained.

2.C Comparison of the Number of Trains

It is readily that based on (2.27), (2.30b) and (2.35), m under average cost regulation and monopoly are in the following order:

$$m_2^{\mathrm{AC}*} > m^{\mathrm{m}*} > m_1^{\mathrm{AC}*} > m_0^{\mathrm{AC}*} = 0 \tag{2.52}$$

Furthermore, since $\frac{\beta}{2\mu}N > B$ under (2.8), (2.19a) indicates that $m^{\mathrm{MC}*}$ satisfies the following relationship:

$$m^{\mathrm{MC}*} > \sqrt{\frac{a\lambda}{s}} N_p^{\mathrm{MC}*}. \tag{2.53}$$

Therefore, $m^{\mathrm{MC}*} > m_2^{\mathrm{AC}*}$ holds and (2.36) is obtained.

3

Multi-modal Commute with Bottleneck Congestion: Carpooling and Solo Driving

3.1 Departure Time and Mode Choice Model

We develop a commuters' departure time and travel mode choice model considering scale economies in carpooling based on standard bottleneck model (Vickrey, 1969).

3.1.1 Basic Assumptions

We consider a city consisted of a CBD and a residential area connected by a road (Figure 3.1). The N commuters are *ex ante* identical. Each chooses his or her CBD arrival time $t \in \mathbb{R}$ and travel mode from solo driving and two-person carpooling (hereafter, subscripts s and p denote solo driving and carpooling, respectively). The numbers of respective users are denoted by N_s and N_p. We then assume that commuters have the same work start time t^* and who arrive at CBD at t^* do not have to pay schedule delay costs.

The road has a single bottleneck with capacity μ just before the CBD (commuters arrive at the CBD just after spilling out of the bottleneck). To model queuing congestion, we employ first-in-first-out (FIFO) and a point queue in which vehicles have no physical length as in standard bottleneck models (e.g., Vickrey, 1969; Arnott et al., 1993). Thus, the total travel time for commuters arriving at work at time t is the sum of queuing time $q(t)$ in bottleneck and free-flow travel time T_0 from residential area to bottleneck. In our model, T_0 has no effect on the results of the subsequent analysis, hence, we set $T_0 = 0$ to simplify the notation.

3.1.2 Behavior of Commuters

Commuters choose travel mode $k \in \{s,\ p\}$ and CBD arrival time to minimize their commuting costs. In this section, assumptions for solo commuting and carpool commuting are described in order.

Commuting cost $c_s(t)$ of solo commuters who arrive at work at time t is expressed as the sum of travel time cost $\alpha q(t)$, schedule delay cost $\eta(t)$ and fixed cost C_{car}: [1]

$$c_s(t) = \alpha q(t) + \eta(t) + C_{\text{car}}, \tag{3.1a}$$

$$\eta(t) = \begin{cases} \beta(t^* - t) & \text{if} \quad t \le t^*, \\ \gamma(t - t^*) & \text{if} \quad t \ge t^*. \end{cases} \tag{3.1b}$$

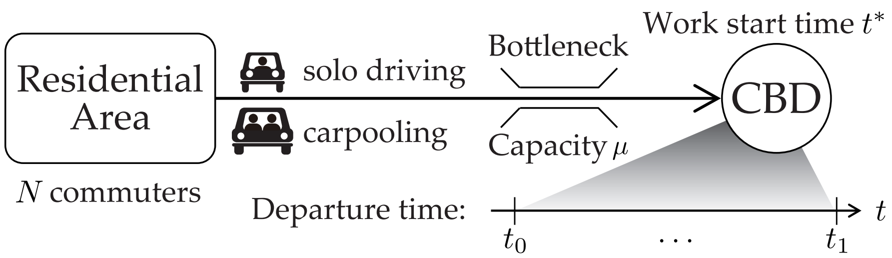


Figure 3.1: The Monocentric City.

[1]See Footnote 1 in Chapter 3.

where $\alpha > 0$ is the value of per unit travel time of commuters. $\beta > 0$, $\gamma > 0$ are early and late delay costs per unit time, respectively. We assume $\alpha > \beta$ so that an equilibrium in our model satisfies the FIFO property.

Commuting cost $c_p(t)$ of carpool commuters who arrive at work at time t is expressed as the sum of travel time cost $\alpha q(t)$, schedule delay cost $\eta(t)$, fare P for carpooling and non-monetary cost $\theta(N_p)$:

$$c_p(t) = \alpha q(t) + \eta(t) + P + \theta(N_p). \tag{3.2}$$

where $\theta(N_p)$ denotes congestion externalities due to increased carpooling demand (i.e., increased temporal and psychological burden), We also assume that $\theta(N_p)$ is a monotonically convex function of N_p.

3.1.3 Behavior of Carpooling Operator

Carpooling operator sets the fare p according to profit π. The profit π is defined as follows:

$$\pi = PN_p - cN_p - F. \tag{3.3}$$

where c is marginal cost, F is fixed cost.

As is clear from the definition of profit π, scale economies operate in the supply of carpooling in this model. Therefore, as discussed in the introduction of Part I, the carpooling sector becomes a monopoly if it is left to market competition. In this case, it is socially desirable to regulate carpooling fares equal to the marginal cost, however, there are drawbacks such as causing deficits for operators. Hence, policies that regulate fares to the level of average cost are generally adopted.

We then consider the following 3 types of situations to investigate the effects of the existence and types of carpooling fare regulations on carpooling operator on equilibrium:

TABLE 3.1: 3 Types of Carpooling Fare Regulations.

Marginal cost regulation	Carpooling fares are regulated equal to the marginal cost.
Average cost regulation	Carpooling fares are regulated equal to the average cost (i.e., zero profit for carpooling operators).
Monopoly	Carpooling fares are set with no regulation.

Marginal Cost Regulation

We consider a case that the regulator sets marginal cost regulation on carpooling fares. In this case, carpooling fare P is set equal to the marginal cost:

$$P = c. \tag{3.4}$$

Average Cost Regulation

We then consider a case that the regulator sets average cost regulation on carpool fares. In this case, carpooling fare P is set equal to the average cost:

$$P = c + \frac{F}{N_p}. \tag{3.5}$$

Equation (3.5) shows that the more the carpooling commuters, the lower the carpooling fare P, reflecting the positive externalities (scale economies) that characterize carpooling commuting.

Monopoly

We finally consider a case where a carpooling operator sets the fare P with no regulation. In this case, the operator anticipates the carpooling demand $N_p(P)$ corresponding to P. That is, P is set to satisfy the first-order condition of the profit maximization problem (i.e., to maximize the operators' profit):

$$N_p(P) + (P - c)\frac{\partial}{\partial P}N_p(P) = 0. \tag{3.6}$$

3.2 Equilibrium Conditions

We consider commuters' choices regarding CBD arrival time and travel mode in the following 2 stages: commuters first decide their travel modes (first stage), and then choose their CBD arrival times (second stage).

In this case, equilibrium can be obtained by solving backward from the second stage. That is, in the first stage, given numbers of commuters $\boldsymbol{N} = \{N_s, N_p\}$ of travel modes in equilibrium (hereafter, we call $\boldsymbol{N}$ travel mode distribution). Commuters choose their CBD arrival time t to minimize their commuting costs c_s, c_p. Hence, equilibrium commuting costs c_s^*, c_p^* are the functions of travel mode distribution $\boldsymbol{N}$. Next, given equilibrium commuting costs c_s^*, c_p^*, commuters choose their travel modes. Hence, the travel mode distribution in equilibrium is determined.

Hereafter, we call the equilibria in the first and second stage described above "short-run equilibrium (equilibrium of mode choice)" and "long-run equilibrium (equilibrium of departure time)", respectively. In preparation for investigating the characteristics of these equilibria in the following sections, this section formulates the equilibrium conditions for each stage.

3.2.1 Short-run Equilibrium Conditions

As mentioned above, CBD arrival time choice for commuters is based on the premise that travel mode is fixed. Therefore, the short-run (departure time choice) equilibrium conditions are consistent with the equilibrium conditions of standard departure time choice model:

$$\begin{cases} c_k^* = c_k(t) & \text{if} \quad n(t) > 0 \\ c_k^* \leq c_k(t) & \text{if} \quad n(t) = 0 \end{cases} \quad \forall k \in \{s,\ p\}, \tag{3.7a}$$

$$\begin{cases} n_s(t) + \frac{n_p(t)}{2} = \mu & \text{if} \quad q(t) > 0, \\ n_s(t) + \frac{n_p(t)}{2} \leq \mu & \text{if} \quad q(t) = 0, \end{cases} \tag{3.7b}$$

$$\int_{-\infty}^{\infty} n_k(t)\,\mathrm{d}t = N_k \quad \forall k \in \{s,\ p\}. \tag{3.7c}$$

where $n_s(t) + \frac{n_p(t)}{2}$ is the number of automobiles arriving at CBD at time t (i.e., CBD arrival rate). This is because we assume that 2 people make a carpooling.

Condition (3.7a) is the no-arbitrage condition for CBD arrival time choices. This condition means that at equilibrium, each commuter has no incentive to change his/her CBD arrival time unilaterally. Condition (3.7b) is the capacity constraint of the bottleneck. It implies that the departure rate $n(t)$ at the bottleneck equals capacity μ if queue occurs at the bottleneck (i.e., $q(t) > 0$) at CBD arrival time (i.e., bottleneck outflow time) t; otherwise, the departure rate is lower than μ. Condition (3.7c) is flow conservation for automobile commuting demand.

Next, let us investigate the uniqueness of short-run equilibrium. Equilibrium conditions (3.7) of departure time choice is consistent with standard departure time choice models, as discussed in the previous section. Hence, as in Daganzo (1985) and Lindsey (2004), commuting costs $c_s^*,\ c_p^*$ in departure time choice equilibrium are uniquely determined, and are given by the following continuous function of $N_c,\ N_p$ (see **Appendix 3.A** for their derivations):

$$c_s^* = \frac{\delta}{\mu}\left(N_s + \frac{1}{2}N_p\right) + C_{\text{car}}, \tag{3.8a}$$

$$c_p^* = \frac{\delta}{\mu}\left(N_s + \frac{1}{2}N_p\right) + \theta(N_p) + P. \tag{3.8b}$$

where $N_s + \frac{1}{2}N_p$ denotes the total number of automobiles, and $\delta = \frac{\beta\gamma}{\beta+\gamma}$.

Condition (3.8a) shows that the total number of commuting automobiles is positively correlated to equilibrium commuting costs, which reflect the traffic congestion externalities during peak hours. We can also confirm that the types of regulations on carpooling operators have no direct effect on departure time choice equilibrium.

3.2.2 Long-run Equilibrium Conditions

Commuters' mode choices are based on the commuting cost $c_s^*,\ c_p^*$ in the departure time choice equilibrium, hence, equilibrium conditions of mode choice are given as follows:

$$\begin{cases} c^* = c_k^* & \text{if} \quad N_k > 0 \\ c^* \leq c_k^* & \text{if} \quad N_k = 0 \end{cases} \quad \forall k \in \{s,\ p\}, \tag{3.9a}$$

$$N_s + N_p = N. \tag{3.9b}$$

Condition (3.9a) is the equilibrium condition for commuters' mode choice, which expresses the following: if there are both solo driving and carpooling commuters, the commuting cost of both modes is the same; if only one of the modes has commuters, the commuting cost of the chosen mode is lower than the other. Condition (3.9b) is the conservation law of the number of commuters.

Since we consider scale economies in carpooling, multiple equilibria may exist that satisfy the long-run equilibrium condition (3.9). As a preparation for investigating the uniqueness and stability of the long-run equilibrium, let us show that a potential function $f(\boldsymbol{N})$ exists in our model.

The function $f(\boldsymbol{N})$ is a potential function if it satisfies the following condition for any $\boldsymbol{N}$ satisfying the population conservation law (3.9b):

$$\frac{\partial}{\partial N_s} f(\boldsymbol{N}) - \frac{\partial}{\partial N_p} f(\boldsymbol{N}) = -c_s^* + c_p^*. \tag{3.10}$$

where $c_s^*,\ c_p^*$ are clearly integrable from (3.8). Therefore, a potential function exists in our model that satisfies condition (3.10).

As in Sandholm (2001, 2010), the long-run equilibrium conditions (3.9a) and (3.9b) are equivalent to Karush-Kuhn-Tucker (KKT) conditions of the following optimization problem:

$$\max_{\boldsymbol{N}} f(\boldsymbol{N}) \tag{3.11a}$$

$$\text{s.t. } N_s + N_p = N,\ N_s \geq 0,\ N_p \geq 0. \tag{3.11b}$$

Therefore, the equilibrium set $\boldsymbol{N}^*$ exactly coincides with the set of KKT points for problem (3.11).

Then, we use the above properties to investigate the uniqueness of the long-run equilibrium. Since the KKT points of problem (3.11) are long-run equilibria, the uniqueness can be investigated by checking the shape of the potential function $f(\boldsymbol{N})$. That is, if $f(\boldsymbol{N})$ is unimodal, the long-run equilibrium is unique; otherwise, it is non-unique. Because the Hessian matrix of the potential function in our model is not necessarily negative definite, the potential function is not necessarily unimodal, which indicates that the equilibrium is not necessarily unique.

We next consider the local stability of long-run equilibria $\boldsymbol{N}^*$ because our model generally includes multiple equilibria as shown above by using the findings in Sandholm (2001). Stable and unstable equilibria have the following properties:

> Equilibria $\boldsymbol{N}^*$ that locally maximize the potential function are (locally) stable under a wide class of adjustment dynamics, [2] whereas other equilibria are unstable.

Hereafter, we characterize the long-run equilibria of each types of regulations.

3.3 Long-run Equilibrium

In this section, we characterize the long-run equilibria under "marginal cost regulation", "average cost regulation" and "monopoly".

[2]This adjustment dynamics includes the *best response dynamic* (Gilboa and Matsui, 1991), the *Brown-von Neumann-Nash* dynamic (Brown and von Neumann, 1950), and the *projection dynamic* (Dupuis and Nagurney, 1993), which have been adopted in many studies. If the equilibrium is an interior solution, replicator dynamic is also included. See Sandholm (2005) for other dynamics.

3.3.1 Marginal Cost Regulation

Under marginal cost regulation, carpooling fare P is set equal to marginal cost c. From this relationship and the equilibrium condition (3.9a) and (3.8), the numbers $N_p^{\mathrm{MC}*}, N_c^{\mathrm{MC}*}$ of carpooling commuters under equilibrium is uniquely determined as follows:

$$N_p^{\mathrm{MC}*} = \theta^{-1}(\Delta C). \tag{3.12}$$

where $\theta^{-1}(\cdot)$ is the inverse function of $\theta(\cdot)$, and $\Delta C = C_{\mathrm{car}} - c$.

3.3.2 Average Cost Regulation

We next characterize the long-run equilibrium in average cost regulation. The carpooling fare P set by carpooling operators equals to average cost (3.5), which is the marginal cost plus a fixed cost per commuter as a service charge. In this case, according to equilibrium conditions (3.9a) and (3.9b), the number $N_p^{\mathrm{AC}*}$ of carpooling commuters in equilibrium satisfies the following condition:

$$N_p^{\mathrm{AC}*} = \theta^{-1}\left(\Delta C - \frac{F}{N_p}\right). \tag{3.13}$$

where $\theta(\cdot)$ is a monotonically increasing function ($\theta^{-1}(\cdot)$ is also a monotonically increasing function), hence, there are at most 3 types of $N_p^{\mathrm{AC}*}$ satisfying condition (3.13) that satisfy the following condition:

$$N_{p1}^{\mathrm{AC}*} > N_{p2}^{\mathrm{AC}*} > N_{p3}^{\mathrm{AC}*} = 0. \tag{3.14}$$

To examine its stability, we show the potential function $f(\boldsymbol{N})$:

$$f(\boldsymbol{N}) = -\left\{\frac{\delta}{4\mu}\left(2N_s^2 + N_p^2 + 4N_s + 2N_p\right) + C_{\mathrm{car}}N_s + \int_0^{N_p} \theta(x)\,\mathrm{d}x + cN_p + F\ln N_p\right\}. \tag{3.15}$$

In this case, $N_p^{\mathrm{AC}*}$ that satisfies the following condition is an equilibrium.

$$\frac{\partial^2}{(\partial N_p^{\mathrm{AC}*})^2} f\left(N_p^{\mathrm{AC}*},\ N - N_p^{\mathrm{AC}*}\right) < 0. \tag{3.16}$$

That is, $N_p^{\mathrm{AC}*} \in [0, N]$ that locally maximize potential function $f\left(N_p^{\mathrm{AC}*},\ N - N_p^{\mathrm{AC}*}\right)$ is a stable equilibrium.

Then by using the fact that the equilibria satisfy the KKT conditions of the maximization problem of potential function, we find that $N_{p1}^{\mathrm{AC}*}$ and $N_{p3}^{\mathrm{AC}*}$ are stable equilibria, and $N_{p2}^{\mathrm{AC}*}$ is an unstable equilibrium. This is because when there are 3 types of N_p satisfying KKT conditions, only the 2 types at both sides will always maximize the potential function.

3.3.3 Monopoly

We finally investigate the long-run equilibrium properties of the case where carpooling operators determine the fare with no regulation on carpooling fare. The number of carpooling commuters $N_p^{\mathrm{m}*}$ and carpooling fare P obtained by solving profit maximization problem are uniquely determined and are as follows:

$$N_p^{\mathrm{m}*} = \theta^{-1}\left(\Delta C - N_p \theta'(N_p)\right). \tag{3.17}$$

$$P^{\mathrm{m}*} = \theta'(N_p) N_p + c. \tag{3.18}$$

3.4 Comparison of Long-run Equilibria

This chapter compares the characteristics of long-run equilibria under marginal cost regulation/ average cost regulation/no regulation obtained so far. We then use these results to clarify the impacts of the existence and types of regulations on carpooling operators on urban transportation system.

3.4.1 Comparison of the Travel Mode Distribution

Firstly, let us compare the number of carpooling commuters. From the results of the previous section, we obtain the following relation (see **Appendix 3.B** for the proof):

$$N_p^{\mathrm{MC}*} > N_{p1}^{\mathrm{AC}*} > N_p^{\mathrm{m}*} > N_{p2}^{\mathrm{AC}*} > N_{p3}^{\mathrm{AC}*} = 0. \tag{3.19}$$

This result is consistent with the intuition that the implementation of marginal cost regulation leads to the maximum use of carpooling. Meanwhile, we can also confirm the following properties of equilibria when the regulator sets carpooling fares equal to the average cost or without regulation on carpooling fares: The number of carpooling commuters $N_p^{\mathrm{m}*}$ in equilibrium with no regulation is more than the unstable equilibrium $N_{p2}^{\mathrm{AC}*}$ and less than the stable equilibrium $N_{p1}^{\mathrm{AC}*}$ under average cost regulation.

These results imply the following equilibrium properties:

1) If the average cost regulation is implemented simultaneously with the development of carpooling (the number of carpooling commuters is zero when the regulation is implemented), the number of carpooling commuters will not increase ($N_{p3}^{\mathrm{AC}*}$). This is because implementing average cost regulation when no one is using carpooling will case high fares, thus, commuters have no incentive to use carpooling.

2) If carpooling operators implement average cost regulation after monopoly (the number of carpooling commuters is $N_p^{\mathrm{m}*}$ when the regulation is implemented), the number of carpooling commuters will increase. This is because carpooling operators, in order to increase their own profits, set their fares lower than the average cost regulation and promote the use of carpooling.

Therefore, a hasty implementation of average cost regulation will hinder the use of carpooling.

In addition, (3.19) also shows the following property: if average cost regulation is implemented as the initial state with $N_p^{\mathrm{m}*}$, the number of carpooling commuters will increase and $N_{p1}^{\mathrm{AC}*}$ will be achieved. This is because evolutionary game theory shows that if the initial state is greater [resp. lower] than $N_{p2}^{\mathrm{AC}*}$ when implementing average cost regulation, the equilibrium changes to $N_{p1}^{\mathrm{AC}*}$ [resp. $N_{p3}^{\mathrm{AC}*}$].

The above results, assuming that the implementation of marginal cost regulation is not realistic, indicate that the following measures can be effective in increasing the number of carpooling commuters in the future: (1) Initially, achieve N_p^{m*} by allowing carpooling operators to be monopolistic without implementing regulation; (2) Then implement average cost regulation to achieve N_{p1}^{AC*}.

3.4.2 Comparison of Equilibrium Commuting Costs

Next, we compare equilibrium commuting costs c^*. From (3.8) and (3.9), equilibrium commuting cost is expressed as follows:

$$c^* = \left(N - \frac{1}{2}N_p^*\right)\frac{\delta}{\mu} + C_{\text{car}}. \tag{3.20}$$

This means that the more carpooling commuters, the lower the equilibrium commuting cost.

Based on the above, we obtain the following relationship for equilibrium commuting costs:

$$c^{\text{MC}*} < c_1^{\text{AC}*} < c^{\text{m}*} < c_2^{\text{AC}*} < c_3^{\text{AC}*}. \tag{3.21}$$

From this result, we can confirm that the marginal cost regulation leads to the lowest commuting cost, whereas the commuting cost under the average cost regulation can be higher than no regulation on carpooling fares.

3.4.3 Numerical Example

In this section, we numerically analyze our model and show the characteristics of the analysis results. Let us assume $\theta(N_p)$ as follows:

$$\theta(N_p) = N_p^2. \tag{3.22}$$

Then we use the following parameter values:

$$F = 10000,\ C_{\text{car}} = 2500,\ c = 1000,\ N = 55. \tag{3.23}$$

As shown in Section 3.4, α, β and γ have no effect on the number of carpooling commuters.

The number of carpooling commuters in different regulations is shown in Figure 3.2. We confirm that the number-magnitude relation of carpooling commuters is consistent with (3.19).

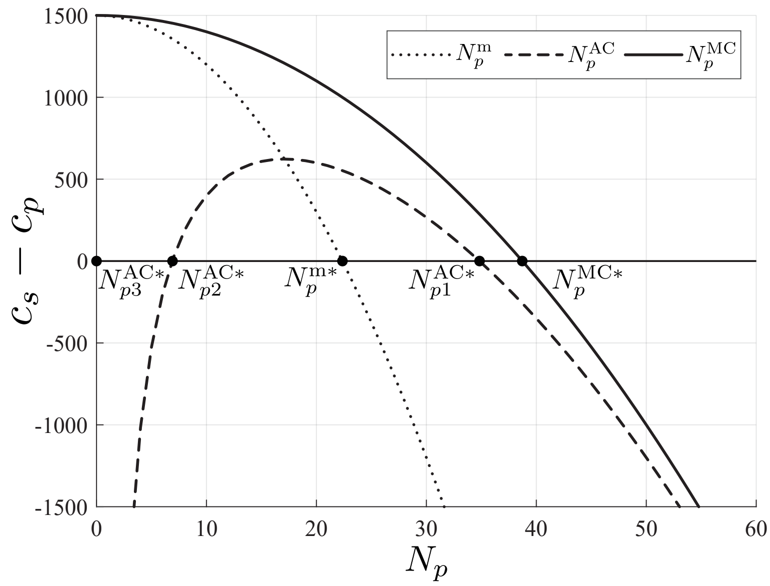


FIGURE 3.2: The number of carpooling commuters in different regulations (numerical example).

3.5 Summary and Discussions

In this chapter, we developed a framework considering commuters' departure time and mode choice behavior and scale economies in carpooling. We then show the properties of equilibria when the regulator sets carpooling fares equal to the marginal cost or average cost and when there is no regulation on carpooling fares.

By comparing these equilibria, we obtained the following findings:

1) The implementation of marginal cost regulation leads to the highest number of carpooling commuters and the lowest equilibrium commuting costs;

2) If average cost regulation is implemented simultaneously with the development of carpooling, the number of carpooling commuters will not increase and equilibrium commuting costs

will not change;

3) If average cost regulation is implemented when carpooling operators are monopolistically competitive, carpooling commuters will increase and equilibrium commuting costs will decrease.

These results imply that when it is difficult to implement marginal cost regulation, a hasty implementation of average cost regulation can lead to a socially undesirable situation (i.e., high carpooling fares and non-increasing in the number of carpooling commuters). The above results also indicate that one way to alleviate the problem is to "allow carpooling operators to be a monopoly to increase the number of carpooling commuters, and then implement average cost regulation." These results are unique to this chapter and have not been presented in previous studies.

Since this chapter focused on the impacts of the marginal cost/average cost regulations on carpooling fares, we only consider the equilibria under those regulations/without regulation. Thus, it is important to clarify the impacts of other regulations such as price-cap regulation (Kidokoro, 2006), compare social (i.e., first-best) optimum and second-best optimum as in Tabuchi (1993) and obtain insights on policies to achieve them (e.g., subsidies, time-of-day-varying fare, congestion pricing). Additionally, one of the main objectives of this paper is to construct the basis of a framework for analyzing commuter choice behavior in the context of economies of scale in carpooling commuting. For this reason, we analyzed the characteristics of equilibrium conditions under the assumption that there are only two travel modes (solo driving and carpooling) and that carpooling is made by 2 commuters. In the future, it will be necessary to relax these assumptions and verify measures to achieve the socially optimum and their effects.

Appendix

3.A Derivation of Equilibrium Commuting Costs

As proven in the studies using standard departure time choice model, the number of automobiles $n_s(t) + \frac{n_p(t)}{2}$ satisfying short-run equilibrium conditions satisfies the following condition:

$$n_s(t) + \frac{n_p(t)}{2} = \begin{cases} \mu & \text{if} \quad t \in [t^{\mathrm{E}}, t^{\mathrm{L}}] \\ 0 & \text{otherwise} \end{cases} \tag{3.24}$$

where, t^{E} [t^{L}] is the earliest [latest] CBD arrival time of commuters and is given as follows based on short-run equilibrium conditions:

$$t^{\mathrm{E}} = t^* - \frac{\gamma}{(\beta+\gamma)\mu}\left(N_s + \frac{N_p}{2}\right), \tag{3.25a}$$

$$t^{\mathrm{L}} = t^* + \frac{\beta}{(\beta+\gamma)\mu}\left(N_s + \frac{N_p}{2}\right). \tag{3.25b}$$

Since we assume that 2 people make a carpooling, the total number of automobiles is $N_s + \frac{N_p}{2}$.

Then, by using equilibrium commuting costs c_s^* and c_p^* expressed as follows, we obtain (3.8).

$$c_s^* = c_s(t^{\mathrm{E}}) = c_s(t^{\mathrm{L}}) = \eta(t^{\mathrm{E}}) + C_{\mathrm{car}}, \tag{3.26a}$$

$$c_p^* = c_p(t^{\mathrm{E}}) = c_p(t^{\mathrm{L}}) = \eta(t^{\mathrm{E}}) + P + \theta(N_p). \tag{3.26b}$$

3.B Derivation of Equilibrium Commuting Cost with no Regulation

We solve the profit maximization problem of the carpooling operator and obtain the fare P which maximizes the profit. In long-run equilibrium, if each travel mode has commuters, equilibrium commuting cost of each mode is equal, hence, based on (3.8a) and (3.8b), carpooling fare P can be expressed as follows:

$$P = C_{\mathrm{car}} - \theta(N_p). \tag{3.27}$$

Substituting (3.27) into (3.3), profit π can be rewritten as follows:

$$\pi = \left[C_{\mathrm{car}} - \theta(N_p)\right] N_p - cN_p - F. \tag{3.28}$$

The first-order condition of the profit maximization problem gives the following:

$$C_{\mathrm{car}} - \theta(N_p) - N_p\theta'(N_p) - c = 0. \tag{3.29}$$

Then using (3.27) and (3.29), carpooling fare $P^{\mathrm{m}*}$ is given by the following:

$$P^{\mathrm{m}*} = c + N_p\theta'(N_p). \tag{3.30}$$

By using (3.30) and (3.9), the number of carpooling commuters with no regulation is expressed as follows:

$$N_p^{\mathrm{m}*} = \theta^{-1}\left(C_{\mathrm{car}} - c + N_p\theta'(N_p)\right). \tag{3.31}$$

3.C Proof of the Number-Magnitude Relation of Carpooling Commuters

We investigate the number-magnitude relation of carpooling commuters. Firstly, based on (3.12), (3.17) and the assumption that $\theta^{-1}(\cdot)$ is monotonically increasing, the magnitude relation of the number of carpooling commuters under marginal cost regulation and without regulation is given by the following:

$$N_p^{\mathrm{MC}*} - N_p^{\mathrm{m}*} = \theta^{-1}(\Delta C) - \theta^{-1}\left(\Delta C - N_p^{\mathrm{m}*}\theta'(N_p^{\mathrm{m}*})\right) > 0. \tag{3.32}$$

Hence, $N_p^{\mathrm{MC}*} > N_p^{\mathrm{m}*}$, that is, the number of carpooling commuters under marginal cost regulation is greater than without regulation.

Next, (3.12) and (3.13) gives the magnitude relation of the number of carpooling commuters under marginal and average cost regulation as follows:

$$N_p^{\mathrm{MC}*} - N_p^{\mathrm{AC}*} = \theta^{-1}(\Delta C) - \theta^{-1}\left(\Delta C - \frac{F}{N_p^{\mathrm{AC}*}}\right) > 0. \tag{3.33}$$

Hence, we obtain that $N_p^{\mathrm{MC}*} > N_p^{\mathrm{AC}*}$.

Finally, we investigate the magnitude relation of the number of carpooling commuters under average cost regulation and without regulation. We assume that carpooling operators' profits are positive in equilibrium with no regulation (∵ if profits are negative, operators will exist):

$$\pi = PN_p^{\mathrm{m}*} - cN_p^{\mathrm{m}*} - F > 0. \tag{3.34}$$

Moreover, it follows from (3.18) that $P = c + N_p^{\mathrm{m}*}\theta'(N_p^{\mathrm{m}*})$, thus, (3.34) can be rewritten as follows:

$$N_p^{\mathrm{m}*}\theta'(N_p^{\mathrm{m}*}) - \frac{F}{N_p^{\mathrm{m}*}} > 0. \tag{3.35}$$

Thus, the number of carpooling commuters with no regulation satisfies the following condition:

$$N_p^{\mathrm{m}*} = \theta^{-1}\left(\Delta C - N_p\theta'(N_p)\right) < \theta^{-1}\left(\Delta C - \frac{F}{N_p}\right). \tag{3.36}$$

The result shows $N_{p2}^{\mathrm{AC}*} < N_p^{\mathrm{m}*} < N_{p1}^{\mathrm{AC}*}$, which is consistent with the result in Figure 3.3.

Therefore, from the above results, we obtain (3.19).

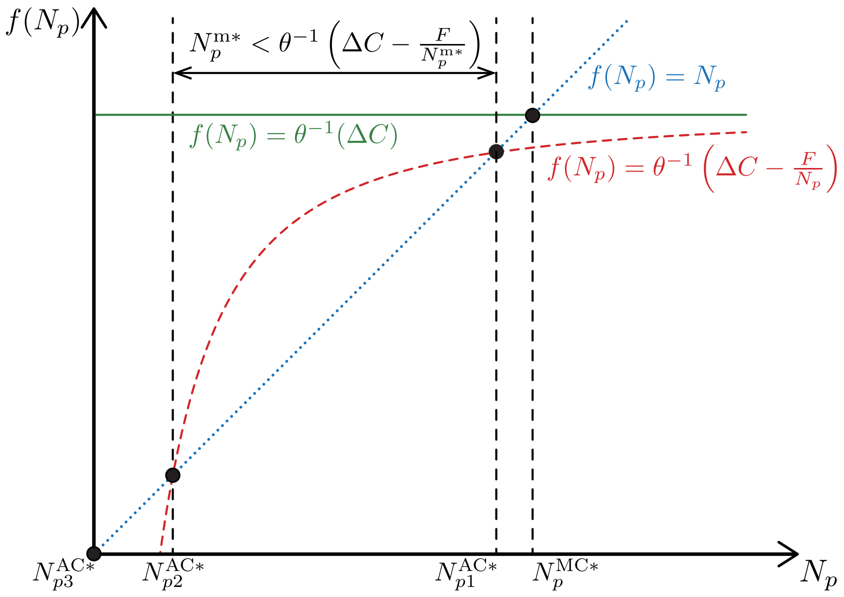


FIGURE 3.3: The number of carpooling commuters in different regulations.

Part II

Spatial and Temporal Distributions of Economic Activities

Introduction

In Part II, we review existing literatures on temporal and spatial externalities.

Studies considering temporal agglomeration economies and diseconomies have been accumulated since the seminal work of Henderson (1981). After Henderson (1981) , a multitude of studies considering WSTs distribution and traffic congestion appeared (e.g., Arnott et al., 2005; Yoshimura and Okumura, 2001; Mun and Yonekawa, 2006; Takayama, 2015; Fosgerau and Small, 2017) . However, these studies assumed the same spatial structure as Henderson (1981) that urban spatial structure is composed of a single CBD, a single residential area and a congestible road connecting the two areas. Thus, these previous studies cannot explain the changes in urban land use of economic activities.

Studies on spatial structure side also have been accumulated over the years. Henderson (1974) spearheaded a system-of-cities theory concerned with optimal city size. Fujishima (2013) was the first to apply potential game approach on system-of-cities theory by considering Pigouvian tax policies in the presence of multiple equilibria in order to achieve an optimal city size distribution. Moreover, studies of urban land use theory (e.g., Fujita, 1989; Kanemoto, 1980; Anas et al., 1998; Brinkman, 2016) that examined the regularity of urban spatial structure (interaction between transportation and land use), traced to Alonso (1964), have been developed. However, these theories did not consider the effect of WSTs distribution on traffic congestion. As a result, previous system-of-cities theory and land use theory cannot be directly applied to investigate the changes in WSTs by implementing TDM measures.

So far, only Wilson (1992) and Takayama (2019) considered the interaction between WSTs distribution and land use pattern in a closed monocentric city. However, Wilson (1992) only considered negative spatial and temporal externalities. Although Takayama (2019) considered negative spatial externalities as well as positive and negative temporal externalities, positive spatial externalities are yet to be discussed.

4

An Open City Model Considering Spatial and Temporal Agglomeration Economies

4.1 The model

4.1.1 Basic assumptions

We consider a spatial structure that consists of one monocentric city and one rural area (Figure 4.1). Let the urban population be $N_u = \sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n_{a,i}$, the rural population be N_r, and the total population of the two regions be fixed at N. The number of residential locations in the city is A. Then, we index the residential locations from the side of CBD and let $\mathcal{A} \equiv \{1, 2, \dots, A\}$ be the set of locations. The area of every residential land is the same at L.

All firms are located in the CBD of their cities and each firm chooses its WST from the feasible set $\{t_1, t_2, \dots, t_I\}$, where $t_i = t_{i-1} + \tau$ for all $i \in \{2, 3, \cdots, I\}$ and τ is a positive constant. The length of a workday is assumed to be identical and fixed at H for all firms; therefore, each firm is characterized by its WST. For convenience, we call the firm that starts work at time t_i "firm i." We further assume the existence of an interval in the workday when all firms begin work (i.e., $t_T < t_1 + H$).

All roads connecting each location are homogeneous, and we further call the road between location $a-1$ and a "road a." The number of workers who start work at t_i passing through road a is denoted by $x_{a,i}$ and expressed by

$$x_{a,i} = \sum_{b=a}^{A} n_{b,i}. \tag{4.1}$$

We assume the transportation cost of workers at firm i passing through road a is $c(x_{a,i})$. Like Henderson (1981), we assume that $c(x_{a,i})$ is a nonnegative, monotonically increasing, and strictly convex function.

Behavior of workers

All workers are *ex ante* identical. Assume that workers commute from their locations to CBD only in their cities. Each worker chooses his or her WST t_i indirectly by choosing an employer (i.e., a firm $i \in \mathcal{I} \equiv \{1, 2, \ldots, I\}$). The number of workers who reside at location a and work at firm i is denoted by $\boldsymbol{n} = (n_{a,i})_{a\in\mathcal{A}, i\in\mathcal{I}}$, and we further call it the *population and WST distribution*. The number N_a of workers working at firm i and the number M_i of workers

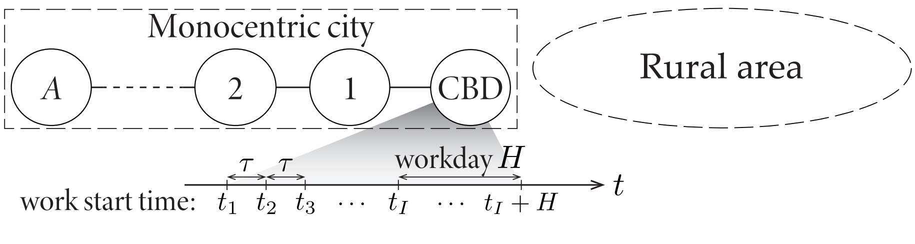


FIGURE 4.1: One Monocentric City and One Rural Area.

residing in location a are expressed, respectively, as follows:

$$N_a = \sum_{i \in \mathcal{I}} n_{a,i}, \tag{4.2a}$$

$$M_i = \sum_{a \in \mathcal{A}} n_{a,i}. \tag{4.2b}$$

The utility of workers who reside at a and start work at t_i is given by the following quasi-linear function:

$$u(z_{a,i}, y_a, t_i) = z_{a,i} + f(y_a) - \delta_i(t_i). \tag{4.3}$$

where $z_{a,i}$ denotes consumption of numéraire goods, y_a is the lot size at a, and $f(y_a)$ is the utility from land consumption. We assume that $f(x)$ is strictly monotonically increasing, concave, and twice differentiable for $x > 0$. Moreover, $\lim_{x\to 0} f'(x) = \infty$, $\lim_{x\to\infty} xf'(x) < \infty$. [1] $\delta_i(t_i)$ denotes schedule delay cost and is given by the following:

$$\delta_i(t_i) = \gamma|t_i - t^*|. \tag{4.4}$$

where $\gamma > 0$ is early/late delay cost per unit time. [2] t^* is the most desired work schedule for commuters that best fits in with their daily activities (e.g., leisure activities with family and friends), thus, commuters with WST t^* do not incur schedule delay cost. [3]

The budget constraint is expressed as

$$w_i = z_{a,i} + r_a y_a + \sum_{b=1}^{a} c(x_{b,i}). \tag{4.5}$$

where w_i denotes the wage from firm i and r_a denotes the land rent at a. Agricultural rent does not qualitatively change subsequent results; hence, we assume it to be zero.

[1] We assume that the WSTs which firms can choose is discrete. That is because most of WSTs are clustered at several points in time such as 8:00, 8:30, 9:00.

[2] In this chapter, based on the empirical findings by Hall (2021), we assume that the marginal schedule delay costs for early and late arrival are equal. Besides, this assumption does not qualitatively change the results obtained from this chapter.

[3] Note that schedule delay cost here is different with that in Part I.

The first-order condition of the utility maximization problem gives the following:

$$\begin{cases} f'(y_a) = r_a & \text{if} \quad y_a > 0 \\ f'(y_a) \leq r_a & \text{if} \quad y_a = 0 \end{cases} \qquad \forall a \in \mathcal{A}. \tag{4.6}$$

where the prime denotes differentiation. The marginal utility of land consumption is infinity at $y_a = 0$; thus, we must have $y_a > 0$ and

$$r_a = f'(y_a) > 0 \qquad \forall a \in \mathcal{A}. \tag{4.7}$$

Let L denote the land supply in every residential location, and land demand of residential land a is given by $N_a y_a$. Thus, according to the supply-demand equilibrium, $y_a = L/N_a$ is obtained.

From (4.3), (4.5), and (4.7), we obtain the following indirect utility function $v_{a,i}$:

$$v_{a,i} = w_i - \sum_{b=1}^{a} c(x_{b,i}) + h(N_a) - \delta_i(t_i). \tag{4.8}$$

where $h(N_a) = f(y_a) - r_a y_a = f(\frac{L}{N_a}) - \frac{L}{N_a} f'(\frac{L}{N_a})$ at location a. $h(N_a)$ can be rewritten as $f(y_a) - r_a y_a$; hence, this represents net utility from land consumption. Furthermore, since

$$h'(N_a) = \frac{L^2 f''(\frac{L}{N_a})}{{N_a}^3} < 0. \tag{4.9}$$

$h(N_a)$ is a strictly decreasing function. That is, the more workers the less net utility from land consumption.

Behavior of firms

All firms produce homogeneous goods under constant returns to scale technology and perfect competition with free entry and exit, which require one unit of labor to produce one unit of output and is chosen as numéraire. We introduce the same productivity effect as in Henderson (1981) and Tabuchi (1986). That is, the longer the overlapping time interval of firms and the greater the number of firms located in one city, the greater the productivity. This implies that a firm's productivity depends on the number of workers $\boldsymbol{M} = (M_i)_{i\in\mathcal{I}}$ of cities and WSTs.

Specifically, we define that G_i is the daily output of a firm i and determined by $\boldsymbol{M} = (M_i)_{i\in\mathcal{I}}$ which shows the distribution of each WST. G_i is expressed as follows:

$$G_i(\boldsymbol{M}) = \alpha \sum_{j\in\mathcal{I}} e^{-|t_i-t_j|} M_j = \alpha \sum_{j\in\mathcal{I}} e^{-\tau|i-j|} M_j. \tag{4.10}$$

Eq. (4.10) shows the effect of positive temporal agglomeration externalities on firms' productivity. That is, the more clustered the distribution of WSTs, the greater the productivity effects. $\alpha > 0$ denotes the magnitude of the productivity effects. Let $1 - \phi \equiv e^{-\tau}$ and denote $\phi \in [1, \infty]$ as a temporal discounting rate that the greater the value of ϕ, the greater the positive temporal agglomeration externality (i.e., the more necessity of synchronizing different firms' work schedules). Thus, (4.10) can be rewritten as follows:

$$G_i(\boldsymbol{M}) = \alpha \sum_{j\in\mathcal{I}} (1-\phi)^{|i-j|} M_j. \tag{4.11}$$

Under the production function defined in (4.11), each firm chooses its city and WST to maximize profit per worker:

$$\max_i \pi_i = G_i(\boldsymbol{M}) - w_i. \tag{4.12}$$

4.1.2 Equilibrium

In our model, each firm chooses a WST, and each person chooses to live in urban area as a worker or in rural area as a farmer. If a person chooses to live in urban area, he/she will choose a residential location, and an employer; if a person chooses to live in rural area, Therefore, the equilibrium distributions of urban population and WSTs $\boldsymbol{n}^*$ can be determined. We then describe these equilibrium conditions that satisfy $\boldsymbol{n}^*$. Hereafter, we use superscript $*$ to distinguish variables relating to equilibrium.

For all $a \in \mathcal{A},\ i \in \mathcal{I}$, equilibrium satisfies the following conditions:

$$\begin{cases} \bar{u} = w_i - \sum_{b=1}^{a} c(x_{b,i}) + h(N_a) - \delta_i(t_i) & \text{if} \quad n_{a,i} > 0, \\ \bar{u} \geq w_i - \sum_{b=1}^{a} c(x_{b,i}) + h(N_a) - \delta_i(t_i) & \text{if} \quad n_{a,i} = 0, \end{cases} \tag{4.13a}$$

$$\begin{cases} \pi^* = G_i(\boldsymbol{M}) - w_i & \text{if} \quad \sum_{a \in \mathcal{A}} n_{a,i} > 0, \\ \pi^* \geq G_i(\boldsymbol{M}) - w_i & \text{if} \quad \sum_{a \in \mathcal{A}} n_{a,i} = 0, \end{cases} \tag{4.13b}$$

$$\sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} n_{a,i} + N_r = N. \tag{4.13c}$$

where $\bar{u}$ is the utility of rural area, and π^* is the equilibrium profit, which equals zero because of free entry and exit of firms.

Conditions (4.13a) and (4.13b) are the equilibrium conditions for workers' choice of firm and firms' choice of WST, respectively. Condition (4.13a) implies that at equilibrium, each worker has no incentive to change employer unilaterally. Condition (4.13b) means that if workers are employed by firm i, the firm earns the equilibrium profit $\pi^* = 0$; otherwise, the profit must be less than zero. Condition (4.13c) is the conservation law of the population of workers.

We easily show that conditions (4.13a) and (4.13b) can be rewritten as the following condition because $\pi^* = 0$.

$$\begin{cases} \bar{u} = v_{a,i}(\boldsymbol{n}) & \text{if} \quad n_{a,i} > 0, \\ \bar{u} \geq v_{a,i}(\boldsymbol{n}) & \text{if} \quad n_{a,i} = 0, \end{cases} \tag{4.14a}$$

$$\sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} n_{a,i} + N_r = N. \tag{4.14b}$$

where $v_{a,i}(\boldsymbol{n})$ denotes the indirect utility of workers living in location a and employed by firm i as

$$v_{a,i}(\boldsymbol{n}) = G_i(\boldsymbol{M}) - \sum_{b=1}^{a} c(x_{b,i}) + h(N_a) - \delta_i(t_i). \tag{4.15}$$

4.2 Equilibria Characterization

We now characterize the equilibrium. First, we indicate the uniqueness and stability of equilibrium by using the properties of a potential game. We next clarify the properties of workers' distributions of different residential locations and WSTs.

4.2.1 Potential game

To characterize the equilibrium, we invoke the properties of *potential game* introduced by Monderer and Shapley (1996) and Sandholm (2001). The equilibrium conditions are represented by (4.14); hence, the equilibrium $\boldsymbol{n}^*$ can be viewed as a population game in which the set of players is $\mathcal{S} \equiv [0, N_u]$, the common action set is $\mathcal{A} \times \mathcal{I}$, and the payoff vector is $\boldsymbol{v}(\boldsymbol{n}) = (v_{a,i}(\boldsymbol{n}))_{a\in\mathcal{A}, i\in\mathcal{I}}$. As is evident from the definition, the equilibrium is a Nash equilibrium of the game. Thus, we denote this game by $S = \{\mathcal{S},\ \mathcal{A} \times \mathcal{I},\ \boldsymbol{v}\}$. We define that S is a potential game if a continuously differentiable function $P(\boldsymbol{n})$ exists, such that

$$\frac{\partial P(\boldsymbol{n})}{\partial n_{a,i}} - \frac{\partial P(\boldsymbol{n})}{\partial n_{b,j}} = v_{a,i}(\boldsymbol{n}) - v_{b,j}(\boldsymbol{n}) \quad \forall a \in \mathcal{A}, i \in \mathcal{I} \text{ and } \boldsymbol{n} \in \Delta. \tag{4.16}$$

where $\Delta \equiv \left\{\boldsymbol{n} \in \mathbb{R}_+^{A\times I} \;\middle|\; \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n_{a,i} = N_u\right\}$ denotes the set of workers' spatial and temporal distributions, and $P(\boldsymbol{n})$ is defined on an open set containing Δ so that its partial derivative is well-defined on Δ. This condition requires the existence of a function in which gradient $\nabla P(\boldsymbol{n})$ equals the payoff vector $\boldsymbol{v}$. Sandholm (2001) demonstrated that, if payoffs $\boldsymbol{v}(\boldsymbol{n})$ are continuously differentiable, this condition is equivalent to the following condition called *externality symmetry*:

$$\frac{\partial v_{a,i}(\boldsymbol{n})}{\partial n_{b,j}} = \frac{\partial v_{b,j}(\boldsymbol{n})}{\partial n_{a,i}} \quad \forall a, b \in \mathcal{A},\ i, j \in \mathcal{I} \text{ and } \boldsymbol{n} \in \Delta. \tag{4.17}$$

From (4.15), $\frac{\partial v_{a,i}(\boldsymbol{n})}{\partial n_{b,j}}$ can be rewritten as follows:

$$\frac{\partial v_{a,i}(\boldsymbol{n})}{\partial n_{b,j}} = \alpha(1-\phi)^{|i-j|} = \frac{\partial v_{b,j}(\boldsymbol{n})}{\partial n_{a,i}}. \tag{4.18}$$

Thus, game S satisfies (4.17). Moreover, rural area satisfies the following condition:

$$\frac{\partial P(\boldsymbol{n})}{\partial N_r} = \bar{u}. \tag{4.19}$$

Therefore, we derive the following proposition.

Proposition 4.1 Game S is a potential game with the potential function

$$P(\boldsymbol{n}) = P_1(\boldsymbol{n}) - P_2(\boldsymbol{n}) - P_3(\boldsymbol{n}) - P_4(\boldsymbol{n}) + P_5(\boldsymbol{n}). \tag{4.20}$$

where $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$ and $P_3(\boldsymbol{n})$ are convex functions that respectively in terms of the effects on productivity, traffic congestion and land consumption; $P_4(\boldsymbol{n})$, $P_5(\boldsymbol{n})$ are linear functions respectively in terms of the effect on schedule delay cost and rural population change such that

$$P_1(\boldsymbol{n}) = \frac{1}{2}\sum_{i\in\mathcal{I}} M_i G_i(\boldsymbol{M}), \tag{4.21a}$$

$$\frac{\partial P_2(\boldsymbol{n})}{\partial n_{a,i}} = \sum_{b=1}^{a} c(x_{b,i}), \tag{4.21b}$$

$$P_3(\boldsymbol{n}) = -\sum_{a\in\mathcal{A}} N_a f\left(\frac{L}{N_a}\right), \tag{4.21c}$$

$$P_4(\boldsymbol{n}) = \sum_{i\in\mathcal{I}} M_i \delta_i(t_i), \tag{4.21d}$$

$$P_5(\boldsymbol{n}) = \bar{u}\left(N - \sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n_{a,i}\right). \tag{4.21e}$$

Proof See Appendix 4.A.1.

The equilibrium of a potential game is characterized by the maximization problem of the potential function. Let us consider the following problem:

$$\max_{\boldsymbol{n}} P(\boldsymbol{n}) \quad \text{s.t.} \sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n_{a,i} \leq N,\ n_{a,i} \geq 0 \quad \forall a \in \mathcal{A}, i \in \mathcal{I}.$$

Let v^* be a Lagrange multiplier for the constraint $\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n_{a,i} \leq N$. The first-order condition is $\frac{\partial P(\boldsymbol{n})}{\partial n_{a,i}} \leq \bar{u}$ in which the equality holds whenever $n_{a,i} > 0$. Then, by (4.16), we have $v_{a,i}(\boldsymbol{n}) = v_{b,j}(\boldsymbol{n})$ for any residential locations a and b, WSTs i and j. We also have $v_{c,k}(\boldsymbol{n}) \leq v_{a,i}(\boldsymbol{n})$ if $n_{c,k} = 0$ and $n_{a,i} > 0$, for all $a, c \in \mathcal{A}, i, k \in \mathcal{I}$. Thus, $\boldsymbol{n}$ is an equilibrium. By similar reasoning, it follows that the converse is also true. That is, if $\boldsymbol{n}$ is an equilibrium, it satisfies the necessary condition for problem (4.22). We then can readily verify that the Karush-Kuhn-Tucker (KKT) conditions of this problem (4.22) are equivalent to equilibrium conditions (4.14). Therefore, the equilibrium set of the game S exactly coincides with the set of KKT points for problem (4.22).

4.2.2 Uniqueness

To characterize the equilibrium, we first examine its uniqueness. The KKT points of problem (4.22) are equilibria; thus, the uniqueness can be investigated by checking the shape of potential function $P(\boldsymbol{n})$. Specifically, if $P(\boldsymbol{n})$ is unimodal, the equilibrium is unique; otherwise, it is non-unique. It follows from this property and the convexity of $P(\boldsymbol{n})$ that we have the following proposition.

Proposition 4.2 The equilibrium is generally non-unique.

Proof $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$, and $P_3(\boldsymbol{n})$ are convex functions; hence, $P(\boldsymbol{n})$ is not generally a concave function but can be a convex function. Therefore, $P(\boldsymbol{n})$ is not generally unimodal.

4.2.3 Stability

We next consider the local asymptotic stability of equilibria because our model generally includes multiple equilibria as shown in Proposition 4.2. Specifically, we examine whether we can justify an equilibrium through the existence of a learning process that makes players settle down in their equilibrium strategies. This chapter describes adjustment dynamics $\dot{\boldsymbol{n}} = \boldsymbol{V}(\boldsymbol{n})$ that maps the distributions of urban population and WSTs $\boldsymbol{n}^0 \in \Delta$ to a set of Lipschitz paths in Δ, which starts from $\boldsymbol{n}^0$. Although we usually consider a specific evolutionary dynamic for stability analysis, we see that a more general analysis is possible due to the existence of a potential function. That is, the stability of equilibria can be characterized under a broad class of dynamics. In particular, we consider the class of admissible dynamics which satisfies the following conditions:

(PC) $\boldsymbol{V}(\boldsymbol{n}) \neq 0$ implies $\boldsymbol{V}(\boldsymbol{n}) \cdot \boldsymbol{v}(\boldsymbol{n}) = \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} V_{a,i}(\boldsymbol{n}) v_{a,i}(\boldsymbol{n}) > 0$.

(NS) $\boldsymbol{V}(\boldsymbol{n}) = 0$ implies that $\boldsymbol{n}$ is a Nash equilibrium of the game S.

Condition (PC), called *positive correlation*, requires a positive correlation between the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$ and the payoffs $\boldsymbol{v}(\boldsymbol{n})$ out of rest points. This implies that, under this condition, all Nash equilibria of the game S are rest points of the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$. [4] Condition (NS), called Nash stationarity, asks that every rest point of the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$ be a Nash equilibrium of game S. Therefore, under the conditions (PC) and (NS), $\dot{\boldsymbol{n}} = \boldsymbol{V}(\boldsymbol{n}) = 0$ if and only if $\boldsymbol{n}$ is a Nash equilibrium of game S. Specific examples of admissible dynamics include the *best response dynamic* (Gilboa and Matsui, 1991), the *Brown-von Neumann-Nash* dynamic (Brown and von Neumann, 1950), and the *projection dynamic* (Dupuis and Nagurney, 1993). [5] Importantly, the replicator dynamics (Taylor and Jonker, 1978), which is often used in spatial economic models (e.g., Fujita et al., 1999), are *not* admissible. Under replicator dynamics, a rest point is always attained on the boundary, but the boundary points are not always Nash equilibria. Therefore, condition (NS) does not hold under replicator dynamics.

As demonstrated by Sandholm (2001) that a Nash equilibrium of a potential game is asymptotically stable under any admissible dynamics if and only if it locally maximizes an associated potential function. We have the following property:

For game S, equilibrium $\boldsymbol{n}^{s*}$, which locally maximizes the potential function $P(\boldsymbol{n})$, is (locally) stable under admissible dynamics. Other equilibria $\boldsymbol{n}^{u*}$ are unstable.

Therefore, we can examine the stability of equilibria only by checking the shape of the potential function.

[4]See Proposition 4.3 of Sandholm (2001).
[5]See Sandholm (2005) for more examples.

Because $P_4(\boldsymbol{n})$ (temporal agglomeration diseconomies due to schedule delay) and $P_5(\boldsymbol{n})$ (economic effects of rural population) are linear functions, the shape of the potential function $P(\boldsymbol{n})$ given by (4.22) depends on $P_1(\boldsymbol{n})$ (spatio-temporal agglomeration economies due to productivity effects), $P_2(\boldsymbol{n})$ (spatio-temporal agglomeration diseconomies due to traffic congestion) and $P_3(\boldsymbol{n})$ (spatial agglomeration diseconomies due to land consumption). Therefore, spatio-temporal agglomeration economies and diseconomies determine stable equilibrium in our model.

In fact, if $P_1(\boldsymbol{n})$ is dominant and the potential function $P(\boldsymbol{n})$ is convex, the state of concentrated intercity population and clustered WSTs is a stable equilibrium. Moreover, if $P_2(\boldsymbol{n})$, $P_3(\boldsymbol{n})$ are dominant and $P(\boldsymbol{n})$ is concave, the state of dispersed urban population and staggered WSTs is the only equilibrium. By using the properties of the potential function, when $\boldsymbol{n}^0$ is an initial state, stable equilibrium can be obtained by searching from $\boldsymbol{n}^0$ to local maximizer $\boldsymbol{n}^*$. That is, stable equilibrium can be easily obtained by locally solving a simple optimization problem. Numerical analysis will be performed by using the characteristics in Section 5.

4.2.4 Distributions of urban population and WSTs

We next characterize the distributions of urban population and WSTs under equilibrium. The utility function (4.15) of the workers in location a can be rewritten as follows:

$$v_{a,i}(\boldsymbol{n}) = v_{a-1,i}(\boldsymbol{n}) - c(x_{a,i}) + h(N_a) - h(N_{a-1}). \tag{4.22}$$

Let $\text{supp}(\boldsymbol{n}^*)$ be the support of equilibrium $\boldsymbol{n}^*$ (i.e., $\text{supp}(\boldsymbol{n}^*) = \left\{(a,i) \,\middle|\, n^*_{a,i} > 0,\ a \in \mathcal{A}, i \in \mathcal{I}\right\}$). By using $\text{supp}(\boldsymbol{n}^*)$, we have

$$v_{a,i}(\boldsymbol{n}^*) \begin{cases} = v^* - c(x^*_{a,i}) + h(N^*_a) - h(N^*_{a-1}) & \text{if} \quad (a-1,i) \in \text{supp}(\boldsymbol{n}^*), \\ \leq v^* - c(x^*_{a,i}) + h(N^*_a) - h(N^*_{a-1}) & \text{if} \quad (a-1,i) \notin \text{supp}(\boldsymbol{n}^*). \end{cases} \tag{4.23}$$

where $v^*(= \bar{u})$ denotes the equilibrium urban utility. From (4.23), we can obtain the following proposition.

Proposition 4.3 Equilibrium $\boldsymbol{n}^*$ has the following properties.

(i) Suppose $N_a^* > 0$ and $(a-1, i), (a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, then, $(a, i), (a, j) \in \text{supp}(\boldsymbol{n}^*)$ and $x_{a,i}^* = x_{a,j}^* > 0$.

(ii) Suppose $N_a^* > 0$, $(a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, then, $n_{a,i}^* \leq n_{a,j}^*$ and $x_{a,i}^* \leq x_{a,j}^*$.

Proof See 4.A.2.

This proposition shows that the nearer the CBD where workers live, the narrower their distribution of WSTs. That is, $\text{supp}((n_{a-1,i}^*)_{i\in\mathcal{I}}) \subseteq \text{supp}((n_{a,i}^*)_{i\in\mathcal{I}})$, and this result exactly coincides with the empirical observation of Fosgerau and Kim (2019).

Moreover, since if $(a, i),\ (a-1, i) \in \text{supp}(\boldsymbol{n}^*)$, then $v_{a,i}(\boldsymbol{n}^*) = v_{a-1,i}(\boldsymbol{n}^*)$. We obtain

$$h(N_a^*) - h(N_{a-1}^*) = c(x_{a,i}^*) > 0. \tag{4.24}$$

Hence, we have the following proposition.

Proposition 4.4 Suppose $N_a^* > 0$ and $N_{a-1}^* > 0$ under equilibrium, then $N_a^* < N_{a-1}^*$. That is, the nearer the residence is from the CBD, the greater the population.

4.3 Optimum

The equilibrium is not generally efficient because of the positive and negative externalities. Therefore, this section discusses TDM policies, such as staggered work hours and taxation, for achieving the optimal distributions of urban population and WSTs. To address this issue, we first define the social welfare function and then analyze Pigouvian policies' effectiveness for achieving local optimum. Hereafter, we use superscript o to distinguish variables relating to local optimum.

4.3.1 Definition of social welfare function

We define the social welfare function as the sum of producer surplus and consumer surplus. Thus, social welfare maximization problem is given by the following:

$$\max_{\boldsymbol{n}} W(\boldsymbol{n}) = W_1(\boldsymbol{n}) - W_2(\boldsymbol{n}) - W_3(\boldsymbol{n}) - W_4(\boldsymbol{n}) + W_5(\boldsymbol{n})$$
$$\text{s.t. } \boldsymbol{n} \in \Delta. \tag{4.25}$$

where, $W_1(\boldsymbol{n})$, $W_2(\boldsymbol{n})$, $W_3(\boldsymbol{n})$, $W_4(\boldsymbol{n})$ and $W_5(\boldsymbol{n})$ are respectively in terms of the effects of productivity, traffic congestion, land consumption, schedule delay cost and rural population, and are expressed as follows:

$$W_1(\boldsymbol{n}) = \sum_{i \in \mathcal{I}} M_i G_i(\boldsymbol{M}) = 2P_1(\boldsymbol{n}), \tag{4.26a}$$
$$W_2(\boldsymbol{n}) = \sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} x_{a,i} c(x_{a,i}), \tag{4.26b}$$
$$W_3(\boldsymbol{n}) = -\sum_{a \in \mathcal{A}} N_a f(\frac{L}{N_a}) = P_3(\boldsymbol{n}), \tag{4.26c}$$
$$W_4(\boldsymbol{n}) = \sum_{i \in \mathcal{I}} M_i \delta_i(t_i) = P_4(\boldsymbol{n}) \tag{4.26d}$$
$$W_5(\boldsymbol{n}) = \bar{u} \left(N - \sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} n_{a,i} \right) = P_5(\boldsymbol{n}). \tag{4.26e}$$

$W_1(\boldsymbol{n}) = 2P_1(\boldsymbol{n})$, $W_3(\boldsymbol{n}) = P_3(\boldsymbol{n})$ are convex; thus, the social welfare function $W(\boldsymbol{n})$ may have multiple maxima. That is, except the global maximizer (i.e., first-best optimum) $\boldsymbol{n}^{so}$ of $W(\boldsymbol{n})$, local optima $\boldsymbol{n}^{o}$ that locally maximize $W(\boldsymbol{n})$ may exist. Moreover, characterizing the global maximum is extremely difficult. Thus, we focus on the policy to achieve local optimum $\boldsymbol{n}^{o}$ from stable equilibrium $\boldsymbol{n}^{*}$.

4.3.2 Distributions of intracity population and WSTs under local optimum

Local optimum $\boldsymbol{n}^o$ is the local maximum point of social welfare function $W(\boldsymbol{n})$; therefore, it satisfies the following KKT conditions of the optimization problem (4.26).

$$\begin{cases} \hat{v}^* = \hat{v}_{a,i}(\boldsymbol{n}^o) & \text{if} \quad n^o_{a,i} > 0, \\ \hat{v}^* \geq \hat{v}_{a,i}(\boldsymbol{n}^o) & \text{if} \quad n^o_{a,i} = 0, \end{cases} \tag{4.27a}$$

$$\sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} n^o_{a,i} = N. \tag{4.27b}$$

where, $\hat{v}_{a,i}(\boldsymbol{n})$ is expressed as follows:

$$\hat{v}_{a,i}(\boldsymbol{n}) = v_{a,i}(\boldsymbol{n}) + G_i(\boldsymbol{M}) - \sum_{b=1}^{a} c'(x_{b,i})x_{b,i}. \tag{4.28}$$

The following proposition is obtained using these KKT conditions and adopting the same procedure as in Section 3.4.

Proposition 4.5 Local optimum $\boldsymbol{n}^o$ has the following properties.

(i) Suppose $N^o_a > 0$ and $(a-1, i), (a-1, j) \in \mathrm{supp}(\boldsymbol{n}^o)$, then, $(a, i),\ (a, j) \in \mathrm{supp}(\boldsymbol{n}^o)$ and $x^o_{a,i} = x^o_{a,j} > 0$. Moreover, suppose $N^o_a > 0$ and $(a-1, i) \notin \mathrm{supp}(\boldsymbol{n}^o)$, $(a-1, j) \in \mathrm{supp}(\boldsymbol{n}^o)$, then, $n^o_{a,i} \leq n^o_{a,j}$ and $x^o_{a,i} \leq x^o_{a,j}$.

(ii) Suppose $N^o_a > 0$ and $N^o_{a-1} > 0$, then, $N^o_a < N^o_{a-1}$.

This proposition shows that properties of the distributions of intracity population and WSTs under local optimum, coincide with equilibrium. That is, the local optimum satisfies the following properties.

(a) The nearer the CBD where workers live, the narrower their distribution of WSTs.

(b) The nearer the residence is from the CBD, the greater the population.

4.3.3 Pigouvian policies

We next discuss tax/subsidy policies that attain the optima as stable equilibria. To achieve the optimum, we consider Pigouvian policies, such as congestion tolls. We do so because the optimal state is supported as an equilibrium by imposing such policies that workers are responsible for their externalities at the optimum. The Pigouvian policy that introduces tax/subsidy $p^d_{a,i}(\boldsymbol{n})$ to workers, is given by

$$p_{a,i}(\boldsymbol{n}) = G_i(\boldsymbol{M}) - \sum_{b=1}^{a} c'(x_{b,i})x_{b,i}. \tag{4.29}$$

Under the Pigouvian policy, our model is viewed as a potential game $\hat{S} = \{\mathcal{S},\ \mathcal{A} \times \mathcal{I},\ \hat{\boldsymbol{v}}\}$, where $\hat{v}_{a,i}(\boldsymbol{n}) = v_{a,i}(\boldsymbol{n}) + p_{a,i}(\boldsymbol{n})$, because the following potential function exists:

$$\hat{P}(\boldsymbol{n}) = P(\boldsymbol{n}) + p_{a,i}(\boldsymbol{n}) \cdot \boldsymbol{n} = W(\boldsymbol{n}). \tag{4.30}$$

The KKT conditions of the maximization problem of the potential function $\hat{P}(\boldsymbol{n})$ subject to $\boldsymbol{n} \in \Delta$ is given by the following:

$$\begin{cases} \hat{v}^* = v_{a,i}(\boldsymbol{n}) + p_{a,i}(\boldsymbol{n}) & \text{if} \quad n_{a,i} > 0 \\ \hat{v}^* \geq v_{a,i}(\boldsymbol{n}) + p_{a,i}(\boldsymbol{n}) & \text{if} \quad n_{a,i} = 0 \end{cases} \quad \forall a \in \mathcal{A}, i \in \mathcal{I}, \tag{4.31a}$$

$$\sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} n_{a,i} \leq N. \tag{4.31b}$$

This implies that the local optimum $\boldsymbol{n}^o$ must be a Nash equilibrium of the game $\hat{S}$.

Effects of policy implementation on the intracity spatial and temporal structure

We next investigate the effects of Pigouvian policy on the distributions of urban population and WSTs. More specifically, we analyze the effects of introducing policies when the equilibrium $\boldsymbol{n}^*$ is given as the initial state.

First, we consider the effects on the distribution of urban population. Suppose $(a-1, i)$,

$(a-1, j) \in \text{supp}(\boldsymbol{n}^o)$, the following always holds:

$$\hat{v}_{a,i}(\boldsymbol{n}^*) - \hat{v}_{a-1,i}(\boldsymbol{n}^*) = -c'(x^*_{a,i})x^*_{a,i} < 0, \tag{4.32}$$

which can be rewritten as $\hat{v}_{a,i}(\boldsymbol{n}^*) < \hat{v}_{a-1,i}(\boldsymbol{n}^*)$.

Second, let us analyze the effects on the distribution of WSTs. Suppose $(a, i), (d, a, i+1) \in \text{supp}(\boldsymbol{n}^*)$, because of $v^d_{a,i}(\boldsymbol{n}^*) = v_{a,i+1}(\boldsymbol{n}^*)$, we have

$$\hat{v}_{a,i}(\boldsymbol{n}^*) - \hat{v}_{a,i+1}(\boldsymbol{n}^*) = \sum_{b=1}^{a} \left\{ \left[c(x^*_{b,i}) - c'(x^*_{b,i})x^*_{b,i}\right] - \left[c(x^*_{b,i+1}) - c'(x^*_{b,i+1})x^*_{b,i+1}\right] \right\}. \tag{4.33}$$

Moreover, since

$$\frac{\partial[c(x) - c'(x)x]}{\partial x} = -c''(x)x < 0, \tag{4.34}$$

if $M^*_i > M^*_{i+1}$, we obtain $\hat{v}_{a,i}(\boldsymbol{n}^*) < \hat{v}_{a,i+1}(\boldsymbol{n}^*)$. That is, according to Proposition 4.3, if $M^*_i > M^*_{i+1}$, we have the following conditions.

$$\begin{cases} x^*_{a,i} \geq x^*_{a,i+1} & \forall a \in \mathcal{A}, \\ \text{there exists } a \in \mathcal{A} \text{ such that } x^*_{a,i} > x^*_{a,i+1}. \end{cases} \tag{4.35}$$

From the these results, we obtain the following proposition.

Proposition 4.6 Consider that Pigouvian policies are introduced in equilibrium $\boldsymbol{n}^*$. We have the following properties.

(i) For all $(a, i), (a-1, i) \in \text{supp}(\boldsymbol{n}^*)$, we have $\hat{v}_{a,i}(\boldsymbol{n}^*) < \hat{v}_{a-1,i}(\boldsymbol{n}^*)$.

(ii) Suppose $M^*_i > M^*_{i+1}$, for all $(a, i), (a, i+1) \in \text{supp}(\boldsymbol{n}^*)$, we have $\hat{v}_{a,i}(\boldsymbol{n}^*) < \hat{v}_{a,i+1}(\boldsymbol{n}^*)$.

In this Proposition, (i) shows that the implementation of the tax/subsidy policy $p_{a,i}(\boldsymbol{n})$ has an increasing effect on the population distribution near CBD when equilibrium $\boldsymbol{n}^*$ is taken as the initial state. (ii) implies that the implementation of the tax/subsidy policy $p_{a,i}(\boldsymbol{n})$ staggers WSTs for each residential location.

It is noteworthy that though the local optimum distribution of WSTs in each residential location is more staggered than stable equilibrium, the local optimum distribution of total WSTs can be more clustered than stable equilibrium. This is because the population density near CBD will be higher after policy implementation, where WSTs distribution is more clustered than suburban. Therefore, staggering WSTs can not only improve but also decrease social welfare.

Effects of Policy Implementation on the Rural-to-Urban Migration

Next, we consider the effects on the distribution of urban population. Based on (4.28), the relationship between $\hat{v}_{a,i}(\boldsymbol{n}^*)$ and $v_{a,i}(\boldsymbol{n}^*)$ is expressed as follows:

$$\begin{cases} \hat{v}_{a,i}(\boldsymbol{n}^*) > v_{a,i}(\boldsymbol{n}^*) & \text{if} \quad p_{a,i}(\boldsymbol{n}^*) > 0, \\ \hat{v}_{a,i}(\boldsymbol{n}^*) < v_{a,i}(\boldsymbol{n}^*) & \text{if} \quad p_{a,i}(\boldsymbol{n}^*) < 0. \end{cases} \tag{4.36}$$

Then we have the following proposition.

Proposition 4.7 Suppose $\boldsymbol{n}^*$ is an equilibrium such that $N_u^* > 0$, then we have the following properties:

(i) If $p_{a,i}(\boldsymbol{n}^*) > 0$ for all $(a, i) \in \text{supp}(\boldsymbol{n}^*)$, $N_u^o > N_u^*$.

(ii) If $p_{a,i}(\boldsymbol{n}^*) < 0$ for all $(a, i) \in \text{supp}(\boldsymbol{n}^*)$, $N_u^o < N_u^*$.

This proposition shows that Pigouvian subsidy (tax) leads to rural-to-urban (urban-to-rural) migration. Since $p_{a,i}(\boldsymbol{n}) = G_i(\boldsymbol{M}) - \sum_{b=1}^{a} c'(x_{b,i})x_{b,i}$, this proposition also implies that high temporal agglomeration economies (high productivity) and low spatial and temporal agglomeration diseconomies (high commuting cost) promote rural-to-urban migration. Besides, since commuting cost $c(x_{a,i})$ is a monotonically increasing, and strictly convex function of $x_{a,i}$, the less population density and the less commuting cost lead to rural-to-urban migration.

4.4 Numerical Examples

We numerically analyze our model and show the distributions of urban population and WSTs. This analysis assumes that the number of residential locations is $A = 10$, and the number of WSTs is $I = 11$. We also assume $f(x) = -\frac{\mu}{x}$, $c(x) = t(1 + x^{\beta})$, and use the following parameter values:

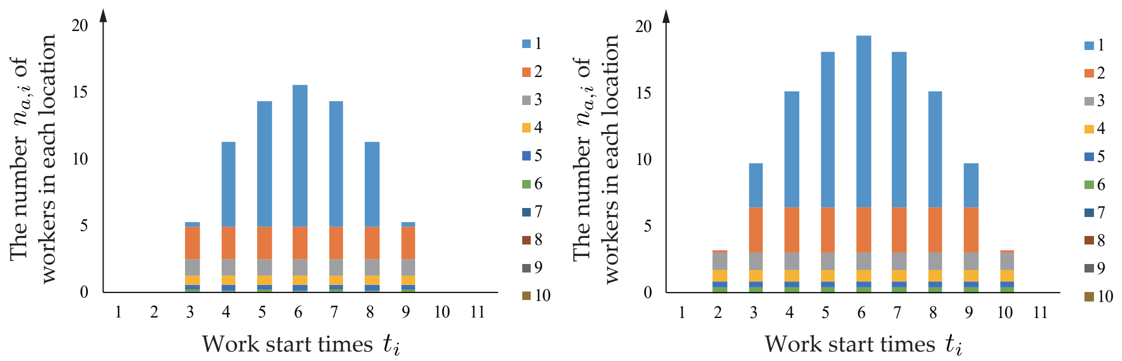
$$N = 1000, \quad \alpha = 10.0, \quad \mu = 2.0, \quad t = 2.0,$$
$$\gamma = 10.0, \quad \bar{u} = 10, \quad L = 3. \tag{4.37}$$

We then investigate the characteristics of stable equilibrium $\boldsymbol{n}^*$ and local optimum $\boldsymbol{n}^o$ with changes in parameters β and ϕ.

4.4.1 Stable Equilibrium

Distributions of Urban Population and WSTs

We assume that $L = 3$ is fixed, and investigate the changes in the distributions of urban population and WSTs under stable equilibrium. Then, we confirm the correspondence between numerical and the theoretical results obtained in Section 4.2. Figure 4.2 shows the number of workers in each WST t_i. We can confirm the patterns from these results, which are consistent with Proposition 4.3 that the nearer the CBD where workers live, the narrower



(A) $\beta = 1.75$.　(B) $\beta = 1.70$.

FIGURE 4.2: The number of workers in each WST ($\phi = 0.6$).

their distribution of WSTs.

Figure 4.3 shows the number $\boldsymbol{n}^* = (N_a^*)_{a\in\mathcal{A}}$ of workers in each residential location under stable equilibrium. This numerical result confirms that "the nearer the residential location is from the CBD, the greater the population" in Proposition 4.4. The above results are consistent with the result in Takayama (2019).

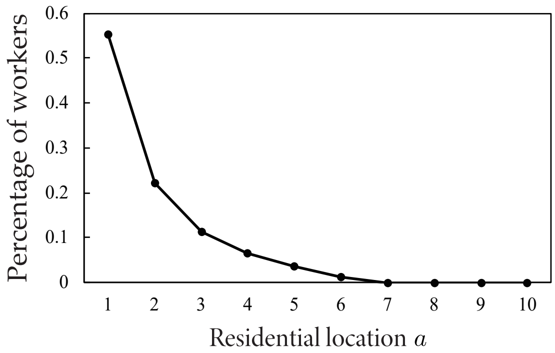
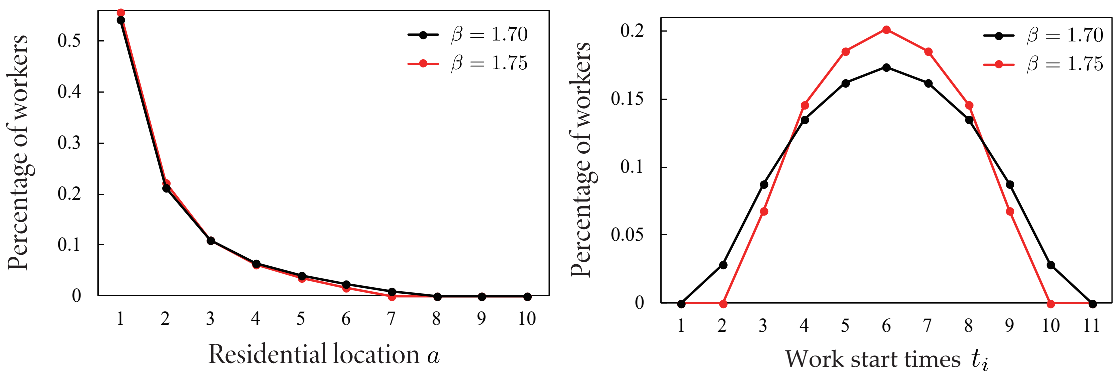


FIGURE 4.3: Percentage of Workers in Each Residential Location.



(A) Distributions of Population Density.

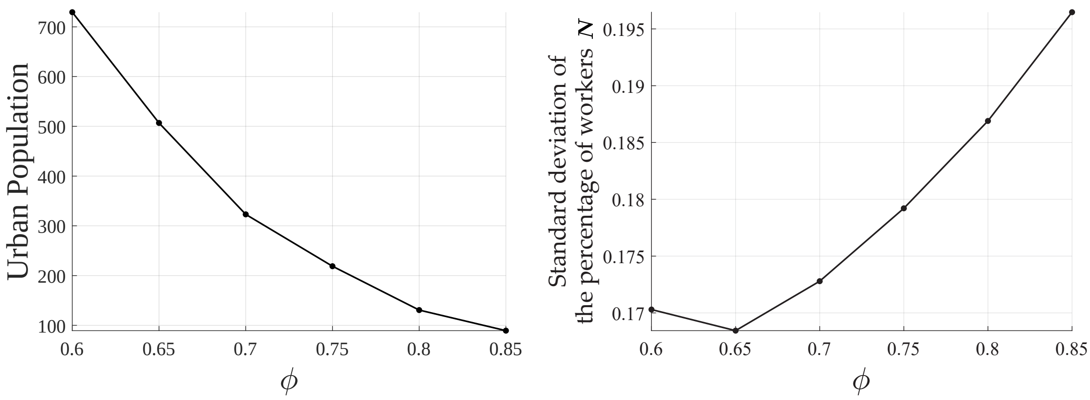
(B) Distributions of WSTs.

FIGURE 4.4: Distributions of WSTs and Population under Different Traffic Situations (ϕ = 0.6).

Next, we investigate the effect of traffic congestion (the value of β). It is apparent from Figure 4.2 that the worse the traffic congestion, the lower the urban population. This is because worse traffic congestion (great β) causes greater effects of spatial and temporal agglomeration diseconomies that leads to urban-to-rural migration.

Figure 4.4a depicts the distributions of urban population when $\beta = 1.70,\ 1.75$. It shows that urban population density increases as traffic congestion increases. That is, reduction in commuting costs has brought about effect that workers choose to live in large houses in suburbs. This result is also consistent with the result in Takayama (2019).

Figure 4.4b depicts the distributions of WSTs under different traffic situations. It shows that the increase in traffic congestion leads to clustered WSTs. This is because increased traffic congestion leads to lower spatial agglomeration economies due to urban-to-rural migration, and as a result, firms choose to cluster their WSTs to improve productivity (temporal agglomeration economies). This result indicates that the effect of traffic situation on WSTs distribution is contrary to the finding of Takayama (2019) when considering the urban spatial structure as an open city.



(A) Urban Population.

(B) Population Density.

FIGURE 4.5: Urban Population and Population Density under Different Temporal Discount Rate
($\beta = 1.75$, $L = 3$).

Next, we clarify the interaction of spatial and temporal distributions. To this end, we investigate the effect of different temporal discount rate (ϕ) on urban population and population density. Figure 4.5a depicts urban population under different temporal discount rate (ϕ) and shows that the less effect of interaction among different WSTs (the lower temporal agglomeration economies) leads to urban-to-rural migration (spatial dispersion). This is because the lower temporal agglomeration economies leads to lower urban utility.

Figure 4.5b depicts urban population density under different temporal discount rate (ϕ) and shows that the less effect of interaction among different WSTs (the lower temporal agglomeration economies) does not necessarily lead to greater urban population density. This is because the lower temporal discount rate (ϕ), the higher the land rent will be due to the increase in urban population. As a result, workers choose to live in large houses in the suburbs.

4.4.2 Comparison between Local Optimum and Stable Equilibrium

This section investigates the local optimum properties and compares them with the properties of stable equilibrium. To this end, we verify the consistency with theoretical analysis results, and qualitatively analyze new properties through numerical analysis. Then through these findings, we clarify the impact of policy implementation.

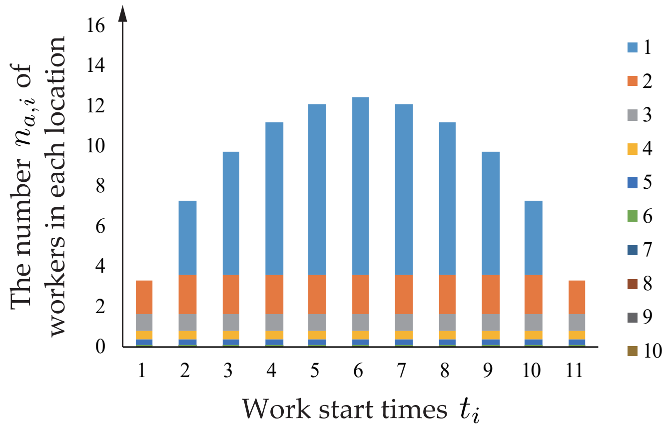
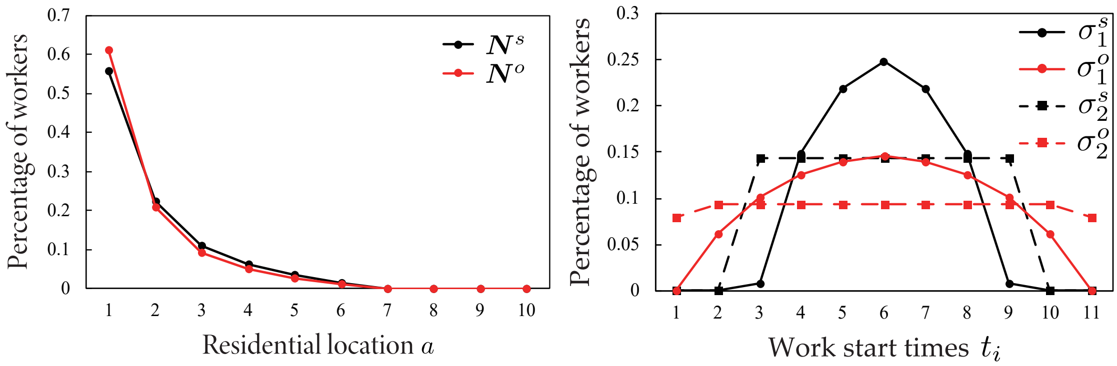


FIGURE 4.6: The number of workers in each WST under local optimum ($\beta = 1.75$, $L = 3$, $\phi = 0.6$).

First, we show the distributions of urban population and WSTs under local optimum. Figure 4.6 shows the number of workers of each WST, and Figure 4.7a shows the stable equilibrium and local optimum distributions of the percentage of workers in each residential location. These results confirm that the distributions of urban population and WSTs are consistent with Proposition 4.5 (similar properties to stable equilibrium). Additionally, we have confirmed that the same results are obtained with other parameters, as in the case of $L = 3,\ \beta = 1.75,\ \phi = 0.6$.

We then compare the distributions of urban population and WSTs between stable equilibrium and local optimum. Figure 5.6a implies that "population density near CBD under local



(A) Distributions of urban population. (B) Distributions of WSTs.

Figure 4.7: Comparison of the distributions of urban population and WSTs under stable equilibrium and local optimum ($L = 3,\ \beta = 1.75,\ \phi = 0.6$).

optimum is higher than that under stable equilibrium," and it coincides with the policy implementation effects shown in Proposition 4.6 (i).

Figure 4.7b shows the percentage of workers of each WST in each residential location under stable equilibrium and local optimum, where $\sigma_a^{\{s,o\}} = \frac{n_{a,i}^{\{s,o\}}}{N_a^{\{s,o\}}}, \forall a \in \mathcal{A}, i \in \mathcal{I}$. Here, we list only the cases of residential locations 1 and 2. The figure implies that "WSTs distribution in each residential location under local optimum is more staggered than that under stable equilibrium," and it coincides with the policy implementation effects shown in Proposition 4.6 (ii).

Urban Population under Stable Equilibrium and Local Optimum

This section investigates the effect of policy implementation on population migration. Specifically, we clarify the conditions under which the policy implementation will lead to urban-to-rural (rural-to-urban) migration by comparing urban population under stable equilibrium and local optimum in each combination of L and β.

Figure 4.8 depicts the comparison of urban population distributions under stable equilibrium and local optimum when $\phi = 0.6,\ \gamma = 100$. This figure shows that implementing Pigouvian policies (tax) in the case of high urban cost (high commuting cost and high land rent) will lead to urban-to-rural migration; implementing Pigouvian policies (subsidy) in the case of low urban cost (low commuting cost and low land rent) will lead to urban-to-rural migration.

These results are consistent with Proposition 4.7 and imply that whether the increase of urban population can improve social welfare depends on urban cost.

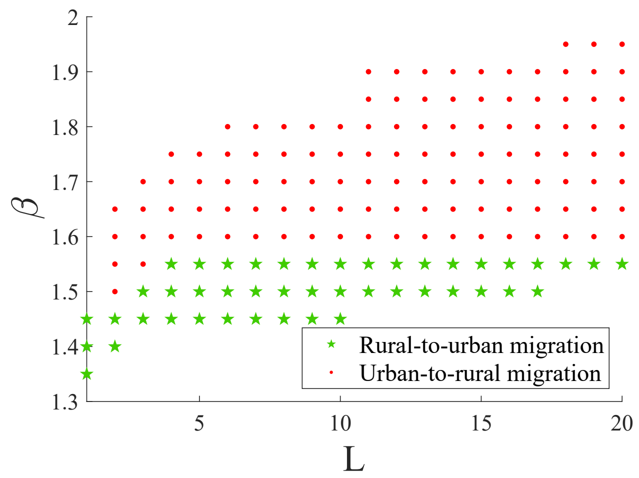


FIGURE 4.8: Comparison of Urban Population Distributions under Stable Equilibrium and Local Optimum ($\phi = 0.6,\ \gamma = 100$).

4.5 Summary and Discussions

This chapter developed an open city model considering spatio-temporal agglomeration economies by introducing urban spatial structure and rural area into Henderson (1981)'s WST choice model. We showed that our model belongs to a class of potential game. Then, by using the properties of the potential game, we show the following properties of the model:

1) The greater interaction among different WSTs leads to spatial agglomeration (i.e., rural-to-urban migration), however, does not necessarily lead to lower population density.

2) The increase of traffic congestion leads to clustered WSTs.

3) The urban population at optimum can be lower than at equilibrium when urban cost is high.

Here, Property 3) implies that it is likely to be socially undesirable if policies (e.g., promotions of urban population inflow) are implemented ineptly.

This chapter aims to establish a basic framework for analyzing the endogenous distributions of urban population and WSTs. Thus, we assumed that "firms are located in one CBD" and "firms and workers are homogeneous." However, these assumptions are considered to have a strong influence on the distributions of urban population and WSTs. Therefore, it would be valuable for future research to extend this model into multi-central urban structure and investigate the effects of heterogeneities in firms and workers.

Appendix

4.A Proofs

4.A.1 Proof of Proposition 4.1

It follows from (4.16), (4.17) that $P(\boldsymbol{n})$ is a potential function. Because $P_4(\boldsymbol{n})$ is a linear function, hereafter, we prove the convexity of $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$, $P_3(\boldsymbol{n})$ to clarify the convexity of potential function $P(\boldsymbol{n})$.

First, let us investigate the convexity of $P_1(\boldsymbol{n})$. Hessian matrix of $P_1(\boldsymbol{n})$ is expressed as follows:

$$\nabla^2 P_1(\boldsymbol{n}) = \alpha \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \\ \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \end{bmatrix} = \alpha(\boldsymbol{E}_A \otimes \boldsymbol{\Phi}). \tag{4.38a}$$

where, $\otimes$ denotes the Kronecker product, $\boldsymbol{E}_A$ is an $A \times A$ matrix with all elements equal to

1, and $\mathbf{\Phi} \equiv (1-\phi)^{\boldsymbol{D}}$, $\boldsymbol{D} = \begin{bmatrix} 0 & 1 & \cdots & I-1 \\ 1 & 0 & \cdots & I-2 \\ \vdots & \vdots & \ddots & \vdots \\ I-1 & I-2 & \cdots & 0 \end{bmatrix}$.

The positive/negative definite of $\mathbf{\Phi}$ coincides with the positive/negative definite of its inverse matrix $\mathbf{\Phi}^{-1}$; hence, we focus on $\mathbf{\Phi}^{-1}$:

$$\mathbf{\Phi}^{-1} = \frac{1}{\iota} \begin{bmatrix} 1 & \phi-1 & & & & \\ \phi-1 & 1+(1-\phi)^2 & \phi-1 & & \boldsymbol{O} & \\ & \phi-1 & 1+(1-\phi)^2 & \phi-1 & & \\ & & \ddots & \ddots & \ddots & \\ & \boldsymbol{O} & & \phi-1 & 1+(1-\phi)^2 & \phi-1 \\ & & & & \phi-1 & 1 \end{bmatrix}. \tag{4.39}$$

where $\iota \equiv \alpha[1-(1-\phi)^2]$.

Let γ_{ij} denote the element (i,j) of $\mathbf{\Phi}^{-1}$, from the Gershgorin circle theorem, λ_i^{-1} which denotes any eigenvalue of $\mathbf{\Phi}^{-1}$ exists in the range of

$$|\lambda_i^{-1} - \gamma_{ii}| \leq R_i \qquad \forall i \in \mathcal{I}. \tag{4.40}$$

where $R_i \equiv \sum_{j=1}^{I} |\gamma_{ij}|$.

Applying (4.40), it is readily that λ_i^{-1} exists in the following range:

$$0 \leq \frac{1}{2\alpha}\frac{\phi}{2-\phi} \leq \lambda_i^{-1} \leq \frac{1}{2\alpha}\frac{2-\phi}{\phi}. \tag{4.41}$$

Therefore, λ_i is nonnegative.

Let μ be the eigenvalue of $\boldsymbol{E}_A$, $\boldsymbol{E}$ be an $A \times A$ identity matrix. Then, since

$$
\begin{aligned}
\boldsymbol{E}_A - \mu \boldsymbol{E} &= \begin{bmatrix} 1-\mu & 1 & \cdots & 1 \\ 1 & 1-\mu & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1-\mu \end{bmatrix} \\
&= (A-\mu)\mu^{A-1},
\end{aligned}
$$

the eigenvalues of $\boldsymbol{E}_A$ are $\mu = 0, A$, the eigenvalue of $\nabla^2 P_1(\boldsymbol{n})$ is $\lambda_i \mu \geq 0$. Thus, $\nabla^2 P_1(\boldsymbol{n})$ is positive-semidefinite, that is, $P_1(\boldsymbol{n})$ is a convex function.

Next, let us focus on the convexity of $P_2(\boldsymbol{n})$. Hessian matrix of $P_2(\boldsymbol{n})$ can be expressed as follows:

$$
\nabla^2 P_2(\boldsymbol{n}) = \begin{bmatrix} \nabla \boldsymbol{c}_{11} & \nabla \boldsymbol{c}_{12} & \nabla \boldsymbol{c}_{13} & \cdots & \nabla \boldsymbol{c}_{1A} \\ & \nabla \boldsymbol{c}_{22} & \nabla \boldsymbol{c}_{23} & \cdots & \nabla \boldsymbol{c}_{2A} \\ & & \nabla \boldsymbol{c}_{33} & \cdots & \nabla \boldsymbol{c}_{3A} \\ & \boldsymbol{O} & & \ddots & \vdots \\ & & & & \nabla \boldsymbol{c}_{AA} \end{bmatrix}, \tag{4.42a}
$$

$$
\nabla \boldsymbol{c}_{a\hat{a}} = \operatorname{diag}\left\{ \left[\sum_{b=1}^{a} c'(x_{b,i}) \right]_{i \in \mathcal{I}} \right\}. \tag{4.42b}
$$

We observe that $\nabla^2 P_2(\boldsymbol{n})$ is a diagonal matrix, $\boldsymbol{\eta}$ is an upper triangular matrix, and all the diagonal elements are positive. Thus, all the eigenvalues are positive, namely, $\nabla^2 P_2(\boldsymbol{n})$ is positive-semidefinite. Moreover, since $c'(x) > 0$, $\nabla P_2(\boldsymbol{n})$ is a convex function.

We next investigate the convexity of $P_3(\boldsymbol{n})$. Hessian matrix of $P_3(\boldsymbol{n})$ is expressed as follows:

$$\nabla^2 P_3(\boldsymbol{n}) = \begin{bmatrix} \boldsymbol{H}_1 & & & & \\ & \boldsymbol{H}_2 & & \boldsymbol{O} & \\ & & \boldsymbol{H}_3 & & \\ & \boldsymbol{O} & & \ddots & \\ & & & & \boldsymbol{H}_A \end{bmatrix}, \tag{4.43a}$$

$$\boldsymbol{H}_a = -h'(N_a)\boldsymbol{I}. \tag{4.43b}$$

where $\boldsymbol{I}$ is a $1 \times I$ matrix with all elements equal to 1. Since $h'(x) < 0$, the eigenvalues of $\nabla^2 P_3(\boldsymbol{n})$ are positive and $\nabla^2 P_3(\boldsymbol{n})$ is positive definite, namely, $P_3(\boldsymbol{n})$ is a convex function.

4.A.2 Proof of Proposition 4.3

Suppose $(a-1, i),\ (a-1, j) \in \text{supp}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i, j \in \mathcal{I}$. From (4.23), we have

$$v_{a,i}(\boldsymbol{n}^*) - v_{a,j}(\boldsymbol{n}^*) = -c(x_{a,i}) + c(x_{a,j}). \tag{4.44}$$

Thus we have the following condition:

$$\begin{cases} v_{a,i}(\boldsymbol{n}^*) > v_{a,j}(\boldsymbol{n}^*) & \text{if} \quad x_{a,i} < x_{a,j}, \\ v_{a,i}(\boldsymbol{n}^*) = v_{a,j}(\boldsymbol{n}^*) & \text{if} \quad x_{a,i} = x_{a,j}, \\ v_{a,i}(\boldsymbol{n}^*) < v_{a,j}(\boldsymbol{n}^*) & \text{if} \quad x_{a,i} > x_{a,j}. \end{cases} \tag{4.45}$$

Suppose $(a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, we have

$$v_{a,i}(\boldsymbol{n}^*) - v_{a,j}(\boldsymbol{n}^*) \leq -c(x_{a,i}) + c(x_{a,j}). \tag{4.46}$$

Therefore, we have the following condition:

$$v_{a,i}(\boldsymbol{n}^*) < v_{a,j}(\boldsymbol{n}^*) \quad \text{if} \quad x_{a,i} > x_{a,j}. \tag{4.47}$$

Suppose $N_a > 0$, thus, there exists i which satisfies $x_{a,i} > 0$. Moreover, from (4.45) and (4.47), we observe that $x_{a,i} > 0$ if $(a-1, i) \in \text{supp}(\boldsymbol{n}^*)$. Combining this result with (4.45), we have Proposition 5.3 (i).

Furthermore, it follows from (4.47) that if $(a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, $x_{a,i} \leq x_{a,j}$ holds. By combining this result with Proposition 4.3 (i), $n_{a,i} \leq n_{a,j}$ holds, thus, we have Proposition 4.3 (ii).

5

A System-of-cities Model Considering Spatio-temporal Agglomeration Economies

5.1 The model

5.1.1 Basic assumptions

We consider a spatial structure that consists of two monocentric cities (Figure 5.1). Let $\mathcal{D} \equiv \{\text{I, II}\}$ be the set of cities and each city is homogeneous. The number of residential locations in each city is the same at A. We index the residential locations from the side of CBD and let $\mathcal{A} \equiv \{1, 2, \ldots, A\}$ be the set of locations. The area of each residential land is the same at L.

All firms are located in the CBD of their cities and each firm chooses its WST from the feasible set $\{t_1, t_2, \ldots, t_I\}$, where $t_i = t_{i-1} + \tau$ for all $i \in \{2, 3, \cdots, I\}$ and τ is a positive

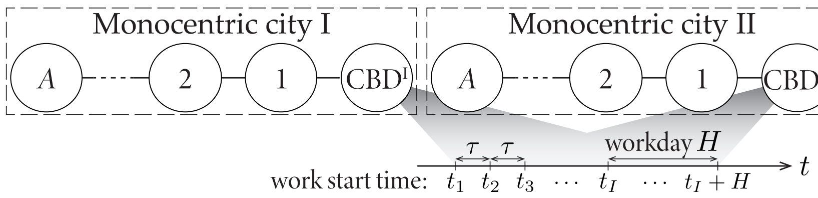


FIGURE 5.1: Two monocentric cities.

constant. The length of a workday is assumed to be identical and fixed at H for all firms; therefore, each firm is characterized by its WST. For convenience, we call the firm that starts work at time t_i "firm i." We further assume the existence of an interval in the workday when all firms begin work (i.e., $t_T < t_1 + H$).

All roads connecting each location are homogeneous, and we further call the road between location $a-1$ and a "road a." The number of workers who start work at t_i passing through road a in city d is denoted by $x^d_{a,i}$ and expressed by

$$x^d_{a,i} = \sum_{b=a}^{A} n^d_{b,i}. \tag{5.1}$$

We assume the transportation cost of workers at firm i in city d passing through road a is $c(x^d_{a,i})$. Like Henderson (1981), we assume that $c(x^d_{a,i})$ is nonnegative, monotonically increasing, and strictly convex function.

Behavior of workers

All workers are *ex ante* identical and the total number of workers of the two cities are fixed at N. Assume that workers commute from their locations to CBD only in their cities. Each worker chooses his or her WST t_i indirectly by choosing an employer (i.e., a firm $i \in \mathcal{I} \equiv \{1, 2, \ldots, I\}$). The number of workers who reside at location a in city d and work at firm i is denoted by $\boldsymbol{n} = (n^d_{a,i})_{d\in\mathcal{D}, a\in\mathcal{A}, i\in\mathcal{I}}$, and we further call it the *distributions of populations and WSTs*. The number N^d_a of workers working at firm i in city d and the number M^d_i of workers residing in location a in city d are expressed, respectively, as follows:

$$N_a^d = \sum_{i\in\mathcal{I}} n_{a,i}^d, \tag{5.2a}$$

$$M_i^d = \sum_{a\in\mathcal{A}} n_{a,i}^d. \tag{5.2b}$$

The utility of workers who reside at a and start work at t_i is given by the following quasi-linear function:

$$u(z_{a,i}^d, y_a^d, t_i) = z_{a,i}^d + f(y_a^d) - \delta_i(t_i). \tag{5.3}$$

where $z_{a,i}^d$ denotes consumption of numéraire goods, y_a^d is the lot size at a in city d, and $f(y_a^d)$ is the utility from land consumption. We assume that $f(x)$ is strictly monotonically increasing, concave, and twice differentiable for $x > 0$. Moreover, $\lim_{x\to 0} f'(x) = \infty$, $\lim_{x\to\infty} xf'(x) < \infty$. [1] $\delta_i(t_i)$ denotes schedule delay cost and is given by the following:

$$\delta_i(t_i) = \gamma|t_i - t^*|. \tag{5.4}$$

where $\gamma > 0$ is early/late delay cost per unit time. [2] t^* is the most desired work schedule for commuters that best fits in with their daily activities (e.g., leisure activities with family and friends), thus, commuters with WST t^* do not incur schedule delay cost. [3]

The budget constraint is expressed as

$$w_i^d = z_{a,i}^d + r_a^d y_a^d + \sum_{b=1}^{a} c(x_{b,i}^d). \tag{5.5}$$

where w_i^d denotes the wage from firm i and r_a^d denotes the land rent at a in city d. Agricultural rent does not qualitatively change subsequent results; hence, we assume it to be zero.

[1]We assume that the WSTs which firms can choose is discrete. That is because most of WSTs are clustered at several points in time such as 8:00, 8:30, 9:00.

[2]In this chapter, based on the empirical findings by Hall (2021), we assume that the marginal schedule delay costs for early and late arrival are equal. Besides, this assumption does not qualitatively change the results obtained from this chapter.

[3]Note that schedule delay cost here is different with that in Part I.

The first-order condition of the utility maximization problem gives the following:

$$\begin{cases} f'(y_a^d) = r_a^d & \text{if} \quad y_a^d > 0 \\ f'(y_a^d) \le r_a^d & \text{if} \quad y_a^d = 0 \end{cases} \quad \forall d \in \mathcal{D}, a \in \mathcal{A}. \tag{5.6}$$

where the prime denotes differentiation. The marginal utility of land consumption is infinity at $y_a^d = 0$; thus, we must have $y_a^d > 0$ and

$$r_a^d = f'(y_a^d) > 0 \qquad \forall d \in \mathcal{D}, a \in \mathcal{A}. \tag{5.7}$$

Let L denote the land supply in each residential location, and land demand of residential land a in city d is given by $N_a^d y_a^d$. Thus, according to the supply-demand equilibrium, $y_a^d = L/N_a^d$ is obtained.

From (5.3), (5.5), and (5.7), we obtain the following indirect utility function $v_{a,i}^d$:

$$v_{a,i}^d = w_i^d - \sum_{b=1}^{a} c(x_{b,i}^d) + h(N_a^d) - \delta_i(t_i). \tag{5.8}$$

where $h(N_a^d) = f(y_a^d) - r_a^d y_a^d = f(\frac{L}{N_a^d}) - \frac{L}{N_a^d} f'(\frac{L}{N_a^d})$ at location a. $h(N_a^d)$ can be rewritten as $f(y_a^d) - r_a^d y_a^d$; hence, this represents net utility from land consumption. Furthermore, since

$$h'(N_a^d) = \frac{L^2 f''(\frac{L}{N_a^d})}{{N_a^d}^3} < 0. \tag{5.9}$$

$h(N_a^d)$ is a strictly decreasing function. That is, the more workers the less net utility from land consumption.

Behavior of firms

All firms produce homogeneous goods under constant returns to scale technology and perfect competition with free entry and exit, which require one unit of labor to produce one unit of output and is chosen as numéraire. We introduce the same productivity effect as in Henderson (1981) and Tabuchi (1986). That is, the longer the overlapping time interval of firms and the greater the number of firms located in one city, the greater the productivity. This implies that a firm's productivity depends on the number of workers $\boldsymbol{m} = (M_i^d)_{d\in\mathcal{D},i\in\mathcal{I}}$ of cities and WSTs.

Specifically, we define that g_i^d is the productivity factor of firm i in city d and determined by $\boldsymbol{m}^d = (M_i^d)_{i\in\mathcal{I}}$ which shows the distribution of each WST. g_i^d is expressed as follows:

$$g_i^d(\boldsymbol{m}^d) = \alpha \sum_{j\in\mathcal{I}} e^{-|t_i - t_j|} M_j^d = \alpha \sum_{j\in\mathcal{I}} e^{-\tau|i-j|} M_j^d. \tag{5.10}$$

Eq. (5.10) shows the effect of positive temporal agglomeration externalities on firms' productivity. That is, the more clustered the distribution of WSTs, the greater the productivity effects. $\alpha > 0$ denotes the magnitude of the productivity effects. Let $1 - \phi \equiv e^{-\tau}$ and denote $\phi \in [1, 0]$ as a temporal discount rate that the greater the value of ϕ, the greater the positive temporal agglomeration externality (i.e., the more necessity of synchronizing different firms' work schedules). Thus, (5.10) can be rewritten as follows:

$$g_i^d(\boldsymbol{m}^d) = \alpha \sum_{j\in\mathcal{I}} (1-\phi)^{|i-j|} M_j^d. \tag{5.11}$$

We then let G_i^d denote the daily output of a firm i in city d which is given by g_i^d as follows:

$$G_i^d(\boldsymbol{m}) = g_i^d(\boldsymbol{m}^d) + (1-\psi) g_i^{\hat{d}}(\boldsymbol{m}^{\hat{d}}). \tag{5.12}$$

Eq. (5.12) shows the effect of positive spatial agglomeration externalities on firms' productivity. $d, \hat{d} \in \mathcal{D}$ $(d \neq \hat{d})$ denotes the two different cities. $\psi \in [0, 1]$ denotes the spatial discount rate: the greater the value of ψ, the greater the positive spatial agglomeration externality (i.e., the more necessity of the intercity spatial agglomeration of different firms).

Under the production function defined in (5.12), each firm chooses its city and WST to maximize profit per worker:

$$\max_{d,i} \pi_i^d = G_i^d(\boldsymbol{m}) - w_i^d. \tag{5.13}$$

5.1.2 Equilibrium conditions

In our model, each firm chooses a city and a WST, and each worker chooses a city, a residential location, and an employer. Therefore, the equilibrium distributions of populations and WSTs $\boldsymbol{n}^*$ can be determined. We then describe these equilibrium conditions that satisfy $\boldsymbol{n}^*$. Hereafter, we use superscript $*$ to distinguish variables relating to equilibrium.

For all $d \in \mathcal{D},\ a \in \mathcal{A},\ i \in \mathcal{I}$, equilibrium satisfies the following conditions:

$$\begin{cases} v^* = w_i^d - \sum_{b=1}^a c(x_{b,i}^d) + h(N_a^d) - \delta_i(t_i) & \text{if} \quad n_{a,i}^d > 0, \\ v^* \geq w_i^d - \sum_{b=1}^a c(x_{b,i}^d) + h(N_a^d) - \delta_i(t_i) & \text{if} \quad n_{a,i}^d = 0, \end{cases} \tag{5.14a}$$

$$\begin{cases} \pi^* = G_i^d(\boldsymbol{m}) - w_i^d & \text{if} \quad \sum_{a\in\mathcal{A}} n_{a,i}^d > 0, \\ \pi^* \geq G_i^d(\boldsymbol{m}) - w_i^d & \text{if} \quad \sum_{a\in\mathcal{A}} n_{a,i}^d = 0, \end{cases} \tag{5.14b}$$

$$\sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n_{a,i}^d = N. \tag{5.14c}$$

where v^* denotes the equilibrium utility, and π^* is the equilibrium profit, which equals zero because of free entry and exit of firms.

Conditions (5.14a) and (5.14b) are the equilibrium conditions for workers' choice of firm and firms' choice of WST, respectively. Condition (5.14a) implies that at equilibrium, each worker has no incentive to change employer unilaterally. Condition (5.14b) means that if workers are employed by firm i, the firm earns the equilibrium profit $\pi^* = 0$; otherwise, the profit must be less than zero. Condition (5.14c) is the conservation law of the population of workers.

We easily show that conditions (5.14a) and (5.14b) can be rewritten as the following condition because $\pi^* = 0$.

$$\begin{cases} v^* = v_{a,i}^d(\boldsymbol{n}) & \text{if} \quad n_{a,i}^d > 0, \\ v^* \geq v_{a,i}^d(\boldsymbol{n}) & \text{if} \quad n_{a,i}^d = 0, \end{cases} \tag{5.15a}$$

$$\sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n_{a,i}^d = N. \tag{5.15b}$$

where $v^d_{a,i}(\boldsymbol{n})$ denotes the indirect utility of workers living in location a and employed by firm i as

$$v^d_{a,i}(\boldsymbol{n}) = G^d_i(\boldsymbol{M}) - \sum_{b=1}^{a} c(x^d_{b,i}) + h(N^d_a) - \delta_i(t_i). \tag{5.16}$$

5.2 Equilibrium

We now characterize the equilibrium. First, we indicate the uniqueness and stability of equilibrium by using the properties of a potential game. We next clarify the properties of workers' distributions of different cities, residential locations, and WSTs.

5.2.1 Potential game

To characterize the equilibrium, we invoke the properties of *potential game* introduced by Monderer and Shapley (1996) and Sandholm (2001). The equilibrium conditions are represented by (5.15); hence, the equilibrium $\boldsymbol{n}^*$ can be viewed as a population game in which the set of players is $\mathcal{S} \equiv [0, N]$, the common action set is $\mathcal{D} \times \mathcal{A} \times \mathcal{I}$, and the payoff vector is $\boldsymbol{V}(\boldsymbol{n}) = \left(v^d_{a,i}(\boldsymbol{n})\right)_{d\in\mathcal{D},a\in\mathcal{A},i\in\mathcal{I}}$. As is evident from the definition, the equilibrium is a Nash equilibrium of the game. Thus, we denote this game by $S = \{\mathcal{S},\ \mathcal{D} \times \mathcal{A} \times \mathcal{I},\ \boldsymbol{V}\}$. We define that S is a potential game if a continuously differentiable function $P(\boldsymbol{n})$ exists, such that

$$\frac{\partial P(\boldsymbol{n})}{\partial n^d_{a,i}} - \frac{\partial P(\boldsymbol{n})}{\partial n^{\hat{d}}_{b,j}} = v^d_{a,i}(\boldsymbol{n}) - v^{\hat{d}}_{b,j}(\boldsymbol{n})$$
$$\forall d \in \mathcal{D}, a \in \mathcal{A}, i \in \mathcal{I} \text{ and } \boldsymbol{n} \in \Delta. \tag{5.17}$$

where $\Delta \equiv \left\{\boldsymbol{n} \in \mathbb{R}^{2\times A\times I}_+ \;\middle|\; \sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n^d_{a,i} = N\right\}$ denotes the set of workers' spatial and temporal distributions, and $P(\boldsymbol{n})$ is defined on an open set containing Δ so that its partial derivative is well-defined on Δ. This condition requires the existence of a function in which gradient $\nabla P(\boldsymbol{n})$ equals the payoff vector $\boldsymbol{V}$. Sandholm (2001) demonstrated that, if payoffs $\boldsymbol{V}(\boldsymbol{n})$ are continuously differentiable, this condition is equivalent to the following condition called *externality symmetry*:

$$\frac{\partial v^d_{a,i}(\boldsymbol{n})}{\partial n^{\hat{d}}_{b,j}} = \frac{\partial v^{\hat{d}}_{b,j}(\boldsymbol{n})}{\partial n^d_{a,i}}$$

$$\forall d, \hat{d} \in \mathcal{D},\ a, b \in \mathcal{A},\ i, j \in \mathcal{I} \text{ and } \boldsymbol{n} \in \Delta. \tag{5.18}$$

From (5.16), $\frac{\partial v^d_{a,i}(\boldsymbol{n})}{\partial n^{\hat{d}}_{b,j}}$ can be rewritten as follows:

$$\frac{\partial v^d_{a,i}(\boldsymbol{n})}{\partial n^{\hat{d}}_{b,j}} = \alpha(1-\psi)(1-\phi)^{|i-j|} = \frac{\partial v^{\hat{d}}_{b,j}(\boldsymbol{n})}{\partial n^d_{a,i}}. \tag{5.19}$$

Game S satisfies (5.18); thus, we derive the following proposition.

Proposition 5.1 Game S is a potential game with the potential function

$$P(\boldsymbol{n}) = P_1(\boldsymbol{n}) - P_2(\boldsymbol{n}) - P_3(\boldsymbol{n}) - P_4(\boldsymbol{n}). \tag{5.20a}$$

where $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$ and $P_3(\boldsymbol{n})$ are convex functions, and $P_4(\boldsymbol{n})$ is a linear function such that

$$P_1(\boldsymbol{n}) = \frac{1}{2}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}} M^d_i G^d_i(\boldsymbol{m}), \tag{5.20b}$$

$$\frac{\partial P_2(\boldsymbol{n})}{\partial n^d_{a,i}} = \sum_{b=1}^{a} c(x^d_{b,i}), \tag{5.20c}$$

$$P_3(\boldsymbol{n}) = -\sum_{d\in\mathcal{D}}\sum_{a\in\mathcal{A}} N^d_a f(\frac{L}{N^d_a}), \tag{5.20d}$$

$$P_4(\boldsymbol{n}) = \sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}} M^d_i \delta_i(t_i). \tag{5.20e}$$

Proof See Appendix 5.A.1.

Here, $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$, $P_3(\boldsymbol{n})$ and $P_4(\boldsymbol{n})$ are respectively in terms of the positive spatio-temporal externalities due to productivity effects, negative spatio-temporal externalities due to traffic congestion, spatial agglomeration diseconomies due to land consumption and temporal agglomeration diseconomies due to schedule delay.

The equilibrium of a potential game is characterized by the maximization problem of the potential function. Let us consider the following problem:

$$\max_{\boldsymbol{n}} P(\boldsymbol{n})$$
$$\text{s.t.} \sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n^d_{a,i} = N, \; n^d_{a,i} \geq 0 \quad \forall d \in \mathcal{D}, a \in \mathcal{A}, i \in \mathcal{I}. \tag{5.21}$$

Let v^* be a Lagrange multiplier for the constraint $\sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n^d_{a,i} = N$. The first-order condition is $\frac{\partial P(\boldsymbol{n})}{\partial n^d_{a,i}} \leq v^*$ in which the equality holds whenever $n^d_{a,i} > 0$. Then, by (5.17), we have $v^d_{a,i}(\boldsymbol{n}) = v^{\hat{d}}_{b,j}(\boldsymbol{n})$ for any cities d and $\hat{d}$, residential locations a and b, WSTs i and j. We also have $v^{\bar{d}}_{c,k}(\boldsymbol{n}) \leq v^d_{a,i}(\boldsymbol{n})$ if $n^{\bar{d}}_{c,k} = 0$ and $n^d_{a,i} > 0$, for all $d, \bar{d} \in \mathcal{D}, a, c \in \mathcal{A}, i, k \in \mathcal{I}$. Thus, $\boldsymbol{n}$ is an equilibrium. By similar reasoning, it follows that the converse is also true. That is, if $\boldsymbol{n}$ is an equilibrium, it satisfies the necessary condition for problem (5.2.1). We then can readily verify that the Karush-Kuhn-Tucker (KKT) conditions of this problem (5.2.1) are equivalent to equilibrium conditions (5.15). Therefore, the equilibrium set of the game S exactly coincides with the set of KKT points for problem (5.2.1).

5.2.2 Uniqueness

To characterize the equilibrium, we first examine its uniqueness. The KKT points of problem (5.2.1) are equilibria; thus, the uniqueness can be investigated by checking the shape of potential function $P(\boldsymbol{n})$. Specifically, if $P(\boldsymbol{n})$ is unimodal, the equilibrium is unique; otherwise, it is non-unique. It follows from this property and the convexity of $P(\boldsymbol{n})$ that we have the following proposition.

Proposition 5.2 The equilibrium is generally non-unique.

Proof $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$, and $P_3(\boldsymbol{n})$ are convex functions; hence, $P(\boldsymbol{n})$ is not generally a concave function but can be a convex function. Therefore, $P(\boldsymbol{n})$ is not generally unimodal.

5.2.3 Stability

We next consider the local asymptotic stability of equilibria because our model generally includes multiple equilibria as shown in Proposition 5.2. Specifically, we examine whether we can justify an equilibrium through the existence of a learning process that makes players settle down in their equilibrium strategies. This chapter describes adjustment dynamics $\dot{\boldsymbol{n}} = \boldsymbol{V}(\boldsymbol{n})$ that maps the distributions of populations and WSTs $\boldsymbol{n}^0 \in \Delta$ to a set of Lipschitz paths in Δ, which starts from $\boldsymbol{n}^0$. Although we usually consider a specific evolutionary dynamic for stability analysis, we see that a more general analysis is possible due to the existence of a potential function. That is, the stability of equilibria can be characterized under a broad class of dynamics. In particular, we consider the class of admissible dynamics which satisfies the following conditions:

(PC) $\boldsymbol{V}(\boldsymbol{n}) \neq 0$ implies $\boldsymbol{V}(\boldsymbol{n}) \cdot \boldsymbol{V}(\boldsymbol{n}) = \sum_{d \in \mathcal{D}} \sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{I}} V^d_{a,i}(\boldsymbol{n}) v^d_{a,i}(\boldsymbol{n}) > 0$.

(NS) $\boldsymbol{V}(\boldsymbol{n}) = 0$ implies that $\boldsymbol{n}$ is a Nash equilibrium of the game S.

Condition (PC), called *positive correlation*, requires a positive correlation between the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$ and the payoffs $\boldsymbol{V}(\boldsymbol{n})$ out of rest points. This implies that, under this condition, all Nash equilibria of the game S are rest points of the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$. [4] Condition (NS), called Nash stationarity, asks that every rest point of the adjustment dynamics $\boldsymbol{V}(\boldsymbol{n})$ be a Nash equilibrium of game S. Therefore, under the conditions (PC) and (NS), $\dot{\boldsymbol{n}} = \boldsymbol{V}(\boldsymbol{n}) = 0$ if and only if $\boldsymbol{n}$ is a Nash equilibrium of game S. Specific examples of admissible dynamics include the *best response dynamic* (Gilboa and Matsui, 1991), the *Brown-von Neumann-Nash* dynamic (Brown and von Neumann, 1950), and the *projection dynamic* (Dupuis and Nagurney, 1993). [5] Importantly, the replicator dynamics (Taylor and Jonker, 1978), which is often used in spatial economic models (e.g., Fujita et al. (1999)), are *not* admissible. Under replicator dynamics, a rest point is always attained on the boundary, but the boundary points are not always Nash equilibria. Therefore, condition (NS) does not hold under replicator dynamics.

[4] See Proposition 4.3 of Sandholm (2001).
[5] See Sandholm (2005) for more examples.

As demonstrated by Sandholm (2001) that a Nash equilibrium of a potential game is asymptotically stable under any admissible dynamics if and only if it locally maximizes an associated potential function. We have the following property:

For game S, equilibrium $\boldsymbol{n}^{s*}$, which locally maximizes the potential function $P(\boldsymbol{n})$, is (locally) stable under admissible dynamics. Other equilibria $\boldsymbol{n}^{u*}$ are unstable.

Therefore, we can examine the stability of equilibria only by checking the shape of the potential function.

Because $P_4(\boldsymbol{n})$ is a linear function, the shape of the potential function $P(\boldsymbol{n})$ given by (5.2.1) depends on $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$ and $P_3(\boldsymbol{n})$. Therefore, spatio-temporal agglomeration economies and diseconomies determine stable equilibrium in our model.

In fact, if $P_1(\boldsymbol{n})$ is dominant and the potential function $P(\boldsymbol{n})$ is convex, the state of concentrated intercity population and clustered WSTs is a stable equilibrium. Moreover, if $P_2(\boldsymbol{n})$, $P_3(\boldsymbol{n})$ are dominant and $P(\boldsymbol{n})$ is concave, the state of dispersed intercity and intracity populations and staggered WSTs is the only equilibrium. By using the properties of the potential function, when $\boldsymbol{n}^0$ is an initial state, stable equilibrium can be obtained by searching from $\boldsymbol{n}^0$ to local maximizer $\boldsymbol{n}^*$. That is, stable equilibrium can be easily obtained by locally solving a simple optimization problem. Numerical analysis will be performed by using the characteristics in Section 5.

5.2.4 Distributions of intracity population and WSTs

We next characterize the distributions of intracity population and WSTs under equilibrium. The utility function (5.16) of the workers in location a city d can be rewritten as follows:

$$v^d_{a,i}(\boldsymbol{n}) = v^d_{a-1,i}(\boldsymbol{n}) - c(x^d_{a,i}) + h(N^d_a) - h(N^d_{a-1}). \tag{5.22}$$

Let $\text{supp}(\boldsymbol{n}^*)$ be the support of equilibrium $\boldsymbol{n}^*$ (i.e., $\text{supp}(\boldsymbol{n}^*) = \{(d, a, i) \mid n^{d*}_{a,i} > 0,\ d \in \mathcal{D},\ a \in \mathcal{A},\ i \in \mathcal{I}\}$). By using $\text{supp}(\boldsymbol{n}^*)$, we have

$$v^d_{a,i}(\boldsymbol{n}^*) \begin{cases} = v^* - c(x^{d*}_{a,i}) + h(N^{d*}_a) - h(N^{d*}_{a-1}) & \text{if} \quad (d, a-1, i) \in \text{supp}(\boldsymbol{n}^*), \\ \leq v^* - c(x^{d*}_{a,i}) + h(N^{d*}_a) - h(N^{d*}_{a-1}) & \text{if} \quad (d, a-1, i) \notin \text{supp}(\boldsymbol{n}^*). \end{cases} \tag{5.23}$$

From (5.23), we can obtain the following proposition.

Proposition 5.3 Equilibrium $\boldsymbol{n}^*$ has the following properties.

(i) Suppose $N^{d*}_a > 0$ and $(d, a-1, i), (d, a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, then, $(d, a, i), (d, a, j) \in \text{supp}(\boldsymbol{n}^*)$ and $x^{d*}_{a,i} = x^{d*}_{a,j} > 0$.

(ii) Suppose $N^{d*}_a > 0$, $(d, a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, then, $n^{d*}_{a,i} \leq n^{d*}_{a,j}$ and $x^{d*}_{a,i} \leq x^{d*}_{a,j}$.

Proof See Appendix 5.A.2.

This proposition shows that the nearer the CBD where workers live, the narrower their distribution of WSTs. That is, $\text{supp}((n^{d*}_{a-1,i})_{d\in\mathcal{D},i\in\mathcal{I}}) \subseteq \text{supp}((n^{d*}_{a,i})_{d\in\mathcal{D},i\in\mathcal{I}})$, and this result exactly coincides with the empirical observation of Fosgerau and Kim (2019).

Moreover, since if $(d, a, i),\ (d, a-1, i) \in \text{supp}(\boldsymbol{n}^*)$, then $v^d_{a,i}(\boldsymbol{n}^*) = v^d_{a-1,i}(\boldsymbol{n}^*)$. We obtain

$$h(N^{d*}_a) - h(N^{d*}_{a-1}) = c(x^{d*}_{a,i}) > 0. \tag{5.24}$$

Hence, we have the following proposition.

Proposition 5.4 Suppose $N^{d*}_a > 0$ and $N^{d*}_{a-1} > 0$ under equilibrium, then $N^{d*}_a < N^{d*}_{a-1}$. That is, the nearer the residence is from the CBD, the greater the population.

5.2.5 Conditions of intercity population concentration

To characterize the equilibrium distribution of intercity population, we investigate the equilibrium condition of intercity population concentration.

Hereafter, let N^d denote the population in city d (i.e., $N^d = \sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n^d_{a,i}$). In addition, we suppose the existence of an equilibrium $\boldsymbol{n}^*$, such that $(\mathrm{II}, a, i) \notin \mathrm{supp}(\boldsymbol{n}^*)$ for any $a \in \mathcal{A}$, $i \in \mathcal{I}$, then

$$v^{\mathrm{I}}_{a,i}(\boldsymbol{n}^*) \geq v^{\mathrm{II}}_{a,i}(\boldsymbol{n}^*) \quad \forall a \in \mathcal{A}, i \in \mathcal{I}. \tag{5.25}$$

It follows from (5.12), (5.15a) and (5.16) that (5.25) can be rewritten as follows:

$$\psi \geq \frac{\zeta_{a,i}(\boldsymbol{n})}{\alpha \sum_{j\in\mathcal{I}}(1-\phi)^{|i-j|}(M^{\mathrm{I}*}_j - M^{\mathrm{II}*}_j)}, \tag{5.26a}$$

$$\zeta_{a,i}(\boldsymbol{n}) = \sum_{b=1}^{a}\left[c(x^{\mathrm{I}}_{b,i}) - c(x^{\mathrm{II}}_{b,i})\right] + h(N^{\mathrm{II}}_a) - h(N^{\mathrm{I}}_a). \tag{5.26b}$$

Let $f_{a,i}(\boldsymbol{n}^*, \phi)$ denote the right-hand side of (5.26a). Then, $\frac{\partial f_{a,i}(\boldsymbol{n}^*,\phi)}{\partial \phi} > 0$, and we have the following proposition.

Proposition 5.5 Suppose $\boldsymbol{n}^*$ is an equilibrium, then these statements are equivalent:

(i) $(\mathrm{II}, a, i) \notin \mathrm{supp}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$.

(ii) $\psi \geq f_{a,i}(\boldsymbol{n}^*, \phi)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$.

Proof See Appendix 5.A.3.

This proposition implies that the distributions of intercity population and WSTs have the following properties:

(a) The lower the positive temporal agglomeration externality (the lower ϕ), the intercity population will be concentrated.

(b) The lower the positive spatial agglomeration externality (the lower ψ), the more clustered the WSTs.

From the aforementioned properties, it is clear that temporal distribution and spatial distribution of economic activities can affect each other. More specifically, property (a) shows that alleviating traffic congestion by staggering the WSTs can lead to intercity population concentration. Property (b) shows that the alleviation of traffic congestion by dispersing the intercity population distribution can lead to more clustered distribution of WSTs.

5.3 Optimum

The equilibrium is not generally efficient because of the positive and negative externalities. Therefore, this section discusses TDM policies, such as staggered work hours and taxation, for achieving the optimal distributions of populations and WSTs. To address this issue, we first define the social welfare function and then analyze Pigouvian policies' effectiveness for achieving local optimum. Hereafter, we use superscript o to distinguish variables relating to local optimum.

5.3.1 Definition of social welfare function

We define the social welfare function as the sum of producer surplus and consumer surplus. Thus, social welfare maximization problem is given by the following:

$$\max_{\boldsymbol{n}} W(\boldsymbol{n}) = W_1(\boldsymbol{n}) - W_2(\boldsymbol{n}) - W_3(\boldsymbol{n}) - W_4(\boldsymbol{n})$$

$$\text{s.t. } \boldsymbol{n} \in \Delta. \tag{5.27}$$

where, $W_1(\boldsymbol{n})$, $W_2(\boldsymbol{n})$, $W_3(\boldsymbol{n})$ and $W_4(\boldsymbol{n})$ are respectively in terms of the effects of productivity, traffic congestion, land consumption and schedule delay cost, and are expressed as follows:

$$W_1(\boldsymbol{n}) = \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}} M_i^d G_i^d(\boldsymbol{m}) = 2P_1(\boldsymbol{n}), \tag{5.28a}$$

$$W_2(\boldsymbol{n}) = \sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} x_{a,i}^d c(x_{a,i}^d), \tag{5.28b}$$

$$W_3(\boldsymbol{n}) = -\sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} N_a^d f(\frac{L}{N_a^d}) = P_3(\boldsymbol{n}), \tag{5.28c}$$

$$W_4(\boldsymbol{n}) = \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}} M_i^d \delta_i(t_i) = P_4(\boldsymbol{n}). \tag{5.28d}$$

$W_1(\boldsymbol{n}) = 2P_1(\boldsymbol{n})$, $W_3(\boldsymbol{n}) = P_3(\boldsymbol{n})$ are convex; thus, the social welfare function $W(\boldsymbol{n})$ may have multiple maxima. That is, except the global maximizer (i.e., first-best optimum) $\boldsymbol{n}^{so}$ of $W(\boldsymbol{n})$, local optima $\boldsymbol{n}^{o}$ that locally maximize $W(\boldsymbol{n})$ may exist. Moreover, characterizing the global maximum is extremely difficult. Thus, we focus on the policy to achieve local optimum $\boldsymbol{n}^{o}$ from stable equilibrium $\boldsymbol{n}^{*}$.

5.3.2 Distributions of intracity population and WSTs under local optimum

Local optimum $\boldsymbol{n}^{o}$ is the local maximum point of social welfare function $W(\boldsymbol{n})$; therefore, it satisfies the following KKT conditions of the optimization problem (5.28).

$$\begin{cases} \hat{v}^{*} = \hat{v}^{d}_{a,i}(\boldsymbol{n}^{o}) & \text{if} \quad n^{do}_{a,i} > 0, \\ \hat{v}^{*} \geq \hat{v}^{d}_{a,i}(\boldsymbol{n}^{o}) & \text{if} \quad n^{do}_{a,i} = 0, \end{cases} \tag{5.29a}$$

$$\sum_{d\in\mathcal{D}}\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{I}} n^{do}_{a,i} = N. \tag{5.29b}$$

where, $\hat{v}^{d}_{a,i}(\boldsymbol{n})$ is expressed as follows:

$$\hat{v}^{d}_{a,i}(\boldsymbol{n}) = v^{d}_{a,i}(\boldsymbol{n}) + G^{d}_{i}(\boldsymbol{m}) - \sum_{b=1}^{a} c'(x^{d}_{b,i})x^{d}_{b,i}. \tag{5.30}$$

The following proposition is obtained using these KKT conditions and adopting the same procedure as in Section 3.4.

Proposition 5.6 Local optimum $\boldsymbol{n}^{o}$ has the following properties.

(i) Suppose $N^{do}_{a} > 0$ and $(d, a-1, i)$, $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^{o})$, then, (d, a, i), $(d, a, j) \in \text{supp}(\boldsymbol{n}^{o})$ and $x^{do}_{a,i} = x^{do}_{a,j} > 0$. Moreover, suppose $N^{do}_{a} > 0$ and $(d, a-1, i) \notin \text{supp}(\boldsymbol{n}^{o})$, $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^{o})$, then, $n^{do}_{a,i} \leq n^{do}_{a,j}$ and $x^{do}_{a,i} \leq x^{do}_{a,j}$.

(ii) Suppose $N^{do}_{a} > 0$ and $N^{do}_{a-1} > 0$, then, $N^{do}_{a} < N^{do}_{a-1}$.

This proposition shows that properties of the distributions of intracity population and WSTs under local optimum, coincide with equilibrium. That is, the local optimum satisfies the following properties.

(a) The nearer the CBD where workers live, the narrower their distribution of WSTs.

(b) The nearer the residence is from the CBD, the greater the population.

5.3.3 Pigouvian policies

We next discuss tax/subsidy policies that attain the optima as stable equilibria. To achieve the optimum, we consider Pigouvian policies, such as congestion tolls. We do so because the optimal state is supported as an equilibrium by imposing such policies that workers are responsible for their externalities at the optimum. The Pigouvian policy that introduces tax/subsidy $p^d_{a,i}(\boldsymbol{n})$ to workers, is given by

$$p^d_{a,i}(\boldsymbol{n}) = G^d_i(\boldsymbol{m}) - \sum_{b=1}^{a} c'(x^d_{b,i})x^d_{b,i}. \tag{5.31}$$

Under the Pigouvian policy, our model is viewed as a potential game $\hat{S} = \{\mathcal{S},\ \mathcal{D} \times \mathcal{A} \times \mathcal{I},\ \hat{\boldsymbol{V}}\}$, where $\hat{v}^d_{a,i}(\boldsymbol{n}) = v^d_{a,i}(\boldsymbol{n}) + p^d_{a,i}(\boldsymbol{n})$, because the following potential function exists:

$$\hat{P}(\boldsymbol{n}) = P(\boldsymbol{n}) + p^d_{a,i}(\boldsymbol{n}) \cdot \boldsymbol{n} = W(\boldsymbol{n}). \tag{5.32}$$

The KKT conditions of the maximization problem of the potential function $\hat{P}(\boldsymbol{n})$ subject to $\boldsymbol{n} \in \Delta$ is given by the following:

$$\begin{cases} \hat{v}^* = v^d_{a,i}(\boldsymbol{n}) + p^d_{a,i}(\boldsymbol{n}) & \text{if} \quad n^d_{a,i} > 0 \\ \hat{v}^* \geq v^d_{a,i}(\boldsymbol{n}) + p^d_{a,i}(\boldsymbol{n}) & \text{if} \quad n^d_{a,i} = 0 \end{cases}$$
$$\forall d \in \mathcal{D}, a \in \mathcal{A}, i \in \mathcal{I}, \tag{5.33a}$$

$$\sum_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \sum_{i\in\mathcal{I}} n^d_{a,i} = N. \tag{5.33b}$$

This implies that the local optimum $\boldsymbol{n}^o$ must be a Nash equilibrium of the game $\hat{S}$.

Effects of policy implementation on the intracity spatial and temporal structure

We next investigate the effects of Pigouvian policy on the distributions of intracity population and WSTs. More specifically, we analyze the effects of introducing policies when the equilibrium $\boldsymbol{n}^*$ is given as the initial state.

First, we consider the effects on the distribution of intracity population. Suppose $(d, a-1, i)$, $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^o)$, the following always holds:

$$\hat{v}^d_{a,i}(\boldsymbol{n}^*) - \hat{v}^d_{a-1,i}(\boldsymbol{n}^*) = -c'(x^{d*}_{a,i})x^{d*}_{a,i} < 0, \tag{5.34}$$

which can be rewritten as $\hat{v}^d_{a,i}(\boldsymbol{n}^*) < \hat{v}^d_{a-1,i}(\boldsymbol{n}^*)$.

Second, let us analyze the effects on the distribution of WSTs. Suppose $(d, a, i), (d, a, i+1) \in \text{supp}(\boldsymbol{n}^*)$, because of $v^d_{a,i}(\boldsymbol{n}^*) = v^d_{a,i+1}(\boldsymbol{n}^*)$, we have

$$\hat{v}^d_{a,i}(\boldsymbol{n}^*) - \hat{v}^d_{a,i+1}(\boldsymbol{n}^*) = \sum_{b=1}^{a} \left\{ \left[c(x^{d*}_{b,i}) - c'(x^{d*}_{b,i})x^{d*}_{b,i} \right] - \left[c(x^{d*}_{b,i+1}) - c'(x^{d*}_{b,i+1})x^{d*}_{b,i+1} \right] \right\}. \tag{5.35}$$

Moreover, since

$$\frac{\partial [c(x) - c'(x)x]}{\partial x} = -c''(x)x < 0, \tag{5.36}$$

if $M^{d*}_i > M^{d*}_{i+1}$, we obtain $\hat{v}^d_{a,i}(\boldsymbol{n}^*) < \hat{v}^d_{a,i+1}(\boldsymbol{n}^*)$. That is, according to Proposition 5.3, if $M^{d*}_i > M^{d*}_{i+1}$, we have the following conditions.

$$\begin{cases} x^{d*}_{a,i} \geq x^{d*}_{a,i+1} \qquad \forall a \in \mathcal{A}, \\ \text{there exists } a \in \mathcal{A} \text{ such that } x^{d*}_{a,i} > x^{d*}_{a,i+1}. \end{cases} \tag{5.37}$$

From the these results, we obtain the following proposition.

Proposition 5.7 Consider that Pigouvian policies are introduced in equilibrium $\boldsymbol{n}^*$. We have the following properties.

(i) For all (d, a, i), $(d, a-1, i) \in \text{supp}(\boldsymbol{n}^*)$, we have $\hat{v}^d_{a,i}(\boldsymbol{n}^*) < \hat{v}^d_{a-1,i}(\boldsymbol{n}^*)$.

(ii) Suppose $M_i^{d*} > M_{i+1}^{d*}$, for all $(d, a, i), (d, a, i+1) \in \text{supp}(\boldsymbol{n}^*)$, we have $\hat{v}_{a,i}^d(\boldsymbol{n}^*) < \hat{v}_{a,i+1}^d(\boldsymbol{n}^*)$.

In this Proposition, (i) shows that the implementation of the tax/subsidy policy $p_{a,i}^d(\boldsymbol{n})$ has an increasing effect on the population distribution near CBD when equilibrium $\boldsymbol{n}^*$ is taken as the initial state. (ii) implies that the implementation of the tax/subsidy policy $p_{a,i}^d(\boldsymbol{n})$ staggers WSTs for each residential location.

It is noteworthy that though the local optimum distribution of WSTs in each residential location is more staggered than stable equilibrium, the local optimum distribution of total WSTs can be more clustered than stable equilibrium. This is because the population density near CBD will be higher after policy implementation, where WSTs distribution is more clustered than suburban. Therefore, staggering WSTs can not only improve but also decrease social welfare.

Conditions of intercity population concentration under local optimum

We now investigate the conditions of intercity population concentration under local optimum. Similar to Section 3.5 that assumes the existence of an equilibrium $\boldsymbol{n}^*$ such that $(\text{II}, a, i) \notin \text{supp}(\boldsymbol{n}^o)$ for any $a \in \mathcal{A}, i \in \mathcal{I}$, we have

$$\hat{v}_{a,i}^{\text{I}}(\boldsymbol{n}^*) \geq \hat{v}_{a,i}^{\text{II}}(\boldsymbol{n}^*) \quad \forall a \in \mathcal{A}, i \in \mathcal{I}. \tag{5.38}$$

Eqs. (5.12), (5.29a) and (5.30) yield the following:

$$\psi \geq \frac{\kappa_{a,i}(\boldsymbol{n}) + \zeta_{a,i}(\boldsymbol{n})}{2\alpha \sum_{j\in\mathcal{I}}(1-\phi)^{|i-j|}(M_j^{\text{I}*} - M_j^{\text{II}*})}, \tag{5.39a}$$

$$\kappa_{a,i}(\boldsymbol{n}) = \sum_{b=1}^{a} \left[c'(x_{b,i}^{\text{I}})x_{b,i}^{\text{I}} - c'(x_{b,i}^{\text{II}})x_{b,i}^{\text{II}} \right]. \tag{5.39b}$$

Let $\hat{f}_{a,i}(\boldsymbol{n}^*, \phi)$ denote the right-hand side of (5.39a). Then, $\frac{\partial \hat{f}_{a,i}(\boldsymbol{n}^*,\phi)}{\partial\phi} > 0$ and we have the following proposition.

Proposition 5.8 Suppose $\boldsymbol{n}^*$ is an equilibrium, then these statements are equivalent:

(i) $(\mathrm{II}, a, i) \notin \mathrm{supp}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$.

(ii) $\psi \geq \hat{f}_{a,i}(\boldsymbol{n}^*, \phi)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$.

This proposition implies that

(a) The lower the positive temporal agglomeration externality, the intercity population will be concentrated.

(b) The lower the positive spatial agglomeration externality, the more clustered the WSTs.

These properties coincide with the properties of Proposition 5.5.

We then compare the conditions of intercity population concentration under equilibrium and local optimum. More specifically, we compare the set of (ϕ, ψ) when intercity population concentrates under equilibrium and local optimum.

Let $\kappa_{a,i}(\boldsymbol{n}) = \sum_{b=1}^{a} \left[c'(x^{\mathrm{I}}_{b,i})x^{\mathrm{I}}_{b,i} - c'(x^{\mathrm{II}}_{b,i})x^{\mathrm{II}}_{b,i} \right]$ and $\zeta_{a,i}(\boldsymbol{n}) = \sum_{b=1}^{a} \left[c(x^{\mathrm{I}}_{b,i}) - c(x^{\mathrm{II}}_{b,i}) \right] + h(N^{\mathrm{II}}_{a}) - h(N^{\mathrm{I}}_{a})$, we then have the following proposition.

Proposition 5.9 Suppose an equilibrium $\boldsymbol{n}^*$ exists such that $N^{\mathrm{I}*} > 0$, then we have the following properties.

(i) If $\kappa_{a,i}(\boldsymbol{n}^*) < \zeta_{a,i}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$, there is no (ϕ, ψ) such that $N^{\mathrm{II}*} = 0$ and $N^{\mathrm{II}o} > 0$.

(ii) If $\kappa_{a,i}(\boldsymbol{n}^*) > \zeta_{a,i}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$, there is no (ϕ, ψ) such that $N^{\mathrm{II}*} > 0$ and $N^{\mathrm{II}o} = 0$.

This proposition shows that whether intercity population concentration can improve social welfare depends on the magnitude relation of $\kappa_{a,i}(\boldsymbol{n}^*)$ and $\zeta_{a,i}(\boldsymbol{n}^*)$ (i.e, urban cost). That is, intercity population concentration does not necessarily improve social welfare.

5.4 Numerical examples

We numerically analyze our model and show the distributions of populations and WSTs. This analysis assumes that the number of residential locations in each city is $A = 10$, and the number of WSTs is $I = 11$. We also assume $f(x) = -\frac{\mu}{x}$, $c(x) = t(1 + x^{\beta})$, and use the following parameter values:

$$N = 1000, \quad \alpha = 10.0, \quad \mu = 2.0,$$
$$t = 8.0, \quad \gamma = 10.0, \quad \psi = 0.05, \quad \phi = 0.05. \tag{5.40}$$

We then investigate the characteristics of stable equilibrium $\boldsymbol{n}^*$ and local optimum $\boldsymbol{n}^o$ with changes in parameters β and L.

5.4.1 Stable equilibrium

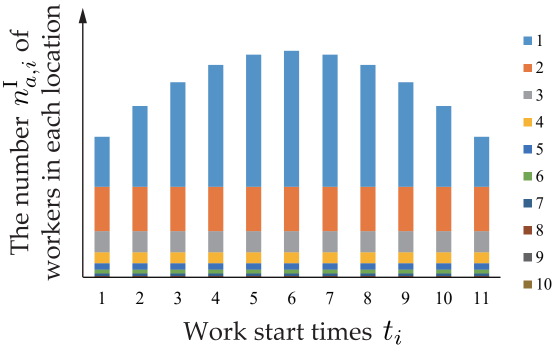
Distributions of intracity population and WSTs

We assume that $L = 1$ is fixed, and investigate the changes in the distributions of intracity population and WSTs under stable equilibrium with the decrease in β (by alleviating traffic congestion). Then, we confirm the correspondence between numerical and the theoretical results obtained in Section 5.2.

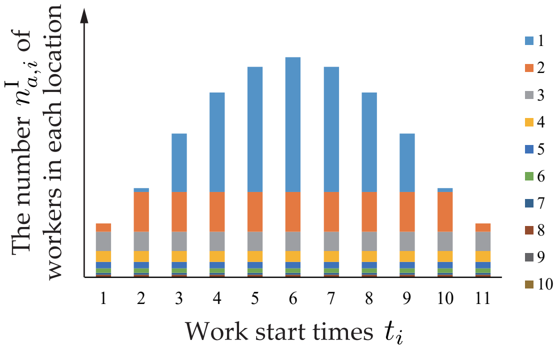
Figure 5.2 shows the number of workers in each WST t_i of city I (the result of city II is the same as city I). We can confirm the patterns from these results, which are consistent with Proposition 5.3 (i) and (ii). Moreover, our numerical results also coincide with the property that the nearer the CBD where workers live, the narrower their distribution of WSTs.

The number $\boldsymbol{n}^{\mathrm{I}*} = (N_a^{\mathrm{I}*})_{a\in\mathcal{A}}$ of workers in each residential location in city I under stable equilibrium is shown in Figure 5.3a. This numerical result confirms that "the nearer the residential location is from the CBD, the greater the population" in Proposition 5.4.

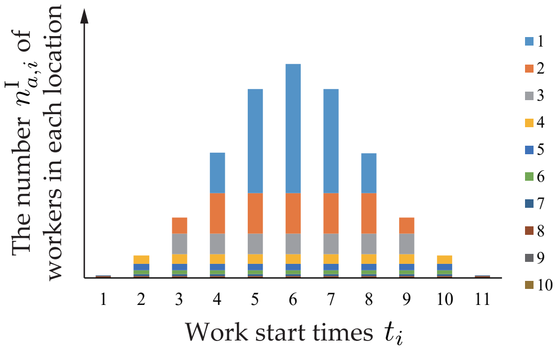
Next, we consider the effect of the alleviation of traffic congestion (the decrease in β). The number $\boldsymbol{m}^{\mathrm{I}*} = (M_i^{\mathrm{I}*})_{i\in\mathcal{I}}$ of workers at each WST in city I under stable equilibrium is shown in Figure 5.3b. As shown in Figures 5.3a and 5.3b, with the alleviation of traffic congestion, the population increases in the suburbs far from CBD, and WSTs are more clustered. This is because the reduction in commuting costs has brought about effects, such as "workers choose to live in large houses in the suburbs" and "firms cluster their WSTs to improve productivity (temporal agglomeration economies)."



(A) β = 1.40.



(B) β = 1.20.



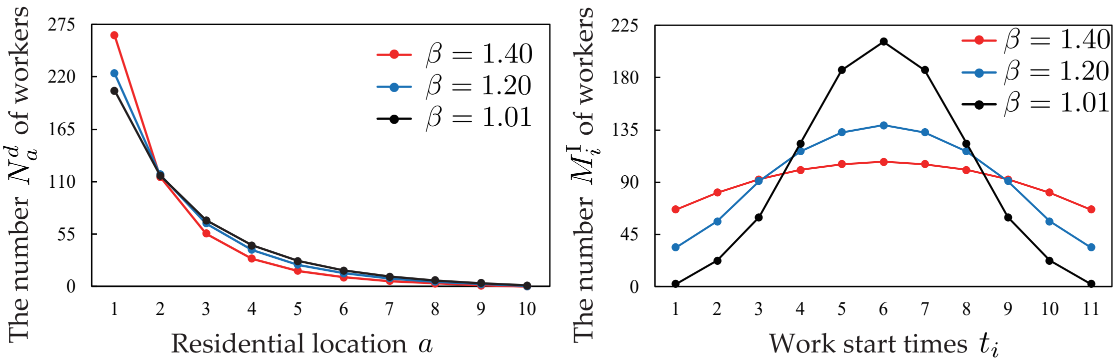
(C) β = 1.01.

FIGURE 5.2: The number of workers in each WST (L = 1).

Distributions of intercity population and WSTs

We investigated the situations when cities I and II are equally inhabited by workers in Section 5.4.1. This section also considers the cases of intercity population concentration, by clarifying the effects of changes in ϕ and ψ on the distributions $\boldsymbol{n}^*$ of populations and WSTs under stable equilibrium.

Figure 5.4 shows the contour plot of the difference in the number of workers at peak hour and off-peak hour (i.e., $\max \sum_{d\in\mathcal{D}} M^{d*}_{i^{\max}} - \min \sum_{d\in\mathcal{D}} M^{d*}_{i^{\min}}$ $i^{\max} = \mathrm{argmax}\boldsymbol{m}$, $i^{\min} = \mathrm{argmin}\boldsymbol{m}$) for each combination of ϕ and ψ. That is, Figure 5.4 shows the effects of different ϕ and ψ on the distributions of intercity population and WSTs. We also change land supply (L) and congestion situation (β) to see how the effects of different ϕ and ψ change. In these figures, the darker the shade of red, the more clustered are the WSTs; the darker the shade of blue, the more staggered are the WSTs. Moreover, the mesh areas show the cases that



(A) Distributions of intracity population.

(B) Distributions of WSTs.

FIGURE 5.3: Distributions of intracity population and WSTs ($L = 1$).

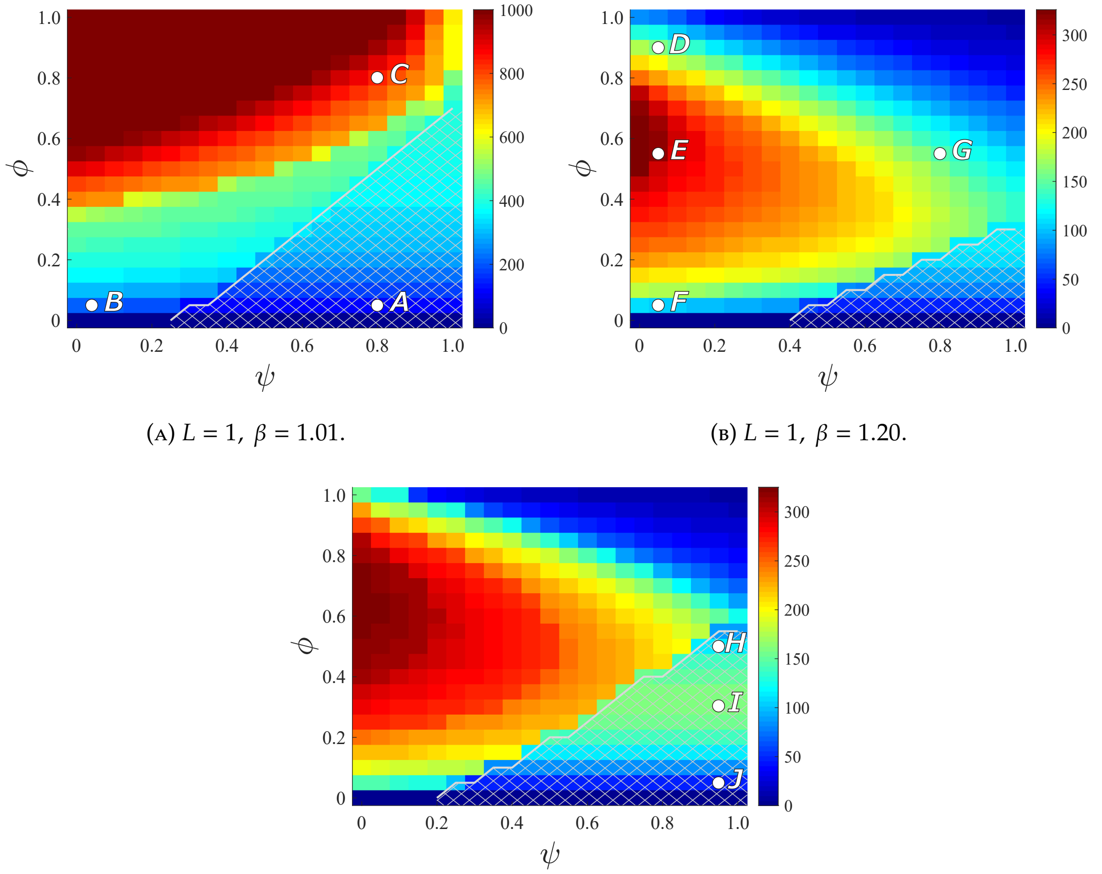
population concentrates in one city.

Figures 5.4 shows that if the value of ψ is fixed, the greater the value of ϕ, intercity population distribution can change from dispersed to concentrated. This result coincides with "alleviation of traffic congestion by staggering WSTs distribution can lead to intercity population concentration" in the property (a) of Proposition 5.5. Moreover, if the value of ϕ is fixed, the greater the value of ψ, the more staggered the WSTs distribution. This result implies that "The lower the positive spatial agglomeration externality, the more clustered the WSTs distribution," and coincides with the property (b) of Proposition 5.5. [6]

Figure 5.4a shows that when $L = 1$, $\beta = 1.01$ (low land supply, low commuting cost) and the value of ψ is fixed, for intercity population dispersion state, the greater the value of ϕ, WSTs distribution changes in the order of "clustered, staggered." This property coincides with the general perception that "low positive temporal agglomeration externality leads to staggered WSTs distribution."

Figure 5.4b shows that when $L = 1$, $\beta = 1.20$ (low land supply, high commuting cost) and the value of ψ is fixed, for intercity population dispersion state, the greater the value of ϕ, WSTs distribution changes in the order of "staggered, clustered, restaggered." This result implies that "low positive temporal agglomeration externality can lead to clustered WSTs distribution." This property occurs when the effect of traffic congestion is large (e.g., low

[6] To deeply understand the interaction between WSTs distribution and intercity population distribution, we compare different conditions in each figure. (see 5.B).



(A) $L = 1,\ \beta = 1.01$.

(B) $L = 1,\ \beta = 1.20$.

(C) $L = 20,\ \beta = 1.20$.

FIGURE 5.4: Distributions of intercity population and WSTs.

road capacity).

So far, we have investigated the effect of changes in ϕ and ψ on WSTs distributions under intercity population dispersion state. Next, let us investigate the WSTs distribution under intercity population concentration state when the value of L increases. Figure 5.4c is the result when $L = 20, \beta = 1.20$ (large land supply, high commuting cost). It is different from the case of $L = 1$: when the intercity population distribution is concentrated and the value of ψ is fixed, the greater the value of ϕ, WSTs distribution changes in the order of "staggered, clustered, restaggered." This result shows that the decrease in temporal agglomeration externality can lead to clustered WSTs distribution, independent of intercity population distribution. This property occurs when the effect of land consumption is low (e.g., large land supply).

5.4.2 Comparison between local optimum and stable equilibrium

This section investigates the local optimum properties and compares them with the properties of stable equilibrium. To this end, we verify the consistency with theoretical analysis results, and qualitatively analyze new properties through numerical analysis. Then through these findings, we clarify the impact of policy implementation.

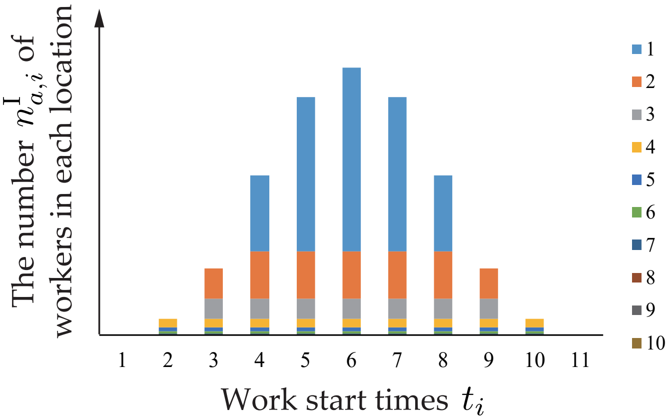


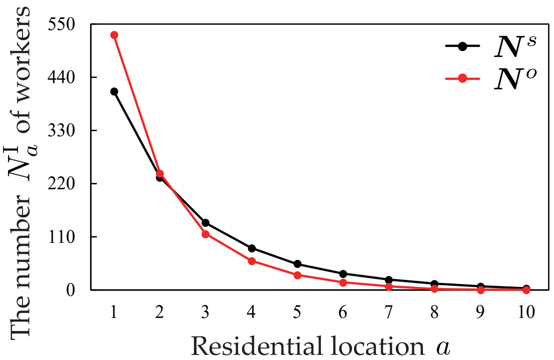
FIGURE 5.5: The number of workers of each WST under local optimum ($L = 1,\ \beta = 1.01$).

Distributions of intracity population and WSTs under local optimum and stable equilibrium

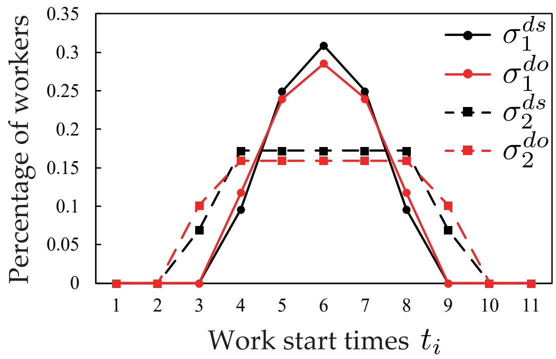
First, we show the distributions of intracity population and WSTs under local optimum. Figure 5.5 shows the number of workers of each WST, and Figure 5.6a shows the stable equilibrium and local optimum distributions of intracity population of city I in case of $L = 1,\ \beta = 1.01$. These results confirm that the distributions of urban population and WSTs are consistent with Proposition 5.6 (i) and (ii) (similar properties to stable equilibrium). Additionally, we have confirmed that the same results are obtained with other parameters, as in the case of $\beta = 1.01$.

We then compare the distributions of intracity population and WSTs between stable equilibrium and local optimum. Figure 5.6a implies that "population density near CBD under local optimum is higher than that under stable equilibrium," and it coincides with the policy implementation effects shown in Proposition 5.7 (i).

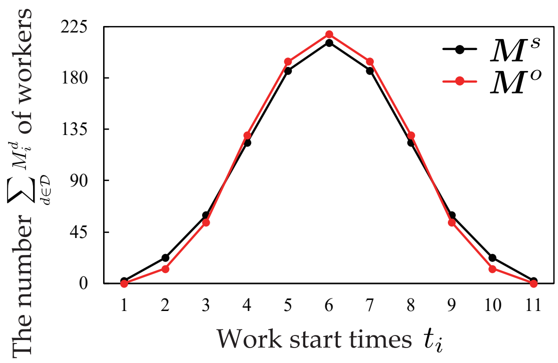
Figure 5.6b shows the percentage of workers of each WST in each residential location under



(A) Distributions of intracity population.



(B) Percentage of workers of each WST in each location.



(C) Distributions of WSTs.

FIGURE 5.6: Comparison of the distributions of populations and WSTs under stable equilibrium and local optimum ($L = 1, \beta = 1.01$).

stable equilibrium and local optimum, where $\sigma_a^{d\{s,o\}} = \frac{\sum_{d\in\mathcal{D}} n_{a,i}^{d\{s,o\}}}{\sum_{d\in\mathcal{D}} N_a^{d\{s,o\}}}$, $\forall a \in \mathcal{A}, i \in \mathcal{I}$. Here, we list only cases of residential locations 1 and 2. The figure implies that "WSTs distribution in each residential location under local optimum is more staggered than that under stable equilibrium," and it coincides with the policy implementation effects shown in Proposition 5.7 (ii).

Figure 5.6c shows the total number of workers of each WST under stable equilibrium and local optimum. It implies that WSTs distribution under local optimum is more clustered than that under stable equilibrium in the case of $L = 1$, $\beta = 1.01$. Thus, "staggering WSTs distribution may decrease social welfare." This result implies that there exist contrary results to Henderson (1981). We also verify that in some cases (e.g., high commuting cost), whole cities' WSTs distribution under local optimum can be more staggered than that under stable

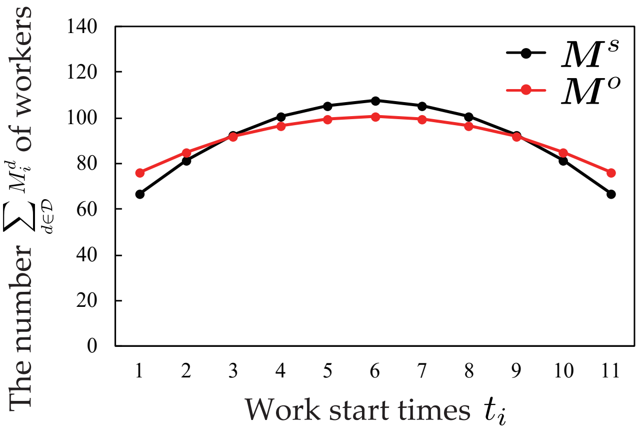


FIGURE 5.7: Distributions of WSTs under stable equilibrium and local optimum (L = 1, β = 1.45).

equilibrium, which are consistent with the results in Henderson (1981) (e.g., Figure 5.7). Based on these results, we find that when the congestion is alleviated (high road capacity), staggering WSTs distribution may decrease social welfare; when the commuting cost is high (e.g, low road capacity), staggering WSTs distribution may improve social welfare. Therefore, "staggering WSTs distribution does not necessarily improve social welfare."

Moreover, Figure 5.8 shows the WSTs distribution under local optimum. It implies that "The lower the positive spatial agglomeration externality, the more clustered the WSTs distribution under local optimum," which coincides with the property (b) of Proposition 5.8.

Distributions of intercity population under local optimum and stable equilibrium

This section investigates the effect of policy implementation on intercity population distribution. Specifically, we clarify how the intercity population distributions under stable equilibrium and local optimum in each combination of ϕ and ψ change, depending on land supply L and the value of the parameter β of BPR function.

The intercity population distributions under stable equilibrium and local optimum are shown in Figure 5.9 where Figures 5.9a, 5.9b and 5.9c show the intercity population distributions in the cases of (L = 20, β = 1.20), (L = 1, β = 1.20) and (L = 20, β = 1.45), respectively. The

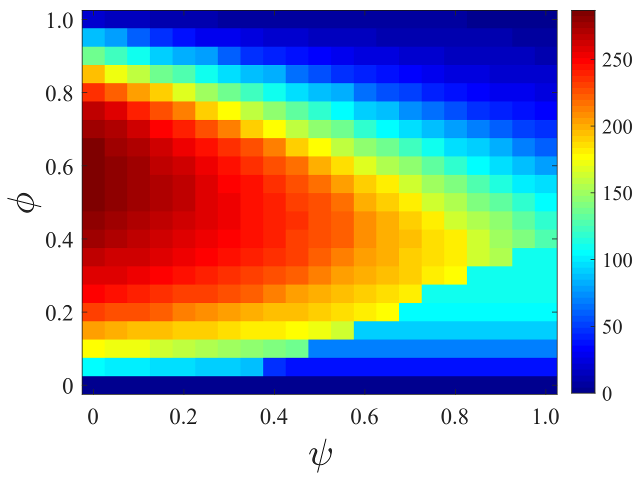
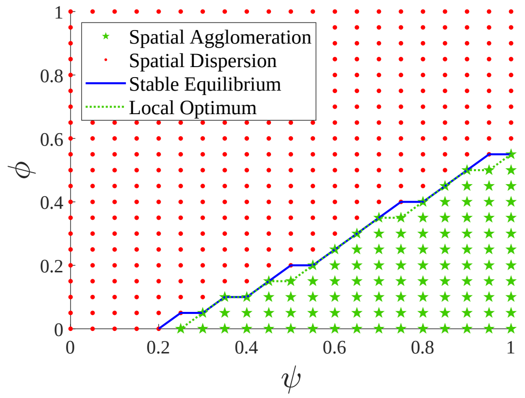


FIGURE 5.8: Distribution of WSTs under local optimum ($L = 1, \beta = 1.20$).

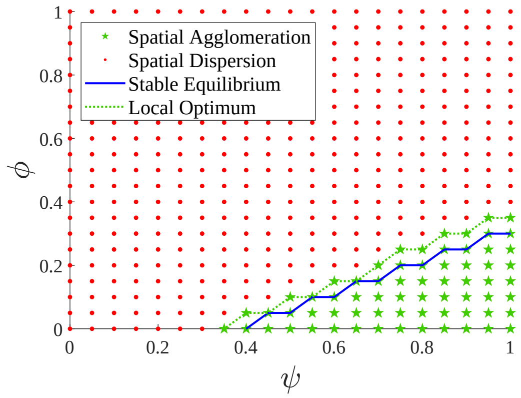
area from the solid blue line to the upper left is the area of intercity population concentration of stable equilibrium; meanwhile, the area from the dashed green line to the upper left is the area of intercity population concentration of local optimum.

Figure 5.9 shows that "the lower the positive temporal agglomeration externality, the intercity population will be concentrated" holds under stable equilibrium and local optimum. These results coincide with property (a) in Proposition 5.5 and property (a) in Proposition 5.8. Moreover, Figure 5.9a, 5.9b and 5.9c imply that intercity population concentration does not necessarily improve social welfare. These results coincide with the property in Proposition 5.9.

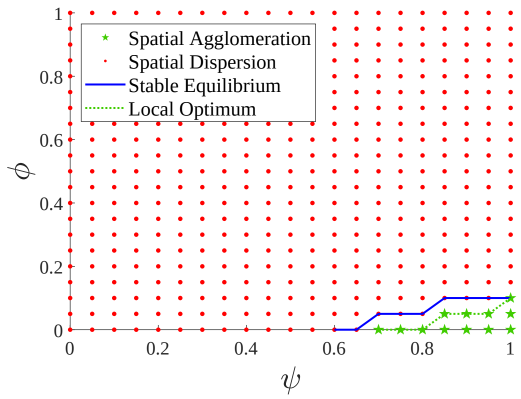
We then examine the impact of intercity population distribution under different urban costs on social welfare by comparing the Figure 5.9a, 5.9b and 5.9c. The comparison of Figures 5.9a and 5.9b implies that in the case of higher urban costs due to land consumption (lower land supply L), intercity population concentration improves social welfare. The comparison of Figures 5.9a and 5.9c implies that in the case of higher urban costs due to commuting cost (worse congestion situation β), intercity population dispersion improves social welfare. Therefore, we observe that for small cities, intercity population concentration may improve social welfare; for large cities, intercity population dispersion may improve social welfare.



(A) $L = 20$, $\beta = 1.20$.



(B) $L = 1$, $\beta = 1.20$.



(C) $L = 20$, $\beta = 1.45$.

FIGURE 5.9: Comparison of the intercity population distributions under stable equilibrium and local optimum.

5.5 Summary and Discussions

This chapter developed a system-of-cities model considering spatio-temporal agglomeration economies by introducing urban spatial structure into Henderson (1981)'s WST choice model. We showed that our model belongs to a class of potential game. Then, by using the properties of the potential game, we show the following properties of the model:

1) The lower the positive spatial agglomeration externality, the more clustered the WSTs distribution.

2) Low positive temporal agglomeration externality leads to intercity population concentration.

3) The equilibrium spatial (temporal) distribution may be less (more) agglomerated than at the optimum.

Here, Properties 3) implies that it is likely to be socially undesirable if policies such as staggered work hours, and promotions of urban population inflow are implemented ineptly.

This chapter aims to establish a basic framework for analyzing the endogenous distributions of populations and WSTs. Thus, we assumed that "firms are located in one CBD" and "firms and workers are homogeneous." However, these assumptions are considered to have a strong influence on the distributions of populations and WSTs. Therefore, it would be valuable for future research to extend this model into multi-central urban structure and investigate the effects of heterogeneities in firms and workers.

Appendix

5.A Proofs

5.A.1 Proof of Proposition 5.1

It follows from (5.17), (5.18) that $P(\boldsymbol{n})$ is a potential function. Because $P_4(\boldsymbol{n})$ is a linear function, hereafter, we prove the convexity of $P_1(\boldsymbol{n})$, $P_2(\boldsymbol{n})$, $P_3(\boldsymbol{n})$ to clarify the convexity of potential function $P(\boldsymbol{n})$.

First, let us investigate the convexity of $P_1(\boldsymbol{n})$. Hessian matrix of $P_1(\boldsymbol{n})$ is expressed as follows:

$$\nabla^2 P_1(\boldsymbol{n}) = \begin{bmatrix} \boldsymbol{F} & \psi\boldsymbol{F} \\ \psi\boldsymbol{F} & \boldsymbol{F} \end{bmatrix}, \tag{5.41a}$$

$$\boldsymbol{F} = \alpha \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \\ \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Phi} & \boldsymbol{\Phi} & \cdots & \boldsymbol{\Phi} \end{bmatrix} = \alpha(\boldsymbol{E}_A \otimes \boldsymbol{\Phi}). \tag{5.41b}$$

where, $\otimes$ denotes the Kronecker product, $\boldsymbol{E}_A$ is an $A \times A$ matrix with all elements equal to 1, and $\mathbf{\Phi} \equiv (1-\phi)^{\boldsymbol{D}}$, $\boldsymbol{D} = \begin{bmatrix} 0 & 1 & \cdots & I-1 \\ 1 & 0 & \cdots & I-2 \\ \vdots & \vdots & \ddots & \vdots \\ I-1 & I-2 & \cdots & 0 \end{bmatrix}$.

The positive/negative definite of $\mathbf{\Phi}$ coincides with the positive/negative definite of its inverse matrix $\mathbf{\Phi}^{-1}$; hence, we focus on $\mathbf{\Phi}^{-1}$:

$$\mathbf{\Phi}^{-1} = \frac{1}{\iota} \begin{bmatrix} 1 & \phi-1 & & & & \\ \phi-1 & 1+(1-\phi)^2 & \phi-1 & & \boldsymbol{O} & \\ & \phi-1 & 1+(1-\phi)^2 & \phi-1 & & \\ & & \ddots & \ddots & \ddots & \\ & \boldsymbol{O} & & \phi-1 & 1+(1-\phi)^2 & \phi-1 \\ & & & & \phi-1 & 1 \end{bmatrix}. \tag{5.42}$$

where $\iota \equiv \alpha[1-(1-\phi)^2]$.

Let γ_{ij} denote the element (i,j) of $\mathbf{\Phi}^{-1}$, from the Gershgorin circle theorem, λ_i^{-1} which denotes any eigenvalue of $\mathbf{\Phi}^{-1}$ exists in the range of

$$|\lambda_i^{-1} - \gamma_{ii}| \leq R_i \qquad \forall i \in \mathcal{I}. \tag{5.43}$$

where $R_i \equiv \sum_{j=1}^{I} |\gamma_{ij}|$.

Applying (5.43), it is readily that λ_i^{-1} exists in the following range:

$$0 \leq \frac{1}{2\alpha}\frac{\phi}{2-\phi} \leq \lambda_i^{-1} \leq \frac{1}{2\alpha}\frac{2-\phi}{\phi}. \tag{5.44}$$

Therefore, λ_i is nonnegative.

Let μ be the eigenvalue of $\boldsymbol{E}_A$, $\boldsymbol{E}$ be an $A \times A$ identity matrix. Then, since

$$
\begin{aligned}
\boldsymbol{E}_A - \mu \boldsymbol{E} &= \begin{bmatrix} 1-\mu & 1 & \cdots & 1 \\ 1 & 1-\mu & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1-\mu \end{bmatrix} \\
&= (A-\mu)\mu^{A-1},
\end{aligned}
$$

the eigenvalues of $\boldsymbol{E}_A$ are $\mu = 0, A$, the eigenvalue of $\nabla^2 P_1(\boldsymbol{n})$ is $\lambda_i \mu \geq 0$. Thus, $\nabla^2 P_1(\boldsymbol{n})$ is positive-semidefinite, that is, $P_1(\boldsymbol{n})$ is a convex function.

Next, let us focus on the convexity of $P_2(\boldsymbol{n})$. Hessian matrix of $P_2(\boldsymbol{n})$ can be expressed as follows:

$$
\nabla^2 P_2(\boldsymbol{n}) = \begin{bmatrix} \boldsymbol{\eta}^{\mathrm{I}} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{\eta}^{\mathrm{II}} \end{bmatrix}, \tag{5.45a}
$$

$$
\boldsymbol{\eta}^d = \begin{bmatrix} \nabla \boldsymbol{c}^d_{11} & \nabla \boldsymbol{c}^d_{12} & \nabla \boldsymbol{c}^d_{13} & \cdots & \nabla \boldsymbol{c}^d_{1A} \\ & \nabla \boldsymbol{c}^d_{22} & \nabla \boldsymbol{c}^d_{23} & \cdots & \nabla \boldsymbol{c}^d_{2A} \\ & & \nabla \boldsymbol{c}^d_{33} & \cdots & \nabla \boldsymbol{c}^d_{3A} \\ & \boldsymbol{O} & & \ddots & \vdots \\ & & & & \nabla \boldsymbol{c}^d_{AA} \end{bmatrix}, \tag{5.45b}
$$

$$
\nabla \boldsymbol{c}^d_{a\hat{a}} = \operatorname{diag}\left\{ \left[\sum_{b=1}^{a} c'(x^d_{b,i}) \right]_{i \in \mathcal{I}} \right\}. \tag{5.45c}
$$

We observe that $\nabla^2 P_2(\boldsymbol{n})$ is a diagonal matrix, $\boldsymbol{\eta}^d$ is an upper triangular matrix, and all the diagonal elements are positive. Thus, all the eigenvalues are positive, namely, $\nabla^2 P_2(\boldsymbol{n})$ is positive-semidefinite. Moreover, since $c'(x) > 0$, $\nabla P_2(\boldsymbol{n})$ is a convex function.

We next investigate the convexity of $P_3(\boldsymbol{n})$. Hessian matrix of $P_3(\boldsymbol{n})$ is expressed as follows:

$$\nabla^2 P_3(\boldsymbol{n}) = \begin{bmatrix} \chi^{\mathrm{I}} & \boldsymbol{O} \\ \boldsymbol{O} & \chi^{\mathrm{II}} \end{bmatrix}, \tag{5.46a}$$

$$\chi^d = \begin{bmatrix} \boldsymbol{H}_1^d & & & & \\ & \boldsymbol{H}_2^d & & \boldsymbol{O} & \\ & & \boldsymbol{H}_3^d & & \\ & \boldsymbol{O} & & \ddots & \\ & & & & \boldsymbol{H}_A^d \end{bmatrix}, \tag{5.46b}$$

$$\boldsymbol{H}_a^d = -h'(N_a^d)\boldsymbol{I}. \tag{5.46c}$$

where $\boldsymbol{I}$ is a $1 \times I$ matrix with all elements equal to 1. Since $h'(x) < 0$, the eigenvalues of $\nabla^2 P_3(\boldsymbol{n})$ are positive and $\nabla^2 P_3(\boldsymbol{n})$ is positive definite, namely, $P_3(\boldsymbol{n})$ is a convex function.

5.A.2 Proof of Proposition 5.3

Suppose $(d, a-1, i)$, $(d, a-1, j) \in \mathrm{supp}(\boldsymbol{n}^*)$ for all $d \in \mathcal{D}$, $a \in \mathcal{A}$, $i, j \in \mathcal{I}$. From (5.23), we have

$$v_{a,i}^d(\boldsymbol{n}^*) - v_{a,j}^d(\boldsymbol{n}^*) = -c(x_{a,i}^{d*}) + c(x_{a,j}^{d*}). \tag{5.47}$$

Thus we have the following condition:

$$\begin{cases} v^d_{a,i}(\boldsymbol{n}^*) > v^d_{a,j}(\boldsymbol{n}^*) \quad \text{if} \quad x^{d*}_{a,i} < x^{d*}_{a,j}, \\ v^d_{a,i}(\boldsymbol{n}^*) = v^d_{a,j}(\boldsymbol{n}^*) \quad \text{if} \quad x^{d*}_{a,i} = x^{d*}_{a,j}, \\ v^d_{a,i}(\boldsymbol{n}^*) < v^d_{a,j}(\boldsymbol{n}^*) \quad \text{if} \quad x^{d*}_{a,i} > x^{d*}_{a,j}. \end{cases} \tag{5.48}$$

Suppose $(d, a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, we have

$$v^d_{a,i}(\boldsymbol{n}^*) - v^d_{a,j}(\boldsymbol{n}^*) \leq -c(x^{d*}_{a,i}) + c(x^{d*}_{a,j}). \tag{5.49}$$

Therefore, we have the following condition:

$$v^d_{a,i}(\boldsymbol{n}^*) < v^d_{a,j}(\boldsymbol{n}^*) \quad \text{if} \quad x^{d*}_{a,i} > x^{d*}_{a,j}. \tag{5.50}$$

Suppose $N^{d*}_a > 0$, thus, there exists i which satisfies $x^{d*}_{a,i} > 0$. Moreover, from (5.48) and (5.50), we observe that $x^{d*}_{a,i} > 0$ if $(d, a-1, i) \in \text{supp}(\boldsymbol{n}^*)$. Combining this result with (5.48), we have Proposition 5.3 (i).

Furthermore, it follows from (5.50) that if $(d, a-1, i) \notin \text{supp}(\boldsymbol{n}^*)$ and $(d, a-1, j) \in \text{supp}(\boldsymbol{n}^*)$, $x^{d*}_{a,i} \leq x^{d*}_{a,j}$ holds. By combining this result with Proposition 3 (i), $n^{d*}_{a,i} \leq n^{d*}_{a,j}$ holds, thus, we have Proposition 5.3 (ii).

5.A.3 Proof of Proposition 5.5

Suppose $\boldsymbol{n}^*$ is an equilibrium, according to (5.15a), we have the following condition:

$$\begin{cases} v^* = v^{\text{I}}_{a,i}(\boldsymbol{n}^*) = v^{\text{II}}_{a,i}(\boldsymbol{n}^*) \;\text{ if }\; n^{\text{I}}_{a,i} > 0,\; n^{\text{II}}_{a,i} > 0, \\ v^* = v^{\text{I}}_{a,i}(\boldsymbol{n}^*) \geq v^{\text{II}}_{a,i}(\boldsymbol{n}^*) \;\text{ if }\; n^{\text{I}}_{a,i} > 0,\; n^{\text{II}}_{a,i} = 0, \\ v^* = v^{\text{II}}_{a,i}(\boldsymbol{n}^*) \geq v^{\text{I}}_{a,i}(\boldsymbol{n}^*) \;\text{ if }\; n^{\text{I}}_{a,i} = 0,\; n^{\text{II}}_{a,i} > 0, \\ v^* \geq v^{\text{I}}_{a,i}(\boldsymbol{n}^*),\; v^* \geq v^{\text{II}}_{a,i}(\boldsymbol{n}^*) \;\text{ if }\; n^{\text{I}}_{a,i} = 0,\; n^{\text{II}}_{a,i} = 0. \end{cases} \tag{5.51}$$

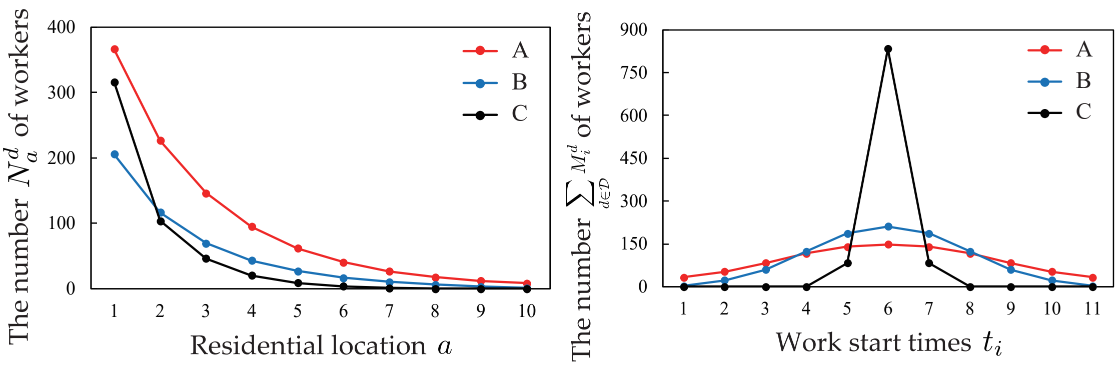
Thus, it is readily to verify that $(\mathrm{II}, a, i) \notin \mathrm{supp}(\boldsymbol{n}^*)$ is equivalent to $v^{\mathrm{I}}_{a,i}(\boldsymbol{n}^*) \geq v^{\mathrm{II}}_{a,i}(\boldsymbol{n}^*)$. Moreover, $\psi \leq f_{a,i}(\boldsymbol{n}^*(1-\phi), \phi)$ is equivalent to $v^{\mathrm{I}}_{a,i}(\boldsymbol{n}^*) \geq v^{\mathrm{II}}_{a,i}(\boldsymbol{n}^*)$ for all $a \in \mathcal{A}$, $i \in \mathcal{I}$; therefore, Proposition 5.5 holds.

5.B Examples of the distributions of populations and WSTs

In this section, we show effects of ϕ and ψ on distributions of populations and WSTs relative to Figure 5.4, under the changes of land supply (L) and congestion situation (β).

5.B.1 Different positive spatio-temporal agglomeration externalities under different intercity population distributions

In Figure 5.4a ($L = 1$, $\beta = 1.01$), point A ($\psi = 0.2$, $\phi = 0.95$) shows the state of intercity population concentration, points B ($\psi = 0.95$, $\phi = 0.95$) and C ($\psi = 0.2$, $\phi = 0.2$) show the states of intercity population dispersion. We use these 3 points to show the effects of different intercity population distributions on the distributions of intracity population and WSTs. Figure 5.10a shows the comparison of intracity population distributions, and Figure 5.10b shows the comparison of WSTs distributions.



(A) Distributions of intracity population. (B) Distributions of WSTs.

FIGURE 5.10: Distributions of intracity population and WSTs ($L = 1$, $\beta = 1.01$).

The followings explain the mechanisms of those distributions' formations:

A Low positive temporal agglomeration externality (high ϕ) (e.g., easy interaction among different WSTs) and high positive spatial agglomeration externality (low ψ) (e.g., difficult interaction between different cities) lead to intercity population concentration. Meanwhile, low positive temporal agglomeration externality yield the less effect of temporal agglomeration economies; thus, the distribution of WSTs will be staggered.

B Low positive temporal agglomeration externality (high ϕ) and low positive spatial agglomeration externality (high ψ) (e.g., easy interaction between different cities) lead to intercity population dispersion and high productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration economies will be less, and the distributions of intracity population and WSTs will be dispersed.

C High positive temporal agglomeration externality (low ϕ) (e.g., difficult interaction among different WSTs) and high positive spatial agglomeration externality (low ψ) lead to intercity population dispersion and low productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration economies will be greater, and the distributions of intracity population and WSTs will be agglomerated.

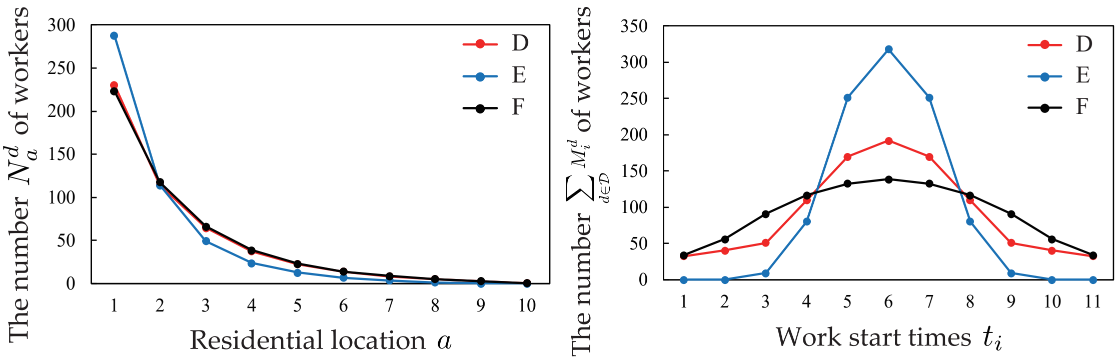
5.B.2 Different positive temporal agglomeration externalities under intercity population dispersion state

In Figure 5.4b ($L = 1$, $\beta = 1.20$), points D ($\psi = 0.95$, $\phi = 0.1$), E ($\psi = 0.95$, $\phi = 0.45$), and F ($\psi = 0.95$, $\phi = 0.95$) show the states of intercity population dispersion. We use these 3 points to show the effects of different positive temporal agglomeration externalities (different ϕ) on the distributions of intracity population and WSTs under intercity population dispersion state. Figure 5.11a shows the comparison of intracity population distributions, and Figure 5.11b shows the comparison of WSTs distributions.

The followings explain the mechanisms of those distributions' formations:

D High positive temporal agglomeration externality (low ϕ) leads to low productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration diseconomies will be greater, and distributions of intracity population and WSTs will be dispersed.

E Relatively low positive temporal agglomeration externality (relatively high ϕ) leads to relatively high productivity of firms (workers' wage). Thus, the effects of spatio-



(A) Distributions of intracity population. (B) Distributions of WSTs.

FIGURE 5.11: Distributions of intracity population and WSTs (L = 1, β = 1.20).

temporal agglomeration diseconomies will be less, and the distributions of intracity population and WSTs will be agglomerated.

F Because of the low positive temporal agglomeration externality (high ϕ), starting work at any time has virtually no effect on the productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration economies will be less, traffic congestion will be alleviated, effects of spatial agglomeration economies will be lower; and the distributions of intracity population and WSTs will be dispersed.

5.B.3 Different positive temporal agglomeration externalities under intercity population concentration state

Next, we consider the cases in Figure 5.4c (L = 20, β = 1.20), points H (ψ = 0.05, ϕ = 0.5), I (ψ = 0.05, ϕ = 0.7), and J (ψ = 0.05, ϕ = 0.95) show the states of intercity population concentration. When L = 20, land supply in each residential location is sufficient, net utility from land consumption of workers is sufficient high, and spatial agglomeration diseconomies due to land consumption will be lower; thus, intracity population distribution of these 3 points are consistent.

We then use these 3 points to show the effects of different positive temporal agglomeration externalities (different ϕ) on intracity population and WSTs distributions under intercity population concentration state. Figure 5.12 shows the comparison of WSTs distributions.

The followings explain the mechanisms of those distributions' formations:

H High positive temporal agglomeration externality (low ϕ) leads to low productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration diseconomies will be greater, and the distribution of WSTs will be staggered.

I Relatively low positive temporal agglomeration externality (relatively high ϕ) leads to relatively high productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration diseconomies will be less, and the distribution of WSTs will be clustered.

J Because of the low positive temporal agglomeration externality (high ϕ), starting work at any time has virtually no effect on the productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration economies will be less, traffic congestion will be alleviated, and the distribution of WSTs will be staggered.

5.B.4 Different positive spatial agglomeration externalities under intercity population dispersion state

In Figure 5.4b ($L = 1$, $\beta = 1.20$), points E ($\psi = 0.95$, $\phi = 0.45$) and G ($\psi = 0.2$, $\phi = 0.45$) show the states of intercity population dispersion. We use these 2 points to show the effects of different positive spatial agglomeration externalities (different ψ) on intracity population and WSTs distributions under intercity population dispersion state. Figure 5.13a shows the comparison of intracity population distributions, and Figure 5.13b shows the comparison of WSTs distributions.

The followings explain the mechanisms of those distributions' formations:

E Low positive spatial agglomeration externality (high ψ) leads to high productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration diseconomies will be less, and the distributions of intracity population and WSTs will be agglomerated.

G High positive spatial agglomeration externality (low ψ) leads to low productivity of firms (workers' wage). Thus, the effects of spatio-temporal agglomeration diseconomies will be greater, and the distributions of intracity population and WSTs will be dispersed.

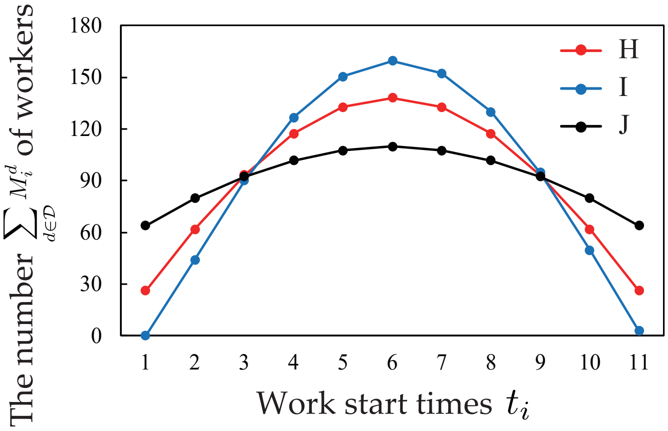
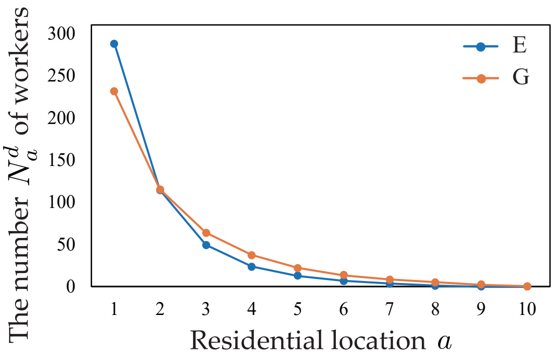
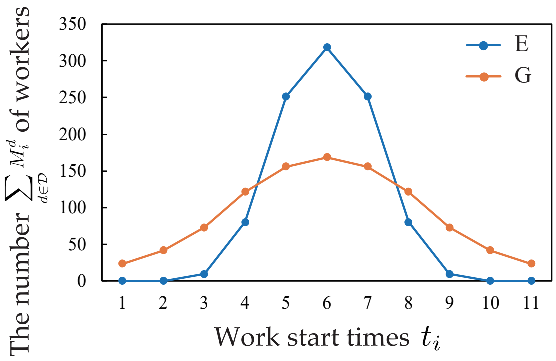


Figure 5.12: Distributions of WSTs (L = 20, β = 1.20).



(A) Distributions of intracity population.



(B) Distributions of WSTs.

Figure 5.13: Distributions of intracity population and WSTs (L = 1, β = 1.20).

6

Conclusions and Future Works

6.1 Summary

In this dissertation, we have investigated the long-term effects of 2 types of TDM measures on structural changes in cities. The long-term effects of reduction in road traffic demand and reduction in spatial and temporal agglomeration of economic activities are verified in Part I and II, respectively.

In Part I, we focused on temporal distribution of economic activities to clarify which departure time and which commuting mode commuters will choose. More specifically, In Chapter 2 and Chapter 3, we developed frameworks considering commuters' departure time and mode choice behavior, and scale economies in rail transit and carpooling, respectively. We then show the properties of equilibria when the regulator sets rail/carpooling fares equal to the marginal cost or average cost and when there is no regulation on rail/carpooling fares.

By comparing these equilibria, we obtained the following findings:

1) The implementation of marginal cost regulation leads to the highest number of rail/carpooling commuters and the lowest equilibrium commuting costs;

2) If average cost regulation is implemented simultaneously with the development of rail transit/carpooling, the number of rail/carpooling commuters will not increase and equilibrium commuting costs will not change;

3) If average cost regulation is implemented when rail/carpooling operators are monopolistically competitive, rail/carpooling commuters will increase and equilibrium commuting costs will decrease.

These results imply that when it is difficult to implement marginal cost regulation, a hasty implementation of average cost regulation can lead to a socially undesirable situation (i.e., high rail/carpooling fares and non-increasing in the number of rail/carpooling commuters). The above results also indicate that one way to alleviate the problem is to "allow rail operators to be a monopoly to increase the number of rail/carpooling commuters, and then implement average cost regulation." These results are unique to this chapter and have not been presented in previous studies.

In Part II, we focused on both spatial and temporal distribution of economic activities to clarify how commuters choose where they reside (urban or rural, which city, which residential location) and their WSTs under equilibrium and after policy implementation.

Specifically, Chapter 4 developed an open city model considering spatial and temporal agglomeration economies by introducing multiple residential locations and rural area into Henderson (1981)'s WST choice model. We showed that our model belongs to a class of potential game. Then, by using the properties of the potential game, we show the following properties of the model:

1) The greater interaction among different WSTs leads to spatial agglomeration (i.e., rural-to-urban migration), however, does not necessarily lead to lower population density.

2) The increase of traffic congestion leads to clustered WSTs.

3) The urban population at optimum can be lower than at equilibrium when urban cost is high.

Note that Property 2) shows that the contrary finding to (Takayama, 2019) and Chapter 5 occurs when the urban spatial structure is considered as an open city. And Property 3) implies that promoting rural-urban migration is not necessarily socially desirable.

Chapter 5 developed a system-of-cities model considering spatio-temporal agglomeration economies by introducing multiple residential locations and multiple cities into Henderson (1981)'s WST choice model. Similar to Chapter 4, potential game also exists in this model. Then by using the properties of the potential game, we show the following properties of the model:

1) The lower the positive spatial agglomeration externality, the more clustered the WSTs distribution.
2) Low positive temporal agglomeration externality leads to intercity population concentration.
3) The equilibrium spatial (temporal) distribution may be less (more) agglomerated than at the optimum.

Here, Properties 3) implies that it is likely to be socially undesirable if policies such as staggered work hours, and promotions of urban population inflow are implemented ineptly.

6.2 Future Works

This dissertation is not the final word on the analysis of long-run structural changes in cities.

Since this Chapter 2 and 3 focused on the impacts of the marginal cost/average cost regulations on rail/carpooling fares, we only consider the equilibria under those regulations/ without regulation. Thus, it is important to clarify the impacts of other regulations such as price-cap regulation (Kidokoro, 2006), compare social (i.e., first-best) optimum and second-best optimum as in Tabuchi (1993) and obtain insights on policies to achieve them (e.g., subsidies, time-of-day-varying fare, congestion pricing).

Chapter 4 and 5 aimed to establish basic frameworks for analyzing the endogenous distributions of populations and WSTs. Thus, we assumed that "firms are located in one CBD" and "firms and workers are homogeneous." However, these assumptions are considered to have a strong influence on the distributions of populations and WSTs. Therefore, it would be valuable for future research to extend this model into multi-central urban structure and investigate the effects of heterogeneities in firms and workers.

Next, we provide further details about three specific directions that are particularly urgent in our opinion.

1. To build a model considering spatial and temporal agglomeration economies that can deal with peak-period congestion.

   To integrate Part I and Part II, and extend to corridor network with multiple bottlenecks.

2. To describe the endogenous formation of CBD and take into account the land use of firms.

   To integrate our model with Fujita and Ogawa (1982) model.

3. To represent the urban structure of the next generation and propose efficient forms of public transportation.

   To consider mobility services, telework and heterogeneities in firms and workers.

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