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# A METHOD TO MINIMIZE TOTAL CAPACITANCE IN CASCADE REALIZATION OF SC FILTERS

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Abstract: A new pole-zero pairing and ordering strategy is proposed, which can provide the minimum total capacitance in cascaded biquad SC filters. Design examples for several kinds of SC filters are demonstrated. The proposed conditions can limit pairing and ordering assignments, by which the total capacitance concentrate within 5% over the minimum value.

## I INTRODUCTION

Switched-capacitor (SC) filters are very attractive for use in integrating many analog circuits on a single monolithic chip. Particularly, they have been applied to communication and signal processing systems, which inherently require analog/digital interface, low-power dissipation, and high-frequency operation [1],[2].

In order to achieve the following properties: (1)miniaturization, (2)low-power dissipation, and (3)high-frequency operation, reduction in the total capacitance is very important.

Capacitance reduction is accomplished through two stages. First, capacitance dispersion is compressed in a process of designing SC circuit configuration. Second, a unit capacitor, corresponding to the minimum capacitance, is reduced in an LSI fabrication process. This reduction is, however, rather limited by capacitance ratio accuracy and noise performance.

In this paper, a design problem in the first stage will be dealt with. Furthermore, cascaded biquad SC filters are taken into account. In the cascade SC filters, the total capacitance, which is a sum of all normalized capacitance, is highly dependent on pole-zero pairing and ordering. Therefore, it is very important to search for an optimum pairing and ordering assignment. This problem was investigated by Xuexiang et al [3],[4]. However, a general strategy for an optimum assignment has not been well accomplished.

In this paper, relations between pole-zero locations, scaling factors and the total capacitance in a biquad section is first investigated. Conditions on scaling factors and amplitude responses of the biquad section, which can lead the minimum total capacitance, are derived. These conditions are further transferred into that on pairing and ordering. The proposed conditions can limit pairing and ordering assignments, by which significant reductions in the total capacitance can be achieved. By searching for the optimum assignment among them, a lot of computing time can be saved. Finally, distribution of the total capacitance for 9th and 11th-order low-pass and high-pass filters are demonstrated.

# II TOTAL CAPACITANCE IN CASCADE SC FILTERS 2.1 Biquad SC Circuit

In our discussion, a biquad SC circuit shown in Fig.1 [5] is taken into account. The resulting conditions for pairing and ordering can be valid for another circuit configuration, such as a family of biquad SC circuits by Fleischer & Laker [6].

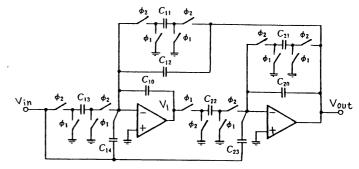


Fig.1 Biquad SC circuit by Martin & Sedra.

Letting  $V_{\text{in}}(z)$ ,  $V_{\text{i}}(z)$  and  $V_{\text{out}}(z)$  be the input, the first and second operational amplifier outputs, respectively, transfer functions are defined as follows:

$$H_1(z) = \frac{V_1(z)}{V_{in}(z)}$$
 (1a)

$$H(z) = \frac{V_{out}(z)}{V_{in}(z)}$$
 (1b)

These transfer function are further expressed using capacitance of the SC circuit shown in Fig.1.

$$\begin{split} H_{1}(z) &= \frac{\left[C_{23}(C_{11} + C_{12}) - (C_{13} + C_{14})(C_{20} + C_{21})\right]}{\left[C_{10}C_{20} + C_{10}C_{21}\right]} \sim \\ &= \frac{+\left[C_{14}C_{21} + C_{20}(C_{13} + 2C_{14}) - C_{23}(C_{11} + 2C_{12})\right]z^{-1}}{+\left[C_{12}C_{22} + C_{11}C_{22} - C_{10}C_{21} - 2C_{10}C_{20}\right]z^{-1}} \sim \\ &= \frac{+\left[C_{12}C_{23} - C_{14}C_{20}\right]z^{-2}}{+\left[C_{10}C_{20} - C_{12}C_{22}\right]z^{-2}} & (2a) \\ H(z) &= -\frac{\left[C_{10}C_{23}\right] + \left[C_{13}C_{22} + C_{14}C_{22} - 2C_{10}C_{23}\right]z^{-1}}{\left[C_{10}C_{20} + C_{10}C_{21}\right] + \left[C_{12}C_{22} + C_{11}C_{22} - C_{10}C_{21}\right]} \sim \\ &= \frac{+\left[C_{10}C_{23} - C_{14}C_{22}\right]z^{-2}}{-2C_{10}C_{20}|z^{-1} + \left[C_{10}C_{20} - C_{12}C_{22}\right]z^{-2}} & (2b) \end{split}$$

# 2.2 Capacitance Expressed with Pole and Zero

Since frequency selective filters usually have wide spread capacitance, elliptic filters are taken into account in this paper. Zero and pole can be expressed as  $\exp(\pm j\theta_B)$ ,  $r_B\exp(\pm j\theta_B)$ , respectively. Using these parameters, the transfer function H(z) is rewritten as follows:

$$H(z) = -h_0 \frac{1 - 2\cos\theta z^{-1} + z^{-2}}{1 - 2r_p \cos\theta z^{-1} + r_p^2 z^{-2}}$$
(3)

Furthermore, coefficients satisfy

$$0 < r_{p} < 1$$

$$0 \le \theta_{E} \le \pi \qquad -2 \le -2 \cos \theta_{E} \le 2$$

$$0 \le \theta_{p} \le \pi \qquad -2 \le -2 r_{p} \cos \theta_{p} \le 2$$

$$(4)$$

In order to obtain relations between the parameters, Eq.(2b) is rewritten as,

$$H(z) = -\frac{C_{10}C_{23}}{C_{10}C_{20} + C_{10}C_{21}} \cdot \frac{1 + \frac{C_{13}C_{22} + C_{14}C_{22} - 2C_{10}C_{23}}{C_{10}C_{23}} z^{-1}}{1 + \frac{C_{12}C_{22} + C_{11}C_{22} - C_{10}C_{21} - 2C_{10}C_{20}}{C_{10}C_{20} + C_{10}C_{21}}}$$

$$\sim \frac{\frac{C_{10}C_{23}-C_{14}C_{22}}{C_{10}C_{23}}z^{-2}}{\frac{C_{10}C_{20}-C_{12}C_{22}}{C_{10}C_{20}+C_{10}C_{21}}z^{-2}}$$
(5)

Comparing Eqs.(3) and (5), the following relations can be obtained.

$$\frac{C_{10}C_{23}}{C_{10}C_{20} + C_{10}C_{21}} = h_0$$
 (6a)

$$\frac{C_{13}C_{22}+C_{14}C_{22}-2C_{10}C_{23}}{C_{10}C_{23}} = -2COS\theta z$$
 (6b)

$$\frac{C_{10}C_{23}-C_{14}C_{22}}{C_{10}C_{23}}=1$$
 (6c)

$$\frac{C_{12}C_{22}+C_{11}C_{22}-C_{10}C_{21}-2C_{10}C_{20}}{C_{10}C_{20}+C_{10}C_{21}} = -2r_{p}COS\theta_{p}$$
 (6d)

$$\frac{C_{10}C_{20}-C_{12}C_{22}}{C_{10}C_{20}+C_{10}C_{21}} = r_{p}^{2}$$
 (6e)

The number of equations is smaller than that of unknown capacitance. Therefore, some of the capacitance should be fixed in advance. The following two cases are available for elliptic filters.

(A) 
$$C_{12}=C_{14}=0$$
 (7a)

(B) 
$$C_{21} = C_{14} = 0$$
 (7b)

Furthermore, the following assumptions are imposed.

$$h_{O} = 1 \tag{8a}$$

$$C_{10} = C_{20} = 1$$
 (8b)

These assumptions do not lose generality, because capacitance will be modified through signal level scaling and capacitance normalization. In the following discussion, the case (A), that is C12=C14=0, is taken into account, due to page limitation. However, the results of this paper can be applied to the case (B).

Now, we represent capacitance using the pole-zero parameters. Substituting Eqs.(7a), (8a) and (8b) for Eqs.(6a)  $\sim$  (6e), the following equations are obtained.

$$\frac{C_{23}}{1+C_{21}} = 1$$
 (9a)

$$\frac{C_{13}C_{22}-2C_{23}}{C_{23}} = -2\cos\theta z \tag{9b}$$

$$\frac{C_{23}}{C_{CO}} = 1 \tag{9c}$$

$$\frac{C_{11}C_{22}-C_{21}-2}{1+C} = -2r_{p}\cos\theta P$$
 (9d)

$$\frac{1}{1+C_{21}} = r_{p}^{2} \tag{9e}$$

Since Eq.(9c) is evident, the number of independent equations is reduced by one. Therefore, assuming

$$C_{22}=1 \tag{10}$$

the remaining capacitance can be expressed with the pole-zero parameters as follows:

$$C_{21} = \frac{1 - r_p^2}{r_p^2} \tag{11a}$$

$$C_{23} = \frac{1}{r^2}$$
 (11b)

$$C_{13} = \frac{2(1-\cos\theta_{2})}{r^{2}}$$
 (11c)

$$C_{13} = \frac{2(1-\cos\theta_{B})}{r_{p}^{2}}$$

$$C_{11} = \frac{1-2r_{p}\cos\theta_{p}+r_{p}^{2}}{r_{p}^{2}}$$
(11d)

# 2.3 Signal Level Scaling

In SC filters, signal level is scaled in order to avoid overload at operational amplifier outputs. Letting  $F_i(z)$  be a transfer function from the filter input to the ith operational amplifier output, it is scaled so as to satisfy

$$\max_{i} |F_i(e^{j\mathbf{w}})| = 1 \tag{14}$$

This is called the L∞ norm scaling [7].

The above scaling is actually carried out by modifying capacitance. When we want to increase an operational amplifier output by k, it can be done by decreasing an integrating capacitance and all capacitance, connected to the output, by 1/k. Let k1 and k2 be scaling factors for V1 and Vout, shown Fig.1, respectively. After signal level scaling, capacitance are modified as follows:

$$C^{-}_{10} = \frac{C_{10}}{k_1} = \frac{1}{k_1} \tag{13a}$$

$$C_{11}^{-1} = \frac{C_{11}}{k_{z}} = \frac{1}{k_{z}} \cdot \frac{1 - 2r_{p}\cos\theta_{p} + r_{p}^{z}}{r_{p}^{z}}$$
(13b)

$$C_{13} = \frac{C_{13}}{k_2'} = \frac{1}{k_2'} \cdot \frac{2(1-\cos\theta_z)}{r_p^2}$$
 (13c)

$$C_{20} = \frac{C_{20}}{k_0} = \frac{1}{k_0}$$
 (14a)

$$C_{21}^{-} = \frac{C_{21}}{k_2} = \frac{1}{k_2} \cdot \frac{1 - r_p^2}{r_p^2}$$
 (14b)

$$C_{20} = \frac{C_{20}}{k_2} = \frac{1}{k_2}$$

$$C_{21} = \frac{C_{21}}{k_2} = \frac{1}{k_2} \cdot \frac{1 - r_p^2}{r_p^2}$$

$$C_{22} = \frac{C_{22}}{k_1} = \frac{1}{k_1}$$
(14a)
$$C_{22} = \frac{C_{22}}{k_1} = \frac{1}{k_1}$$
(14b)

$$C^*_{23} = \frac{C_{23}}{k_2'} = \frac{1}{k_2'} \cdot \frac{1}{r_p^2}$$
 (14d)

where, k2' is a scaling factor for Vout in the preceding section.

# 2.4 Total Capacitance in Biquad SC Circuit

Before calculating the total capacitance, minimum capacitance in capacitance groups is normalized to unity. There are two capacitance groups in the SC circuit shown in Fig.1. One of them includes  $C_{1i}$ ,  $i=0 \sim 4$ , and the other includes  $C_{2i}$ ,  $i=0 \sim 3$ . Total capacitance for each capacitance group are given here before normalization.

$$C^{+}_{T1} = \frac{1}{k_{1}} + \frac{1}{k_{2}} \cdot \frac{1 - 2r_{p}\cos\theta_{p} + r_{p}^{2}}{r_{p}^{2}} + \frac{1}{k_{2}!} \cdot \frac{2(1 - \cos\theta_{g})}{r_{p}^{2}}$$
(15a)  
$$C^{+}_{T2} = \frac{1}{k_{2}} + \frac{1}{k_{2}} \cdot \frac{1 - r_{p}^{2}}{r_{p}^{2}} + \frac{1}{k_{1}} + \frac{1}{k_{2}!} \cdot \frac{1}{r_{p}^{2}}$$
(15b)

$$C_{T2}^* = \frac{1}{k_r} + \frac{1}{k_r} \cdot \frac{1 - r_p^2}{r^2} + \frac{1}{k_r} + \frac{1}{k_r} \cdot \frac{1}{r^2}$$
 (15b)

After capacitance normalization, C'T1 and C'T2 are further denoted by CT1 and CT2, respectively. The total capacitance for the biquad SC circuit is given by  $C_T = C_{T1} + C_{T2}$ 

 $C_{\text{T1}}$  and  $C_{\text{T2}}$  are highly dependent on capacitance spread.

# III TOTAL CAPACITANCE REDUCTION IN BIQUAD SC CIRCUIT 3.1 Conditions on Scaling Factors

In this section, relation between the parameters and the total capacitance is discussed. From Eq.(14b), when rp is close to unity, C^21 is very small. This causes very large  $C_{\mbox{\scriptsize TZ}}$ . Therefore, the total capacitance of a high-Q section, where  $r_p = 1$ , is dominant in the total capacitance of the cascade SC filter. Therefore, reductions in the total capacitance of a high-Q section should take first priority.

Assuming

$$r_p = 1 \tag{17}$$

 $C_{T1}$  and  $C_{T2}$  in Eqs.(15a) and (15b), can be approximately expressed as follows:

$$C_{T_1} = \frac{1}{k_1} + \frac{1}{k_2} 2(1 - \cos \theta_p) + \frac{1}{k_2} 2(1 - \cos \theta_z)$$
 (18a)

$$C_{TZ} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_2} + \frac{1}{k_2} (1 - r^2_p) + \frac{1}{k_2}$$
 (18b)

In  $C^{T_1}$ ,  $2(1-\cos\theta_{P})/k_{Z}$  is assumed to be the minimum. That is,

$$\frac{1}{k_{z}}2(1-\cos\theta_{p}) < \frac{1}{k_{1}}, \frac{1}{k_{z}}2(1-\cos\theta_{z})$$
 (19)

are assumed. After normalizing  $2(1-\cos\theta_{\rm p})/k_{\rm z}$  to be unity,  $C_{T1}$  is given by

$$C_{\text{T1}} = \frac{k_{\text{Z}}}{k_{\text{1}}} \cdot \frac{1}{2(1-\cos\theta_{\text{P}})} + 1 + \frac{k_{\text{Z}}}{k_{\text{Z}}} \cdot \frac{1-\cos\theta_{\text{Z}}}{1-\cos\theta_{\text{P}}}$$
 (20) Conditions for reduction in  $C_{\text{T1}}$  can be given by

$$\frac{k_2}{k_1} \rightarrow \text{small} \tag{21a}$$

$$\frac{k_1}{k_2} \rightarrow \text{small} \tag{21b}$$

When  $2(1-\cos\theta_z)/k_z'$  is minimum in  $C^*_{T1}$ ,  $C_{T1}$  in Eq.(20)

$$C_{T1} = \frac{k_{Z}'}{k_{1}} \cdot \frac{1}{2(1 - \cos \theta_{E})} + \frac{k_{Z}'}{k_{Z}} \cdot \frac{(1 - \cos \theta_{E})}{(1 - \cos \theta_{E})} + 1$$
 (22)

In this case, the first term is dominant. Therefore, the condition Eq.(21a) is still valid. Furthermore, as will be described in the following,  $C_{TZ}$  is dominant in  $C_{T}$ , Eqs. (21a) and (21b) could be conditions for reductions in the total capacitance.

Since  $r_p$  is close to unity,  $(1-r_p^2)/k_2$  is the minimum in CTR. That is,

$$\frac{1}{k_2}(1-r_p^2) < \frac{1}{k_1}, \frac{1}{k_2}$$
 (23)

$$C_{TZ} = \frac{1}{1 - r_p^2} + 1 + \frac{1}{1 - r_p^2} \left( \frac{k_z}{k_1} + \frac{k_z}{k_2'} \right)$$
 (24)

From the above expression, conditions on  $k_1$  and  $k_2$  are the same as Eqs.(21a) and (21b).

# 3.2 Condition on Amplitude Responses

Equation (21a) is further transferred into conditions on amplitude responses.  $H_1(z)$  and H(z) in Eqs.(2a) and (2b) are expressed using pole and zero as follows:

$$H_{1}(z) = h_{10} \frac{1 - z'_{z} z^{-1}}{(1 - z_{p} z^{-1})(1 - z^{*}_{p} z^{-1})}$$
(25a)  

$$H(z) = -h_{0} \frac{(1 - z_{p} z^{-1})(1 - z^{*}_{z} z^{-1})}{(1 - z_{p} z^{-1})(1 - z^{*}_{p} z^{-1})}$$
(25b)

$$H(z) = -h_0 \frac{(1-z_z z^{-1})(1-z_z z^{-1})}{(1-z_p z^{-1})(1-z_p^* z^{-1})}$$
(25b)

z'z, h10, h0 can be expressed using capacitance as follows:

$$z'_{z} = \frac{C_{23}C_{11} - C_{20}C_{13}}{C_{23}C_{11} - C_{20}C_{13} - C_{13}C_{21}}$$
(26a)

$$h_{10} = \frac{C_{23}C_{11} - C_{13}(C_{20} + C_{21})}{C_{10}(C_{20} + C_{21})}$$
(26b)

$$z'_{z} = \frac{C_{23}C_{11} - C_{20}C_{13}}{C_{23}C_{11} - C_{20}C_{13} - C_{13}C_{21}}$$

$$h_{10} = \frac{C_{23}C_{11} - C_{13}(C_{20} + C_{21})}{C_{10}(C_{20} + C_{21})}$$

$$h_{0} = \frac{C_{23}}{(C_{20} + C_{21})}$$
(26b)

z' z is rewritten as

$$z'_{z} = \frac{1}{1 - \frac{C_{13}C_{21}}{C_{23}C_{11} - C_{20}C_{13}}}$$
(27)

Using Eqs. (13) and (14),

$$\frac{C_{13}C_{21}}{C_{23}C_{11}-C_{20}C_{13}} = \frac{2(1-\cos\theta \cdot \mathbf{z})(1-r_{p}^{2})}{1-r_{p}^{2}+2r_{p}(-\cos\theta_{p}+r_{p}\cos\theta_{z})} = 0 \quad (28)$$
is approximately held, then, z'z satisfies.

$$z'_{z} = 1 \tag{29}$$

hio and ho are also approximately expressed by

$$h_{10} = 2\frac{k_1}{k_2}(\cos\theta e^{-\cos\theta})$$
 (30a)

$$h_0 = \frac{k_2}{k_2} \tag{30b}$$

Let  $H_{1max}$  and  $H_{max}$  be maximum amplitude values for  $H_1(z)$  and H(z), respectively,

$$II_{1\text{max}} = \max_{\{i\}} \left\{ \left| II_{1}(e^{jw}) \right| \right\}$$
 (31b)

$$H_{\text{mex}} = \max_{\omega} \{ \mid H(e^{Jw}) \mid \}$$
 (31b)

Since they appear approximately at  $z=\exp(j\theta_p)$ , they can be expressed as

$$II_{1\text{max}} = \left[ h_{1} \frac{\exp(j\theta_{p}) - z'_{z}}{(\exp(j\theta_{p}) - z_{p})(\exp(j\theta_{p}) - z^{*}_{p})} \right]$$
(32a)

$$H_{\text{max}} = \left| h_{\frac{\text{cxp}(j\theta_{p}) - z_{z}}{\text{exp}(j\theta_{p}) - z_{p}}} \left( \frac{\exp(j\theta_{p}) - z_{z}}{\exp(j\theta_{p}) - z_{p}} \right) \right|$$
(32b)

The above expressions are further rewritten as

$$H_{1\text{max}} = \left| h_{10} \frac{2\sin(\theta_{P}/2)}{(1-r_{P})(1-r_{P}\exp(-j2\theta_{P}))} \right|$$
 (33a)

$$H_{\text{max}} = \left| h_0 \frac{2 \sin\{(\theta_z - \theta_p)/2\} \cdot 2 \sin\{(\theta_z + \theta_p)/2\}}{(1 - r_p)(1 - r_p \exp(-j2\theta_p))} \right| (33b)$$

Taking Eqs.(30a) and (30b) into account, the following result can be obtained.

$$\frac{k_2}{k_1} = \left| 2\sin\frac{\theta_p}{2} \right| \frac{H_{\text{max}}}{H_{1\text{max}}}$$
 (34)

As shown in Eq.(20),  $C_{T1}$  is inversely proportional to  $\theta$  <sub>P</sub>. The first term in Eq.(20) is rewritten as,

$$\frac{k_{2}}{k_{1}} \cdot \frac{1}{2(1-\cos\theta_{p})} = \frac{k_{2}}{k_{1}} \cdot \frac{1}{4\sin^{2}(\theta_{p}/2)}$$
(35)

Using Eq.(34), it can be expressed as

$$\frac{H_{\text{max}}}{H_{\text{lmax}}} \cdot \frac{1}{|2\sin\frac{\theta_{\text{P}}}{2}|}$$
 (36)

On the other hand, in  $C_{TZ}$  given by Eq.(24), the third term is modified into

$$\frac{k_{z}}{k_{1}} \cdot \frac{1}{1 - r_{p}^{z}} = \frac{H_{max}}{H_{1max}} \cdot \frac{|2\sin\frac{\theta_{p}}{2}|}{1 - r_{p}^{z}}$$
(37)

Therefore, in order to reduce the total capacitance,  $II_{max}/II_{1max}$  should be decreased.

$$\frac{H_{\text{max}}}{H_{\text{lmax}}} \rightarrow \text{small} \tag{38}$$

Next, the condition Eq.(21b) is considered here. Since k'z is a scaling factor for Vout in the preceding section, it is determined independently of kz. In order to satisfy Eq.(21b), k'2 should be large. Supposing  $H_{max}$  in Eq.(33b) be that in the preceding section, the following conditions are required, in order to increase  $k'_{2}$  under the condition  $H_{mex}=1$ .

$$r_p \rightarrow small$$
 (39a)

$$\mid \theta_{z} - \theta_{p} \mid \rightarrow \text{small}$$
 (39b)

This means low-Q poles and zeros close to them should be assigned to the preceding sections.

# IV CONDITIONS ON PAIRING AND ORDERING

#### 4.1 Conditions on Pairing and Ordering

Now, we are ready to derive general conditions on pairing and ordering based on Eqs.(38), (39a) and (39b).

First, we consider Eq.(38). Letting  $f_{1\text{max}}$  and  $f_{\text{max}}$  be frequencies at which  $Il_{1\text{max}}$  and  $Il_{\text{max}}$  appear, respectively,  $f_{\text{max}}$  slightly shifts to the passband compared with  $f_{1\text{max}}$ . Figure 2 shows this relation. Although both functions  $Il_1(z)$  and Il(z) have the same pole. a zero of Il(z) locates close to the passband compared with that of  $Il_1(z)$ . Because a zero of  $Il_1(z)$  locates around z=1, as shown in Eq.(29). Based on this relation and Eq.(38), a sub-transfer function from the filter input to the the preceding section output should has a steeply slanting amplitude response as shown in Fig.2.

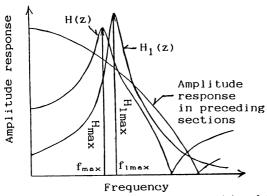


Fig.2 Amplitude responses for  $H_1(z)$ , H(z) and the preceding section.

As described in Sec. III, since the total capacitance in a high-Q section is dominant, it should take priority to satisfy the above condition. This can be rephrased as, (1) Zeros close to the high-Q pole should be assigned to the preceding sections. Next, from Eqs. (39a) and (39b), (2) low-Q poles and zeros close to them should be assigned the preceding sections. The condition on zeros are the same as in (1).

# 4.2 Searching Procedure for Optimum Assignment

In high-order SC filters, the number of all possible pairing and ordering assignments is very large. Furthermore, an occurrence rate of optimum assignments is low. Therefore, a searching process for an optimum assignment, without any constraint on pairing and ordering, requires a lot of computing time.

On the contrary, the proposed conditions can limit pairing and ordering assignments, which can provide significantly reduced total capacitance. Among them, the optimum solution can be easily searched for with fewer searching steps. Numerical examples for several kinds of SC filters are provided in the next section.

# V NUMERICAL EXAMPLES

# Filter Responses

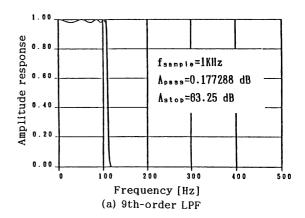
9th and 11th-order low-pass (LPF) and high-pass (HPF) filters were designed. Poles and zeros are listed in Tables 1 and 2, and amplitude responses are shown in Figs. 3(a) and 3(b).

Table 1. Zeros and poles for 9th-order LPF

	rz	θ z(deg.)		rp	$\theta_{p}(\deg.)$
N1	1	180.0	D1	0.760	0.0
N2	1	79.66	D2	0.814	21.06
N3	1	53.23	D3	0.902	32.70
N4	. 1	45.48	D4	0.959	37.54
N5	1	43.12	D5	0.989	39.20

Table 2. Zeros and poles for 11th-order HPF

	rz	θ z(deg.)		rp	θ p(deg.)
N1	1	0.0	D1	0.669	180.0
N2	1	58.3	<b>D2</b>	0.729	149.0
N3	1	88.7	D3	0.837	130.0
N4	1	101.7	<b>D4</b>	0.917	120.7
N5	1	107.2	D5	0.963	116.4
N6	1	109.2	D6	0.990	114.8



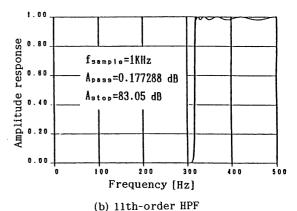
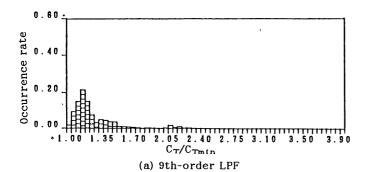


Fig.3 Amplitude responses for SC filters.

# Distribution of Total Capacitance

Distribution of the total capacitance for all possible pairing and ordering assignments, without any constraint, are shown in Figs.4(a) and 4(b). In these figures, the total capacitance ( $C_{\rm T}$ ) is normalized by the minimum value ( $C_{\rm Tmin}$ ). For instance, 1.35 on the horizontal axis means the total capacitance is 35% over the minimum value. From these figures, it can be observed that an occurrence rate for assignments, by which the total capacitance is within 5% over the minimum is very low.

Next, the constraints shown in Figs.5(a) and 5(b) are imposed on pairing and ordering. For instance, the constraint in Fig.5(a) means that the highest-Q pole D5



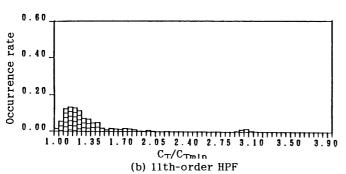
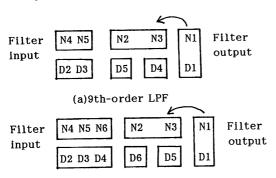


Fig. 4 Distributions of the total capacitance without any constraint.



(b)11th-order HPF Fig.5. Constraints on pairing and ordering

is fixed to the third section, and zeros N4 and N5, which are close to D5, are assigned to the first or second section. The first-order pole D1 is paired with the zero N1 in advance, and is assigned to any stage. Distribution of the total capacitance is illustrated in Figs.6(a) and 6(b). The total capacitance is mostly concentrated within 5% over the minimum. The same results were obtained for 11th-order LPF, 9th-order HPF having different cutoff frequencies.

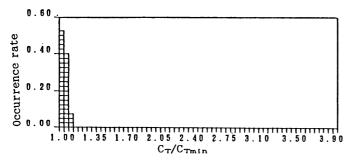
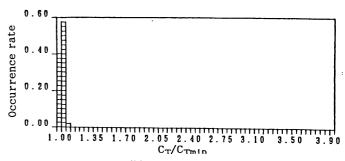


Fig.6(a) 9th-order LPF



(b) 11th-order HPF Fig.6 Distributions of the total capacitance with the constraints shown in Fig.5.

#### VI CONCLUSIONS

A new strategy of searching for the optimum pairing and ordering assignment has been proposed. Design examples for several kinds of SC filters have been demonstrated. The total capacitance for the selected pairing and ordering assignments mostly concentrates within 5% over the minimum. The optimum assignment can be easily searched for among the limited assignments.

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